

PATROL ALLOCATION METHODOLOGY FOR POLICE DEPARTMENTS

PREPARED FOR THE DEPARTMENT OF
HOUSING AND URBAN DEVELOPMENT

JAN M. CHAIKEN R-1852-HUD SEPTEMBER 1975

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RESEARCH, DEPARTMENT OF HOUSING AND URBAN DEVELOPMENT



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PREFACE

This report was written primarily for police department administrators and planners. It is intended to assist them in determining which computer models, if any, among those available would be useful in resolving their patrol allocation problems. Analysts in the field of urban emergency services may also be interested in this report as a review of current knowledge.

Preparation of the report was sponsored by the Office of Policy Development and Research, U.S. Department of Housing and Urban Development. Other reports in this series include models for the deployment of emergency services, users' manuals for such models, case studies of applications in selected cities, and lecture notes for a training course in the deployment of emergency services. A companion report to the present one has also been prepared for fire departments:

Jan M. Chaiken, Edward J. Ignall, and Warren E. Walker,
Deployment Methodology for Fire Departments, The Rand
Corporation, R-1853-HUD, forthcoming.

Further information on any of these materials can be obtained from the author by writing to the address shown in Appendix B.

SUMMARY

This report reviews mathematical modeling methods that have been developed to assist police departments in allocating patrol resources. Models are described and compared, but not discussed in detail. Instead, the reader is directed to appropriate source documents.

The key issues of patrol allocation are:

- Determining the number of patrol units to have on duty in each of the department's geographical commands; this may vary by time of day, day of week, or season of the year.
- Designing patrol beats.
- Developing policy for dispatching and redeployment of patrol units.
- Scheduling manpower to match the variations in the number of units needed on duty.

Traditional approaches to the first issue are based on hazard formulas or workload formulas. Hazard formulas have serious failings and are not recommended for any purpose. Workload formulas have limited utility, which is to find allocations that balance workload. Better than either of these are computer programs that calculate a variety of performance measures and recommend allocations that meet the objectives established by the department. Only one general-purpose model for allocation of patrol units is currently available for installation on a police department's computer system.

For designing patrol beats, two models are available. These require much more detailed data than a model for allocating patrol units and should generally be used after a department has determined the appropriate number of units to have on duty. Either model will guide the planner to substantially better beat designs than he can obtain using only a map and manual calculations.

Issues of dispatching and redeployment policy are best resolved using simulation models, which are much more expensive and difficult to

use than either allocation or beat design models. Most departments would require outside analytical assistance to analyze such issues.

Manpower scheduling issues are of several types, and the choice of a suitable model depends on understanding the distinctions among them. Some of these models are very flexible, and most are relatively easy to use. The schedules they generate can be considerable improvements over those now in use in most departments.

Steps in a well-managed study of patrol allocation include collection of data, processing the data for the purpose of identifying the policy issue to be addressed, selection of a methodology, finding people with the relevant analytical capabilities, assembling a project team, acquiring and running the computer programs, and developing policy recommendations based on the analysis.

ACKNOWLEDGMENTS

In 1971, Richard Larson (consultant to The Rand Corporation) and the author prepared a survey of methods for allocating urban emergency units, parts of which have been adapted, updated, and included in this report. The author is indebted to Professor Larson for permission to extract sections of the earlier study that may have been his contribution without specific attribution of the source in each instance. Work by Thomas Crabill has also provided guidance in descriptions of some of the models.

An earlier draft of this report was circulated to many of the designers of the models described here and to researchers working on related topics. Valuable comments and suggestions for changes, which were incorporated in the final text, were received from Robert Baumgardner, Deepak Bammi, Joseph Ferreira, Nelson Heller, Edward Ignall, Rick Jarvis, Peter Kolesar, Richard Larson, Thomas McEwen, Spencer Smith, Keith Stevenson, Warren Walker, and Thomas Willemain.

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I. INTRODUCTION

This report describes various mathematical models that are available to assist police departments in allocating patrol resources, the principles on which the models operate, what they can be used for, and whether they require any special data collection or programming skills. Such information should help managers and planners discover whether they have a need for any of the models and, if so, which ones would be most appropriate in their circumstances.

The term "patrol" is used in a somewhat restricted sense here; it refers to uniformed officers who ride in mobile vehicles and engage in such activities as responding to calls from the public for emergency services. Although most police departments devote well over half their total budget dollars to this function, top police administrators rarely consider the question of possible deficiencies or improvements in patrol allocation. Primarily this is because patrol is conducted routinely and continuously by the lowest-level officers in the department and is unlikely to be the subject of public praise or concern, whether it operates efficiently or not.

Indeed, issues that should and do occupy the attention of police administrators--such as community support for the police, control of crime, and relations between labor and management--may well outweigh any considerations of efficiency that arise from viewing the patrol function in a narrow technical or mathematical sense. Thus, planning officers and patrol commanders who are concerned with patrol allocation must address operational questions in a context of both explicit and implicit administrative, legal, and political constraints. They cannot expect mathematical methods and models, such as those described in this report, to tell them the "best" solution to whatever problem their department faces, and they must be prepared to use all such methods with caution, imagination, and common sense.

When used with appropriate care, mathematical models provide several benefits to the planner. First, they can help identify deficiencies in current allocation policies by calculating performance statistics that

were not previously available to the department. Second, they permit the planner to consider and analyze numerous alternative allocation policies, whereas only one or two alternatives can be considered seriously if tedious manual calculations are required. Third, some of the models will suggest alternatives that may be preferable to any of the ones that were initially under consideration.

WHAT IS "PATROL ALLOCATION"?

The general subject of patrol allocation includes a variety of topics, some of which can be addressed using mathematical models and others not. The primary allocation issues that will be considered in this report are the following:

1. *How many patrol units* should be on duty?* This may be a long-term planning decision related to the department's budget and the relative amount of resources to be devoted to the patrol function, or it may concern the appropriate variation in patrol levels by time of day, day of the week, or season of the year.
2. *How many patrol units should be assigned to each geographical command?* This question refers to the location of patrol units in a general sense. The term "geographical command" refers to an organizational unit that is variously called a precinct, division, district, or area. For departments that have such geographical subdivisions, the number of patrol units to be assigned to each of them is an important allocation question.
3. *Design of patrol beats.* Ordinarily, each patrol unit is assigned to cover a small geographical area referred to as its patrol beat. The patrol beats of different units may be distinct, or several units can share a single beat, or the

* The expression "patrol unit" means the mobile vehicle used by patrol officers. Typical names given to patrol units by police departments are: patrol car, cruiser, RMP unit, black-and-white, and squad car. In addition, some officers may be deployed in scooters or other vehicles that can also be considered patrol units.

beats may overlap in various ways. Most police departments specify the geographical distribution of their patrol units by "designing" the patrol beats, that is, by drawing the boundaries of each unit's area on a map and determining the relative emphasis to be given to patrolling different parts of each area.

4. *Dispatching policy.* This is a very important aspect of patrol allocation and includes the following topics:
 - a. *How many patrol units will be sent to each reported incident?* Ordinarily, the answer to this question is that either one or two patrol units are sent to an incident. The answer may vary among departments according to the number of officers in each patrol unit, and it may also vary according to the reported nature of the incident and the availability of patrol units. In addition, most departments have a policy of not dispatching any patrol unit to certain types of unimportant incidents. Which calls are *screened out* in this way constitutes part of the department's dispatching policy.
 - b. *Which particular patrol unit(s) will be dispatched?* Departmental policy may specify that the closest unit to the location of an incident should be dispatched, or that the patrol unit assigned to the beat in which the incident occurs will be dispatched if available, or that the unit assigned to the beat will *always* be dispatched, even if the call must wait until that unit is available, etc. In addition, technical considerations such as the radio frequencies that can be received in each unit may affect which units can be dispatched to an incident.
 - c. *When will calls be queued (stacked, backlogged)?* At some level of saturation of the patrol force, new calls will be placed in queue to await the availability of an appropriate patrol unit. At a minimum this will have to happen if an incident occurs when all the patrol units in the city are busy. However, departmental policy may prescribe that calls

will be queued when all the units under the control of a single dispatcher are busy, or even if just the unit assigned to the incident's patrol beat is busy. In addition, it is possible to establish a policy of queuing calls under certain circumstances, even though some units are available. Such a policy can preserve the department's capability to respond rapidly to urgent calls that may occur in the near future.

- d. *Priority structure.* Once calls are placed in queue, a choice must be made as to which call will be handled first when a patrol unit becomes available. Ordinarily, the dispatcher categorizes the calls in his mind according to their relative importance, or priority, and when a unit becomes available he dispatches it to the highest priority call that has been waiting the longest. Departmental policy can affect what the dispatcher's priority structure is. In some cities, the patrol officer is informed of all calls waiting in his beat, and he decides the order in which to handle them. (This is termed "stacking calls on units.")
5. *Redeployment policy.* Patrol units can be reassigned to new beats, even outside their command, as unavailabilities develop. If a department adopts this policy, which we call redeployment, then rules must be developed as to when and how it is to be accomplished.
6. *Manpower scheduling.* Even if a department knows how many patrol units it wants to have on duty in each geographical command during each hour of the day, it may still be difficult to arrange for the appropriate number of officers to be on duty each hour to man the desired number of patrol units. There are actually three separate but interrelated questions here. One is how to determine the hours of the day at which tours of duty for the officers will begin. For example, in some departments all officers start work at one of three specified times, such as midnight, 0800, or 1600 hours, and they work for eight hours.

In other departments there are more than three tours, and they overlap in some way.

The second question is how to schedule the work days, days off, vacation days, and patterns of rotation among tours for the patrol officers. Such a schedule will determine the number of officers who are supposed to appear for work at the beginning of each tour. But then meal breaks, court appearances, and other events will remove patrol officers from patrol work even though they are on duty, and the third question is how such unavailabilities should be scheduled (to the extent that this is possible).

The discussion in this report is limited to the issues listed above primarily because we wish to emphasize the use of mathematical models that have been designed to suggest or analyze answers to the questions raised. These same models can in some instances be used to study equipment-oriented questions, such as whether a department should purchase a computer-assisted dispatching system or install car locators in its patrol units. But such topics fall outside the domain of "patrol allocation," and therefore will be omitted. Also omitted are a variety of issues of considerable concern to patrol commanders but for which suitable modeling approaches have not yet been developed. These include the appropriate role of patrol officers in conflict resolution, investigation of crimes, and field interrogation; mobilization of patrol resources for special events or major emergencies; and improvement of selection, training, and management policies so as to enhance the effectiveness of the patrol force.

