PATROL TACTICS TO MINIMIZE RESPONSE TIME

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Abstract

For a random distribution of service calls over an area, the average response time of a service unit, such as a police patrol car, is minimized if the unit remains stationary at the centroid of the distribution while awaiting calls for service.

INTRODUCTION

Several different kinds of activities may be described, in general terms, as involving a service unit which is summoned in some fashion, which transits from an initial location to a more or less distant site, and which performs some function at that site. A significant portion of the service unit's operating time is spent waiting for a summons, and one measure of the unit's effectiveness is the speed with which it arrives at the designated service site. Some examples of this general class of activities include the dispatch of ambulances to the scene of an accident, the dispatch of fire engines to the scene of a fire, the dispatch of police cars to the scene of a crime, and the dispatch of coastal patrol boats to inspect small craft at sea. In the police case, it has been shown that the likelihood of closing a case by an arrest is negatively correlated with response time - the quicker the police arrive at the scene of the crime, the more likely it becomes that the criminal will be apprehended. An important operational question concerns the deployment of the service unit while it is awaiting a call for service, and the effect which mode of operation has upon response time.

A two-year study was conducted by one police department[1] in which the normal police patrols were contrasted with a computer-generated random patrol pattern. It was reported that the response time decreased markedly both for the randomly patrolling group and for the control group during the course of the trials. However, the experimenters concluded that randomness of patrol minimized response time.

This paper presents an analytic study of the expected value of response time for calls which follow a uniform random distribution over an urban service area. It concludes that response time is minimized if the service unit remains stationary at the center of the service area until called, response time is maximized if the service unit remains stationary at the farthest corner away from the center of the service area until called, and random patrol within the service area results in an average response time which is intermediate between those two values. Similar conclusions are valid for random call distributions which are not Uniform and for service areas other than street networks.

