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SCHOOL OF URBAN AND PUBLIC AFFAIRS

THE DYNAMICS OF A HOMEOSTATIC PUNISHMENT PROCESS

by

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Urban Systems Institute

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I. INTRODUCTION

In his now classic analysis of crime, Durkheim argues that some level of crime is "an integral part of all healthy societies . . . provided that it attains and does not exceed a certain level for each social type" (Durkheim, 1964, pp. 66-67). He argues that crime is an unavoidable consequence of the very processes which contribute to the maintenance of social cohesion. As the set of standards and beliefs which define and bound a society are specified, some types of behavior will be prohibited,and those engaging in these behaviors will be considered criminals. Furthermore, the public condemnation and punishment that follows a criminal act serves to articulate and reinforce the common set of norms and sentiments which ultimately guide the actions of the members of the society, thereby further enhancing social cohesion. Thus, while crime is a natural outgrowth of the processes generating social solidarity, it is the social response to crime that particularly serves to consolidate and reinforce that solidarity.

Blumstein and Cohen (1973) have re-examined Durkheim's theory of a stable level of crime and pose an alternative position emphasizing the stability of punishment. Their argument is that the standards or thresholds that define punishable behavior are adjusted in response to overall shifts in the behavior of the members of a society so that roughly a constant proportion of the population is always undergoing punishment. Thus, if many more individuals engage in behavior defined as punishable, the demarcation between criminal and non-criminal behavior would be adjusted to re-designate at least part of the previously criminal behavior as non-criminal, or the intensity or duration of punishment for those convicted would be reduced. A similar, but opposite reassessment would occur when fewer people committed currently punishable acts. Their principal evidence in support of this hypothesis is the stability of imprisonment rates in the U.S.A. over the period 1930-1970 and in Norway over the period 1880-1964 (Figure 1). Canadian imprisonment rates over the period 1880-1959 have been obtained subsequently, and these (Figure 1c) show the same stability behavior.

In this paper, we extend the theoretical structure and the empirical basis of this earlier work, and we hypothesize some processes that might generate the stable level of punishment. First, the time series of the imprisonment data for the U.S.A., Norway and Canada are analyzed to provide an empirical description of the structure of the data. These results indicate a striking similarity in the data structures in the three countries studied. Different models of the crime and imprisonment process are then explored in an effort to characterize an underlying process that would generate the kinds of time series observed. A sensitivity analysis is then performed to identify how the different parameters of one such model contributes to national differences in observed levels of punishment.

II. THE BASIC HOMEOSTATIC HYPOTHESIS

We first review the stability of punishment theory. Blumstein and Cohen (1973) posit a statistical density function $f_B(x)$, representing the distribution of behavior in a society. The basic concept of such a distribution is that there exists a range of behavior which may be viewed at one extreme as being compulsively moralistic and at the other as being severely criminally deviant (see Figure 2), and with all shades in between. It is then hypothesized that society establishes a boundary, B_0, defining the limits of legitimate behavior. Individuals who engage in behavior $B > B_0$ are deemed punishable.

A punishment probability function, $g(B)$, is introduced which reflects the probability that a person engaging in behavior beyond B_0 will be punished, and a punishment intensity function, $I(B)$, reflects the intensity of punishment applied to a punished individual at B. Thus, α, the aggregate amount of punishment delivered by society, is given by:

$$\alpha = \int_{B_0}^{\infty} f_B(x) g(x) I(x) dx$$

It is then hypothesized that α will be relatively stable over time in a given society, even though it may deviate somewhat for severely disruptive periods like wars or depressions. One means of maintaining the stable value of α in the face of changing behavior in the society is through redefinition of the boundary, B_0, between the criminal and the non-criminal

Under this homeostatic hypothesis, if behavior were to become less criminally deviant, that is, if $f_B(x)$ were to shift to the left, B_0 would be adjusted to $B_0' < B_0$, so that $\alpha(B_0) = \alpha'(B_0') = \alpha$. In terms of the integral formulation, the hypothesis can be represented by:

$$\alpha'(B_0') = \int_{B_0'}^{\infty} f_B'(x)\ g'(x)\ I'(x)\ dx = \int_{B_0}^{\infty} f_B(x)\ g(x)\ I(x)\ dx = \alpha$$

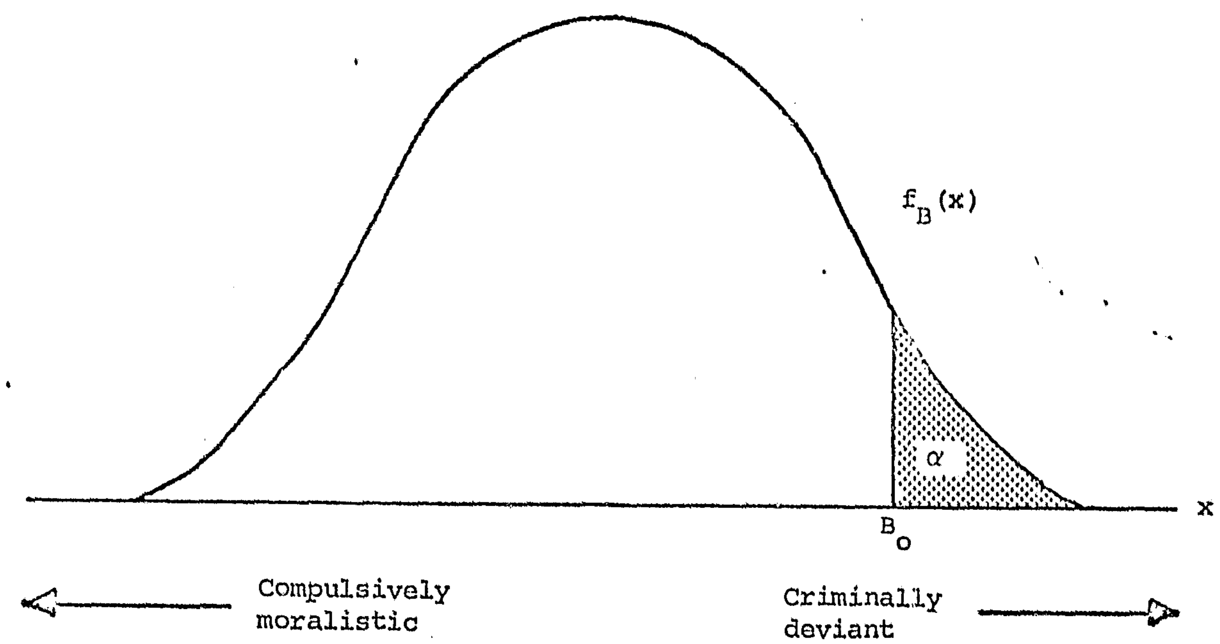

FIGURE 2

THE BEHAVIOR DISTRIBUTION

It is argued that the social forces accounting for stability include more than simple prison-cell capacity, or even the limited willingness of society to accept the economic burden of processing individuals through the criminal justice system, confining them and foregoing their productivity. Such an explanation does not account for the tendency of downward movements in imprisonment rates to reverse themselves and return to the mean. More fundamental considerations of social structure are probably at work. If too large a portion of the society is declared deviant, then the fundamental stability of the society may well be disrupted. Likewise, if too few are punished, the basic identifying values of the society will not be adequately articulated and re-enforced, again leading to social instability. In the former case there will be pressures toward decriminalizing some behavior, while in the latter, there will be pressures for stricter law enforcement and perhaps more severe punishments.

III. TIME-SERIES ANALYSIS

Time-series analysis is often directed at a sequence of observations such as those of Figure 1 in order to discover structures in the data, particularly relationships between an observation in period t and those in prior periods. In time-series analyses, two basic types of structures are typically explored - autoregression and moving averages. These can be studied either separately or in combination, and in many instances, can explain the systematic behavior of the time-series. The autoregressive structure is defined by:

$$Y_t = \delta + \sum_{i=1}^{T} \phi_i Y_{t-1} + \epsilon_t \qquad (1)$$

where

Y_t is the observation in period t

δ, $\emptyset_i$ are the fixed parameters of the generating process

ϵ_t's are independent and identically distributed random variables with zero mean and variance σ^2

Equation (1) states that the observation at t (Y_t) is a weighted linear function of a constant and the observations of T prior periods, plus an independent stochastic error, ϵ_t. The time series analysis provides a means for estimating the number of prior periods, if any, for which the $\emptyset$'s are significantly different from zero. The "order" of the autoregressive process is equal to largest subscript of the non-zero $\emptyset$'s. For example, if $\emptyset_3 > 0$ and $\emptyset_i = 0$ for all $i > 3$, the process is called a "third-order" autoregression.