MEASURES OF PERFORMANCE

Although there is widespread agreement that the primary objectives of the patrol function are crime deterrence, apprehension of criminal offenders, recovery of stolen property, providing the community with a sense of security from crime, and satisfaction of public demands for noncrime emergency services,⁽¹⁾ analysts who concern themselves with patrol allocation are in the peculiar position of not being able to base

their studies directly on any of these. One reason for this situation is the unavailability of suitable measures of the degree to which some of the objectives, such as citizen satisfaction, are being met. But more important is the lack of reliable methods for estimating the effect of changes in patrol allocation on any of the objectives.

For example, if the number of patrol units on duty is doubled, or halved, no one can state with any degree of certainty what will happen to crime rates or to the fraction of incidents that result in a patrol arrest. Indeed, if the number of patrol units is doubled, one effect will be an increase in the time available for preventive patrol, but recent studies⁽²⁾ cast doubt on whether this would lead to any change in crime rates. A second effect would be that patrol units arrive at the scene of incidents faster than previously. This may have value for deterring crime, but, if it does, the exact nature of the effect is unknown. It presumably also produces an increase in the number of on-scene apprehensions, but current knowledge of the extent of the change in Los Angeles⁽³⁾ and Seattle⁽⁴⁾ is not adequate to permit quantitative estimates for other cities.

As a result of these uncertainties, analysis of patrol allocation must rest on limited information about performance (whatever can be calculated reliably), together with informed judgment. Certain general principles are clear. First, a department must have enough patrol units to handle (sooner or later) all the calls for service that it wants to respond to. For example, if during an eight-hour tour the calls arriving from the public will entail 18 unit-hours of on-scene work, then two patrol units (i.e., at most 16 unit-hours) will certainly not be enough.

Second, extremely long delays (say three hours) in responding to even relatively minor incidents will not be considered satisfactory in most communities. Third, some kinds of changes can be identified as desirable even if we don't know exactly how much better they are. For example, if a change in allocation policy that uses the existing patrol resources can reduce response times to all incidents that are of concern to the department, this is a desirable change. Or, if the existing level of response times can be achieved at lower cost, such a change is also desirable. Although such "ideal" changes cannot often be achieved in

practice, they do illustrate the fact that the desirability of a specific change can often be identified without knowing its precise effect on police performance.

Considerations like these permit the listing of measures that are both practical to calculate and relevant for patrol allocation.

1. *Response time.* This is the length of time from the moment the caller reaches the police department until a patrol unit arrives at the scene. Response time includes a *dispatcher's processing delay*, which is not usually affected in any important way by changes in patrol allocation, a *queuing delay*, which is the length of time the call must wait until a unit is available to begin its trip to the incident, and a *travel time*, which is the length of time between the start of the patrol unit's trip and its arrival at the scene.
2. *Fraction of dispatches that take a unit outside its assigned patrol beat.* This is often deemed important because one reason for assigning patrol units to beats is to encourage the patrol officer to establish a "neighborhood identity" with a particular part of the city. This identity, which arises from patrolling and from citizen contacts made while responding to calls for service, is supposed to help the officer know the best way to handle incidents in his area and feel responsible for public order there. Should it happen that the officer is actually responding for the most part to incidents outside his patrol beat, this objective is undermined.
3. *Time available for other activities.* Since patrol officers have duties other than responding to calls for service--such as investigating suspicious circumstances, providing traffic control, engaging in patrol on foot in selected areas, executing warrants, searching for wanted persons and stolen vehicles, and interacting with citizens--the amount of time available for such activities is relevant to the performance of the patrol

force. In addition, patrol officers need some free time for meals, maintenance of their vehicle, and various administrative duties. Thus, it is ordinarily desirable to allocate patrol units in such a way that at least a certain minimum amount of time is available for activities other than handling calls for services.

Moreover, the relative balance of free time among patrol units is of interest in relation to the morale of patrol officers. For example, if one patrol unit has substantially more call-for-service work than any of the other nearby units, and thus less time for other activities, the patrol allocation policy will not be viewed as equitable by the officers.

4. *Cost.* It goes without saying that the three measures of performance discussed above can always be improved by assigning more officers to patrol duty. Therefore, in comparing various possible patrol allocation policies, it is necessary either to compare equal-cost alternatives or to identify clearly the difference in cost among alternatives. Often the cost of an allocation policy is clearly represented by specifying the number of officer-hours it entails, and there is no need to calculate the actual dollar values.

In this context it is important to note that the cost of the vehicle itself is completely insignificant compared with the cost of manning it. For example, for a city to field one extra patrol car, manned by two officers, around the clock involves incurring the salary and benefit costs for approximately ten officers, which in many large cities amounts to over \$150,000 per year. The cost of the patrol car itself, assuming it can be used for at least two years, accounts for about 2 percent of the cost.

OVERVIEW

This section described the patrol allocation issues to be discussed and the measures of performance to be used in comparing different

allocation policies; the remainder of this report is devoted to the methodologies that analysts have developed to assist police departments in resolving such issues. The next section begins with a non-technical description of the mathematical principles that underlie all the models. Then we discuss various individual methodologies in turn.

II. GENERAL PRINCIPLES FOR PATROL ALLOCATION

This chapter describes the basic principles that underlie analysis of patrol allocation. Some of these principles are well-known facts about the operations of patrol units that are mentioned here only because it is important to keep them in mind. Others are simple mathematical formulas that have been found useful for calculating performance measures. The equations are simple enough to be stated in words, using only a few mathematical symbols.

A few expressions will be used throughout the remainder of this report. The term *call for service (cfs)* means an incident to which patrol units are dispatched.* Most of these arise from telephone calls made by the public, which is why they have the name calls for service, but some arise from direct requests to the dispatcher by radio.** The *call rate* is the number of calls for service per hour, and a patrol unit that is unavailable for dispatch because it has responded to a call for service is said to be *busy on cfs work*. The *service time* of a patrol unit on a call for service is the length of time it is unavailable for dispatch while handling the call. This includes the unit's travel time and its *on-scene service time*. If more than one unit is dispatched to a call for service, then the number of *unit-hours* spent at the incident is the sum of the service times (in hours) of all the units.

- 1. *Number of units busy on cfs work.* At any given moment, a dispatcher of patrol units can easily determine whether none of the fielded patrol units is busy on cfs work, one unit is busy, two are busy, or whatever. If he periodically wrote down the number of patrol units busy on cfs work (say every minute) and then averaged these numbers at the end of an hour, an eight-hour

* Other common names for calls for service are *radio runs*, *radio jobs*, and *dispatch jobs*.

** For example, the fire department may be able to communicate directly with the dispatcher by radio.

four, a day, or some other period of time, he would have the *average number of units busy on cfs work* during that period.

The first general principle is an equation relating the average number of units busy to the call rate and the amount of work done on each call, as follows:

$$\left(\begin{array}{l} \text{average number} \\ \text{of units busy} \\ \text{on cfs work} \end{array} \right) = \left(\begin{array}{l} \text{average} \\ \text{call rate} \end{array} \right) \times \left(\begin{array}{l} \text{average} \\ \text{unit-hours} \\ \text{per call} \end{array} \right) \quad (1)$$

As an example, if an average of two calls are received per hour, one unit responds to each, and the average service time for a call is 30 minutes (= 1/2 hour), then on the average 1 unit would be busy on cfs work. If 2 units are on duty, on the average each of them would spend half its time busy on cfs work; if 3 units are on duty, each would spend an average of one-third its time on cfs work; and so forth.

While this equation may appear to be a meaningless manipulation of two numbers to calculate a third number, its value lies in the fact that it is possible to *predict*, from past experience, the approximate call rate and average number of unit-hours per call. Once these predictions have been made, the equation tells how many units will be busy on cfs work, on the average.

Consider a geographical command where, on Friday evenings between 4 p.m. and midnight, an average of 12 calls for service will be expected per hour. Suppose that half the calls will require one unit for 30 minutes and the other half will require two units for 30 minutes each. Then the average number of unit-hours per call is,

$$\begin{aligned} & 1/2 \times (1 \text{ unit}) \times (1/2 \text{ hour}) + 1/2 \times (2 \text{ units}) \times (1/2 \text{ hour}) \\ & = 3/4 \text{ unit-hours,} \end{aligned}$$

and the average number of units busy will be $12 \times 3/4 = 9$ units.

Now it is apparent that the absolute minimum number of units that can be assigned to this command on Friday nights is 9, since a smaller number of units will not be able to handle all the call-for-service work.

In general, the number of units fielded must at least equal the average number of units expected to be busy on cfs work; otherwise, it is impossible to respond to all the calls for service. We will see that sensible performance standards will require a substantially larger number of units than this to be fielded.

- 2. *Unpredictable characteristics of calls for service.* It is common experience that calls for service do not occur at orderly, predictable times and that the length of time required to handle a call is not exactly the same for all calls. Were it not for these unpredictable aspects of calls for service, it might be possible to allocate a smaller number of patrol cars than are needed in the real world.

An example will clarify this observation. Suppose that one call occurred regularly on the hour and half-hour, and each one took exactly 30 minutes to handle (including travel time). Then one patrol unit could handle all the calls, and no caller would have to wait for a unit to be dispatched. If there were two patrol units on duty, not only would no callers wait but also one of the units would always be available on patrol.

But in the real world, calls do not occur on the hour and the half hour. Indeed, if an average of two calls per hour is expected during a certain period of time (say, Friday from 4 p.m. to midnight), some of the hours may have no calls at all, while others may have five or more calls. Studies have shown that the number of calls will have what is known as a *Poisson distribution*.^{*} In the case of an average call rate of two calls

^{*}For a description of the Poisson distribution and its widespread applicability to random events such as accidents and radioactive decay of particles, see any textbook on queuing theory or probability theory, for example Refs. 5 or 6. Reference 7 shows that calls for service to the police are well described by the Poisson distribution.

per hour, the Poisson distribution tells us that

- 14% of hours will have no calls,
- 27% of hours will have 1 call,
- 27% of hours will have 2 calls,
- 18% of hours will have 3 calls,
- 9% of hours will have 4 calls,
- 4% of hours will have 5 calls, and
- 1% of hours will have 6 or more calls.

