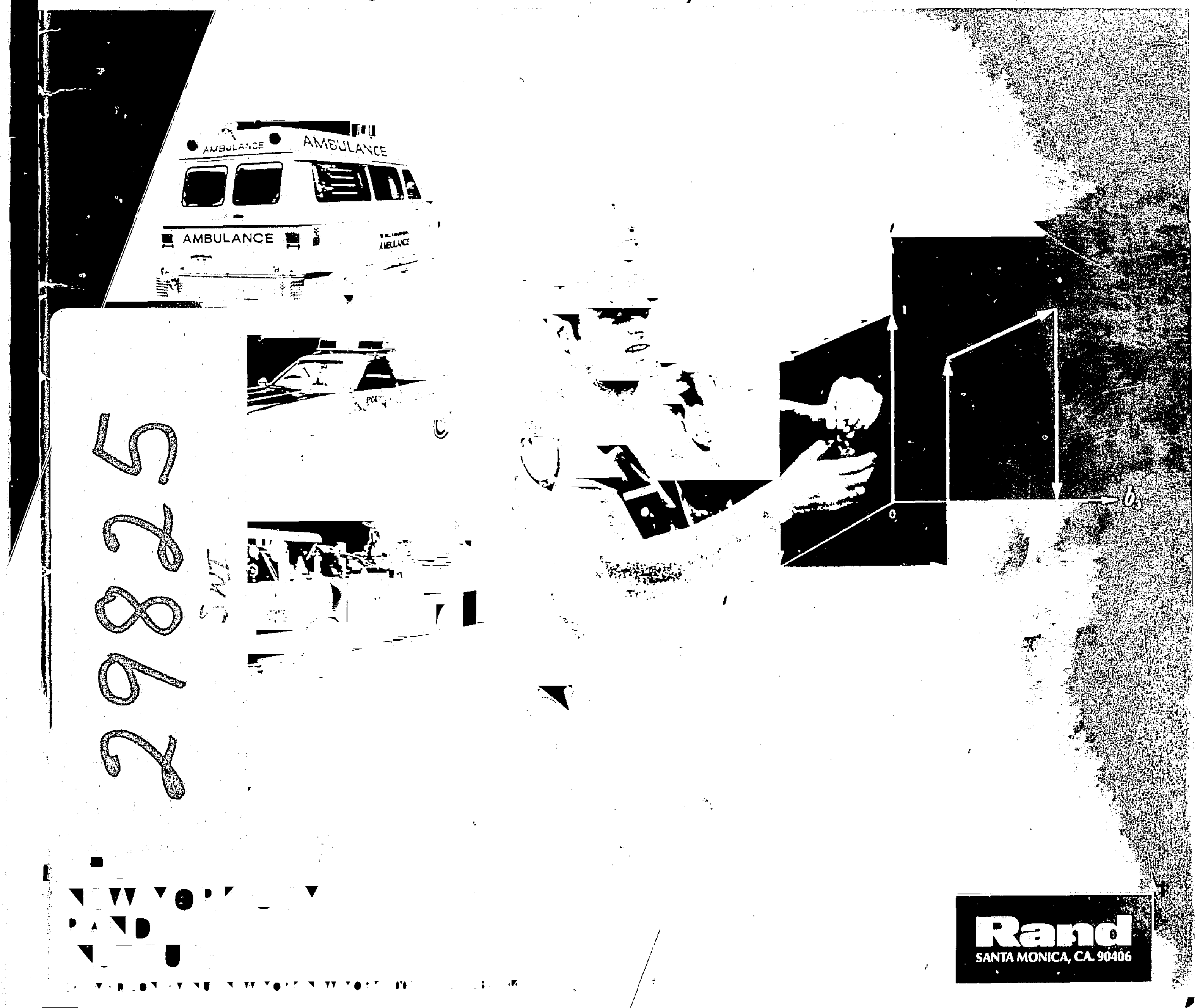
HYPERCUBE QUEUING MODEL: EXECUTIVE SUMMARY

PREPARED FOR THE DEPARTMENT OF HOUSING AND URBAN DEVELOPMENT

JAN M. CHAIKEN R-1688/1-HUD JULY 1975



The New York City-Rand Institute is a nonprofit research institution founded, as its Articles of Incorporation state, "... primarily to conduct programs of scientific research and study, and provide reports and recommendations, relevant to the operations, planning or administration of the City of New York." The Institute was established in 1969 by the City of New York and The Rand Corporation as a center for the continuing application of scientific and analytic techniques to problems of urban life and local government. It is governed by a Board of Trustees appointed jointly by the City and Rand.