The autoregressive structure assumes the stochastic component, ϵ_t, to be independent of the stochastic components of prior observations. In time-series data, this is often not the case and the ϵ_t's may be serially correlated over one or many periods.

A moving-average process is defined by:

$$Y_t = u + \epsilon_t \quad (2)$$

where, now:

$$\epsilon_t = \mu_t + \sum_{i=1}^{T} \gamma_i \mu_{t-1} \quad (3)$$

where:

u, γ_i are fixed parameters of the generating process

μ_t are independent and identically distributed random variables with mean zero and variance σ^2.

The analyses provide a means for estimating u and the γ_i which are different from zero. As with autoregressive processes, the "order" of the moving average is defined by the maximum subscript of the γ_i's which are different from zero.

Thus, in the moving-average processes, the relationship between an observation at time t and prior observations occurs through the serial correlation of successive realizations of the stochastic component, ϵ_t. In autoregressive processes, this occurs through serial correlation of the observations, Y_t. While the difference between these two processes in terms of the behavior of the induced time-series may not be obvious, their properties are very different. These differences permit the wide variety of time-series which are encountered in practice to be estimated by making judicious use of autoregressive, moving average, or mixed (autoregressive and moving average) processes of proper order.

In order to gain further insight into the dynamics of the imprisonment process, time-series analysis was performed on the annual imprisonment rate data for the U.S.A., Norway and Canada. Briefly, the analysis involves the following steps:

1) Using ordinary least squares, estimate an autoregressive function of arbitrarily high order, say T. If the autoregressive coefficient of the T^{th} subscript is statistically insignificant, estimate an autoregressive relationship of order T-1. Continue this process until a statistically significant autoregressive coefficient is found.

2) Two methods are available to determine if there is serial correlation of the stochastic component, ϵ_t (i.e., a moving-average process.) First, an insignificant Durban-Watson statistic suggests

no serial correlation. One can also run autoregressions on the deviations of the actual data from those predicted by the estimated autoregression. If no significant autoregression coefficients are then found, and if the Durban-Watson statistic is not significant, there is strong evidence of no serial correlation in the stochastic component.

In the time-series analysis for each country, we began by estimating autoregression functions of order 4 (T = 4) and found no significant coefficient ϕ_T until we estimated the second-order autoregression. When checking for serial correlations among the stochastic components, we found no significant autoregression relationships among the deviations and none of the Durban-Watson statistics were significant. (Figure 3 is a plot of the actual Canadian data against the values predicted by the second-order autoregression. A visual inspection reveals both the high explanatory power of the regression and the seemingly random nature of the deviations.) Thus, because the time-series of the imprisonment rates for the U.S.A., Norway, and Canada each followed a second-order autoregressive process with no moving average component, we can write:

$$r_t = \delta + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \epsilon_t \quad . \qquad (4)$$

where r_t is the imprisonment rate (prisoners/100,000 population) in year t. Table 1 presents the estimated autoregression parameters for each country.

Given the wide range of possible structures for these data, the finding that the imprisonment rates in the three different countries follows a second-order

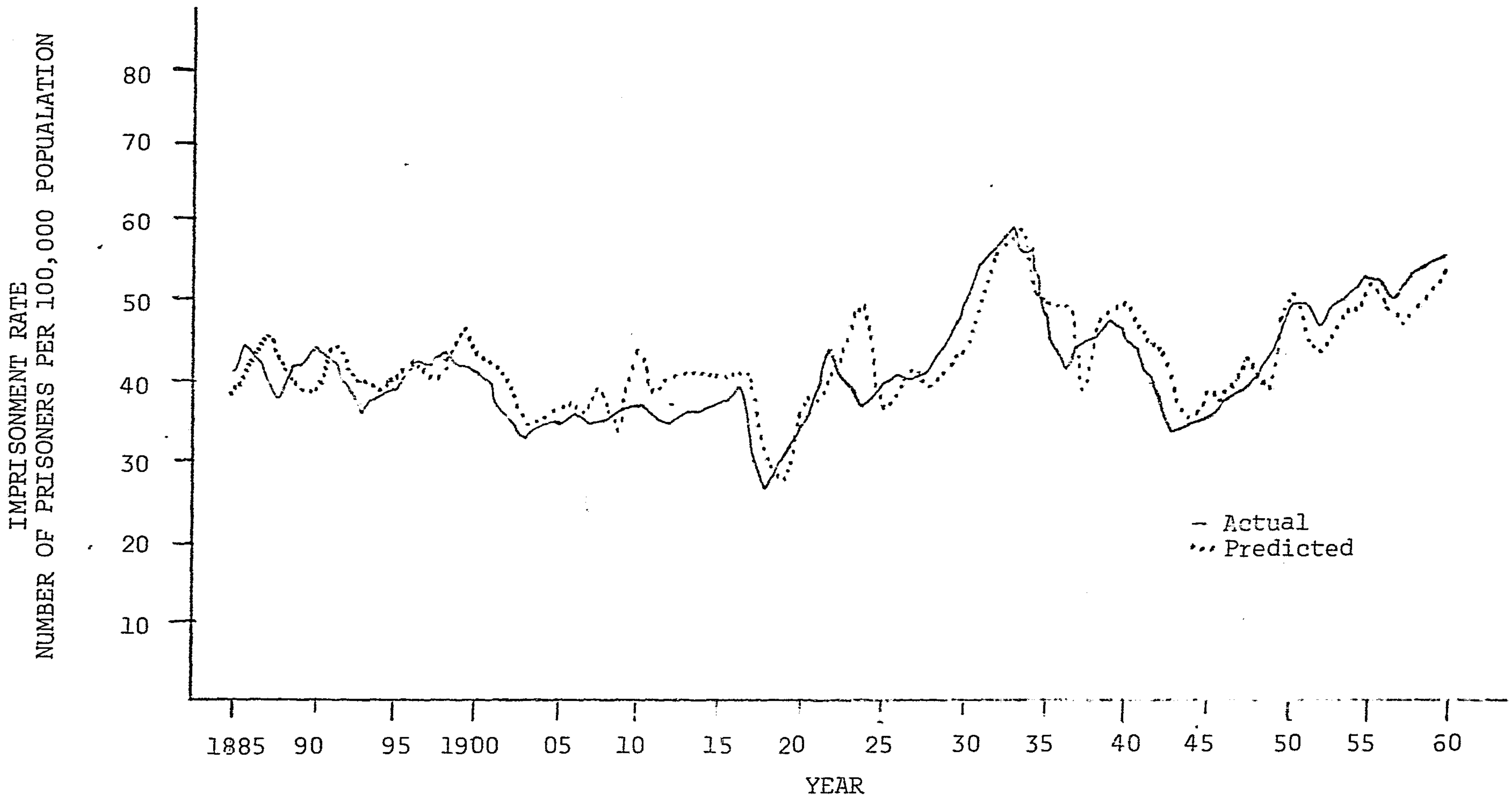

FIGURE 3

ACTUAL VS PREDICTED IMPRISONMENT RATE IN CANADA: 1885-1959

Table 1

Estimated Autoregression Parameters for the Annual Imprisonment Rate* (r_t) in the USA, Norway and Canada

$$r_t = \delta + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t$$

PARAMETER	USA	NORWAY	CANADA
ϕ_1	1.42 (10.35)	1.17 (10.47)	1.25 (11.58)
ϕ_2	-.63 (-4.41)	-.35 (-3.13)	-.42 (-3.83)
δ	22.74 (2.76)	9.34 (3.15)	7.42 (3.04)

*The imprisonment rate is the average daily prison population per 100,000 general population. In the USA and Norway the rate base is 100,000 total population, while in Canada it is 100,000 population 16 years of age or older.

autoregression strongly suggests that a similar mechanism may be generating each, albeit with different driving parameters. It would be desirable to be able to identify such a mechanism consistent with these empirical findings.

Differential Equations

Processes following a second-order linear differential equation (not necessarily with constant coefficients) generate second-order autoregressive functions. This connection may be shown by approximating the derivatives in the differential equation by difference equations, i.e., if r_t is the imprisonment rate at time t, and its first two time derivatives are denoted by $\dot{r}_t$ and $\ddot{r}_t$, then we approximate $\dot{r}_t$ and $\ddot{r}_t$ by:

$$\dot{r}_t = r_t - r_{t-1}$$

$$\ddot{r}_t = (r_t - r_{t-1}) - (r_{t-1} - r_{t-2})$$

The general second-order differential equation with constant coefficients is $\ddot{r}_t + c\dot{r}_t + dr_t = F$, and, in the approximating difference equation, we have:

$$\ddot{r}_t + c\dot{r}_t + dr_t = (r_t - r_{t-1}) - (r_{t-1} - r_{t-2}) + c\,(r_t - r_{t-1}) + d\,r_t = F \quad (5)$$

Equation (5), put into the form of Equation (4), defines the following second-order autoregressive function:

$$r_t = [\frac{2+c}{1+c+d}]r_{t-1} + [\frac{-1}{1+c+d}]r_{t-2} + \frac{F}{1+c+d}$$

where ϕ_1, ϕ_2, and δ are expressed in terms of c, d and F.