Adding together all the percents beginning with 3 calls, we now see that a single patrol unit would not even be adequate to handle all the calls in 32 percent of the hours. More important, calculations that take into account typical variations in service times^{*} show that even with two patrol units on duty there will be no unit available one-third of the time, and the average length of time a caller would have to wait for a car to be dispatched will be 10 minutes. (This includes calls that do not wait.) Seventeen percent of calls would have to wait more than 20 minutes, a situation that most departments would not consider satisfactory.

Although the absolute minimum number of cars needed to handle cfs work is 1 in this example, the unpredictable characteristics of calls result in a need for more than twice this minimum, if acceptable levels of performance are to be achieved.

- 3. *Patrol units are sometimes unavailable for reasons other than cfs work.* All dispatchers know that patrol units may be unavailable even though they have not been dispatched to a previous call for service. This may be discovered either by the patrol officers' announcement of unavailability by radio or by the dispatcher's inability to reach a patrol unit by radio when he wishes to assign a dispatch job to it.

What are the patrol officers doing during these unavailabilities? In some departments, patrol units are permitted to be unavailable during meal times for the officer(s) in the

^{*}See Appendix A.

unit. In all departments, patrol units may be unavailable for self-initiated anticrime activities (which may result in a lengthy unavailability for arrest processing), maintenance or repair for the vehicle, special assignments by a superior officer, execution of warrants, and authorized or unauthorized personal activities. These activities that make a patrol unit unavailable for dispatch are called *non-cfs work*.*

While the existence of non-cfs work is well known, what is important for patrol allocation analysis is the fact that non-cfs work may consume as much time of patrol units as cfs work, or even more. Studies in New York,⁽⁸⁾ Kansas City,⁽²⁾ and Los Angeles⁽⁹⁾ have shown that ordinarily more than 30 percent of a patrol unit's time is spent on non-cfs work, and typically it is somewhere between 45 and 65 percent of the time.

This means that if acceptable levels of performance are to be achieved, even more patrol units need to be on duty than would be suggested by considering only cfs work. Returning to the example above, where having two units on duty would lead to average delays around ten minutes before a call could be dispatched, suppose that the patrol units spend half their time on non-cfs work. Then the department would actually have to field four patrol units to achieve the performance levels characteristic of two units. To achieve better performance levels, five or more patrol units would have to be fielded.

To summarize, considering both the unpredictable nature of calls and the existence of non-cfs work, a command that appears to need one patrol unit to handle the cfs work actually needs five or more patrol units to achieve acceptable levels of performance in the real world.

* This is sometimes called *downtime*, incorrectly suggesting that no useful police function is performed during this time.

4. *Number of units needed to meet desired performance levels does not increase proportionately with the call rate.* For analysis of patrol allocation, it is very important to realize that when the call rate doubles, it is *not* necessary to double the number of patrol units to achieve the same levels of performance. In general, any percentage increase in the call rate leads to a need for a *smaller* percentage increase in the number of patrol units.

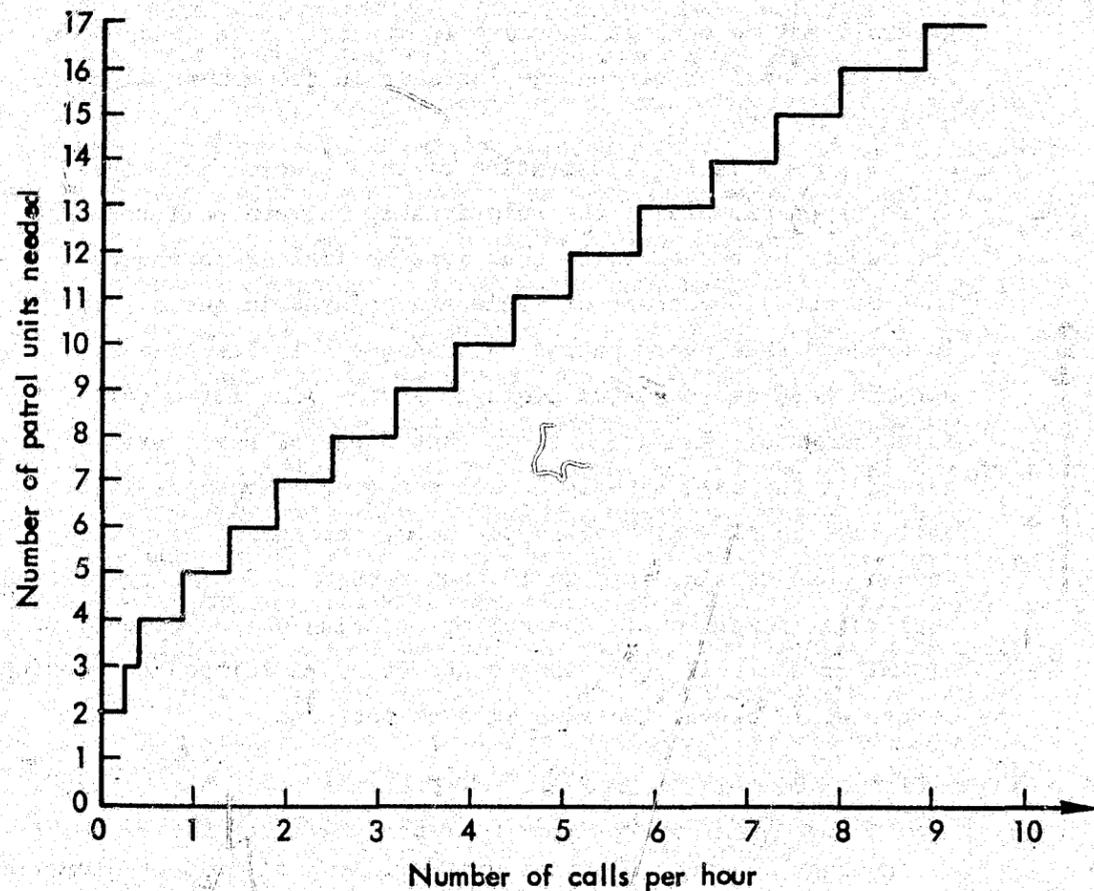
Figure 1 is an illustration of this general principle. It shows an example of the relationship between call rate and the number of patrol units that must be fielded to assure that at most 10 percent of calls are delayed in queue. It is assumed that every patrol unit spends half its time on non-cfs work and that the average service time for a call is 30 minutes. The graph shows that 5 patrol units are needed if the call rate is 1 call per hour. However, if the call rate doubles to 2 calls per hour, the number of units needed does not increase to 10, but rather to 7. At triple the call rate (3 per hour), the number of units needed is only 8. A similar analysis using some other standard related to queuing delays would reveal the same kind of pattern.

5. *Average travel time.* A very simple relationship, called the *square-root law*, has been found to give accurate estimates of the average *distance* a patrol unit will travel from its location at the moment of dispatch to the scene of the incident. The square-root law for average travel distances is important because, if we know how to calculate travel distances, then we will be able to estimate travel *times*. As mentioned in Chapter I, travel time is one component of response time, an important measure of performance.

If a geographical command has area A (in square miles) and there are N patrol units available* at the moment of dispatch,

* Available means not busy on either cfs work or non-cfs work.

Number of patrol units needed so that at most 10% of calls delayed



Assumptions: 30 minute service time per call
50% of each car's time spent on non-cfs work

Fig. 1—Example relationship between call rate and number of units needed

then the square-root law states that the average travel distance (in miles) is given by the equation

$$\left(\begin{array}{c} \text{average} \\ \text{travel} \\ \text{distance} \end{array} \right) = (\text{constant}) \times \sqrt{\frac{A}{N}} \quad (2)$$

The constant is approximately equal to 2/3. This relationship was derived through mathematical modeling^(10,11) and was validated by analysis of data constructed by using a patrol car simulation model (to be described in Chapter V).⁽¹²⁾

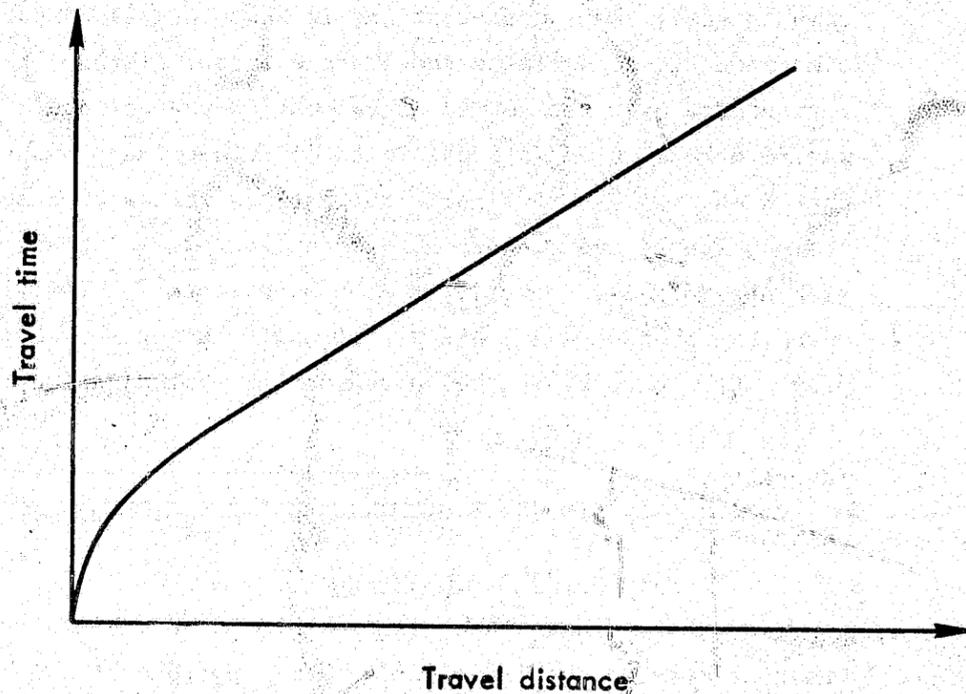
Since the number of available patrol units will vary from time to time, the square-root law in Equation (2) cannot be used directly to estimate the average travel distance to calls for service over the course of an hour or several hours. One way to make such an estimate is to calculate the probability that N cars will be available for various values of N and to use a weighted average of the distances shown in equation (2) (see Appendix A). However, it has been found⁽¹¹⁾ that if the average number of available patrol units is not too small (say, it is over 2.0), then a reasonable approximation is

$$\left(\begin{array}{c} \text{average} \\ \text{travel} \\ \text{distance} \end{array} \right) = (\text{constant}) \times \sqrt{\frac{\text{area}}{\text{avg. number of units avail.}}} \quad (3)$$

Now that we have this relationship, we need only convert travel distance into travel time. Two approaches have been suggested for this. In one of them, the patrol units have been assumed to travel at a constant speed when responding to calls.* However, studies of the travel characteristics of fire engines⁽¹³⁻¹⁵⁾ and ambulances⁽¹⁶⁾ have shown that this

*Or, in a slight variant, patrol units are assumed to travel at one speed when traveling in one direction (say, east-west) and another speed in the other direction (north-south).

is not a very good assumption, and in fact travel time is related to travel distance according to a curve such as the one shown in Fig. 2. This curve is equivalent to travel at a constant speed after a certain distance has been covered, but for small distances the speed is not constant. To my knowledge, no study has been done to date that shows whether the first approach (constant speed) is better or worse than the second approach (varying speed) for estimating the travel times of police patrol units.



