CENTER FOR THE ECONOMETRIC STUDIES OF CRIME AND THE CRIMINAL JUSTICE SYSTEM
HOOVER INSTITUTION
STANFORD UNIVERSITY

ECONOMIC MODELS OF CRIMINAL BEHAVIOR:
AN OVERVIEW

by

J.M. Heineke

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Economic Models of Criminal Behavior:

An Overview

Over the past six to eight years, economists have shown increasing interest in modeling the choice problem confronting individuals engaged in illegal activities. A number of factors are responsible for this new found interest, not the least of which are the lack of progress on the part of criminologists in providing a systematic framework for analyzing criminal activity, and the belated recognition by economists that the choice theoretic models of microeconomics afford a particularly useful structure for such an analysis.

Criminologists have approached the task of explaining illegal activity by attempting to determine those psychological and/or physiological factors that are unique to criminals. This has led criminologists to study the social backgrounds and behavior patterns of individual criminals in the hope of identifying a common set of characteristics which underpin criminal behavior. Such an essentially inductive approach to model building will not in general lead to testable models of criminal behavior.

On the other hand, economic models of criminal behavior take as given those influences in the personal and social backgrounds of individuals that determine "respect for law," proclivities to violence, preferences for risk and other behavioral characteristics held to be determinants of criminality. These models are based upon characteristics of

individuals which are alleged to be common not only to large classes of offenders, but to large classes of economic agents in general. In a sentence, the models of economic choice theory, of which the criminal choice is a special case, hypothesize that all individuals, criminal and non-criminal alike, respond to incentives; and if the costs and benefits associated with an action change, the agent's choices are also likely to change. More specifically, these models postulate that the decision to commit an illegal act is reached via an egocentric cost-benefit analysis. As is implicit in this statement, the expected benefits and costs associated with an illegal act may contain both monetary and psychic elements. But by treating the individual's "taste for crime" as a datum, one may build a theory of criminal behavior based upon the opportunities confronting the potential offender.

In what follows we construct four rather broad classes of models of criminal behavior and analyze the properties of each class with special emphasis on testable implications. The usefulness of this approach lies in the fact that all models of the economic literature with which we are familiar belong to one of the classes.[1] We find rather dramatic differences in implications across classes with what at first blush may appear to be small differences in model structure.

A BRIEF SURVEY OF THE LITERATURE

Perusal of the economic literature indicates two distinct approaches to modeling the offense decision. The first approach is essentially a

[1] We are referring here to theoretical models based upon the individual as the decision unit, not empirical models. By far the greatest number of papers dealing with criminal behavior have been empirical in focus. These papers usually begin by postulating the existence of aggregate offense functions with certain plausible, but nonetheless *ad hoc* properties.

portfolio approach in which the agent makes a decision as to what portion of his wealth to put at risk in a criminal activity. The second approach has been to view the offense decision as a time allocation problem. The papers of Allingham and Sandmo (1972), Kolm (1973), and Singh (1973) have treated the offense decision as portfolio decisions.[2] Such a tack is permissable only in so far as <u>all</u> consequences of the illegal activity in question may be expressed in purely monetary terms. Because each of these papers addresses the question of income tax evasion, there would seem to be little doubt that benefits from the illegal activity are purely monetary in nature. But although the penalty for unsuccessful evasion is almost inevitably a fine, it is doubtful whether the total cost of unsuccessful evasion is the fine, since the convicted evader may experience significant non-monetary costs in the form of loss of respectability, reputation, etc. To the extent that this is the case, it will be inappropriate to employ the portfolio specification.[3] In addition, to the extent that the illegal activity in question is time consuming, it again will be inappropriate to model the decision problem as a choice over wealth orderings. The fact that an illegal activity is time consuming, means that the offense decision problem is formally a labor supply problem with uncertain consequences.[4] And given the set of time consuming illegal activities,

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[2] In this paper we use the terminology <u>portfolio problem</u> or <u>portfolio decision</u> to designate a decision problem with uncertain consequences in which all "costs" and all "benefits" are pecuniary.

[3] Allingham and Sandmo acknowledge this point and devote a section of their paper to a model which incorporates non-monetary attributes of unsuccessful tax evasion.