Table 2 presents the parameters of the differential equation (c,d, and F) derived from the autoregression parameters for each country.

Table 2

Parameters for the Second-Order Differential Equation
which Generates the Estimated Autoregressive
Process for the Imprisonment Rate Time-Series

$$\ddot{r}_t + c\dot{r}_t + dr_t = F$$

$$c = \frac{-\phi_1 - 2\phi_2}{\phi_2}$$

$$d = \frac{\phi_1 + \phi_2 - 1}{\phi_2}$$

$$F = \frac{-\delta}{\phi_2}$$

$$\Pi = \text{periodicity} = \frac{4\pi}{\sqrt{4d - c^2}}$$

PARAMETER	USA	NORWAY	CANADA
c	.25	1.34	.98
d	.33	.51	.40
F	36.10	26.69	17.62
Π	11.2 yrs.	25.4 yrs.	15.7 yrs.

characteristic time period (Π) of the cycles for each equation.*

Thus, the differential equation (5) is the mathematical characterization of a dynamic process that would generate the time series that were observed. In its present form, equation (5) is only an abstract representation that could describe any number of physical or social processes. We would now like to posit a flow process in and out of prison that would generate a differential equation consistent with (5). Such a model will allow a sociological interpretation of the stability of imprisonment rates in terms of conceptually meaningful characteristics of a society (e.g., the degree of punitiveness and the level of conformity). Our first formulation is quite simple and requires only that the prison population remain stable through a simple balancing of receptions and releases. This formulation will be shown to be inconsistent with the observed behavior of the Canadian data. A second, more elaborate model which incorporates the homeostatic principles will be shown to be much more satisfactory and consistent with the Canadian data.

IV. EXPLORATION OF POSSIBLE EXPLANATORY MODELS

In this section, models of the social mechanism generating imprisonment rates are developed and their consistency with the observed stability and second-order autoregressive movement of the time series are explored. The models are developed by partitioning the total population of a society into three groups, one of which is the prison population. The flow rates of individuals among these groups is then examined.

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*A differential equation of the specified form results in cyclical behavior when $c^2-4d < 0$ and the period Π, is obtained from:

$$\Pi = \frac{4\pi}{\sqrt{4d-c^2}}$$

These simultaneous flows generate a system of simultaneous first-order differential equations. Such systems can be solved so that each population is defined solely as a function of its own derivatives (see Appendix I). The result for any population group is in general a second-order differential equation (although in some systems, the second-order term vanishes, leaving only a first-order equation). We can judge the adequacy of each hypothesized structure by comparing the parameters of the autoregressive process implicit in the differential equation generated by the model with the autoregressive parameters estimated from the observed time series.

A. Prisoner, Ex-Convict, and Virgin Model

The first model to be examined partitions the total population T(t), into a prison population P(t), an ex-convict population M(t), and a population of individuals who have never been to prison (virgins) V(t). The possible flows in this structure are shown in Figure 4. Within this structure, the only mechanism for maintaining a stable imprisonment rate would be the balancing of releases from P(t) with receptions from V(t) and M(t).

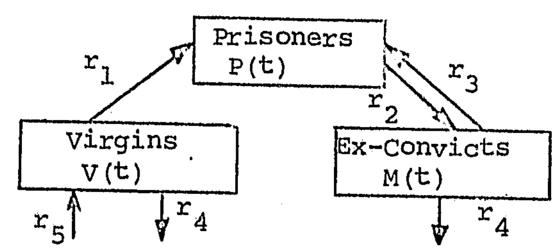

Figure 4

Model I

The relationship among the flows may be formalized as follows:

$$\dot{P}(t) = -r_2P(t) + r_3M(t) + r_1V(t)$$

$$\dot{M}(t) = r_2P(t) - (r_3 + r_4)M(t) \qquad (7)$$

$$\dot{V}(t) = -(r_1+r_4)V(t)+r_5T(t)$$

where

$\dot{P}(t)$, $\dot{M}(t)$, $\dot{V}(t)$ = rate of change at t of the respective populations

r_1 = imprisonment rate of virgins

r_2 = release rate from prison

r_3 = imprisonment rate of ex-convicts

r_4 = death rate*

r_5 = birth rate

Since the sum of P(t), M(t) and V(t) is the total population at time t,T(t), then V(t) may be replaced in the first equation of (7) by:

$$V(t) = T(t) - P(t) - M(t)$$

The dynamic behavior of P(t) can now be expressed by a system of two flow equations where:

$$\begin{aligned} \dot{P}(t) &= -(r_2 + r_1)P(t) + (r_3 - r_1)M(t) + r_1T(t) \\ \dot{M}(t) &= r_2P(t) \quad -(r_3 + r_4)M(t) \end{aligned} \qquad (8)$$

or, in matrix form

$$\dot{Y} = AY + F$$

where

$$Y = \begin{bmatrix} P(t) \\ M(t) \end{bmatrix} \qquad A = \begin{bmatrix} -(r_1+r_2) & (r_3-r_1) \\ r_2 & -(r_3+r_4) \end{bmatrix}$$

$$F = \begin{bmatrix} r_1T(t) \\ 0 \end{bmatrix}$$

*For the purpose of simplicity we have ignored the differences between the death rate of ex-cons and of virgins and the small number of deaths of prisoners.

Using the procedure outlined in Appendix I, P(t) may be translated to:

$$\ddot{P}(t) + a\dot{P}(t) + bP(t) = F_p \qquad (9)$$

where

$$a = (r_1+r_2+r_3+r_4)$$

$$b = (r_3+r_4)\ (r_1+r_2) - r_2(r_3-r_1) = r_1(r_2+r_3) + r_4(r_1+r_2)$$

$$F_p = -(r_1+r_2)\ r_1T(t) + (r_1+r_2+r_3+r_4)\ r_1T(t) + r_1\dot{T}(t) = (r_3+r_4)r_1T(t) + r_1\dot{T}(t)$$

Equation (9) is a differential equation describing the dynamic behavior of the total prison population, P(t), whereas the autoregressions and their implied differential equations are expressed in terms of a <u>rate</u> of imprisonment per population. However, a translation between the two can be made; when r(t) is the imprisonment rate per <u>unit</u> of population:

$$P(t) = r(t)T(t) \qquad (10)$$

then:

$$\dot{P}(t) = \dot{r}(t)T(t) + r(t)\dot{T}(t) \qquad (10a)$$

$$\ddot{P}(t) = \ddot{r}(t)T(t) + 2\dot{r}(t)\dot{T}(t) + r(t)\ddot{T}(t) \qquad (10b)$$

As a first estimate, we assume that after accounting for "deaths", T(t) grows exponentially. Then:

$$P(t) = T_0\epsilon^{gt}r(t) \qquad (10c)$$

$$\dot{P}(t) = T_0\epsilon^{gt}(\dot{r}(t) + gr(t)) \qquad (10d)$$

$$\ddot{P}(t) = T_0\epsilon^{gt}(\ddot{r}(t) + 2g\dot{r}(t) + g^2r(t)) \qquad (10e)$$

We then substitute (10c), (10d) and (10e) into (9) and divide the equation by T(t). Then:

$$[\ddot{r}(t) + 2g\dot{r}(t) + g^2r(t)] + a[\dot{r}(t) + gr(t)] + br(t) = \frac{F_p}{T_0}\epsilon^{-gt} \qquad (11)$$

Rearranging terms,

$$\ddot{r}(t) + c\dot{r}(t) + dr(t) = F' \qquad (12)$$

where

$$c = a + 2g = r_1 + r_2 + r_3 + r_4 + 2g$$

$$d = b + ag + g^2 = r_1(r_2+r_3) + r_4(r_1+r_2) + ag + g^2$$

$$F' = r_3 r_1 + r_4 r_1 + r_1 g$$

To assess the adequacy of this model, estimates of c and d generated by the model can be compared with the estimates from the observed Canadian time-series reported in Table 2.* The imprisonment rate of virgins, r_1, is exceedingly small. In Canada, for example, even if we were to assume that all receptions in prison in a year are of first-time offenders, r_1 would be no larger than .0004 and $(r_2 + r_3)$ no larger than .73. For the period 1880 to 1960, g, the exponential growth rate of the Canadian population, was about 0.019 and r_4, the death rate, about .017. Therefore, according to Equation (12), d is about 0.027, while c is about .79. In this model, therefore, c must be more than twenty-five times larger than d.