Published by The Rand Corporation

HYPERCUBE QUEUING MODEL: EXECUTIVE SUMMARY

PREPARED FOR THE OFFICE OF POLICY DEVELOPMENT AND RESEARCH, DEPARTMENT OF HOUSING AND URBAN DEVELOPMENT

JAN M. CHAIKEN

R-1688/1-HUD
JULY 1975

THE NEW YORK CITY RAND INSTITUTE
545 MADISON AVENUE NEW YORK NEW YORK 10022 (212) 758-2244

Rand
SANTA MONICA, CA. 90406

PREFACE

This report describes in nontechnical terms a computer program called the Hypercube Queuing Model. It was written to help emergency service agency administrators and other local government officials understand how the model can be used to design response districts for ambulances, police patrol cars, and fire-fighting units.

Preparation of this report was funded under contract H-2164 between The New York City-Rand Institute and the Office of Policy Development and Research of the U.S. Department of Housing and Urban Development (HUD). Among the objectives of this HUD contract are the development, field testing, and documentation of methods for improving the deployment of municipal emergency services.

The Hypercube Model was designed, programmed, and documented by Richard Larson, partly under the HUD contract mentioned above and partly under a grant from the National Science Foundation (Research Applied to National Needs) to the Innovative Resource Planning Project of the Massachusetts Institute of Technology. For more detailed information about the model, readers should consult Larson's publications cited in the text.

Documentation of the Hypercube Model constitutes part of a series of HUD-funded reports describing several different emergency service deployment models and their applications in several cities. Further information about these reports can be obtained from the author at the address shown in the Appendix.

CONTENTS

I. INTRODUCTION

The Hypercube Queuing Model is a computer program that can help police departments, fire departments, and emergency medical agencies plan the locations and response districts of their mobile units.* The program stores geographical information that can be pictured as representing a map to be used by a dispatcher.

If the units have fixed "home" locations when they are not busy (e.g., fire stations or ambulance garages), such a map would typically show these locations. It might also show the *response district* of each unit, which consists of all places in the city where the unit is the dispatcher's first choice to respond to incidents. For patrol cars, the map would usually show their *patrol areas* (also called *sectors* or *beats*), which are the areas the cars cruise around when not otherwise busy. Elaborate maps of this type could have colored overlays showing the regions covered by special units such as sergeants' cars or cardiac care units.

For planning purposes, a variety of changes in the operations of the agency might be considered. Any changes that could possibly be shown on a dispatcher's map are suitable for analysis using the Hypercube Model. Examples of such changes include:

- Adding new units
- Eliminating an existing unit
- Moving units from one location to another
- Redrawing the boundaries of patrol areas
- Changing response districts without moving any units
- Moving the overlays representing special units.

---

*The word *hypercube* in the model's name comes from the way the emergency units are described in the computer program. If a city happened to have exactly three emergency units, the program would describe them mathematically in terms of the corners of a cube; so more than three units are represented as the corners of a "hypercube." The word *queuing* refers to the type of equations used in the model. For brevity, we will use the name Hypercube Model.

The Hypercube Model assists in the planning process by describing the consequences of a proposed change in terms of performance measures such as workloads of units and travel times to emergency incidents. If several possible changes are under consideration, the output from the computer program may indicate that one of them is best. More commonly, the information generated by the model shows that no one proposal is best in regard to all performance measures, and this leads the planner to invent new proposals that may achieve a better balance among conflicting objectives than any of the ones originally presented.

The value of the model lies in the fact that most agencies' operations involve sufficient complications that it is nearly impossible for a planner to look at a map and make accurate "guesses" regarding the workloads of the units or the parts of the city where travel times may be high. In many cases, when the *existing* configuration is described to the model, the resulting output is illuminating, because the agency may never have collected data permitting it to calculate the performance measures generated by the computer program.

It should be noted that the Hypercube Model does not *suggest* any kind of change as being desirable. It simply helps the agency evaluate alternative plans that agency personnel create themselves. The model provides detailed quantitative information about each of the designs they create, thereby permitting careful analysis of which ones appear best. It should also be noted that it is not actually necessary to draw maps showing each design to be tested by the model, although often it will be useful to do so.