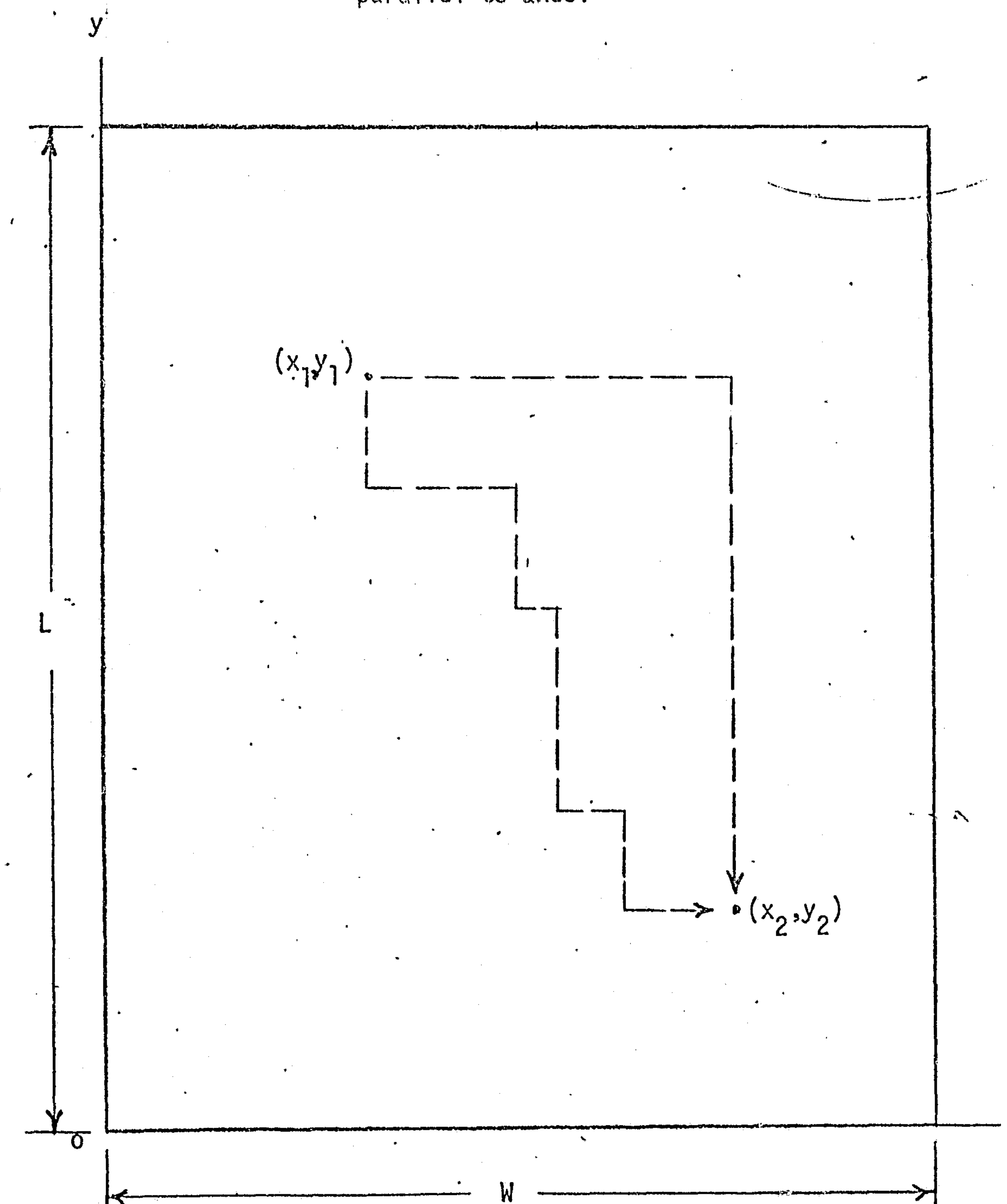
THE MODEL

A hypothetical case is illustrated in figure 1. Let us assume that there is a rectangular service area of width W, and length L, and let us impose a Cartesian coordinate system with its origin at the lower left hand corner. In an urban environment, the network of streets generally forms a rectangular pattern; as a first approximation to that pattern, motion between any two points

1. W. Bennet and J. R. DuBois, "The Use of Probability Theory in the Assignment of Police Patrol Areas," PR 70-2, U.S. Dept. of Justice, LEAA, NILECJ, Jul. 1970.

Figure 1. Hypothetical Patrol Area

All travel only in directions parallel to axes.



Minimum path length between any two points:

$$P = \left| x_2 - x_1 \right| + \left| y_2 - y_1 \right|$$

in this hypothetical region will only be allowed in directions parallel to the x - axis and the y - axis of the figure. (With this constraint, the minimum path length P between two points is the sum of the distance travelled parallel to the x - axis plus the distance travelled parallel to the y - axis regardless of the order or number of segments over which the travel is conducted, provided that there is no reversal of direction). We shall further postulate that only one service unit operates within the rectangular boundary and that a call for service is just as likely to come from one point within the region as from any other (the occurrence of incidents requiring service is random, with a Uniform probability distribution.) Additional assumptions made for the hypothetical model are that the time required to travel from one point to another within the service region is strictly proportional only to the length of the path travelled (there are no important delays due to starting and stopping), and that the speed of travel is constant over the entire path; thus, minimizing average path length is equivalent to minimizing average response time.

THE RANDOM PATROL CASE

When a service unit (e.g. police car) patrols its service area in a truly random fashion, its location at the time of receiving a service call is a random variable which, like the location of the service site, has a probability distribution that is uniformly distributed over the service area. If the service unit is at location (x_1, y_1,) and the call is for service at location (x_2, y_2), the path length travelled is the sum of the distance from x_1 to x_2 plus the distance from y_1 to y_2. For a random patrol, the average path length $E(P)_r$ is given by

Equation (1):

$$E(P)_r = 2\int_0^W\int_0^{x_2}(x_2-x_1)\frac{dx_1}{W}\frac{dx_2}{W} + 2\int_0^L\int_0^{y_2}(y_2-y_1)\frac{dy_1}{L}\frac{dy_2}{L} \qquad (1)$$

This integrates to:

$$E(P)_r = \frac{1}{3}(W+L) \qquad (2)$$

THE CORNER LOCATION CASE

One alternative to the random moving patrol is the stationary post -- the service unit remains in a fixed location until it is dispatched. If call locations are uniformly distributed over the service area and the service unit is stationed at the corner which is designated as the origin of our Cartesian coordinate system, the average path length $E(P)_c$ is given by Equation (3):

$$E(P)_c = \int_0^W x\frac{dx}{W} + \int_0^L y\frac{dy}{L} \qquad (3)$$

This integrates to:

$$E(P)_c = \frac{1}{2}(W+L) \qquad (4)$$

THE CENTER LOCATION CASE

When the service unit is stationed in the middle of the service area, the expected value of the path length to uniformly distributed service calls, $E(P)_m$, is given by Equation (5):

$$E(P)_m = \int_{W/2}^W x\frac{dx}{W} + \int_{L/2}^L y\frac{dy}{L} \qquad (5)$$

This integrates to:

$$E(P)_m = \frac{1}{4}(W+L) \qquad (6)$$

CONCLUSION

Comparison of Equations (2), (4), and (6) indicates that average response distance (and thus average response time) is minimized when the waiting service unit is kept in a fixed location in the middle of the service area, it is maximized when the service unit is stationed at a corner, and random patrol within the area gives an intermediate value.