The values of c and d (Table 2) estimated from Canadian autoregression parameters are .98 and .40 respectively. Thus, for Canada, this model yields only a fair estimate of c and dramatically underestimates d.** The very low estimate of d will result in the model predicting non-oscillatory

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*It should be noted that eqn. (12) is based on the imprisonment rate per unit of population, while the estimated differential equations in Table 2 are based on the rate per 100,000 population. Although the rates differ by a factor of 10^5, the coefficients c and d are unaffected and may be directly compared. The constant term F', however, must be multiplied by 10^5 when it is compared to the constant term F in Table 2.

**When the predicted values of c and d are transformed into autoregressive form (eqn. (6)), the respective values of ϕ_1 and ϕ_2 are 1.54 and -.55. The predicted value of ϕ_1 1.54, is outside a 95% confidence interval of the value 1.25 estimated from the actual data.

behavior in $r(t)^*$. This is, however, completely contrary to the strong cyclical behavior actually observed. It thus appears that Model I, which considers only a steady-state balance of receptions and releases, does not adequately explain the observed dynamics of the imprisonment rate. A more elaborate flow structure is required.

B. Prisoner, Criminal, Law-Abider Model

We now propose an alternative parititoning of the population into three subsets (Figure 5), now identified as "law-abiders", "criminals", and "prisoners", with the numbers in each group varying over time. In the context of the behavior distribution of Figure 2, the number of law-abiders at time t, $L(t)$, are those individuals whose behavior $B(t) < B_0(t)$. Likewise, the criminal population, $C(t)$, are those individuals with behavior $B(t) > B_0(t)$. The prison population, $P(t)$, are those individuals drawn from the criminal population who are confined in institutions at t.

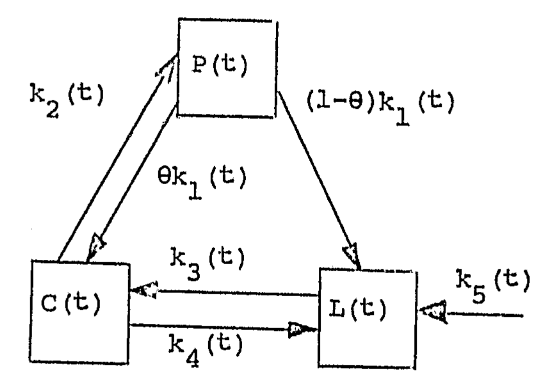

Figure 5
Model II - Stable Imprisonment as a Homeostatic Process

*A necessary condition for oscillatory behavior is that $(c^2 - 4d) < 0$.

The composition of populations changes continuously, as shown in the flow diagram of Figure 5. Some criminals are arrested, convicted, and sent to prison. Prisoners are regularly released from prison, with some returning to the criminal group and others becoming law-abiders. There is also an important two-way flow between the criminal and law-abiding populations. As $f_B(x)$, the behavior distribution in Figure 2, shifts to the right, for example, C(t) increases and L(t) decreases correspondingly. Similarly, a shift to the left, i.e., to a population that is more law-abiding, results in a net flow from C(t) to L(t). These changes in the population compositions are reflected in changes in the normal flow rates, $k_i(t)$, among the population groups.

The possibility of flows between the criminal and law-abiding population are important elements of the model because these flows permit the incorporation of a central theme of the homeostatic notion, namely the redefinition of criminal behavior. Suppose, for example, that at time t_o the system were in equilibrium and $P(t_o)/T(t_o)$ was the average long-term imprisonment rate. Now, suppose that at t_1 the behavior distribution, $f_B(x)$ were to shift to the right (i.e., the population becomes more criminal by current standards). This shift would be reflected in an increase in $k_3(t)$ to $k_3(t_1) > k_3(t_o)$. The increase in $k_3(t)$, would result in a net increase in the flow from L(t) to C(t). That increase would perturb the system from equilibrium and holding all other $k_i(t)$ constant, would increase P(t)/T(t) and C(t)/T(t).

An increase in P(t)/T(t), according to the homeostatic model would set in motion the de-criminalization of certain behavior by shifting the demarcation between criminal and non-criminal behavior, B_0. This shift would be reflected by readjustments in $k_3(t)$ and $k_4(t)$ such that C(t)/T(t) and L(t)/T(t) would return toward the equilibrium values.

Even when $f_B(x)$ and B_0 are stable, there is a regular flow between C(t) and L(t). A previously law-abiding college student begins dealing in drugs or a businessman finds that profits are substantially improved by criminal collusion with competitors. An occasional burglar gets married or gets a better job, and decides to cease his criminal activity. Thus, each population is continuously feeding the others. We can formalize the description of these flows as follows:

$$\begin{aligned} \dot{P}(t) &= -k_1(t)\ P(t) + k_2(t)\ C(t) \\ \dot{C}(t) &= \theta k_1(t)\ P(t) - k_2(t)\ C(t) - k_4(t)\ C(t) + k_3(t)\ L(t) \\ \dot{L}(t) &= (1-\theta)k_1(t)\ P(t) + k_4(t)\ C(t) - k_3(t)\ L(t) + k_5(t)T(t) \end{aligned} \tag{16}$$

where

$\dot{P}(t)$, $\dot{C}(t)$, $\dot{L}(t)$ = rate of change at t of the respective populations (i.e., their first derivatives)

$k_1(t)$ = release rate from prison at t

$k_2(t)$ = imprisonment rate of the criminal population at t

$k_3(t)$ = rate at which law-abiders become criminals at t

$k_4(t)$ = rate at which criminals become law-abiders at t

$k_5(t)$ = net population growth rate at t

θ = portion of the persons released from prison who return to criminal activity

Since the sum of P(t), C(t) and L(t) is the total population at t, T(t), we can replace L(t) by

$$L(t) = T(t) - C(t) - P(t)$$

and the dynamic behavior of P(t) can be expressed by the two flow equations:

$$\dot{P}(t) = -k_1(t)\ P(t) + k_2(t)\ C(t) \qquad (17)$$

$$\dot{C}(t) = [\theta k_1(t) - k_3(t)]\ P(t) - [k_2(t) + k_3(t) + k_4(t)]\ C(t) + k_3(t)\ T(t)$$

In matrix form:

$$\dot{Y} = AY + F$$

where:

$$\dot{Y} = \begin{bmatrix} \dot{P}(t) \\ \dot{C}(t) \end{bmatrix} \qquad Y = \begin{bmatrix} P(t) \\ C(t) \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ k_3(t)\ T(t) \end{bmatrix}$$

$$A = \begin{bmatrix} -k_1(t) & k_2(t) \\ (\theta k_1(t) - k_3(t)) & -(k_2(t) + k_3(t) + k_4(t)) \end{bmatrix}$$

Equation (17) is a first-order system of simultaneous differential equations like those examined in the discussion of Model I, but here the coefficients are not necessarily constant. In the case of constant coefficients each population was defined solely in terms of its own derivatives, for instance:

$$\ddot{P}(t) + a\dot{P}(t) + bP(t) = F_p\ .$$

and a, b and F_p were determined from the matrix A (Appendix I). A similar solution in terms of its own derivatives also exists for each population when the coefficients are not constant, namely:

$$\ddot{Y} + a(t)\ \dot{Y} + b(t)\ Y = F(t) \qquad (18)$$

However, now the time-varying coefficients, a(t), b(t) and F(t) are in general complicated, and in this case elusive, functions of the $k_i(t)$ Nevertheless, as a point of departure we can explore the dynamic character of this model by assuming the $k_i(t)$ are approximately constant.*

Under the assumption of constant k_i, the differential equation governing the behavior of P(t), the prison population, is:

$$\ddot{P}(t) + a\dot{P}(t) + b\,P(t) = F_p \qquad (19)$$

where:

$$a = k_1 + k_2 + k_3 + k_4$$

$$b = k_1[(1-\theta)\,k_2 + k_3 + k_4] + k_2k_3$$

$$F_p = k_2k_3\,T(t)$$

We can change (19) into a differential equation describing the behavior of the <u>rate</u> of imprisonment per unit of population, r(t) using the procedure outlined in Equations (9) through (12) to yield:

$$\ddot{r} + c\dot{r} + dr = F' \qquad (20)$$

where

$$c = a + 2g$$

$$d = b + ag + g^2$$

$$F' = k_2k_3$$

The dynamic behavior of (20) is determined by the relative magnitudes of c, d, and g. It is therefore important to establish some reasonable bounds on their values to determine whether (20) is consistent with the dynamic behavior of the actual time series for imprisonment rates. Toward this end, the model will be analyzed using rates associated with Canadian penitentiaries. Visual inspection of the series in Figure 1c indicates that there is no obvious trend from 1880-1959. However, there does appear to

*This assumption of constant $k_i(t)$ disregards a central element of the stability of punishment theory, namely the changes in $k_3(t)$ and $k_4(t)$ that accompany the adjustment of the standards defining punishable behavior in response to shifts in objective behavior. The static nature of this representation results in serious limitation in the development and empirical analysis which follow. It does not, however, lend it vacuous. If the model, even under the restriction of constant $k_i(t)$ can generate coefficients which are plausibly close to the actual values, then a rationale for exploring more complicated forms where the $k_i(t)$ vary will be established.

have been a marked change in its dynamic behavior after 1925. To reduce the time variation in the k's (and, therefore, in c, d and F'), we restrict this analysis to the post-1925 series.