The computer program that constitutes the Hypercube Queuing Model is now available to any emergency service agency for the cost of duplication, by request to any of the addresses shown in the Appendix. It is supplied with a complete user's manual: Richard Larson, *Hypercube Queuing Model: User's Manual*, R-1688/2-HUD. Also available are two reports that describe the mathematical principles that underlie the Hypercube Model:

11696

- Richard Larson, *A Hypercube Queuing Model for Facility Location and Redistricting in Urban Emergency Services*, R-1238-HUD

- Richard Larson, *Urban Emergency Service Systems: An Iterative Procedure for Approximating Performance Characteristics*, R-1493-HUD.

This executive summary is a companion to the technical reports and user's manual. It tells how the Hypercube Model differs from other computer programs available to emergency service agencies for analysis of station location and response district design, how the model works, how it is used, and the amount of effort and expertise required to use it.

II. WHEN TO USE THE HYPERCUBE MODEL

Emergency service agencies have a choice of several different computer models that can be used for similar purposes. Depending on the planning issues to be addressed and certain details of the agency's operations, the Hypercube Queuing Model may or may not be the most suitable one. This section discusses how to make this choice, first for fire and ambulance agencies, and then for police departments.

FIRE AND AMBULANCE AGENCIES

For emergency services whose units have fixed home locations (primarily ambulance agencies and fire departments), there are several computer programs other than the Hypercube Model that have been designed specifically to help an agency decide how many units should be on duty and where they should be located.

One of these, the *Parametric Allocation Model*,(1) provides the user with a general picture of the number of units needed in different parts of the city. It is very quick and inexpensive to use, primarily because it requires very little data to be collected. But it cannot evaluate specific locations for the units in any detail. Its primary purpose is for assistance in the initial steps of a study of the locations of fire stations or ambulance garages.

For detailed evaluation of locations of units, the following computer programs are suitable for ambulance agencies and fire departments, although they were designed primarily for fire departments:

- The Fire Station Location Package, designed by Public Technology, Incorporated,(2)
- The Firehouse Site Evaluation Model, developed by The New York City-Rand Institute,(3) and
- The Station Configuration Information Model, developed by the Denver fire services research project.(4)

All three of these site evaluation models operate on the assumption that units will nearly always be available at their home locations to respond to emergencies. This is a reasonable assumption for fire-fighting units

in most cities, but it may be a poor approximation for emergency medical services, whether provided by a fire department or by some other agency.

By contrast, the Hypercube Model specifically takes into account the fact that units may be unavailable when needed, because of a previous dispatch to another incident. Therefore, the Hypercube Model will provide more accurate estimates of average travel times and workloads than any of the site evaluation models, if an agency often has 10 percent or more of its units busy at one time. Fire departments and ambulance agencies that do not encounter this situation have no reason to use the Hypercube Model, or indeed any model more complicated than the ones mentioned above, for analysis of alternative configurations of locations for their units.

An *ambulance agency* that does frequently have many units unavailable would be well advised to use the Hypercube Model for analysis of alternative configurations of its ambulance locations. The Hypercube Model is just as easy to use as the site evaluation models, requires approximately the same kinds of data, and gives a more accurate picture of the operations of the agency.

For a *fire department* that falls in the category of having 10 percent or more of its fire-fighting units busy at one time, whether or not to use the Hypercube Model is a more difficult choice than in the case of ambulance agencies. This is because the Hypercube Model operates on the assumption that only one fire-fighting unit is ordinarily dispatched to each incident. If, by considering engine (pumper) companies and ladder (truck) companies separately, this is a reasonable assumption, then the Hypercube Model is appropriate. Otherwise, a fire department in which many units are busy at once would have to use a more complicated model, such as the Simulation Model of Fire Department Operations,(5,6) for final evaluation of a proposed configuration of stations. In any event, if a fire department is considering changes in dispatch policy or in policies related to the relocation (move up) of units when coverage is inadequate, as well as changes in station locations, the Simulation Model will be needed, and the Hypercube Model should not be used.