[4] See Block and Heineke (1973) for an analysis of the labor supply decision when returns are stochastic.

the more interesting questions, both from the point of view of economic theory and of social policy, would seem to be those concerned with the factors responsible for the individual's time allocation between legal and illegal activities and how responsive the individual is to changes in these factors. The point is that, except for carefully selected illegal acts, the offense decision is most appropriately modeled as a time allocation problem into which the psychic costs and benefits associated with criminal activity have been explicitly incorporated.

A second group of papers addressing the criminal choice, the papers of Becker (1968), Block and Heineke (1975a), Ehrlich (1970, 1973), and Sjoquist (1973), all view the criminal choice problem as a time allocation problem and to one degree or another acknowledge the role of non-monetary costs and returns in the offender's decision problem. But although each of these authors claims to recognize both the time allocative aspects of the problem and the non-monetary aspects of the penalty if unsuccessful, the qualitative implications of these models differ substantially. The cause of such variation between models is of considerable interest both theoretically and practically and is examined at some length in what follows. Briefly, the differences between these models are a result of specialized assumptions (some explicit, some implicit) concerning either the amount of time devoted to leisure or the role of non-monetary (psychic) attributes, or both. We proceed by presenting a series of models into which an increasing number of characteristics of the criminal choice are incorporated. Shortcomings of the various specifications and differences in implications are noted at each step. We begin with a simple "portfolio" model.

MODEL I - THE SIMPLE PORTFOLIO MODEL

Consider an individual with an exogenous income confronted with the problem of deciding what portion of this income to allocate to illegal activity (the risky asset). The following definitions will be used:

W: actual income

W^o: wealth or exogenous income

$U(W)$: the individual's von Neumann-Morgenstern utility function, $U_W > 0$, $U_{WW} < 0$

x : the proportion of W^o to be allocated to illegal activity, $0 \leq x \leq 1$

$g(x;\alpha)$: the increase in income if the illegal endeavor is successful; α is a shift parameter.

$f(x;\beta)$: the monetary penalty if the illegal endeavor is unsuccessful; β is a shift parameter.

p : the probability that the illegal endeavor is unsuccessful

W_s : the individual's income if the illegal endeavor is successful; $W_s \equiv W^o + g(x;\alpha)$

W_u : the individual's income if the illegal endeavor is unsuccessful; $W_u \equiv W^o + g(x;\alpha) - f(x;\beta)$

If apprehended the individual's income is reduced by the amount $f(x;\beta)$, where $f(x;\beta) \gtrless g(x;\alpha)$. To carry out an analysis of the agent's decision it is necessary to adopt certain conventions concerning the functions $g(\cdot)$ and $f(\cdot)$. These are

$$g(\cdot) > 0, \quad x > 0 \quad ; \quad g(\cdot) = 0, \quad x = 0$$
$$f(\cdot) > 0, \quad x > 0 \quad ; \quad f(\cdot) = 0, \quad x = 0$$
$$g_x > 0 \quad ; \quad g_\alpha > 0; \quad g_{x\alpha} > 0$$
$$f_x > 0 \quad ; \quad f_\beta > 0; \quad f_{x\beta} > 0$$

These conditions are obvious: Gains and losses from illegal activity (i) are non-negative; (ii) are increasing functions of the amount at risk; and (iii) are increasing functions of the shift parameters α and β, for given values of x. Finally, increases in the shift parameters α and β are defined to increase not only total gains and total losses, $g_\alpha > 0$, $f_\beta > 0$, but also marginal gains and marginal losses, $g_{x\alpha} > 0$, $f_{x\beta} > 0$.

Adopting this framework, the agents' expected utility is:[5]

(1) $EU(W) = (1-p)U(W_s) + pU(W_u)$

For the agent to devote some, but not all, of his income to illegal activity there must be an x^o such that

(2) $(1-p)U'(W_s)g_x + pU'(W_u)(g_x - f_x) = 0$

It is straight forward to interpret these conditions when (2) holds as a strict inequality and either $x^o = 0$ or $x^o = 1$. We leave this to the interested reader and assume $0 < x^o < 1$.