Note: The scale on each axis will vary from city to city

Fig. 2— Schematic representation of the relationship between travel time and travel distance

If we use the first approach and measure travel speed in miles per hour, it is easy to see from equation (3) that

$$\left(\begin{array}{l} \text{average} \\ \text{travel time} \\ \text{in minutes} \end{array} \right) = \frac{60 \times (\text{constant})}{(\text{travel speed})} \times \sqrt{\frac{\text{area}}{\text{avg. number of units avail.}}} \quad (4)$$

The number 60 comes from the number of minutes in an hour. Assuming an average travel speed of 20 miles per hour (which is realistic, although patrol officers often estimate higher speeds) and a constant equal to 2/3, equation (4) reduces to

$$\left(\begin{array}{l} \text{average} \\ \text{travel time} \end{array} \right) = (2 \text{ minutes}) \times \sqrt{\frac{\text{area}}{\text{avg. number of units avail.}}} \quad (5)$$

Although we have made a series of simplifying assumptions in deriving this equation, it is nonetheless surprisingly accurate and is quite useful for comparing travel times in geographical commands of different sizes.

To illustrate its use, consider two geographical commands called Precinct 1 and Precinct 2. Suppose their characteristics are as shown in Table 1, and one patrol unit responds to every call. In Precinct 1, the average number of units busy on cfs work is (from equation (1)) $3 \times 1/2 = 1.5$, and the average number of units busy on non-cfs work is $1/2 \times 10 = 5$. Thus, the average number of units available is $10 - 5 - 1.5 = 3.5$. So, from equation (5) we know that the average travel time in Precinct 1 is approximately $2 \times \sqrt{\frac{8}{3.5}} = 2 \times \sqrt{2.29} = 3.0$ minutes.

In Precinct 2, on the average 5.25 cars are available, and the average travel time is approximately $2 \times \sqrt{\frac{18}{5.25}} = 3.7$ minutes.

Table 1

CHARACTERISTICS OF ILLUSTRATIVE PRECINCTS

Characteristic	Precinct 1	Precinct 2
Area (square miles)	8	18
Number of units on duty	10	9
Call rate	3/hr	1/hr
Average service time	30 min	45 min
Fraction of time on non-cfs work	1/2	1/3
Average travel speed	20 mph	20 mph

When the curve shown in Fig. 2 is used to relate travel time to travel distance, the equation for average travel time is only slightly more complicated than equation (4), and it can be used similarly to obtain good estimates. (17)

6. *Patrol frequency.* As mentioned in Chapter I, there is doubt in some quarters as to whether the amount of preventive patrol provided by a police department has a significant effect on crime rates or the number of apprehensions made by patrol officers. However, a substantial amount of analytical attention has been devoted to designing strategies for preventive patrol, and we shall mention here one general principle that underlies this work.

An appropriate measure for the amount of preventive patrol at any location is the number of times per hour that a patrol unit on preventive patrol will pass by. This is called the *patrol frequency*. The general principle is that the average patrol frequency in a geographical command increases in direct proportion to the speed of the patrol units and the average number of available units, and it decreases in inverse proportion to the number of street miles in the command. Expressed as an equation, the relationship is

$$\left(\begin{array}{l} \text{average} \\ \text{patrol} \\ \text{frequency} \end{array} \right) = \frac{(\text{patrol speed}) \times (\text{avg. number of units avail.})}{(\text{street miles})}$$

7. *Calls can be distinguished by priority.* Some calls for service received by the police require the response of a patrol unit, but not immediately. This means that even if there is a delay in response by the patrol officers, they can accomplish just as much as they could if they responded rapidly. A typical example of such a call, which may be termed *low priority*, is a report of a burglary that occurred several days in the past.

At the other extreme are calls that are universally recognized as being of *high priority*, meaning that the department would like to respond as rapidly as possible. A call for help by a police officer would fall in this category.

Between low priority calls and high priority calls are a variety of incidents that might be classified differently by different police departments, or even by different dispatchers within a single department. Practices differ as to how priorities are assigned. In some departments, the notion of priority is not formally recognized, and the dispatcher classifies calls by priority as he sees fit. In other departments, guidelines are provided for certain types of calls, while the remainder, often the vast majority, are left to the judgment of the dispatcher. Still other departments have established a formal system of priority levels, usually three or more, and have established procedures for determining the priority of each call.

Whether a department has a formal priority structure for calls for service or not, it is nonetheless likely that dispatchers will treat some calls as more important than other calls. Therefore, from the point of view of analysis of patrol allocation, there are two facts to be noted in regard to priorities. First, if a model of patrol operations does not take into account the actual practices of dispatchers in regard to priorities, then it may not result in an accurate description of the performance of the patrol units. Second, the performance of patrol units might be improved if the dispatchers modified their use of priorities.

How can the differences in priorities among calls be exploited? One possibility, which is common in many departments, is to use special procedures for eliminating the delay between receipt of a telephone call and transmittal of the information to the radio dispatcher in the case of high priority calls. This may be done by transferring the telephone call directly to the dispatcher, relaying the information to the dispatcher by intercom, or providing specially colored cards or other devices to alert the dispatcher to the fact that a particular call is important.

A second policy, also adopted in most departments, applies when no patrol units are available in a geographical command and ordinary calls are placed in queue. Very high priority calls will often be dispatched in these circumstances by sending a patrol unit from a neighboring command, dispatching a supervisor or special-purpose unit (such as a traffic car, a scooter, or an investigator's car), or by some other means. Moderately high priority calls will be placed in queue, but then will be dispatched prior to other calls that arrived earlier, as soon as a patrol unit is available. When this practice is adopted, high priority calls wait a shorter time in queue than low priority calls, but the average waiting time for all calls together is the same as if priorities were ignored.

A third possibility, which is much less common, is to hold one or two units in reserve in each geographical command for high priority calls. These can be specially designated units that respond only to high priority calls, or the department can simply establish the policy that no calls other than high priority ones are to be dispatched when only two units are available. In the latter case, the units held in reserve will be those units that happened to be available at the time. Either of these policies will result in longer average delays in queue, for all calls taken together, than the usual practice of not holding units in reserve. However, the possibility of having to place high priority calls in queue is practically eliminated.

A fourth possibility for exploiting priority distinctions among calls might be called "adaptive screening." This policy amounts to refusing to send a patrol unit to certain types of low priority incidents at times when a large fraction of the patrol force is busy.⁽¹⁸⁾ Dispatchers in certain cities currently find themselves forced to drop some calls from queue, although this practice is contrary to departmental policy and cannot please those who awaited the arrival of a patrol unit. By contrast, under a policy of adaptive screening, callers would be told that no patrol unit will be dispatched because of a temporarily high workload for the patrol force.

Finally, it is possible to schedule low priority calls to be handled at a more convenient time, when the number of calls for service is lower. Such a policy is under consideration in several cities, but we do not know of any department that has as yet adopted it.

III. DETERMINING THE NUMBER OF UNITS TO HAVE ON DUTY

This chapter discusses methods for determining the total number of patrol units a department should have on duty at various times and how they should be divided among geographical commands. These are the first two issues of patrol allocation mentioned in the Introduction.

A common error made in analysis of patrol allocation is to treat the question of how many *patrol officers* should be assigned to patrol unit duty as if it were separate from the question of how many patrol *units* should be on duty at various times and locations. However, if one method is used to determine the number of officers to be assigned to each geographical command and an independent method is used to specify the number of patrol units to have on duty, the results are not likely to be compatible. Some patrol commanders will find they are incapable of fielding as many units as they are instructed to field, while others will have extra officers without any guidance as to the best way to use them.

Therefore, it is important for a department to determine the relationship between patrol units and patrol officers and to assure that allocation decisions regarding each of these are compatible. This is not at all difficult to do. Every patrol unit that is on duty for an hour implies a requirement for one man-hour in the case of one-man units and two man-hours in the case of two-man units. Adding these requirements together over a week results in the total man-hours required. To determine how many patrol officers are needed to provide this number of man-hours, one only has to know the average number of hours per week that an officer devotes to duty in a patrol unit. This may be found by adjusting an officer's total work hours in a week to account for the average amount of vacation time, sick leave, court appearances, training, and the like.

If the department collects data showing the number of units currently fielded by time of day and day of the week, then it can simply divide the number of unit-hours provided per week by the number of officers assigned to patrol unit duty. This will give directly the necessary

relationship between officers and units. For example, suppose a department has 150 officers assigned to patrol unit duty and they work in one-man cars on eight-hour tours. If this department fields 20 patrol units during the AM tour, 28 during the Midday tour, and 32 during the PM tour, then it is providing $8 \times (20 + 28 + 32) = 640$ unit-hours per day, which is $7 \times 640 = 4480$ unit-hours per week. Rounding off to 4500 unit-hours (for simplicity), this amounts to $4500/150 = 30$ unit-hours per officer. Thus, on the average each officer works 30 hours per week in his patrol unit. If this department wants to add one patrol unit on the Midday tour and two on the PM tour, 168 unit-hours will be needed. Since $168/30 = 5.6$, the department will have to hire six additional officers (or transfer them from other duties).