EXTENSIONS

The average response time E(T) for any patrol tactic can be calculated by dividing the appropriate expected response path length E(P), by the average speed of response $\bar{S}$:

$$E(T) = \frac{E(P)}{\bar{S}} \qquad (7)$$

It is intuitively apparent that systematic (i.e., non-random) patrols within a service area will yield average response times intermediate between those obtained from stationary corner and center locations, but a general expression cannot be calculated.

If the service unit's travel is not restricted to a network of roads (e.g., the service area is a body of water, the service unit is a helicopter which flies directly from one point to another), the expressions for average path length become much more complex, but the conclusion remains unchanged. Expressions for the stationary cases were not calculated; the expected path length $E(P)_{rs}$ in the random patrol case at sea is given by equation (8):

$$E(P)_{rs} = 4\int_0^L \int_0^{y_2} \int_0^W \int_0^{x_2} \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\;\frac{dx_1}{W}\,\frac{dx_2}{W}\,\frac{dy_1}{L}\,\frac{dy_2}{L} \qquad (8)$$

This integrates to:

$$E(P)_{rs} = \frac{1}{15}\left[\frac{L^3}{W^2} + \frac{W^3}{L^2} - \sqrt{L^2+W^2}\left(\frac{L^2}{W^2} + \frac{W^2}{L^2} - 3\right)\right] + \frac{1}{6}\left[\frac{W^2}{L}\left\{\log\left(L+\sqrt{L^2+W^2}\right) - \log W\right\} + \frac{L^2}{W}\left\{\log\left(W+\sqrt{L^2+W^2}\right) - \log L\right\}\right] \qquad (9)$$

The approximate value is:

$$E(P)_{rs} \simeq \frac{1}{3}\sqrt{W^2 + L^2} \qquad (10)$$

(Equation (10) is useful in the analysis of searches and patrols between points selected at random from a uniform distribution. The maximum error occurs when W = L; the approximate value is about 9% too low.)

For any convex service area and for any random distribution of service calls, it is intuitively apparent that minimum response time will be obtained when the waiting service unit is stationed at the centroid of the distribution of service calls.

TACTICS TO MINIMIZE RESPONSE TIME

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NATIONAL INSTITUTE OF LAW ENFORCEMENT
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LAW ENFORCEMENT ASSISTANCE
ADMINISTRATION

U. S. DEPARTMENT OF JUSTICE

SERVICE UNITS

POLICE PATROL CARS

AMBULANCES

FIRE ENGINES

COASTAL PATROL BOATS

RESCUE HELICOPTERS

DESCRIPTION OF ACTIVITY

SERVICE UNIT:

- RECEIVES CALL FOR SERVICE
- TRAVELS TO SITE
- PERFORMS SERVICE
- WAITS FOR NEXT CALL

DURING WAITING PERIOD, UNIT MAY:

- REMAIN STATIONARY
- RETURN TO ORIGIN AND THEN REMAIN STATIONARY
- ENGAGE IN A MOVING "PATROL"