The questions of interest here are the responses of the equilibrium portion of income devoted to illegal activity, x^o, to changes in the several parameters in the model. These are listed next:

(3) $$\frac{\partial x^o}{\partial W^o} = - ((1-p)\, U''(W_s)g_x + pU''(W_u)(g_x - f_x)) / J_1^o$$

(4) $$\frac{\partial x^o}{\partial \alpha} = - g_{x\alpha}(\partial EU/\partial W^o)/J_1^o + g_\alpha\, (\partial x^o/\partial W^o)$$

(5) $$\frac{\partial x^o}{\partial \beta} = (p((g_x - f_x)\, U''(W_u)f_\beta + U'(W_u)f_{x\beta}))/J_1^o$$

(6) $$\frac{\partial x^o}{\partial p} = (U'(W_s)g_x - U'(W_u)(g_x - f_x)) / J_1^o$$

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[5] In what follows we assume that all functions possess continuous derivatives of sufficient order to permit the analysis and that regular, internal maxima exist for each model.

Finally define $\partial x^o/\partial\gamma$ to be the change in x^o due to a shift in the penalty function and a corresponding change in p such that the expected loss remains unchanged. That is, $\partial x^o/\partial\gamma \equiv (\partial x^o/\partial\beta)$, given $d(pf) = 0$. Now $d(pf) = p(f_x dx + f_\beta d\beta) + fdp = 0$, so that $\partial(pf)/\partial\beta = pf_\beta + f(\partial p/\partial\beta) = 0$; which implies $\partial p/\partial\beta = -(pf_\beta/f)$. Therefore,

(7) $$\frac{\partial x^o}{\partial\gamma} = \frac{\partial x^o}{\partial\beta} + \frac{\partial x^o}{\partial p}\frac{\partial p}{\partial\beta}$$

$$= \frac{\partial x^o}{\partial\beta} - \frac{\partial x^o}{\partial p}(pf_\beta/f)$$

In equations (3) - (7) the symbol J_1^o represents the Jacobian associated with equilibrium condition (2), evaluated at x^o, and is negative by hypothesis. Defining the Arrow-Pratt measure of absolute risk aversion as $R(W) \equiv -U''/U'$ and keeping in mind that we have assumed the potential offender to be risk averse, we adopt the usual assumption that $\partial R/\partial W < 0$. It can be shown (see Appendix) that the model possesses the following qualitative properties:

(3') $$\frac{\partial x^o}{\partial W^o} > 0$$

The individual invests a larger portion of his income in illegal endeavors the wealthier he is.

(4') $$\frac{\partial x^o}{\partial\alpha} > 0$$

Increases in the returns to illegal activity, increase the income allocation to these activities.

(5') $$\frac{\partial x^o}{\partial\beta} < 0$$

Increases in the costs of engaging in illegal activity cause decreases in the allocation to these activities.

(6') $$\frac{\partial x^o}{\partial p} < 0$$

Increases in the probability of "failure" cause decreases in the allocation to illegal activity. And finally, if $f(x;\beta)$ is separable[6]

(7') $$\frac{\partial x^o}{\partial \gamma} < 0$$

Compensated increases in the penalty which leave expected losses unchanged, decrease the allocation to illegal activity. This is equivalent, by equation (7), to saying that proportional increases in punishment (loss) deter illegal activity to a greater extent then do equi-proportional increases in the probability of apprehension. It can also be shown [see Block and Heineke (1975a)], that equation (7) is equivalent to measuring the allocative effect of a mean preserving change in the dispersion of returns. Since mean preserving increases in β increase the dispersion of returns to illegal endeavors, equation (7') may be interpreted as implying that increases in the amount of uncertainty surrounding returns to illegal activity will decrease the income allocated to these activities.

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[6] Here we use "separable" in the sense that $f(x;\beta) = f_1(x)f_2(\beta)$. It does not seem to be possible to establish (7') without restricting $f(\cdot)$. Results of this type reported in the literature are usually obtained under the strong assumption that $f(x;\beta) = \beta x$, i.e., $f_1(x) = x$ and $f_2(\beta) = \beta$. Also note that if we define $\eta = f_x(x/f)$ as the elasticity of the penalty w.r.t. changes in the income allocation, then $\partial\eta/\partial\beta = 0$ is qualitatively equivalent to the condition $f(x;\beta) = f_1(x)f_2(\beta)$. In fact, as long as shifts in the penalty function do not result in decreases in η, inequality (7') will hold.