Once a department has the capability to convert numbers of patrol officers into numbers of patrol units, and vice versa, a variety of policy questions can be addressed by the methods described in this chapter. At the broadest level, one might ask *how many patrol officers does the department need?* This question arises in preparing budgets and concerns the level of resources to be devoted to the patrol function.

Ordinarily, however, allocation questions are posed in narrower scope, with part of the answer already determined by budgetary or administrative considerations. These questions are related to distributing fixed resources by time or geography. For example, the total number of patrol officers may be known. In this case, the question might be *how many of the officers should be assigned to each geographical command?* Or, the number of officers in each command may be considered fixed, and the question is *how many of the officers should be on duty during each tour (or watch)?*

Conversely, the department may have no flexibility to change the number of officers on duty at different times of day, but be free to move them around among geographical areas. Then a typical allocation question would be *how many of the patrol units on duty during the AM tour should be assigned to each geographical command?*

WORKLOAD FORMULAS

The traditional method for allocating police patrol units to geographical commands is to use a *hazard* or *workload* formula. Here we shall describe why such formulas are inadequate and then suggest a substitute in the next section. Perhaps the best known hazard formula was developed by O. W. Wilson in the late 1930s. (19-21) In this method, the user identifies "factors" that are thought to be relevant for man-power allocation. Factors frequently used include measures of the crime rate in each command (total number of Part I crimes, total outside crime, etc.), measures of activity (arrests, calls for service, traffic accidents, etc.), and measures of implicit requirements for police service (number of street miles, number of doors and windows to be checked, etc.). A simple example with only three factors is shown in Fig. 3.

Once an administrator has determined what factors he considers relevant, the next step is to assign a "weight" to each factor. If a *hazard* formula is being constructed, the weights will specify the relative "importance" attached to each factor. Thus, violent outside crimes might be given a weight of 5, other FBI index crimes a weight of 3, and other calls for service a weight of 2. This means that outside violent crime is judged to be 2½ times as important as a noncrime call for service.

On the other hand, if a *workload* formula is being constructed, the weights reflect the number of man-hours required to handle each factor. In this case the weight for a noncrime call for service might be larger than the weight for a violent outside crime if the data show that non-crime calls for service require more time to handle them.

The next step in the method is to combine the weights and the factors to obtain a *hazard index* or *workload index* for each geographical command. The workload index is found by multiplying the weight for each factor by the amount of the factor in the command and adding the products for all the factors. To calculate the hazard index, one first finds the total amount of each factor in the entire city and calculates the fraction of the factor that occurs in the command in question. Then, this fraction is multiplied by the weight, and the products are added together as in the case of a workload index.*

*Mathematically, every formula for calculating a hazard index is equivalent to some workload formula with different weights.

Factor	Precinct A	Precinct B	Precinct C	Total
1. Violent outside crimes	280	500	900	1680
2. Other FBI index crimes	700	1400	2500	4600
3. Other calls for service	6000	20000	56000	82000

Hazard formula

h_1 = hazard weight for violent outside crimes

h_2 = hazard weight for other FBI index crimes

h_3 = hazard weight for other calls for service

$$H_A = \text{hazard index for Precinct A} = h_1 \frac{280}{1680} + h_2 \frac{700}{4600} + h_3 \frac{6000}{82000}$$

$$\text{Hazard assignment to Precinct A} = (\text{total number of officers}) \times \frac{H_A}{(h_1 + h_2 + h_3)}$$

Workload formula

w_1 = workload weight for violent outside crimes

w_2 = workload weight for other FBI index crimes

w_3 = workload weight for other calls for service

$$W_A = \text{workload index for Precinct A} = w_1 \cdot 280 + w_2 \cdot 700 + w_3 \cdot 6000$$

$$W = \text{total workload} = w_1 \cdot 1680 + w_2 \cdot 4600 + w_3 \cdot 82000$$

$$\text{Workload assignment to Precinct A} = (\text{total number of officers}) \times \frac{W_A}{W}$$

Fig. 3--Hazard and workload formulas for example city

The final step is to determine the total hazard or workload index in the entire city and to allocate patrol officers or patrol units among geographical commands in direct proportion to their indexes. Thus, a command whose index is 7 percent of the total index for the city would receive 7 percent of the patrol officers.

In principle, if the workload index could be calculated very accurately, it would also tell the total number of patrol officers needed in the city, but as a practical matter it is not possible to account for every man-hour of the patrol force in the index. More reasonably, it might be claimed that an increase in the workload index by 4 percent over last year implies a need to increase the size of the patrol force by 4 percent. Primarily, however, both hazard formulas and workload formulas are used to allocate a specified total number of officers or units among commands.

Many of the factors in a hazard or workload formula vary by time of day. By calculating the index separately for each hour or tour, it is possible to use the index for allocating officers across time as well as across geographical commands.

The widespread use of hazard or workload formulas in police departments attests to the fact that they have considerable appeal. They are easy to understand, and the required calculations can be done manually (although this may be a tedious process). They also appear to take into account everything that might possibly be relevant for allocating patrol units.

However, the apparent virtues of these formulas are basically deceptive, since they do not accomplish what most users think they do. In the case of a workload formula, the allocation accomplishes only one objective. It equalizes the workload of patrol units, without regard to any other measures of patrol performance. Thus, it is possible for two geographical commands which are nearly identical in all respects other than size to obtain allocations that result in substantially different queuing delays for calls for service, travel times, patrol frequencies, and other measures of performance. These disparities can be easily predicted in advance, as has been done in a recent study⁽²²⁾ that compares allocations derived from a workload formula with allocations derived by other methods to be described in the next section.

Allocation by hazard formula suffers from even worse difficulties than allocation by workload formula, because it is not possible to identify any objective that is met by the allocation. The main problem is that an administrator may give some factor a low weight (even zero) because he does not consider it "important," but this in no way relieves the patrol officers of the obligation to handle the associated activity. The unintended consequence is that when officers spend time on the "unimportant" activity they are unavailable to respond to important, as well as unimportant, incidents.

To illustrate the difficulty, consider the example city described in Fig. 3. If the administrator considers all calls for service to be equal in importance, he would assign the hazard weights $h_1 = h_2 = h_3 = 1$. In this case, the hazard formula assignment to Precinct C would be

$$\begin{aligned} & \text{(total number of officers)} \times \frac{\frac{900}{1680} + \frac{2500}{4600} + \frac{56000}{82000}}{3} \\ & = \text{(total number of officers)} \times (0.587). \end{aligned}$$

Thus, Precinct C would receive 58.7 percent of all the officers.

On the other hand, if the administrator considers violent outside crimes very important, he might assign weights $h_1 = 7$, $h_2 = 2$, $h_3 = 1$. Then the assignment to Precinct C is

$$\text{(total number of officers)} \times \frac{7 \cdot \frac{900}{1680} + 2 \cdot \frac{2500}{4600} + \frac{56000}{82000}}{10},$$

which turns out to be 55.2 percent of all the officers. Note that Precinct C has the largest number of outside crimes, but the higher the weight given to violent outside crimes in the hazard formula, the smaller is the number of patrol officers assigned to Precinct C by the hazard formula.*

*This is because the ratio of the third factor (other calls for service) to the first factor (violent outside crimes) is larger in Precinct C than in the other precincts, while the ratio of the second factor

As a result, by *increasing* the weight for violent outside crimes, the administrator in the example city brings about an allocation that *increases* the fraction of time that officers in high crime commands spend on unimportant incidents. In short, the consequences of changing weights in a hazard formula are not necessarily what they appear to be.

Many hours have been spent in police department conference rooms around the country in an attempt to agree on the "right" importance weightings for a hazard formula. The answer to this problem is simple: There are no "right" weights. The use of a hazard formula cannot be recommended for any purpose.

Returning now to workload formulas, certain practical difficulties should be noted. First, it may not be easy to determine the workload weight for some factors. What, for example, is the appropriate number of man-hours to assign to a door or window that must be checked, or to a street-mile? Second, great care must be exercised to avoid double-counting (or multiple-counting) of man-hours. For example, one factor may be "calls for service," another may be "Part I crimes," and another may be "arrests." How should one count the man-hours spent on a call for service that is a Part I crime? What if the officer responding to a call for service makes two arrests at the scene? Although in principle it is not difficult to decide how such occurrences should be handled, the data may not be available in appropriate form.

Third, a workload formula may have the perverse effect of indicating a need for additional personnel in areas which already are relatively overallocated. This can arise because factors such as number of arrests and reported crimes depend on the number of personnel currently allocated to an area. For example, crime suspects are more likely to be apprehended in an area that is sufficiently staffed than in one that is not. The workload index of the former area would be artificially inflated by the high arrest rate of that area, thereby "substantiating" the need for more officers. The real need for personnel would be in the latter area, but a workload formula would not fully reveal this need.

(other index crimes) to the first factor is about the same for all precincts. Since it is not unusual to find that high-crime commands have proportionately more noncrime calls for service than low-crime commands, the example illustrates a peculiar property of hazard formulas that could occur in many cities.

Even worse, if a patrol commander succeeds in reducing the crime rate in his district in comparison with crime rates elsewhere, he will be "rewarded" the next time the workload formula is used by receiving a smaller allocation of patrol officers.

If all these difficulties are kept in mind, successful applications of workload formulas are possible. An important point is not to accept the allocations suggested by the formula blindly, but rather to make appropriate adjustments after determining the effect of the proposed allocation on performance measures. Also, if a workload formula is to be used, the department should realize that the calculation can be easily programmed on a computer. If it does so, it will save many days or weeks of tedious manual calculations and will avoid the numerical errors that inevitably occur during the course of such work.

METHODS BASED ON PERFORMANCE MEASURES

Allocation methods based on performance measures have been developed during the past decade. These methods depend on the ability of computers to calculate performance measures rapidly under a variety of assumptions about the number of patrol units on duty. This capability permits approaches of the following types:

- The department devises several different plans for the number of units to have on duty and uses the performance measures to determine which plan appears best.
- The department establishes *standards of performance* and assures that at least enough units are on duty in every tour in every command to meet the standards. Typical standards in use are that no more than 10 percent of calls for service encounter a queue, or the average waiting time in queue does not exceed 5 minutes for high-priority calls, or the total response time averages no more than 8 minutes.
- The department decides to allocate its existing unit-hours in such a way as to achieve the best possible value of some performance measure. For example, the total unit-hours that can be fielded on a Monday night might be allocated among tours so as to

yield the lowest possible average response time, where the average is taken over all calls that occur during the day.