To test the sufficiency of (20), estimates of the k_i must be made to generate the theoretical values for c, d, and F'. Equation (20) will then be translated into an autoregressive relationship (e.g., $r_t = \delta + \sum_{i=1}^{n} \phi_i r_{t-1}$) by the approximation shown previously in (6). An autoregression can then be run on the actual data to determine whether the parameters estimated from the data are comparable to those generated by the theoretical model.

The known values of the system characterized by equations (16) (and hence by (20)) are k_1 (the release rate), T(t) and P(t). Their values at five-year intervals from 1925 to 1960 are given in Table 3. We chose the year 1940 to generate estimates for the model parameters. That year is about mid-way through the series, and its release rate, k_1, and imprisonment rate/100,000 ($P/T \times 10^{-5}$) are the same as the means for the series.

Table 3

The Release Rate ($k_1(t)$), Average Daily Prison Population ($P(t)$) and Total Population ($T(t)$) for Canada: 1925-1960*

Year	$k_1(t)$	$P(t)$	$T(t)$**
1925	.37	2266	5,100,000
1930	.43	2868	6,700,000
1935	.55	3895	7,350,000
1940	.50	3736	7,850,000
1945	.46	3063	8,500,000
1950	.45	4380	9,400,000
1955	.52	5204	10,400,000
1960	.73	6141	11,500,000

*Prisoner statistics were obtained from unpublished statistics provided by the Office of Statistics, Secretariat of the Ministry of the Solicitor General, Government of Canada.

**The total population includes only persons 16 years of age or older. Urquhart, M.C. and K.A.H. Buckley (1965), Historical Statistics of Canada (Toronto: Cambridge-Macmillan).

The unknown values are: k_2 (the imprisonment rate of criminals); k_3 (the rate at which law abiders become criminals); k_4 (the rate at which criminals become law-abiders); (1-θ) (rehabilitation rate);and C (the size of the criminal population). Estimates for k_2, k_3, k_4 are made for equilibrium estimates of C/T as 1.5%, 1.0%, and 0.5%. Since individuals do not continuously behave in a criminal manner, a reasonable convention must be established to operationalize the idea of an individual belonging to the criminal population. A reasonable definition might categorize a person as a criminal in year t if he has committed an act for which he would have been imprisoned if caught and convicted.* Then k_2, the rate of imprisonment of the criminal population, is the ratio of prison receptions (a known value) to the estimate of the size of the criminal population.

The analysis is relatively insensitive to the value of θ, the portion of released prisoners returning directly to the criminal population. A plausible estimate is 0.33. Given our definition of membership in the criminal population, θ includes all those released prisoners who commit at least one crime within a year of their release. In a study of parole success Gottfredson (1959) reported that during a two year follow-up period 38% of released prisoners returned to prison. In another study cited by Robison and Smith (1971) 51% of released prisoners returned to prison during the three years immediately following their release. Since recidivism rates decline with each additional year following release and not all releasees who return to crime are apprehended, it is not unreasonable to assume that 33% of released prisoners return immediately to the criminal population.

*Note that this definition restricts the minimum time spent in the criminal population to 1 year.

The value of k_4 is calculated somewhat differently. If τ is the average time spent in C, then k_4, the rate at which criminals leave C, is the reciprocal of τ. τ is assigned a value of 2 years for C/T = 1.5%. For the other values of C/T, 1.0% and .5%, τ is taken to be successively larger. The smaller C is assumed to be associated with a larger τ to reflect a more "hard core" criminal population in C. Thus, for C/T = 1.0%, we let τ = 3 years and for C/T = 0.5%, we let τ = 4 years.

The remaining parameter to be estimated is k_3. This parameter may be specified as the value which will maintain C(t) at a constant level given the values of k_1, k_2, and k_4. This is equivalent to assuming $\dot{C}(t)$ to be zero, so that from the second equation in system (16), we have:

$$k_3 = \frac{-\theta k_1 P(t) + (k_2 + k_4)C(t)}{T(t) - P(t) - C(t)}$$

The values of the k's and the resulting differential equation and autoregression coefficients are given in Table 4 for the three assumed values of C/T. For comparison, the second-order autoregression function estimated from the annual Canadian imprisonment rate from 1925-1960 is as follows:

$$r_t = \underset{(8.26)}{1.23\, r_{t-1}} - \underset{(-2.89)}{.43\, r_{t-2}} + \underset{(2.25)}{9.17} \qquad (21)$$

where the values in parentheses are the t-values associated with each of the coefficients. A comparison of the parameter estimates (21) with the corresponding autoregression parameters theoretically derived from the k_i in Table 4 show them to be roughly equivalent.* The coefficient of r_{t-1}, $\emptyset_1$, is overestimated by about 5% to 15%, whereas $\emptyset_2$ is underestimated

*See first footnote, p. 20.

Table 4

Estimates of Flow Parameters (k_i) for Model II
and the Associated Coefficients for the Autoregression
and Differential Equations Generated by Model II Using Annual
Canadian Imprisonment Rates from 1925-1960

$$\ddot{r}(t) + \dot{r}(t) + d\dot{r}(t) = F' \qquad \text{(i)}$$

$$r_t = \phi_1 r_{t-1} + \phi_2 r_{t-2} + \delta' \qquad \text{(ii)}$$

$C/T = 0.5\%$, $\tau = 4$	$C/T = 1.0\%$, $\tau = 3$	$C/T = 1.5\%$, $\tau = 2$
FLOW PARAMETERS:		
$k_1 = .50$	$k_1 = .50$	$k_1 = .50$
$k_2 = .046$	$k_2 = .023$	$k_2 = .015$
$k_3 = .0014$	$k_3 = .0035$	$k_3 = .0078$
$k_4 = .25$	$k_4 = .33$	$k_4 = .50$
$(1-\theta) = .67$	$(1-\theta) = .67$	$(1-\theta) = .67$
DIFFERENTIAL EQUATION COEFFICIENTS:*		
$c = .84$	$c = .89$	$c = 1.06$
$d = .16$	$d = .19$	$d = .28$
$F' = 6.4 \times 10^{-5}$	$F' = 8.1 \times 10^{-5}$	$F' = 11.7 \times 10^{-5}$
AUTOREGRESSION COEFFICIENTS:**		
$\phi_1 = 1.42$	$\phi_1 = 1.39$	$\phi_1 = 1.31$
$\phi_2 = -.50$	$\phi_2 = -.48$	$\phi_2 = -.43$
$\delta' = 3.2 \times 10^{-5}$	$\delta' = 3.9 \times 10^{-5}$	$\delta' = 5.0 \times 10^{-5}$

C/T = average criminal population/total population

τ = mean stay in criminal population

*These coefficients are estimated for (i) above using (20).

**These coefficients are estimated for (ii) above using (6) and the results for differential equation (i).

by about the same amount in each case. The relative direction of these differences is consistent with the high negative correlation (-.82) between the coefficients of r_{t-1} and r_{t-2} in the autoregression.

The value of the constant term is underestimated by as much as 60% in the theoretical estimates, $\delta' \times 10^5$. However, all of the estimates of $\delta' \times 10^5$ are within a 90% confidence interval of the regression value (2.57,15.77).

Overall, despite the speculative, albeit plausible, nature of some of the parameter estimates, the model appears to do remarkably well in generating parameters consistent with those estimated from the actual data. The encouraging nature of these results indicates the potential merit of this approach to modeling the imprisonment process and justifies further work in this direction, especially efforts to examine the process without the restrictive assumption of constant flow rates. Furthermore, while acknowledging the tentative nature of Model II, we can cautiously begin to interpret the flow rates in the model in an effort to characterize those features of a society which contribute to its particular imprisonment rate.