Tables 1 and 2 summarize the choices of models to be used by fire and ambulance agencies. For a more detailed description of deployment policies for fire departments, see the executive summary of the Simulation Model.(5)

Table 1

CHOICES OF MODELS FOR AMBULANCE AGENCIES

Problem and Situation	Recommendation
• To choose the number of ambulances to have on duty, their locations and response districts	
-- Rarely more than 10 percent of ambulances busy at one time	Use Parametric Allocation Model and one of the site evaluation models.
-- Often more than 10 percent of ambulances busy at one time	Use Parametric Allocation Model and Hypercube Model.
• To choose response districts for mobile (cruising) ambulances	Use Hypercube Model.

Table 2

CHOICES OF MODELS FOR FIRE DEPARTMENTS

Problem and Situation	Recommendation
• To choose the number of fire-fighting units to have on duty, their locations and response districts	
-- Rarely more than 10 percent of units busy at one time	Use Parametric Allocation Model and one of the site evaluation models.
-- Often more than 10 percent of units busy at one time	
- Usually one engine and/or one ladder dispatched	Use Parametric Allocation Model and Hypercube Model
- Usually several engines and ladders dispatched	Use Parametric Allocation Model, a site evaluation model, and the Simulation Model.
• To evaluate alternative station locations, plus changes in other policies such as dispatching or relocation (move up)	Use Parametric Allocation Model, a site evaluation model, and the Simulation Model.

POLICE DEPARTMENTS

The Hypercube Model was developed specifically to be useful to police departments for design of patrol areas. For this purpose it is a substitute for more complex computer programs such as simulation models.[7,8] Compared to a police patrol simulation model, the Hypercube Model is much less expensive to operate, requires somewhat less data to be collected, needs fewer statistical skills for interpretation of output, and requires only such programming capabilities and software packages as would ordinarily be available at a city's data-processing unit.

While a simulation model gives a more accurate representation of the operations of patrol cars, the differences will generally not be large enough that they should be of practical concern to any police department. However, patrol car simulation models have many applications for studies not related to design of response districts and patrol areas, and these are described in the executive summary of Rand's simulation model.[8]

Although the Hypercube Model can be used to analyze how many patrol cars should be on duty in various parts of the city at different times of day, other computer programs are preferable for this purpose. (The Rand Corporation provides such a patrol car allocation model.[9]) It is best for a police department to know, before using the Hypercube Model for design of patrol areas, how many patrol cars it wants to have on duty.

III. HOW THE HYPERCUBE MODEL WORKS

The Hypercube Model distinguishes two conditions for each emergency unit: it is either available for dispatch or unavailable. The *state* of the entire collection of emergency units is described by specifying the condition of each unit. For example, if there are three units, one possible state is that all three units are available, another is that unit 1 is unavailable and units 2 and 3 are available, and there are six other possible states. (This is a total of eight states, corresponding to the eight corners of a cube.)

When an incident occurs, the unit that is the dispatcher's first choice to respond to the location of the incident will be dispatched if it is available. The computer program knows the patrol area or location of the unit and therefore can calculate the expected travel time of the unit to the emergency. If the first-choice unit is unavailable when an emergency occurs, the program figures out which unit will respond (from information provided by the user) and calculates the expected travel time. If all units are unavailable, the user has a choice of having the program assume either that the incident will wait until a unit is available (typical for police and some ambulance agencies) or that some unit from another agency will handle the incident (typical for fire departments and some ambulance agencies).

The program calculates how often each state will occur by using equations from a field of mathematics known as *queuing* theory. In one mode of operation, the program solves the queuing equations *exactly*. This is practical for up to 15 units. In a second mode, the program solves the equations *approximately*. For 15 units or less the approximate mode is less expensive to operate on the computer than the exact mode; for more than 15 units it is necessary to use the approximate mode. The errors introduced by using the approximate mode are almost always under 5 percent, and typically under 2 percent. The user has his choice of the mode of operation, so for 15 or fewer units he can check the accuracy of the approximate mode in his application directly.

Once the program knows the chances that each state will occur and what happens in terms of travel time and which unit responds for each state, it can calculate all the performance measures described in the next section.

IV. HOW TO USE THE HYPERCUBE MODEL

To use the model, the city or part of the city to be studied must be divided into small "reporting areas." These are about the size of several city blocks or, in the case of a fire department, can correspond to the area covered by a single alarm box. A reporting area must be considerably smaller than a patrol area or a unit's response district.

The number of emergencies per hour that are expected to occur in each reporting area must be estimated by the user from past data. Also, the average length of time it takes for a unit to handle an incident must be estimated for the part of the city being studied.* For police patrol cars, the user must determine, in addition, the speed of units while on patrol and the number of patrollable street-miles in each reporting area.