Some of the computer programs that have been devised for this purpose will simply display the performance measures for a given allocation plan, leaving it to the user to try enough plans to see which one meets the department's objectives. For example, if the program shows the fraction of calls that will encounter a queue under the assumption that two units are on duty, three units are on duty, and so forth, then it is not difficult to scan down the printout and determine the smallest number of units needed to keep under 10 percent of calls from being queued.

Other computer programs generate the trial allocations themselves and only display the ones that meet objectives specified by the user. Not only does this reduce the volume of printout, but also the computer program may find an allocation that is better than any that would have been devised by the user.

However, even a computer cannot consider a large number of potential allocations at reasonable cost if the method used to calculate the performance measures is exceptionally complicated. For this reason, any program that is practical to use for determining the number of units to have on duty will necessarily be based on simplifying assumptions about the operations of patrol units and the characteristics of calls for service. The general principles discussed in Chapter II are examples of such simplifying assumptions.

Whether the assumptions incorporated in a computer program lead to sufficiently accurate estimates of performance measures can only be determined individually in each city. This is best done by using the computer program to estimate the performance of the *existing* allocation and at the same time collecting enough data about the actual queuing delays, workloads, and other characteristics to make a comparison. If the disparities are large, blind acceptance of the allocations suggested by the computer program is no more sensible than blind acceptance of allocations from a workload formula. However, experience in several cities has shown that large disparities can usually be traced to errors

in data preparation, and appropriate adjustments eventually lead to a suitable fit of the estimates to reality.

At the present time, police departments do not have a large choice of computer programs for allocating patrol units by time of day and geographical command. At one time, the IBM Corporation provided a program called the Law Enforcement Manpower Resource Allocation System (LEMTRAS),⁽²³⁾ which was based on a program initially designed for the St. Louis Police Department.⁽²⁴⁻²⁷⁾ However, LEMTRAS was withdrawn at the end of 1974 because only a small number of departments had ever used the program, and it was not compatible with the operating systems then being marketed by IBM. In addition, most customers were interested in an on-line interactive program, while LEMTRAS operated in batch mode.*

LEMTRAS estimated future call rates and service times from past data by using a mathematical technique known as *exponential smoothing*. It also calculated the probability that calls of various priority levels would be delayed in queue for 5 minutes, 10 minutes, and so forth, depending on the number of units on duty. Users could establish standards of performance related to these particular performance measures and rapidly determine the minimum number of units that had to be on duty during each tour in each geographical command to meet the standards.

Later, Richard Larson⁽¹⁰⁾ developed an interactive program that permitted users to establish standards related to performance measures other than queuing (namely, workload, travel time, and preventive patrol frequency) and to allocate a fixed total number of units so as to achieve the lowest possible average waiting time in queue. But this program did not predict call rates and service times, and it was written in a programming language that cannot be used on current computer systems. Urban Sciences, Inc., subsequently rewrote Larson's program and greatly enhanced its interactive capabilities.⁽²⁸⁾ This version of the program may be accessed by a contract with Urban Sciences for time-sharing services, but it is not available for a police department to install on its own system.

* In *batch* mode, all instructions to the program are prepared on cards or similar input device prior to program execution, and output is received later on a high-speed printer. In *interactive* mode, the user enters instructions at a terminal and receives output immediately at the same terminal.

The most recent program, the Patrol Car Allocation Model (PCAM), (29) was developed by The New York City-Rand Institute. It incorporates most of the features of previous programs and improves upon them in some details. It will operate in either batch or interactive mode. However, it does not assist the user in estimating call rates and service times. Thus, with the withdrawal of LEMRAS, a police department that wants to use a computer program for allocating patrol units must either use averages of past data as substitutes for estimates of future call rates and service times or acquire a separate program for projecting into the future. Since several departments have designed such projection programs, it may be possible to "borrow" one.*

PCAM calculates performance measures according to the principles outlined in Chapter II. For each geographical command, the user must provide the following information:

- Call rates and service times by hour of the day and day of the week**
- Area of the command (square miles)
- Street miles in the command
- Response speed and patrol speed of patrol units
- Crime rates
- Parameters permitting a determination of the amount of non-cfs work.

Then the PCAM program will estimate all of the following performance measures (if the number of units on duty is known):

- Average number of units available (i.e., not busy on cfs work or non-cfs work)
- Preventive patrol frequency
- Average travel time to incidents

*The author will assist departments interested in locating such a program. See the address given in Appendix B.

**Calls may be broken down into three priority levels.

- Fraction of calls that will encounter a queue
- Average waiting time in queue for calls of each priority level
- Average total response time.

PCAM will display all of these performance measures for any allocation proposed by the user. In addition, it will determine the minimum number of units needed to meet standards of performance for any of these measures. The user may also choose any of the performance measures displayed with a double bullet (••), and PCAM will allocate a specified total number of unit-hours so as to minimize the chosen measure. This capability permits allocation across time or geography or both. In other words, the user can specify the total number of units on duty in the city at a particular time of day, and the program will allocate them among geographical commands. Or the user can specify the total number of unit-hours that can be fielded in a week (in one command, several commands, or all commands together), and the program will allocate patrol units to tours so as to add up to the total number of unit-hours.

These capabilities are about the best that can be accomplished at the present time. PCAM permits a department to specify its objectives and then to find allocations of patrol units that meet those objectives. To the extent that performance measures of importance to police administrators (e.g., deterrence of crime, apprehension of criminal offenders) are omitted from the program, it is not because they are deemed unimportant but rather because there is no known way to estimate them. However, we have already pointed out that methods that appear to take such objectives into account, namely hazard formulas, do not actually do so.

IV. DESIGN OF PATROL BEATS

The models to be discussed in this chapter are suited for designing patrol beats and analyzing other questions related to geographical details of patrol operations within a command. Previously discussed models, such as the Patrol Car Allocation Model, cannot be used for these purposes because they calculate *average* performance measures for an entire command. Moreover, they estimate these averages from approximate formulas that ignore variations within a command.

Thus, for example, PCAM will estimate the average response time in a command when 10 patrol units are on duty. But it cannot determine the average response time in each unit's patrol beat, nor can it estimate how the average response time in the whole command will change if the patrol beats are modified.

On the other hand, a model that takes into account geographical details within each command is too cumbersome and expensive to use routinely for allocating patrol units among tours and commands. Such a model requires much more extensive data than PCAM and provides detailed output that would overwhelm the user with information he does not need to decide how many units to have on duty.

While a geographically detailed model can calculate better estimates of performance measures than PCAM can, the improved accuracy is not important for allocating units among tours and commands. This is because the number of units assigned to a command has a much larger influence on performance measures than the locations or patrol beats of the units, so long as they are more or less sensibly located. For example, different geographical arrangements of 10 units in a command will lead to different average response times for the command as a whole. But all of these averages will be much closer to PCAM's estimate for 10 units than to PCAM's estimate for 12 units. To decide whether to have 10 units or 12 units on duty, it is not necessary to know the exact response time.

In many circumstances a department will want to use both types of models. First, it determines how many patrol cars to have on duty in each command, using PCAM. Then it designs patrol beats for the commands,

using a geographically detailed model. These models help the planner to identify beat designs that accomplish one or more of the following objectives:

- Balancing workloads among units
- Equalizing response times among different parts of a command
- Minimizing average response time for the entire command
- Minimizing the extent to which patrol units are dispatched outside their assigned beats.

In general, it is impossible to achieve all of these objectives simultaneously, so the models also assist in finding acceptable compromises.

The models will permit analysis of designs in which beats overlap, as well as traditional nonoverlapping designs. This capability is particularly important to departments that wish to minimize or reduce the extent of out-of-beat dispatching. With nonoverlapping beats, it is inevitable that a substantial fraction of dispatches will take units outside their assigned beats. In fact, if patrol units are assigned to nonoverlapping beats and are busy (on cfs work or non-cfs work) about 60 percent of the time, then typically somewhere between 50 and 70 percent of calls for service will be handled by a unit other than the local beat unit. In these circumstances, it is extremely difficult for the patrol officer to establish a "neighborhood identity."

However, with overlapping beats the extent of out-of-beat dispatching can be substantially reduced. Many departments have recently introduced "team policing" or other allocation plans in which several units share responsibility for an area that is larger than a traditional patrol beat. These plans constitute various forms of overlapping beats, and the areas of responsibility for each team can be designed using beat design models.

In the absence of a mathematical model, most departments design patrol beats in such a way that they are "reasonably" shaped, lie wholly on one side of any natural barriers to travel that may exist (limited-access highways, railroads, rivers, and the like), and correspond as closely as possible to recognizable "neighborhoods" (in the sense of

commonality of spoken language, demographic characteristics, and land use). In addition, planners usually attempt to equalize the numbers of calls for service that can be expected in each beat. However, most other measures of performance are too difficult to estimate by looking at a map, so they are not considered in beat design.

When using a mathematical model, the planner must still be familiar with barriers to travel, "neighborhoods," main streets, etc., but the model provides him with detailed information about performance measures. This information permits the planner to identify the failings of any proposed beat design, and leads him to construct a sequence of improvements, ultimately resulting in an acceptable design. When calculating the cfs workload of units, a model takes out-of-beat dispatching into account, and thereby gives much better estimates than can be obtained by counting the number of calls for service in each beat.

The model "understands" that the burden of out-of-beat dispatches falls more heavily on some units than on others, depending on their locations. For example, it is apparent that a patrol unit whose beat is in the center of a command will be the dispatcher's second choice (after the local beat unit) for more locations than will a unit whose beat is on the boundary. If the beat in the center and the beat on the boundary both have the same call rate, the workload of the centrally located unit will nonetheless be higher, because it will have more out-of-beat dispatches.