V. VARIATIONS IN IMPRISONMENT RATES

As a corollary to the hypothesis of the stability of crime, Durkheim also conjectured that the particular level of crime would vary among different "social types" and that it might be possible to specify the level appropriate to each "social type".* Two of the authors have argued elsewhere (Blumstein and Cohen [1973], p. 199) that Durkheim was not speaking of the the level of actual criminal behavior that occurs,** but rather the level of punished criminal acts. Hence, it is the level of punishment meted out

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*Durkheim (1964), pp. 66-67. A "social type" is simply a collection of similar societies. More formally, "social types" may be thought of as equivalence classes within the set of societies.

**This would include any act that is a violation of some criminal statute.

which remains stable, but varies in magnitude among different classes of societies.

A brief inspection of Figure 1 provides visual evidence for this corollary. While there is a stable process in each country with the annual imprisonment rate fluctuating around the mean, there are substantial differences among those means. The mean imprisonment rate for the U.S.A. is 2-3 times greater than either the rate in Norway or Canada.* In an effort to account for these differences, Model II will be interpreted in terms of some general societal characteristics. The ways in which these characteristics generate different imprisonment rates can then be examined within the framework identified by the model.

Two characteristics of societies important to the phenomena of crime and punishment are the level of conformity within a society and the degree of punitiveness. The parameters k_1 and k_2 in Figure 5 reflect two aspects of the degree of punitiveness that are often cited, the severity and certainty of punishment. When other forms of punishment are ignored and only imprisonment is considered, the severity of punishment varies with the time actually served in prison. Since increases in the time served result in decreases in the release rate from prison, k_1 (the release rate) may be regarded as an inverse measure of the severity of punishment. The lower the value of k_1, the more severe the punishment meted out. Alternatively, the flow rate of criminals to prison, k_2, reflects the certainty of punishment for criminal behavior. The higher

*The definition of and institutional arrangements for prison populations vary considerably from country to country. The Canadian and U.S. data include only individuals in prisons and penetentiaries which are largely restricted to persons serving sentences of one year or more. In Norway, on the other hand, the typical sentence for the prison population rarely exceeds two months. Nevertheless, despite these differences, the selected prison statistics refer to the most severe penalty imposed in each country, aside from capital punishment. Our intention is to gain insight into the reasons for differences in the level of only the most severe form of punishment. From this perspective, then, the differences in definition allow cautious comparison of the rates while always keeping in mind the potential incompatibilities.

k_2, reflects the certainty of punishment for criminal behavior. The higher the value of k_2, the more criminals are imprisoned.

Parameters k_3 and k_4 in Figure 5 are the flows between the law-abiding and criminal populations and together they reflect the overall level of conformity in a society. The magnitude of the flow from law-abiders to criminals, k_3, provides some indication of the strength of the commitment to conformity within a society; the stronger the commitment, the smaller the flow out of law-abiders. The level of commitment to conformity in any society is probably a complex product of a number of different contributing factors, among them the successful internalization of the normative code, the deterrent effects associated with penalties, and the heterogeneity of the society.

These factors affect the commitment to conformity differently and operate on very different dimensions of an individual's motivation. The more deeply rooted the norms and values of a society in the individual consciences of its members, the stronger will be their commitment to conformity. In this case the members conform out of a sense of duty or obligation. Deterrence, on the other hand, captures the extent to which individuals respond to the costs associated with the penalty structure. Effective deterrence will increase the strength of commitment to conformity.

Alternatively, greater heterogeneity in a society, be it cultural, ethnic, racial, or religious, can weaken the overall commitment to conformity through the existence of competing normative systems which may be at odds with the institutionalized standards. As the members of a society respond to the behavioral codes of different sub-cultures, there will be a larger variance in actual behavior and more chances of deviance. While Model II does not permit distinguishing the contributions of these different factors, the effect of the resulting commitment to conformity can be examined through parameter k_3.

Parameter k_4 is the flow from the criminal population to the law-abiding population. It reflects the endurance of the criminal role, or the extent to which individuals remain active criminals after committing a single crime. Thus, k_4 may be thought of as an inverse measure of the prevalence of hard-core criminality in a society. As k_4 gets smaller, fewer criminals return to the law-abiding population and the more enduring the criminal role.

The endurance of the criminal role is undoubtedly the result of a complicated process involving both the availability of opportunities to return to the law-abiding population and the existence of disincentives to remain a criminal. The opportunities to return are a function of the permanence of of the stigma attached to being labeled a criminal, as well as any institutionalized barriers which explicitly exclude former criminals from various aspects of a law-abiding life (e.g., laws which bar known criminals from certain types of employment). The disincentives to remaining a criminal vary with the effectiveness of deterrents. The only deterrent explicitly identified in Model II is imprisonment. Nevertheless, a host of other unspecified deterrents (e.g., arrest and conviction) may also operate on the criminal population and be reflected in variations in the value of k_4. In general, increases in both legitimate opportunities and criminal disincentives will be associated with decreases in the endurance of the criminal role and increases in k_4.

Having identified each parameter in terms of punitiveness and conformity, the differential impact of these characteristics on the imprisonment rate and the level of criminality in a society can be explored. The flow process in Figure 5 can easily be translated into a Markov process in which the populations are the states of the process and the flow rates become the transition probabilities of moving from one state to another. Assuming the $k_i(t)$ are constant over time, the transition matrix for Model II is:

$$M = \begin{array}{c} \\ P(t) \\ C(t) \\ L(t) \end{array} \begin{array}{ccc} P(t+1) & C(t+1) & L(t+1) \\ \end{array}$$

$$M = \begin{matrix} P(t) \\ C(t) \\ L(t) \end{matrix} \begin{bmatrix} 1-k_1 & \theta k_1 & (1-\theta)k_1 \\ k_2 & 1-k_2-k_4 & k_4 \\ 0 & k_3 & 1-k_3 \end{bmatrix}$$

Since this matrix is regular*, the equilibrium probability distribution among the three states can be obtained by raising the matrix to successive powers, M^n. As n becomes large, each row of M will approach the same equilibrium vector and any row of the matrix gives the equilibrium distribution.

This feature of matrix M permits the use of simulation techniques to examine the equilibrium distribution for different assigned values of the k_i and θ in M. By systematically changing the value of one parameter at a time, one can investigate the effect of that parameter alone on the equilibrium distribution. Each parameter is assigned five values, while holding all other parameters constant. The entries in Table 5 are the equilibrium rates/100,000 total population for each of the three sub-populations of interest.**

*A transition matrix is regular if there is at least one path, perhaps multi-step, from each state to every other state.

**The rates in Table 5 were found by multiplying the equilibrium probability of each state by 10^5.

Table 5

The Equilibrium Distribution Among Prisoners (P
Criminals (C) and Law-Abiders (L) Associated
With Different Values of the Parameters of Model II

Parameter Values					Rates/100,000 Total Population		
k_1	k_2	k_3	k_4	θ	(P) Prisoners	(C) Criminals	(L) Law-Abiders
I							
.200					175.8	1406.1	98416.3
.250					140.6	1406.5	98452.4
.333	.025	.005	.333	.333	105.5	1407.0	98486.8
.500					70.4	1407.5	98521.8
.000					35.2	1408.1	98556.6
II							
	.010				29.0	1449.0	98521.7
	.025				70.4	1407.5	98521.8
.500	.050	.005	.333	.333	134.3	1343.6	98521.8
	.075				192.8	1285.2	98522.0
	.100				246.3	1231.6	98521.8
III							
		.001			14.2	284.9	99700.7
		.003			42.5	849.5	99107.9
.500	.025	.005	.333	.333	70.4	1407.5	98521.8
		.008			111.7	2232.4	97657.5
		.010			138.7	2774.2	97087.1
IV							
			.200		112.6	2253.1	97633.7
			.250		91.9	1838.8	98068.9
.500	.025	.005	.333	.333	70.4	1407.5	98521.8
			.500		47.9	958.0	98993.9
			1.000		24.5	489.3	99486.2
				.100	69.2	1384.9	98545.7
				.250	70.0	1399.4	98530.4
500	.025	.005	.333	.333	70.4	1407.5	98521.8
				.500	71.2	1424.3	98504.2
				.750	72.5	1450.1	98477.1

Section I of Table 5 indicates the effects of varying the severity of punishment, $1/k_1$. As k_1 increases, punishments become less severe and the average imprisonment rate decreases sharply. In fact, as the average time served drops from 5 years to 1 year, the imprisonment rate also decreases five-fold. However, the proportion of criminals among the total population is virtually unaffected by changes in k_1. This is largely due to P's comparatively small size with respect to both C and, more obviously, L. In fact, for all values of k_1 in the table, P is never even 0.2% of the total population and it represents at most only 12.5% of the criminal population.* Thus, changes in k_1, which affect the flow out of P, will have very little effect on the size of C. Any variations in the deterrent effect associated with changes in the release rate, k_1, will be manifested in changes in k_3 and k_4, the flows between criminals and law-abiders. Since these flows are held constant as k_1 varies, this effect cannot be detected in this analysis.