A number of points are of interest here: First, qualitative results (5'), (6'), and (7') depend only upon risk aversion and the fact that the individual allocates some but not all of his income to illegal activities, i.e., $0 < x^o < 1$.[7] Results (3') and (4') require in addition the hypothesis of decreasing absolute risk aversion. Second, inequalities (4') - (7') are the formal underpinning of any unambiguous economic theory of deterrence. These inequalities tell us that increases in gains always increase criminal activity, while increases in costs always decrease criminal activity. In addition, either increases in the probability of failure or increases in the amount of uncertainty surrounding returns will assuredly decrease the resources being allocated to criminal activity. Third, although the return and loss functions of Model I are quite general, it must be kept in mind that these functions contain only monetary gains and losses and hence the model will be strictly applicable only when all returns and all costs from engaging in the illegal activity are monetary in nature. This implies that there are no non-monetary consequences of the penalty if a failure occurs and also that the activity in question does not entail a significant "labor" input, which would introduce elements of a time allocation problem.

One interesting application of this model has been to the problem of optimal under-reporting of income to the tax authorities. In this case the labor input tends to be insignificant and the psychic costs as-

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[7] In addition to risk aversion, (7') requires the penalty function to be separable in the sense of f.n.6.

sociated with conviction for tax fraud may, in many groups, be relatively small. As we noted above, this is the problem treated in the Allingham and Sandmo, Kolm, and Singh papers. But, if we are to have a broadly applicable theory, non-monetary characteristics of illegal activity must be accounted for.[8]

MODEL II - PORTFOLIO MODELS OF TIME ALLOCATION

The models presented in this section address in a particular manner, the question of the determinants of the allocation of time between legal and illegal activity. As we noted at the outset, the term "portfolio model" is used in this paper to denote that class of models in which all returns and costs are monetary. So "a portfolio model of time allocation" is a <u>non sequitor</u> to the extent that

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[8] If the agent prefers risk and the symmetric hypothesis of increasing risk preference is adopted, $-\partial R/\partial W>0$, it can be shown (see Appendix) that

$$(3'')\quad \frac{\partial x^o}{\partial W^o} > 0 \qquad (4'')\quad \frac{\partial x^o}{\partial \alpha} > 0 \qquad (6'')\quad \frac{\partial x^o}{\partial p} < 0$$

as before; and that

$$(7'')\quad \frac{\partial x^o}{\partial \gamma} > 0$$

whenever the penalty function is separable. In fact, under the conditions of model I, $\partial x^o/\partial\gamma > 0$ <u>iff</u> $U'' > 0$ and $\partial x^o/\partial\gamma < 0$ <u>iff</u> $U'' < 0$. It is interesting to observe that whether the agent is risk averse or risk preferring, increases in wealth result in an increased portion of that wealth being devoted to illegal activity. Increased pay offs also result in increased allocations to illegal endeavors independent of the agent's behavior toward risk. In addition, increases in the probability of failure results in decreased illegal allocations independent of risk behavior. The only result which does not carry over from the risk aversion case is the response of x^o to changes in the penalty, $\partial x^o/\partial\beta$. The reason is obvious: Positive shifts in the penalty function decrease mean returns and increase the dispersion of returns; on the one hand making the agent worse off, and on the other hand better off. It is not possible to determine the net effect.

"work," be it legal or illegal, is disagreeable, i.e., involves psychic costs. This fact helps explain why authors who have utilized models of this sort (see Becker (1968), Ehrlich (1970, 1973) and Sjoquist (1973)), have justified their approach by including in the gain and loss functions of their models both monetary returns and the "monetary or wealth equivalent" of any psychic gains or losses. It is shown below that implicit in the models of these authors are rather strong restrictions on the functional form of the monetary equivalents of effort and penalties and hence on the preferences of offenders. We first digress to explore the formal structure of monetary equivalence and then establish the precise nature of these restrictions. A generalized version of the Becker-Ehrlich-Sjoquist models is then presented.