EFFECTIVENESS TENDS TO INCREASE AS RESPONSE TIME DECREASES

PROBABILITY OF ARREST IS CORRELATED WITH RESPONSE TIME

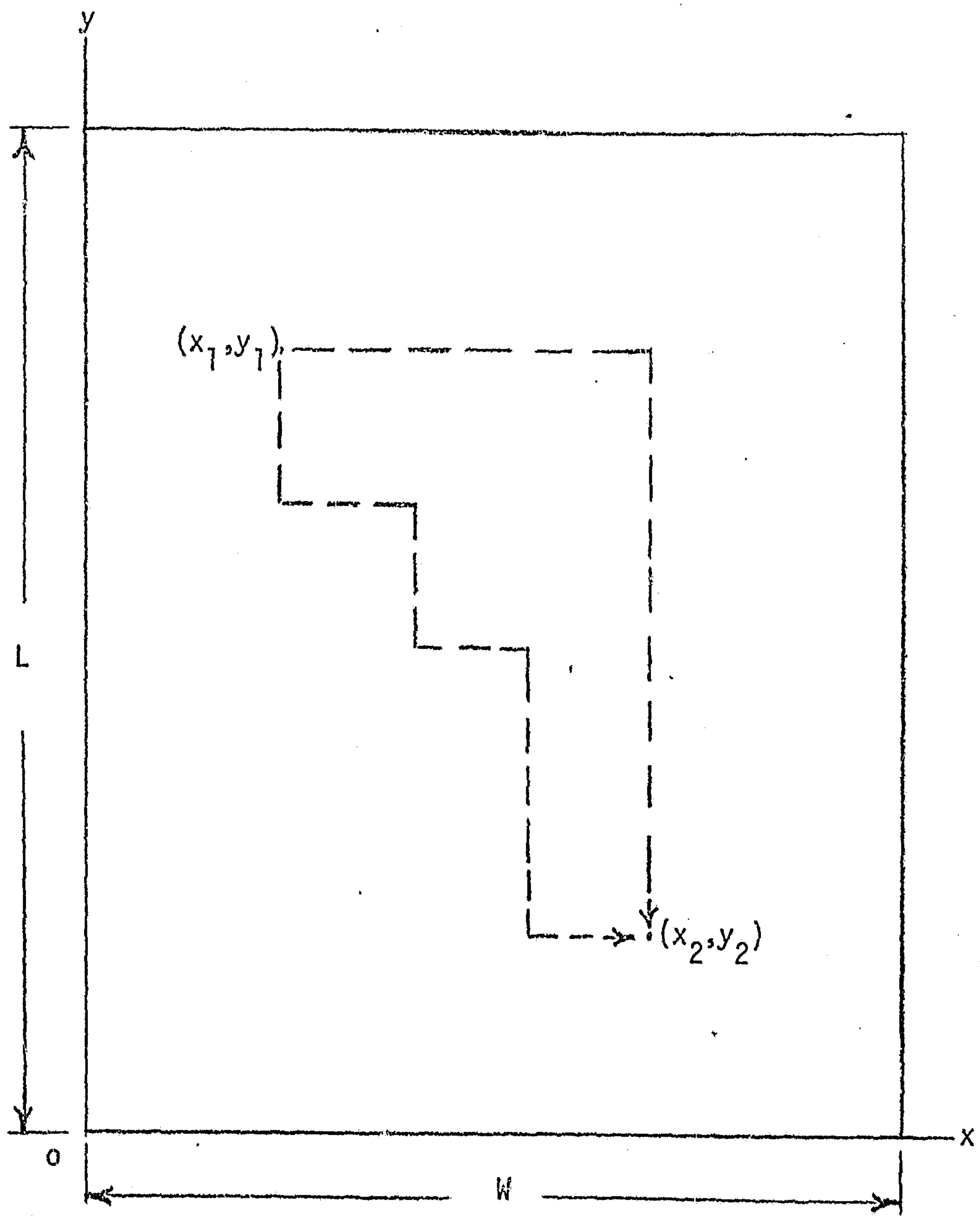
(Task Force Report: Science and Technology, A Report to the President's Commission on Law Enforcement and Administration of Justice, pp. 9-10, the Institute for Defense Analysis, 1967)

RANDOM PATROL MINIMIZES RESPONSE TIME

(W. Bennet and J. R. DuBois, The Use of Probability Theory in the Assignment of Police Patrol Areas, PR 70-2, U. S. Dept. of Justice, LEAA, NILECJ, July, 1970)

MODEL PATROL AREA

All travel only in directions parallel to axes.