The variations in k_2 (section II, Table 5) reflect changes in the certainty of punishment. As k_2 increases, a higher proportion of criminals are imprisoned and the imprisonment rate increases. There is also some change in the relative size of the criminal population which decreases by 15% from 1449 to 1232 criminals/100,000 population as k_2 increases from .01 to .10. To the extent that the level of crime is a function of the number of criminals, the response of the criminal population to changes in k_1 and k_2 is consistent with the currently popular notion that it is the certainty of punishment and not its

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*These are not unreasonable bounds on the relative size of P. In the U.S.A. in 1970, for example, there were slightly less than 200,000 state and federal prisoners, or about 0.1% of the total population. (National Prisoner Statistics, 1968, 69 & 70). During the same year there were 1,273,783 reported arrests for Index Crimes (Uniform Crime Reports, 1970). Since the arrests of all police agencies are not contained in the reported figures and not all criminals are arrested, 2,500,000 is not an unreasonable estimate of the size of the criminal population. In this case the prisoner population is only 8% of the criminal population.

severity, which has the greatest deterrent effect on crime (Wilson [1975a] and [1975b]).

Parameter k_3 is assumed to vary with the strength of the commitment to conformity in a society. The larger k_3, the weaker that commitment and the more frequently law-abiders commit crimes. As section III of Table 5 reveals, increases in k_3 are accompanied by similar increases in both the relative size of the criminal population and the imprisonment rate.

The magnitude of parameter k_4 reflects the prevalence of "occasional" criminals as opposed to hard-core "professionals" in the criminal population. As k_4 increases more criminals return to the law-abiding population indicating criminality of a more transitory nature. It is thus no surprise that as k_4 increases (section IV, Table 5), both the relative size of the criminal population and the imprisonment rate decrease. In fact, a five-fold increase in k_4 from .2 to 1.0 is accompanied by a five-fold decrease in the rates of criminals and prisoners in the population.

The last section of Table 5 presents the effects of changes in θ, the recidivism rate of released prisoners. It is clear that the populations are virtually insensitive to changes in recidivism. Sizeable increases in θ have very little effect on the size of the criminal and prison populations. As with parameter k_1, the lack of effect on the criminal population is due to the extremely small size of P, which in section V of the Table is less than 0.1% of the total population and represents only 5% of the criminal population. The variations in the number flowing from this small P to C that result from changes in θ will hardly be noticed in C. Furthermore, since θ determines the distribution of the flow out of P and not the magnitude of that flow, changes in θ have virtually no effect on the size of P.

With the exception of θ, changes in any one parameter of the model result in important differences in the imprisonment rate. The most striking consequence of the model, however, is the predominant effect of k_3 or k_4 alone on the criminal population. This has important policy implications for the control of crime. If Model II is an accurate representation of the flow process among law-abiders, criminals, and prisoners, the results in Table 5 suggest that the activities of the criminal justice system reflected in isolated changes in parameters k_1, k_2 or θ alone have very little impact on the size of the criminal population.

According to Model II manipulations of only the time served in prison ($1/k_1$) or the various efforts in prisons to reduce recidivism (θ) will not affect the incidence of criminals. Furthermore, singly increasing the rate at which criminals go to prison (k_2) has only a marginal effect on the criminal population, while greatly expanding the prison population. So, although the imprisonment policies of a society are clearly important in determining the imprisonment rate, it is much more difficult to relate them to the extent of criminality in a society.

The size of the criminal population is most responsive to the parameters reflecting the level of conformity, namely k_3 and k_4. To the extent that conformity is a function of an effective socialization process and/or the homeogeneity of a society, very little, in the form of implementable policies, can be done to reduce the proportion of criminals. However, to the extent that deterrence and opportunities for return to the law-abiders are operating, more reasonable attempts can be made to reduce criminality. Certainly, any efforts to remove barriers to a return to the law-abiding population which increase the value of k_4 will decrease the level of criminality. The more interesting policy implication, however, is the important role of deterrence in reducing crime. Inasmuch as effective general deterrence increases incentives to remain a law-abider (decreases k_3), while effective special deterrence increases incentives to leave the criminal population (increases k_4), the level of conformity increases and the proportion of criminals decreases. The exact mechanisms involved in optimizing these deterrence effects are then vital to efforts to reduce crime.

The results in Table 5 identify only the effects of "pure" changes in the parameters and as such they are necessarily artificial. Undoubtedly, several of the parameters will vary at the same time , and the actual population distributions will reflect the cumulative effect of these different parameters, as well as any interactive effects due to functional relationships among the parameters. Nevertheless, looking at the effects of each parameter alone does provide some opportunity for accounting for observed differences in imprisonment rates.

Within the constraints of Model II we can conclude from Table 5 that more punitive societies, either in terms of the severity or certainty of punishment, have higher imprisonment rates. Also, more conforming societies have lower imprisonment rates. We will now explore the success of these factors in accounting for the reported differences in imprisonment rates in the U.S.A., Norway and Canada. Three attributes of these societies have been chosen for comparison, the average time served in prison, the probability of a prison sentence for convicted individuals and the homogeneity/heterogeneity of the society.

Table 6 reports each country's values for the punishment variables. The U.S.A. and Canada are quite similar in their use of prison sentences for convicted offenders, although the length of time served is somewhat higher in the U.S.A. Norway is quite distinct from the two North American countries. The average sentence served in Norway is much shorter (only about 3 months) and the imprisonment option is used more often. Despite the dramatic differences on the individual punishment variables, the expected sentences for convicted individuals are more similar. Furthermore, to the extent that this expected sentence varies with the overall level of punitiveness, Model II would predict that the imprisonment rates in these three countries would be ordered from highest to lowest as follows: U.S.A. > Norway > Canada.* This is in fact the order which is observed.

The prevalence of ethnic and religious differences in a society will be used as a measure of the degree of heterogeneity (cultural differences) of that society. Table 7 reports data on these variables for the three countries of interest. With respect to the religious variables, the U.S.A. is the most heterogeneous, followed by Canada and then Norway, which is strikingly homogeneous. The evidence on ethnic differences is less apparent. The differences among the proportion of immigrants to each country are quite small. However, since the populations of all three countries are predominently Northern European in origin, immigrants of Southern or Eastern European, Asian, African or South American origin represent a sharper contrast to the dominant culture. Here the immigrants to the U.S.A. contribute more to the heterogeneity, again followed by Canada and then Norway. Furthermore, although exact figures are not reported in the Table, the population of the U.S.A. is also more racially heterogeneous than either Canada or Norway.

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*The Canadian imprisonment rate is computed fir the population 16 years of age or older. Using the total population would only decrease the imprisonment rate and therefore not change the result.

Table 6

Variations in Punitiveness in the USA, Norway and Canada

VARIABLE	USA* (1970)	NORWAY** (1971)	CANADA*** (1960)
Average time served in prison (S) (Avg. prison pop'n / # Receptions in Prison)	2.5 yrs. (3.5 yrs)	.26 yrs.	1.9 yrs.
Probability of a prison sentence given a conviction (p) (# Receptions in Prison / # Convictions)	(.062)	.567	.051
Expected sentence given a conviction (pS)	(.22 yrs.)	.15 yrs.	.10 yrs.

*Data for convictions are not available for the USA. Therefore, the figures for the State of Pennsylvania (whose imprisonment rate is comparable to the USA's) were used as proxies for the national figures. These numbers are reported in parentheses and refer to individuals convicted in criminal court and/or sentenced to state prisons. Sources: Statistical Abstract of the United States:1972 (U.S. Department of Commerce) and the Pennsylvania Statistical Abstract: 1969 (Commonwealth of Pennsylvania, Bureau of Publications).

**The figures for Norway refer to individuals convicted of "crimes" (excludes minor offenses and misdemeanors) and/or sentenced to prison for "crimes". Source: Statistical Yearbook of Norway: 1973 (Norway's Central Bureau of Statistics).

***The Canadian figures refer to individuals ≥ 16 years old who were convicted of indictable offenses and/or sentenced to penetentiaries. Source: Historical Statistics of Canada, M.C. Urquhart & K.A.H. Buckley (eds) (Cambridge-MacMillan, 1965).