A Digression on Monetary Equivalences

Two points are of interest here: (1) Questions concerning the existence of monetary equivalents of the psychic costs of the effort and penalty attributes of an offense; and (2) questions concerning the form of "total" (monetary plus psychic) return and "total" cost functions, assuming the appropriate monetary equivalents exist. The first question has been discussed in some detail in Block and Heineke (1975a) and in Block and Lind (1975). For our purposes here it will suffice to merely sketch the monetary equivalent argument in enough detail to indicate that it is not generally true that monetary equivalents exist to labor and penalty attributes of an offense.

To begin, it should be noted that there is agreement in the literature that models of the offense decision must in general account for non-monetary costs in both the time allocation and penalty aspects of the decision. In other words, there is agreement that the underlying von Neumann-Morgenstern utility function is of the form $Z(t_1, t_2, S, W)$ where t_1 and t_2 represent the time allocated to legal and illegal activity, respectively, and S represents a vector of attributes of the penalty (the length of sentence, loss of reputation, and so on).

To proceed, consider an individual with income W, who allocates t_1 "hours" to legal activity, t_2 "hours" to illegal activity and suffers penalty S if unsuccessful. For the individual in question, a monetary equivalent to this effort allocation and penalty exists if and only if there exists an income level sufficiently low, say W*, so that the individual is indifferent between this income with no penalty and no "work" and the given effort allocation, income and penalty. Formally, if there exists a wealth level, W*, such that

$$(8) \qquad Z(t_1,t_2,S,W) = Z(0,0,0,W^*)$$

then W - W* is the monetary equivalent of t_1 "hours" of legal activity, t_2 "hours" of illegal activity, and a penalty of severity S. Clearly, existence of such an equivalence will depend upon the tastes and preferences of the particular offender and there is no reason to expect it to exist in general. If, for example, for a particular effort allocation and penalty the marginal rate of substitution between income and either t_1, or t_2 or S is infinite, then no monetary equivalent exists at that point. Or one could ask whether for any given effort allocation there exists a reduction in income to say $\hat{W}$ such that the agent is indifferent between $(t_1,t_2,0,\hat{W})$ and (t_1, t_2, S,W). Of course this depends upon the given effort allocation, the severity of the penalty and the agent's income. If the penalty is sufficiently severe and/or the discounted value of the agent's lifetime income is sufficently low, a monetary equivalent to the penalty will not exist. If $\overline{W}$ represents discounted lifetime earnings, then no monetary equivalent to the penalty S exists whenever $W = \hat{W} > \overline{W}$. As the discussion and examples indicate, monetary equivalents to psychic costs may not exist.

From equality (8), if an income level W* exists such that $Z(t_1,t_2,S,W) = Z(0,0,0,W^*)$ then $W - W^*$ is the monetary equivalent of the "state of the world" (t_1,t_2,S,W) and is a function of t_1,t_2,S and W. Designating this function as $C(\cdot)$, we may write $W^* \equiv W - C(t_1,t_2,S,W)$. Defining $Z(0,0,0,W^*) \equiv V(W^*)$, we have $V(W^*) \equiv V(W - C(t_1,t_2,S,W))$ which is the formal justification for collapsing all arguments of the multi-attibute utility function $Z(\cdot)$ into one attribute. To summarize, the monetary equivalent approach to modeling the offense decision implies that "return" and "cost" functions into which both moneary and non-monetary returns have been aggregated (via monetary equivalents) will be functions of t_1, t_2, S and W. That is, the function $W - W^* \equiv C(\cdot)$ is in general a function of each argument entering the utility function $Z(\cdot)$.

To draw out the implications of this discussion for modeling the criminal choice we define the following functions:

$G(t_2;\alpha)$: the monetary return resulting from t_2 "hours" of illegal activity; $G_2 > 0$, $G_\alpha > 0$ and $G_{2\alpha} > 0$.