The two basic models that have been applied to beat design are Richard Larson's Hypercube Queuing Model^(30,31) and a model developed by Deepak Bammi.^{(32-35)*} These share the common feature that the command to be studied must be divided into small "reporting areas," which are approximately the size of a few city blocks. Beats may be defined in any way desired by the user as collections of (usually adjacent) reporting areas, and they may overlap partially or fully. In addition, the user can specify the relative amount of preventive patrol in each reporting area. This is a particularly important feature for departments

* Some earlier models of this type (e.g., Refs. 36 and 37) did not attempt to account for out-of-beat dispatches and will not be discussed here.

that have policies of "directed patrol," in which officers are instructed to spend their free time on specified anticrime activities in selected locations.

The call rate in each reporting area must be determined from historical data (possibly projected into the future), and some means of calculating the travel time from one reporting area to another must be provided. In Larson's model, the user has a choice of specifying a travel speed and the coordinates of the center of each reporting area, in which case the program estimates travel times, or the user can calculate the travel times between reporting areas by some other means and provide them as input to the program. Bammi's program permits the user to specify travel distances and travel speeds between *adjacent* reporting areas only, and then the program calculates all the travel times, a convenience to the user.

Both programs then estimate the workloads and travel times for all of the units. The main differences between the programs are as follows:

- Bammi's program explicitly takes non-cfs work into account, while in Larson's program non-cfs work must be included in the call rate if it is to be considered at all.
- Bammi's program permits two priority levels for calls, whereas in Larson's program all calls have the same priority but some calls may preferentially be served by certain units (e.g., units with Spanish-speaking officers).
- Bammi's program will calculate a beat design that minimizes average response time in the command. The version of Larson's program that is currently available will not *suggest* any beat designs but will simply calculate performance measures for designs proposed by the users. However, programs that include Larson's model and generate beat designs that minimize workload imbalance or minimize travel imbalance are nearing completion and will be available in the future.⁽³⁸⁾

The two models will not make identical estimates of performance measures, because they are based on different mathematical assumptions.

If the workloads of units are either very low (e.g., the typical unit is busy less than 15 percent of the time) or very high (e.g., busy more than 50 percent of the time), both models should give about the same values for performance measures. However, most departments' workloads lie between these extremes, where the models will differ. The magnitude of these differences is not known, but perhaps some comparative test will be made in the future.

Larson's model pays particular attention to the fact that when one patrol unit is busy, nearby patrol units are also likely to be busy (because they will respond to calls not only from their own beat but also from the beat where the unit is busy). Bammi's model assumes that the unavailability of one unit has no bearing on the unavailability of nearby units (i.e., unavailabilities are assumed to be independent). If most unavailabilities arise from calls for service, then Larson's model will give a better representation of the situation in the field. But if most unavailabilities arise from non-cfs work, Bammi's independence assumption may be closer to reality.

Both models have been tested for accuracy, Bammi's test⁽³²⁾ using simulated data,* and the test of Larson's model using real data from New Haven, Connecticut.⁽³⁹⁾ Either model appears to be adequately accurate for the purposes of beat design and will lead to substantially better designs than a planner can produce using a map and manual calculations. Both models are well documented and can be installed on computer systems available to most police departments, but currently only Larson's model has a complete step-by-step user's manual⁽³¹⁾ to aid in implementation. Assistance in installing and using either model is available, and the author will refer interested readers to an appropriate source of assistance.**

* See Chapter V for a description of simulation models.

** Write to the address given in Appendix B.

V. DISPATCHING AND REDEPLOYMENT POLICY

The Patrol Car Allocation Model and the beat design models, discussed in the previous two chapters, were programmed to be inexpensive and easy to use. For this reason they include only such details of patrol operations as are likely to have a major impact on the policy decisions that can be made using those models. However, police departments may wish to take into account the omitted details, either to assure themselves that policies developed with the assistance of the models will actually work out as anticipated, or to study policies that are directly related to the omitted details. For these purposes, which would include most studies of dispatching and redeployment policy, more complicated models are required.

Typical details that are not taken into account by PCAM or a beat design model are the following:

- More than one unit may be dispatched to certain types of incidents, or a backup unit may respond even though the dispatcher has not specifically requested it to do so.
- Units may be dispatched across command boundaries for high-priority calls. (If such cross-command dispatches occur commonly for all types of calls, this can be taken into account easily by reinterpreting the meaning of the term "command" to apply to larger regions.)
- Dispatchers may queue low-priority calls to await the availability of the local beat unit, even when nearby units in the command are available. (This practice will increase queuing delays above those estimated by the previously discussed models.)
- Dispatchers may "stack calls on units." This means that several calls are assigned simultaneously to a patrol unit, which is then unavailable until it completes service on all of them. (This practice will also increase queuing delays. Although it will appear that a call is no longer in queue after it has been assigned by the dispatcher, the call is actually in queue until the moment the patrol unit begins its response to the scene.)

- Some kinds of non-cfs work may occur at predictable times, rather than at random. Typical examples would be meals and school-crossing duty.
- One or two units in each command may be held in reserve for high-priority calls.
- The dispatcher's choice of which unit to dispatch may depend on the availability of other units. For example, the closest available unit may be surrounded by beats in which the units are busy, and the dispatcher might then dispatch a more distant unit. For a complete discussion, see Refs. 40, 41, and 42.
- Service on low-priority calls may be preempted for dispatch to high-priority calls. This means the patrol officers interrupt what they are doing to respond to a more important call.
- Patrol units may be redeployed to new beats or new commands as unavailabilities develop.
- Patrol units may become available for dispatch prior to the time they return to patrolling, for example when they are completing paperwork.

The only types of models that can take all such complexities into account are *simulation models*. These imitate step-by-step the operations of patrol units. A large number of real or imaginary incidents are tracked from the moment the call is received through the dispatch of one or more units, their arrival at the scene, work at the incident, completion of the job, and return to patrol.

Simulation models are able to provide much more complete summary statistics than can be obtained from simpler models. For example, the fraction of calls that received a response time over 20 minutes can be determined, or the largest response time that occurred in all the simulated incidents can be displayed. In addition, the accurate representation of patrol operations that is possible with a simulation model leads to much more reliable estimates of those performance statistics that are also generated by simpler models.

For anyone with the requisite skills, it is relatively easy to design a simulation model, and therefore there are many more of them

than we can possibly describe here. However, experience has shown that it is not easy to design a *good* simulation model, because this requires close familiarity with the department's operations. Also, simulation models are expensive to run on a computer, need large amounts of detailed data as input, and require considerable analytical skill to interpret the output correctly. Therefore, they are not recommended for any short-term policy analysis. They should only be considered for incorporation in a sustained study of a comprehensive collection of policy options, to be conducted over a period of a year or more. However, the cost of such a study may well be substantially less than the cost of a real-world test of a new policy, especially if it turns out not to have been a good idea.

Two general-purpose simulation models of patrol operations that are available to police departments were designed by Larson⁽¹⁰⁾ and by Kolesar and Walker.⁽⁴³⁾ Larson's model was rewritten by Urban Sciences, Inc., and is available by contract for time-sharing services.⁽⁴⁴⁾ The Kolesar-Walker model is available from The Rand Corporation for installation on a department's own computer system. The primary differences between them are as follows:

- The Kolesar-Walker model will accept a historical stream of calls for service as input, thereby permitting an exact duplication of demands placed on the department, together with their actual variations of service times. Alternatively, the input stream can consist of imaginary incidents derived from average call rates and service times. Larson's model uses imaginary incidents only, and they are generated internally based on average call rates and service times provided by the user.
- The Larson model permits specifying the relative amount of free time spent in various locations by patrol units. In the Kolesar-Walker model the patrol units are imagined to remain at the scene of the last incident until they are dispatched to another one.*

* There are exceptions to this for out-of-beat dispatches.

- The Larson model allows for preemption of calls. In the Kolesar-Walker model a call is preempted only if the assigned patrol unit has not yet arrived at the scene of a lower-priority call.
- Scheduled non-cfs work can be included in the Kolesar-Walker model, but not in the Larson model.
- Only one unit is dispatched to incidents in the Larson model.

The process of understanding an existing simulation model, modifying it to reflect unique aspects of departmental policy, and applying it successfully may be more difficult than writing one's own simulation model. Therefore, a department that is convinced of the need for a simulation model should carefully review the design concepts of both of the models described here. Then it can make an informed judgment whether to adopt one of them or to design a new model that incorporates their desirable features.

VI. MANPOWER SCHEDULING

The three related issues of manpower scheduling described in the Introduction can each be resolved rather easily with the assistance of available computer programs.

The first issue concerns the times of day at which patrol officers should begin and end their tours of duty. Traditional practice has been for police officers to work one of three eight-hour *tours* (also called *watches* or *shifts*). In this arrangement, if tours begin on the hour, there are only eight choices for the starting times of tours. Namely, the first tour of the day can begin at midnight, 0100, 0200, 0300, 0400, 0500, 0600, or 0700. (The starting times of the other tours are determined by the starting time of the first tour.)

Some of these starting times may be far superior to others in terms of the match between manpower requirements and manpower on duty. For example, if the largest number of calls for service occur during the hours from 1800 to 0200, followed by a period with few calls for service, a tour that begins at midnight will either have too few units on duty for the first two hours or too many for the last six. Beginning the tour at 0200 will permit a better match.

Some departments have a fourth eight-hour *overlay tour* that begins during one of the regular tours and ends during the next. In considering whether to adopt such a practice, a department would want to know the benefits that could be achieved from an overlay tour and the best choice of a starting time. More complicated possibilities, such as overlapping ten-hour tours or a large number of overlaid eight-hour tours may also be proposed or already adopted in some departments.

The simplest types of tour-scheduling problems can be easily resolved using a model such as the Patrol Car Allocation Model. For example, in the case of three eight-hour tours, the eight possibilities for starting times mentioned above can be compared by operating PCAM with eight different data bases, each shifted by one hour compared with the preceding one. PCAM will also permit displaying performance measures for one overlay tour and comparing the results with the situation

without an overlay tour. The seven possible on-the-hour starting times for the overlay tour can also be rapidly compared with this model.

To study more complicated arrangements, it is necessary to determine the desired number of patrol units on duty during each hour of each day of the week. In many cases, this can be accomplished by preparing a data base for PCAM that imagines each hour to be a separate tour, but if the requirements for patrol units are unrelated to calls for service (e.g., they perform traffic control or school-crossing functions), some other method may be needed. Next, a computer program specially designed for this purpose⁽⁴⁵⁾ will determine the number of patrol units that should start on duty during each hour of the week. (Of course, many hours will not be the starting times for any units. However, there may be tours beginning at midnight, 0200, 0500, 0900, and so forth.) The schedule generated by this program will have the smallest number of unit-tours needed to meet or exceed all the stated requirements. The version of the program that is currently available assumes all tours last eight hours.