The program also needs to know the travel time between reporting areas. If the user wishes, he may specify the location of the center of each reporting area on a grid map of the city and specify the travel speed of the units; then the program will estimate the travel times. Alternatively, the user can find out some or all of the travel times by another method, such as by experiment[10] or use of a computerized road network.[2] These times are then input directly to the program.

The user specifies a possible configuration of the emergency units by telling the program how many units there are, the patrol areas or locations for each one, and the relative amounts of time the unit patrols each reporting area in its patrol area when not otherwise busy. (If the unit has a fixed location, it stays in one reporting area 100 percent of the time when not busy.) To do this, the user simply draws on a map the patrol areas to be tried out and sees which reporting areas fall in each one.

The program also needs to know which unit will be the dispatcher's first choice to respond into each reporting area, which will be his

*The exact hypercube model permits this service time to vary according to which unit responds to the incident.

second choice if that unit is busy, and so on. The user can input this information for each reporting area or let the program calculate the dispatcher's choices according to the length of time it would take each unit to travel to the incident.*

Since the purpose of the Hypercube Model is to *compare* configurations, the user might want to prepare descriptions of several alternative configurations. Or, he can just enter one configuration and then later make changes to it, in response to the output information provided to him by the model.

The Hypercube Model will describe all the following characteristics of a trial configuration:

- For the entire city or part of the city under study:
  - -- average travel time to an incident
  - -- the difference in workload** between the busiest and least busy unit
  - -- percent of dispatches that take units outside their response districts (for units with fixed locations) or patrol areas (for mobile units)
- For each emergency unit:
  - -- average travel time to the incidents it responds to
  - -- its workload
  - -- percent of its dispatches that take it outside its response district or patrol area
- For each response district or patrol area:
  - -- average travel time to incidents in the district
  - -- percent of incidents handled by a unit assigned to the district
- For each reporting area:
  - -- average travel time to incidents in the area

*The general idea is that the program assumes the dispatcher will choose the closest available unit (*closest* in the sense of travel time), but there are several variations permitted.

***Workload* is the fraction of time the unit is busy handling incidents.

-- percent of incidents there that are handled by each of the units

-- (in the case of patrol cars) the average number of times per hour that a car passes a randomly chosen point in the area while on patrol.

From this information it might be found, for example, that the busiest unit in a trial configuration will be unavailable 80 percent of the time and will be able to respond to only 20 percent of the calls in its district. This would suggest that its response district should be made smaller. A new trial configuration would then be designed, and the model would indicate whether an adequate improvement has been made. Case studies have been written showing the step-by-step process by which patrol areas for police cars were designed in several cities using the model.[11-13] An agency wishing to use the Hypercube Model (whether it is a police department or not) is urged to read one or more of these case studies so as to see the steps involved in a typical application.

V. WHAT RESOURCES ARE NEEDED

The computer program for the Hypercube Model is written in a language called PL/I, so an agency wishing to use the program must have access to a compiler for this language. It is not necessary for any of the agency's staff to understand the PL/I language. All options available with the program are chosen by means of input cards described in the user's manual,[14] so there is never any need for an agency to make changes to the program statements themselves before using the model.

The cost for each run of the program on a computer system will vary from installation to installation, depending on the price structure, but a special feature of the program permits rapid determination of the costs of each stage of the calculations. The primary influences on cost are:

* Whether the user chooses the exact hypercube model, which can be quite expensive (but less expensive than a simulation model), or the approximate model, which is inexpensive,
* The number of emergency units to be considered (which cannot be more than 15 when using the exact model, but is essentially unlimited in the approximate model), and
* The number of reporting areas in the city or part of the city to be modeled.

Using the MIT Information Processing Center's IBM 370/165 computer to model a city with 120 reporting areas, the exact hypercube program required core storage ranging from 120K for a small number of emergency units up to 500K for 15 units. The cost for one run of the exact model ranged as high as $100. However, the approximate model never required more than 200K core storage, and the cost for each run was under $10 in all realistic cases tried. For most typical runs, the cost is about $1.00.