$F(t_2;\beta)$: the monetary penalty resulting from t_2 "hours" of illegal activity, if the individual is apprehended and convicted; $F_2 > 0$, $F_\beta > 0$ and $F_{2\beta} > 0$.

$L(t_1;\delta)$: the monetary return resulting from t_1 "hours" of legal activity; $L_1 > 0$, $L_\delta > 0$ and $L_{1\delta} > 0$.

W_s: $W^o + L(t_1;\delta) + G(t_2;\alpha)$

W_u: $W_s - F(t_2;\beta)$

where the symbols α, β and δ represent shift parameters in the respective functions.[9] It is also helpful to "disaggregate" $C(t_1, t_2, S, W)$ into the functions $C^1(t_1, t_2, S, W)$, $C^2(t_1, t_2, S, W)$ and $C^3(t_1, t_2, S, W)$, the monetary equivalents of the psychic costs of legal activity, illegal activity and the penalty, respectively; and to define $\bar{L}(t_1, t_2, S, W) \equiv L(t_1;\delta) - C^1(\cdot)$, $\bar{G}(t_1,t_2,S,W) \equiv G(t_2;\alpha) - C^2(\cdot)$ and $\bar{F}(t_1,t_2,S,W) \equiv F(t_2;\beta) + C^3(\cdot)$ as the "total" return functions for legal and illegal activity and the "total" cost of the penalty, respectively. These are "total" return and cost functions in the sense that the monetary equivalents of the psychic costs of "labor" have been netted out of $L(\cdot)$ and $G(\cdot)$ and the monetary equivalent of psychic costs of the penalty has been added to the monetary penalty, $F(\cdot)$. Once this has been accomplished the problem

$$(9) \quad \max_{t_1,t_2} \{(1-p)Z(t_1,t_2,0,W_s) + pZ(t_1,t_2,S,W_u)\}$$

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[9] We should indicate here that the "failure state," W_u, might be characterized either as $\{W^o + L+G-F;p\}$ or as $\{W^o+L-F;p\}$, depending upon the disposition of G when the individual is captured. A more general failure state can be obtained by defining the random variable Y, $0 \leq y \leq 1$, with distribution function $K(y)$, to be the portion of G the offender manages to retain if captured. Then W_u becomes $\{W^o+L + YG-F;p\}$, which reduces to the above special cases when $y \equiv 1$ and when $y \equiv 0$. See Heineke (1975) for more detail.

is equivalent to the problem[10]

$$(10) \quad \max_{t_1, t_2} \quad \{(1-p)V(W^o+\bar{L}+\bar{G}) + pV(W^o+\bar{L}+\bar{G}-\bar{F})\}$$

PORTFOLIO MODELS OF TIME ALLOCATION - CONTINUED

In this section two models are analyzed. Both are special cases of the model given as (10) above and are essentially generalized versions of the models presented by Becker, Ehrlich and Sjoquist. The first case of interest occurs when the monetary equivalent of legal activity is restricted to depend only upon t_1 and the monetary equivalents of illegal activity and the penalty are restricted to depend only upon t_2. Formally, this means that the functions $C^1(\cdot)$, $C^2(\cdot)$ and $C^3(\cdot)$ above reduce to $C^1(t_1)$, $C^2(t_2)$ and $C^3(t_2)$ and hence "total" return and "total" cost functions are $\bar{L}(t_1;\delta)$, $\bar{G}(t_2;\alpha)$ and $\bar{F}(t_2;\beta)$.[11] This will be the case when, for example, the monetary equivalent of t_2 "hours" of illegal activity is independent of (i) the amount of time the agent spends in legal activity, (ii) the attributes of the penalty, S, and (iii) the wealth position of the agent.

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[10]See Block and Heineke (1974, 1975a) for more detail.

[11]More precisely, $\bar{L}(t_1, \delta) \equiv L(t_1, \delta) - C^1(t_1)$, $\bar{G}(t_2; \alpha) \equiv G(t_2; \alpha) - C^2(t_2)$, $\bar{F}(t_2; \beta) \equiv F(t_2; \beta) - C^3(t_2)$.