Minimum path length between any two points:

$$P = |x_2 - x_1| + |y_2 - y_1|$$

EXPECTED MINIMUM PATH LENGTH, E(P), FOR UNIFORM RANDOM DISTRIBUTION OF SERVICE CALLS

SERVICE UNIT ON RANDOM PATROL

$$E(P)_r = 2\int_0^W \int_0^{x_2} (x_2 - x_1)\,\frac{dx_1}{W}\,\frac{dx_2}{W} + 2\int_0^L \int_0^{y_2} (y_2 - y_1)\,\frac{dy_1}{L}\,\frac{dy_2}{L} \qquad (1A)$$

SERVICE UNIT STATIONARY AT CORNER

$$E(P)_c = \int_0^W x\,\frac{dx}{W} + \int_0^L y\,\frac{dy}{L} \qquad (2A)$$

SERVICE UNIT STATIONARY AT CENTER

$$E(P)_m = \int_{W/2}^W x\,\frac{dx}{W} + \int_{L/2}^L y\,\frac{dy}{L} \qquad (3A)$$

EXPECTED MINIMUM PATH LENGTH, E(P), FOR

UNIFORM RANDOM DISTRIBUTION OF SERVICE CALLS

SERVICE UNIT ON RANDOM PATROL

$$E(P)_r = \frac{1}{3}(W + L) \qquad (1B)$$

SERVICE UNIT STATIONARY AT CORNER

$$E(P)_c = \frac{1}{2}(W + L) \qquad (2B)$$

SERVICE UNIT STATIONARY AT CENTER

$$E(P)_m = \frac{1}{4}(W + L) \qquad (3B)$$

EXPECTED MINIMUM PATH LENGTH, E(P), FOR

UNIFORM RANDOM DISTRIBUTION OF SERVICE CALLS

(TRAVEL DIRECTLY POINT-TO-POINT)

SERVICE UNIT ON RANDOM PATROL

$$E(P)_r = 4\int_0^L\int_0^{y_2}\int_0^W\int_0^{x_2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\;\frac{dx_1}{W}\,\frac{dx_2}{W}\,\frac{dy_1}{L}\,\frac{dy_2}{L} \qquad (4A)$$

$$= \frac{1}{15}\left[\frac{L^3}{W^2}+\frac{W^3}{L^2}-\sqrt{L^2+W^2}\left(\frac{L^2}{W^2}+\frac{W^2}{L^2}-3\right)\right]$$

$$+\frac{1}{6}\left[\frac{W^2}{L}\left\{\log\left(L+\sqrt{L^2+W^2}\right)-\log W\right\}\right.$$

$$\left.+\frac{L^2}{W}\left\{\log\left(W+\sqrt{L^2+W^2}\right)-\log L\right\}\right] \qquad (4B)$$

APPROXIMATE VALUE

$$E(P)_r \simeq \frac{1}{3}\sqrt{L^2+W^2} \qquad (4C)$$

CONCLUSION: TO MINIMIZE RESPONSE TIME TO RANDOMLY DISTRIBUTED SERVICE CALLS, STATION THE SERVICE UNIT AT THE CENTROID OF THE DISTRIBUTION.

DETERRENCE

$$D.\ E. = 1 - \frac{C\ (\text{actual})}{C\ (\text{potential})}$$

D. E. = DETERRENT EFFECT

C (ACTUAL) = NUMBER OF CRIMES COMMITTED WHEN DETERRENT ACTION IS TAKEN.

C (POTENTIAL) = NUMBER OF CRIMES WHICH WOULD HAVE BEEN COMMITTED IF DETERRENT ACTION HAD NOT BEEN TAKEN.

$$-\infty \leqslant D.\ E. \leqslant 1$$

(IF D. E. IS NEGATIVE, THE "DETERRENT" ACTION ENCOURAGES CRIME.)

DETERRENT EFFECT:

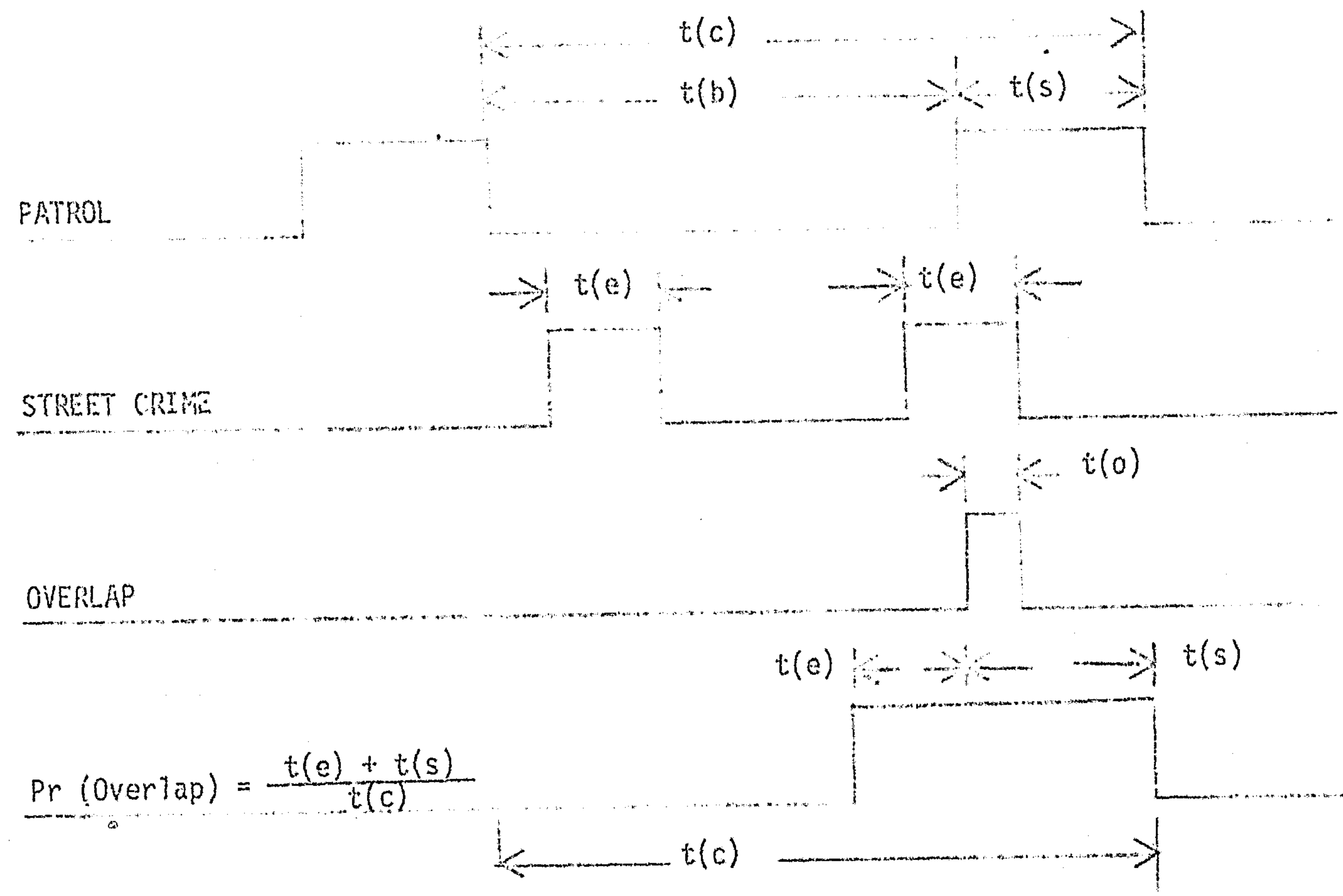
IS A FUNCTION OF POTENTIAL CRIMINALS' PERCEPTIONS

IS A FUNCTION OF POTENTIAL CRIMINALS' ACTIONS AND
RESPONSES TO THEIR PERCEPTIONS

CANNOT BE CALCULATED A PRIORI

DETERRENCE MAY BE RELATED TO ACTUAL RISK OF APPREHENSION

PATROL DETECTION MODEL



$$Pr\ (Overlap) = \frac{t(e) + t(s)}{t(c)}$$

t(b) = blind time (patrol)
t(c) = cycle time (patrol)
t(e) - exposure time (criminal)
t(o) = overlap time (both)
t(s) = surveillance time (patrol)

ACTUAL RISK DUE TO PATROL

RANDOM CRIMINAL ACTION

Pr(DETECTION) = Pr(OVERLAP) X Pr(DETECTION/OVERLAP)

$$= \frac{t(e) + t(s)}{t(c)} \times Pr(D/O)$$

"INTELLIGENT CRIMINAL"

OBJECTIVE: Pr(OVERLAP) = 0

SUFFICIENT CONDITION: $t(e) = 0$

NECESSARY CONDITIONS:

$t(e) < t(b)$, and

CRIMINALS CAN IDENTIFY START OF BLIND TIME, OR

CRIMINALS CAN PREDICT START OF SURVEILLANCE PERIOD IN ENOUGH TIME TO ESCAPE.

EFFICACY OF PATROL AS A DETERRENT MUST BE DETERMINED ON A CASE-BY-CASE BASIS.

END