This same program can be used to answer another scheduling issue mentioned in the Introduction, namely the times at which schedulable non-cfs work (usually meal times) should begin. The program specifies the number of units in each tour that should have their meal during each hour of the tour. The user is permitted to place various conditions on meal-starting times, such as forbidding any meals to begin during the first or second hour of a tour.

To estimate performance measures accurately when the number of units on duty is changing from hour to hour, a simulation model will ordinarily be required. However, many queuing characteristics can be determined from a dynamic queuing model developed at The New York City-Rand Institute.⁽⁴⁵⁾

The last manpower scheduling issue concerns the days of the week on which officers work and each officer's pattern of rotation from one tour to another. For example, one officer might work the night tour on Monday-Friday of one week and Sunday-Thursday the next week, after which he changes to the morning tour and begins work on Monday. The primary objective in designing such schedules is to match the number of officers

on duty to the number that the department would like to have on duty as closely as possible. A second objective is to make the schedule "fair" in the sense that no officer would feel that some other officer had a better schedule.

One program that has been designed for this purpose is part of a package of models known as Superbeat.⁽⁴⁶⁾ It is specifically oriented to a pattern of schedules in which each officer works five days a week with two consecutive days off. Some of the officers rotate from one tour to another on a monthly basis, while others work a fixed tour.

A second program of this type was initially designed by Heller and was perfected with the assistance of McEwen and Stenzel.⁽⁴⁷⁾ This is an extraordinarily flexible and well-designed program that can handle substantially more complicated patterns of rotation than the Superbeat model.

Heller's method involves dividing the entire pool of manpower into a number of squads, which are of different sizes. All the officers in a single squad have the same working days, days off, and tour rotation days. The schedule for some other squad is identical, except that it is shifted by a certain number of weeks. Thus, in the long run all officers experience exactly the same sequence of days on, days off, and rotations.

Certain features of a schedule are considered desirable, while others are undesirable, and the program attempts to produce schedules with the largest possible number of the former and the least possible number of the latter. Examples of desirable features are:

- days off on Saturday and Sunday (a "recreation weekend")
- large number of days off in a row.

Examples of undesirable features (some of which can be forbidden by the user) are:

- large number of weeks in a row without a recreation weekend
- large number of days worked consecutively

- single day off before beginning work again
- rotation to a new tour on a day that does not follow a day off.

Scheduling programs can be used to schedule officers in any division of a police department, not just the patrol force.

VII. STEPS IN AN ALLOCATION STUDY

Nearly any study of patrol allocation will require summary statistics concerning the amount of cfs work and non-cfs work of patrol units, broken down by time of day, geography, and priority. Therefore, it is important for a department to develop the capability to obtain computer-processable data from dispatchers showing the time of arrival of each call for service, the time the call was assigned to a patrol unit, the number of patrol units assigned, the times at which the units were once again available, the type of the call, and its location. The times at which units arrive at the scene will also be useful, but it may not be feasible to record this for every call (depending on radio traffic). Arrival times for suitable samples of calls will be satisfactory.* The times at which non-cfs unavailabilities begin and end should also be obtained.

The *type* of a call for service should be recorded in a form that is suitable for determining its priority. If the possibility of changing the priority structure is not under consideration, then the easiest and most accurate way to do this is to have the dispatchers indicate the priority of each call. Otherwise, 20 or more categories of calls may be needed. Existing crime codes are unlikely to be satisfactory for this purpose, except for crime types whose priority is obvious.

The *location* of the call must be specified to a level of detail appropriate to the studies that are being considered. At a minimum, the geographical command in which the call is located should be known. Recording the beat in which the call occurs is ordinarily of no use, since this permits study of the *current* beat structure only. If beat design is under consideration, then finer geographical information is needed, namely the calls must be counted by reporting area, as described in Chapter IV.

* For example, arrival times can be recorded for every dispatch during a one-week period or for every fifth dispatch during a one-month period.

One approach is to define reporting areas in advance of data collection and have the dispatcher record the reporting area of each incident. A second is to record the address of each incident and use some computerized address-matching facility to determine the reporting area for each address. The latter is likely to lead to some 20 percent of incidents that fail to match, due to misspelling of street names, special locations (e.g., Community Hospital), and incomplete address-matching files. Some of these unmatched addresses can be ignored because they are randomly located, while a tedious manual process will be needed to determine the reporting areas for locations that fail to match in a consistent fashion. (For example, one street name may be frequently misspelled, resulting in a failure to match.)

Having collected appropriate summary statistics, there is a tendency for the planners to want to send them to patrol commanders to prove that they have accomplished a major task. But performance statistics in the absence of suggestions for changes in allocation policy are practically useless and may lead the patrol commanders to doubt the sanity of the planners. Therefore, an important next step is to *do something* with the data.

It is rare indeed for the statistics to indicate that the department has no patrol allocation problems. Usually they will clearly show a need to reallocate by time of day or by geography, or to modify the design of beats. It may happen that the data were collected with beat design in mind, but they indicate clearly a misallocation of patrol units by time of day. *The policy issue to be addressed first should be determined from the statistics.*

The next step is to determine a suitable *method* to use for addressing the selected policy issue. While this report gives general guidance, the planners should conduct a careful *search* of the available options. This will involve reading the appropriate references given in this report and also reviewing related summary documents. ^(48,49) Site visits to other departments that have already installed one of the models may be the best source of frank opinions. Selecting the most appropriate existing computer program for a particular application, or deciding to develop one's own programs, is an important choice that should not be

treated casually. If reallocation by time of day is under consideration, the method to be used eventually for scheduling manpower to meet the desired allocations should be resolved during the initial stages also.

Next, the department will have to determine whether its own personnel are capable of conducting the study or whether outside assistance is required. Fully documented simple models such as the Patrol Car Allocation Model and the Hypercube Queuing Model can be utilized without special programming or statistical skills; the department must merely have access to a computer system. Other models may be too complex to operate properly without outside assistance, or they may be available only as part of a "package deal" that includes consulting services. If the department requires outside assistance, local consulting firms or university students and professors may prove to be the best choice, because regular interaction between the analysts and the department's staff is a prerequisite for a successful project.

A project team should be assembled, consisting of the department's planners, outside analysts, administrators who will be called upon to make the final policy decisions, patrol commanders who will have to implement them, and possibly representatives of labor unions if negotiable issues are under study. No policy change, however logically related to performance measures, can succeed if it is infeasible for reasons not taken into account. Assembling an appropriate project team will avoid producing the "perfect" allocation plan that is never implemented.

Next comes the step of operating the computer programs. While in most instances complete users' manuals are available to instruct the project team in this process, experienced planners know that unanticipated problems may arise before the program operates properly with the department's data base. It is important to try out the *base case* (namely, the department's current operations) and to compare the estimates provided by the model with actual data to the greatest extent possible. This may reveal errors in data collection, especially if some of the data were "guessed." For example, a department may have trouble believing that the response speed of its units is 19 m.p.h.

But if this speed, when entered as data in a model, causes the estimates of travel times to conform to reality, it should be used.

The final step, developing policy recommendations, may be the most difficult, but little advice can be given. Only local administrators will know what appears "best" in light of budgetary, political, and other constraints.

Appendix A

MATHEMATICAL FORMULATION OF QUEUING

Nearly all analytical queuing models for police patrol allocation assume that calls for service arrive according to a Poisson process at rate λ , the service times of all calls are independently exponentially distributed with mean $1/\mu$, and the system is in steady state. In this case, if there are n patrol units not busy on non-cfs work, the probability that k units will be busy on cfs work is

$$P_k(n) = P_0(n) \rho^k / k! \quad \text{for } k < n$$

and

$$P_n(n) = P_0(n) \rho^n / (n! (1 - \rho/n)),$$

where $\rho = \lambda/\mu$ and

$$P_0(n) = \left[\sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \frac{\rho^n}{n! (1 - \rho/n)} \right]^{-1}.$$

$P_n(n)$ is the probability that all units are busy, and is therefore the probability that an incoming call for service will have to enter a queue. $P_{n-m}(n)$ is the probability that m units are available, which can be used as a weight for estimating average travel times, as mentioned in Chapter II.

The average length of time that a call will wait in queue (including calls that do not wait) is

$$\bar{W}(n) = P_n(n) / (n\mu(1 - \rho/n)).$$

If calls are divided into priority classes, it is assumed that calls are dispatched in a first-come first-served manner within priority class, with priority 1 calls dispatched before priority 2 calls, and so forth.

Then, denoting by α_p the fraction of calls that are of priority p, the average wait for a priority p call is ⁽⁵⁰⁾

$$\bar{W}_p(n) = P_n(n) / \left(n\mu \left(1 - \frac{\sum_{k=1}^p \alpha_k \lambda}{n\mu} \right) \left(1 - \frac{\sum_{k=1}^{p-1} \alpha_k \lambda}{n\mu} \right) \right)$$

(The sum $\sum_{k=1}^{p-1}$ is zero if p = 1.)

Now if N patrol units are on duty, the number n of units not busy on non-cfs work will vary from time to time. An approximate method for taking this into account is to calculate the average number of units not busy on non-cfs work. If this average is an integer, it can be used in the above equations to replace n. If it is not an integer, the equations must be interpolated. See the PCAM user's manual ⁽²⁹⁾ for further details. More accurate treatment of non-cfs work requires a dynamic queuing model ⁽⁴⁵⁾ or a simulation model.

Appendix B
ADDRESSES FOR FURTHER INFORMATION

1. For copies of publications and computer programs distributed by The Rand Corporation, or inquiries regarding this report:

Jan M. Chaiken
The Rand Corporation
1700 Main Street
Santa Monica, California 90406
(213) 393-0411

2. Research sponsor:

U.S. Department of Housing and Urban Development

Alan Siegel, Director
Hartley Campbell Fitts, Program Manager
Office of Policy Development and Research
Community Development and Management
Research Division
451 Seventh Street, S.W.
Washington, D.C. 20410
(202) 755-6970

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