Under these conditions the problem is to maximize $(1-p)V(W_s) + pV(W_u)$ with respect to t_1 and t_2 subject to the constraint $t_1 + t_2 \leq \overline{t}$.[12] Necessary conditions for an internal maximum are

$$(11) \quad \begin{aligned} &(1-p)V'(W_s)\overline{L}_1 + pV'(W_u)\overline{L}_1 = 0 \\ &(1-p)V'(W_s)\overline{G}_2 + pV'(W_u)(\overline{G}_2 - \overline{F}_2) = 0 \end{aligned}$$

The first equation in (11) provides a hint as to the consequences of the specialized monetary equivalents. In particular, notice that this equation holds only if $\overline{L}_1 = 0$. Therefore if $C^1(\cdot)$ depends only upon t_1 and $C^2(\cdot)$ and $C^3(\cdot)$ depend only upon t_2, the individual's time allocation to legal activities will be independent of his wealth and independent of all attributes of the penalty. It is also clear from this equation that the uncertainty surrounding returns to illegal activities has absolutely no effect on the time allocated to legal endeavors. So no matter what the agent's wealth may be, no matter how high returns to illegal endeavors, how low is the penalty or how unlikely is apprehension, model II always yields the same allocation of time to legal activity. Since these properties of model II are of a global nature in that, as long as $0 < t_1^o < \overline{t}$, t_1^o remains unchanged <u>whatever</u> the values of p, W^o, $\overline{G}(\cdot)$ and $\overline{F}(\cdot)$, it follows that the analagous marginal effects are zero. These results plus other comparative static properties of the

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[12] And of course $t_i \geq 0$, $i = 1,2$.

model are presented next. The symbol J_2^o is used to represent the Jacobian associated with system (11) evaluated at (t_1^o, t_2^o), the equilibrium allocation. The elements of J_2^o are denoted D_{ij}, $i,j = 1,2$.

(12) $\partial t_1^o/\partial p = 0$

(12') $\partial t_2^o/\partial p = D_{11}(V'(W_s)\overline{G}_2 - V'(W_u)(\overline{G}_2 - \overline{F}_2))/\ J_2^o$

(12") $\partial t_2^o/\partial p < 0$

Changes in the probability of apprehension have no effect on the time allocated to legal activity, while increases in this parameter will deter participation in illegal activities.

(13) $\partial t_1^o/\partial W^o = 0$

(13') $\partial t_2^o/\partial W^o = -D_{11}((1-p)V''(W_s)\overline{G}_2 + pV''(W_u)(\overline{G}_2 - \overline{F}_2))/\ J_2^o$

(13") $\partial t_2^o/\partial W^o > 0$

Whether the individual is risk averse, risk neutral or prefers risk, exogenous changes in wealth will have no effect on the time allocated to legal activities. On the other hand, if the individual is risk averse and displays decreasing absolute risk aversion or prefers risk and displays increasing absolute risk preference, $(-\partial R/\partial W > 0)$, participation rates in illegal activities will increase with wealth levels.

(14) $\partial t_1^o/\partial \alpha = 0$.

(14') $\partial t_2^o/\partial \alpha = -\ D_{11}(\partial EV/\partial W^o)G_{2\alpha}/\ J_2^o\ + \overline{G}_2(\partial t_2^o/\partial W^o)$

(14") $\partial t_2^o/\partial \alpha > 0$

Changes in the returns to illegal endeavors have no effect on labor force participation rates, although decreasing absolute risk aversion implies the participation rate in illegal endeavors will increase with increases in returns.

(15) $\partial t_1^o/\partial \beta = 0$

(15') $\partial t_2^o/\partial \beta = pD_{11}(V'(W_u)\overline{F}_{2\beta} + \overline{F}_\beta V''(W_u)(\overline{G}_2 - \overline{F}_2))/\ J_2^o$

(15") $\partial t_2^o/\partial \beta < 0$

Increasing the severity of the penalty for unsuccessful illegal acts will not affect the t_1 decision, but will deter criminal activity. It should be kept in mind here, that the penalty function $\overline{F}(t_2\,;\beta)$ measures only the level of monetary costs plus those non-monetary costs that depend upon t_2 alone. All other attributes of the punishment, S, are treated as parameters in $F(\cdot)$.