The length of time required to collect data for use in the model will depend on the following:

- Whether the agency has previously recorded the reporting area (or equivalent geographical information) for each incident in computer-readable form. (This is needed to calculate the expected number of incidents in each reporting area.)
- Whether the coordinates of reporting areas on a grid map of the city are known, or, as a substitute, the time required to travel between each pair of reporting areas is known.

If both of these have been done previously, the Hypercube Model can be used after at most two man-weeks of data preparation. Otherwise, the agency should plan on about four man-months for data collection.

Emergency service agencies that wish to consider using the Hypercube Model may obtain a deck of the program and a copy of the user's manual from any of the addresses shown in the Appendix. The charge for this service will be the cost of duplication. Questions about the program may be directed to Professor Larson, but it is not possible for him to provide routine user's services.

REFERENCES

1. Kenneth Rider, *A Parametric Model for the Allocation of Fire Companies*, The Rand Corporation, R-1615-NYC/HUD, 1975.

2. *Fire Station Location Package*, Public Technology, Incorporated, 1140 Connecticut Avenue, N.W., Washington, D.C. 20036.

3. Warren Walker, *Firehouse Site Evaluation Model: Executive Summary*, The Rand Corporation, R-1618/1-HUD, 1975. Peter Dormont, Jack Hausner, and Warren Walker, *Firehouse Site Evaluation Model: Description and User's Manual*, The Rand Corporation, R-1618/2-HUD, 1975.

4. Thomas E. Hendrick, Donald R. Plane, et al., *Denver Fire Services Research Project Report; Feasibility Test of Applying Emergency Service Deployment and Facility Location Methods to Assist in Municipal Budget Decisions in the Fire Service*, Denver Urban Observatory, Box 2483, University Center, 4200 East 9th Street, Denver, Colorado 80220, 1974. (Also available from The Rand Corporation as R-1566/3-HUD.)

5. Grace Carter, Jan Chaiken, and Edward Ignall, *Simulation Model of Fire Department Operations: Executive Summary*, The Rand Corporation, R-1188/1-HUD, 1974.

6. Grace Carter, *Simulation Model of Fire Department Operations: Program Description*, The Rand Corporation, R-1188/2-HUD/NYC, 1974.

7. Richard Larson, *Urban Police Patrol Analysis*, MIT Press, Cambridge, Massachusetts, 1972.

8. Peter Kolesar and Warren Walker, *A Simulation Model of Police Patrol Operations*, The Rand Corporation, R-1625/1-HUD/NYC (*Executive Summary*), R-1625/2-HUD (*Program Description*), 1975.

9. Jan Chaiken and Peter Dormont, *Patrol Car Allocation Model*, The Rand Corporation, R-1786/1-HUD/DOJ (*Executive Summary*), R-1786/2-HUD/DOJ (*User's Manual*), R-1786/3-HUD/DOJ (*Program Description*), 1975.

10. Peter Kolesar and Warren Walker, *Measuring the Travel Characteristics of New York City's Fire Companies*, The Rand Corporation, R-1449-NYC, 1974.

11. Richard Larson, "Illustrative Police Sector Redesign in District 4 in Boston," *Urban Analysis*, Vol. 2, 1974, pp. 51-91.

12. Kenneth Chelst, *Implementing the Hypercube Queuing Model in the New Haven Department of Police Services: A Case Study in Technology Transfer*, The Rand Corporation, R-1566/6-HUD, 1975.

13. James Jarvis, Mark McKnew, and Larry Deetjen, *Data Collection and Computer Analysis for Police Manpower Allocation in Arlington, Massachusetts*, Innovative Resource Planning in Urban Public Safety Systems, Massachusetts Institute of Technology, Cambridge, Massachusetts (to appear).

14. Richard Larson, *Hypercube Queuing Model: User's Manual*, The Rand Corporation, R-1688/2-HUD, 1975.

Appendix

ADDRESSES FOR FURTHER INFORMATION

1. For documentation of the Hypercube Model, copies of the program on card or tape, or answers to questions about the program:

Dr. Jan Chaiken
The Rand Corporation
1700 Main Street
Santa Monica, California 90406
(213) 393-0411

Professor Richard Larson
Room 4-209
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139
(617) 253-1358

2. Research sponsor:

U.S. Department of Housing and Urban Development
Alan Siegel, Director
Hartley Campbell Fitts, Program Manager

Office of Policy Development and Research
Community Development Research Division
Room 8166
451 Seventh Street, S.W.
Washington, D. C. 20410
(202) 755-5580

END