Table 7

Variations in Cultural Heterogeneity in the USA, Norway and Canada

VARIABLE	USA	NORWAY	CANADA
% of Population in dominant religion	36.8% (Roman Catholic) (1971)	97% (Norwegian Church) (1972)	43% (Roman Catholic) (1951)
# Religions including at least 75% of the population	9	1 (1972)	3 (1951)
% of Population that is foreign born	4.7% (1970)	---	8% (1960) (excluding those born in British Commonwealth Countries)
Total Immigrants as a % of Total Population	0.2% (1970)	0.5% (1971)	0.6% (1960)
% of Immigrants of Southern or Eastern European, Asian, or African origin	81% (1970)	17% (1971)	44% (1960)

Sources: Statistical Abstract of the United States: 1973 (U.S. Department of Commerce), Statistical Yearbook of Norway: 1973 (Norway Central Bureau of Statistics), Historical Statistics of Canada, M.C. Urquhart & K.A.H. Buckley (eds.) Cambridge-Macmillan 1965)

Now using heterogeneity as an indication of the level of conformity in a society, Model II would predict that the U.S.A. would have the highest imprisonment rate followed by Canada and then Norway. This conflicts with the observed order of rates for Norway and Canada. In part this disagreement may be a reflection of real differences in the punishments compared. The rate for Canada includes only the most serious criminals who are sentenced to penetentiaries and excludes sentences to other penal institutions. No such discrimination is made in Norway's data, which includes sentences to any type of prison.* It is also possible, however, that homogeneity is simply a poor indicator of the level of conformity, especially if homogeneity is generated by effective socialization or if conformity is dominated by other factors like deterrence.

All things considered, the interpreted model provides a satisfactory account of differences in imprisonment rates. The model is interpretable in terms of punitiveness and conformity, and leads to predictions about the magnitude of imprisonment rates which are consistent with prior intuitions on the matter. However, the model is less adequate when it comes to actual differences in observed imprisonment rates. This is largely due to difficulties in identifying and measuring the level of conformity. The model can be improved by further refining and clarifying the processes involved in the k_3 and k_4 flows between criminals and law-abiders.

*If the Canadian data were changed to include other penal institutions, the measures ofpunitiveness would also change as the average daily population and receptions to other institutions are considered. This may well alter the predicted order of the imprisonment rates based on the level of punitiveness. Such a comparison is not possible with the data available.

VI SUMMARY

It has been conjectured that a homeostatic process operates within a society to maintain a stable level of punishment. This process is presumed to work through adaptive responses to changes in criminal behavior. In the short run these responses might involve changes in sentencing policies (e.g., an increase in the number of persons sentenced to prison, or a decrease in the length of sentences imposed). In the long run, the limits of criminal behavior may actually be redefined through changes in law and/or in practice. The result is either the decriminalization of previously criminal acts or the addition of newly prohibited acts to the criminal code.

Evidence of the stability of punishment, especially imprisonment, has been presented. The national imprisonment rates in three countries were shown to be trendless time-series, each generated by a second-order autoregressive process. Two models specifying the flow of individuals among different population groups were specified in an effort to identify the underlying dynamic process responsible for this stability.

Model I which requires only a simple balancing of prison receptions and releases was shown to be inadequate. For reasonable estimates of the parameter values this process does not yield the observed cyclical behavior in imprisonment rates. A second model, which includes movements between the law-abiding and criminal populations, results in a better fit between the predicted and actual time-series. Furthermore, Model II can be interpreted in terms of the levels of punitiveness and conformity in a society, thereby integrating the model into the existing body of work on deviance and social control.

The model, however, requires further development if its adequacy is to be fully explored. The major limitation in the development presented here is the assumption of constant flow rates among the populations. A central feature of the stability of punishment theory is adaptive behavior. In the context of our model, incorporation of adaptive behavior would require time-varying k's. The incorporation of time-varying k_i's into the model in a manner that is consistent with the theory would represent a major extension to our work. Also, the model does not explicitly incorporate deterrent effects. A further elaboration of the relationship of the flow rates to the deterrence process would further enhance the generality of the model by providing some synthesis of the stability of punishment with the notion of deterrence.

APPENDIX I

Suppose we have a system of simultaneous flows among three populations, A(t), B(t), C(t), where:

$$\dot{A}(t) = a_{11}A(t) + a_{12}B(t) + a_{13}C(t) \qquad \text{(a)}$$

$$\dot{B}(t) = a_{21}A(t) + a_{22}B(t) + a_{23}C(t) \qquad (1) \quad \text{(b)}$$

$$\dot{C}(t) = a_{31}A(t) + a_{32}B(t) + a_{33}C(t) \qquad \text{(c)}$$

such that

$$A(t) + B(t) + C(t) = T(t) \qquad (2)$$

with:

T(t) = total population at t

a_{ij} may possibly be zero.

Since C(t) = T(t) - A(t) - B(t), system (1) may be re-written as:

$$\dot{A}(t) = (a_{11}-a_{13})A(t) + (a_{12}-a_{13})B(t) + a_{13}T(t)$$

$$\dot{B}(t) = (a_{21}-a_{23})A(t) + (a_{22}-a_{23})B(t) + a_{23}T(t) \qquad (3)$$

or in matrix notation:

$$\dot{Y} = AY + F \qquad (4)$$

where:

$$Y = \begin{bmatrix} A(t) \\ B(t) \end{bmatrix} \qquad \dot{Y} = \begin{bmatrix} \dot{A}(t) \\ \dot{B}(t) \end{bmatrix} \qquad F = \begin{bmatrix} a_{13}T(t) \\ a_{23}T(t) \end{bmatrix}$$

$$A = \begin{bmatrix} (a_{11}-a_{13}) & (a_{12}-a_{13}) \\ (a_{21}-a_{23}) & (a_{22}-a_{23}) \end{bmatrix}$$

Taking the derivative of (4), we get:

$$\ddot{Y} = A\dot{Y} + \dot{F} \qquad (5)$$

Substituting (4) for $\dot{Y}$

$$\ddot{Y} = A^2Y + AF + \dot{F} \qquad (6)$$

Let a and b be the coefficients of the quadratic equation resulting from taking the determinant of $[A - \lambda I]$:

$$\begin{vmatrix} (a_{11}-a_{13}) - \lambda & (a_{12}-a_{13}) \\ (a_{21}-a_{23}) & (a_{22}-a_{23}) - \lambda \end{vmatrix} = \begin{vmatrix} c_{11} - \lambda & c_{12} \\ c_{21} & c_{22} - \lambda \end{vmatrix} = 0 \qquad (7)$$

or

$$(c_{11} - \lambda)(c_{22} - \lambda) - c_{21}c_{12} = 0$$

$$\lambda^2 - (c_{11} + c_{22})\lambda + (c_{11}c_{22} - c_{21}c_{12}) = 0^* \qquad (8)$$

Thus,

$$a = -(c_{11} + c_{22})$$

$$b = c_{11}c_{22} - c_{21}c_{12}$$

Adding the sum $(a\dot{Y} + bY)$ to both sides of (6)

$$\begin{aligned} \ddot{Y} + a\dot{Y} + bY &= (A^2Y + a\dot{Y} + bY) + AF + \dot{F} \\ &= (A^2Y + aAY + bY) + aF + AF + \dot{F} \\ &= [A^2 + aA + bI)Y + aF + AF + \dot{F} \\ &= aF + AF + \dot{F} \end{aligned} \qquad (9)$$

since $A^2 + aA + bI = 0$ and eqn. (9) are no longer simultaneous.

*Note: the values of λ which satisfy (8) are the eigenvalues of A.

REFERENCES

Blumstein, A., and J. Cohen (1973), "A Theory of the Stability of Punishment", Journal of Criminal Law and Criminology, Vol. 64, No. 2.

Durkheim, E., (1964) The Rules of the Sociological Method, S. Solovay & J. Mueller translators (New York: The Free Press).

Gottfredson, D., (1959) "The Role of Base Expectancies in the Study of Treatments", California Department of Corrections, Research Division, 1959.

National Prisoner Statistics: Prisoners in State and Federal Institutions for Adult Felons: 1968, 1969, 1970, National Prisoner Statistics Bulletin #47, U. S. Department of Justice, Bureau of Prisons, (Washington, D.C.; U. S. Government Publications, April 1972).

C. R. Nelson (1973) Applied Time Series Analysis for Managerial Forecasting (San Francisco, Holden-Day, Inc.)

Robison, J. and C. Smith (1971) "The Effectiveness of Correctional Programs", Crime and Delinquence, Vol. 17, No. 1.

Uniform Crime Reports: 1970, U. S. Department of Justice, Federal Bureau of Investigation, (Washington, D. C.; U. S. Government Publications)

Wilson, James Q. (1975a), quoted in "The Purpose of Prisons", Newsweek, February 10, 1975.

Wilson, James Q. (1975b), "Lock 'em Up and Other Thoughts on Crime", New York Times Magazine, March 9, 1975.

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