(16) $\partial t_1^o/\partial \gamma = 0$

(16') $\partial t_2^o/\partial \gamma = \partial t_2{}^o/\partial \beta - (\partial t_2{}^o/\partial p)(p\overline{F}_\beta/\overline{F})$ [13]

(16") $\partial t_2^o/\partial \gamma \gtreqless 0 \quad \text{iff} \quad U''(W) \gtreqless 0$ [14]

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[13] See equation (7) above.

[14] This follows if $\overline{F}(\cdot)$ is separable in the sense of footnote 6 above.

Mean preserving increases in the dispersion of returns to illegal activity will have no affect on the t_1 decision. But if the penalty function is separable, such changes decrease, leave unchanged or increase participation in illegitimate activities if and only if the agent is risk averse, risk neutral or prefers risk, respectively.

(17) $\partial t_1^o/\partial\delta = - D_{22}(\partial EV/\partial W^o)\bar{L}_{1\delta}/ J_2^o$

(17') $\partial t_1^o/\partial\delta > 0$

(18) $\partial t_2^o/\partial\delta = \bar{L}_\delta(\partial t_2^o/\partial W^o)$

(18') $\partial t_2^o/\partial\delta > 0$ [15]

Finally, increases in the returns to legal activity increase participation rates in both legal and illegal activity. Legal and illegal activities are gross complements!

To be sure we are not accustomed to finding so many unambiguous qualitative results in the models of economic choice theory. These results stem from the independence of the markets for legal and illegal activities which is implied by the special nature of the monetary equivalences we have used. Of course system (11) is not a system of simultaneous equations, but rather a recursive system in which legal activity decisions are made and then, given t_1^o, the allocation to illegal activities is determined. Comparison of (12"), (13"), (14"),

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[15] The proofs of these propositions are entirely analogous to those presented for model I once one notes that $\bar{L}_1 = 0$ implies $D_{12} = D_{21} = 0$.

(15") and (16") with inequalities (3')-(7') above indicates that this specification of monetary equivalence functions has the effect of reducing the time allocation model given as (10) (or (9)), to an analog of the simple portfolio model.[16]

The question remains as to whether or not it is useful to restrict the preferences of offenders to such an extent. Only confronting the model with data can provide the answer. And unlike many of the models of economic theory, the large number of unambiguous predictions yielded by model II afford an excellent opportunity for empirical testing. This is particularly true due to the rather unorthodox predictions that the time spent in legal opportunities is independent of the structure of returns to illegal activity and that legal and illegal activity are gross complements. These results alone provide a strong basis for testing the model.

It is of interest to note that if $\bar{L}_1 < 0$ then $t_1^o = 0$ and again (as with the "internal" solution) the allocation to legal activities is invariant to the changes in returns and costs in the market for illegal activity. So the model predicts that there is no diminution in returns to illegal activity nor increase in the uncertainty of returns that will cause "professional criminals" ($t_1^o = 0$) to enter legal occupations.

general version of the Becker, Ehrlich and Sjoquist models.[18] Yet both Ehr..lich and Sjoquist report that legal and illegal activities are substitutes in their models, which is clearly inconsistent with model II in its present form.[19] The explanation lies in one additional assumption that was adopted by these authors, viz., that the time allocated to leisure is fixed and independent of the level of returns and costs in the markets for legal and illegal activities. In this case equations (11) above reduce to

$$(19) \quad (1-p)V'(W_s)(-\overline{L}_1 + \overline{G}_2) + pV'(W_u)(-\overline{L}_1 + \overline{G}_2 - \overline{F}_2) = 0$$

which will have an internal solution for $\overline{G}_2 > \overline{L}_1$ and $\overline{F}_2 > \overline{G}_2 - \overline{L}_1$. Then

$$(20) \quad \partial t_2^o/\partial p = ((V'(W_s)(-\overline{L}_1 + \overline{G}_2) - V'(W_u)(-\overline{L}_1 + \overline{G}_2 - \overline{F}_2))/J_3^o$$

$$(20') \quad \partial t_2^o/\partial p < 0$$

$$(21) \quad \partial t_2^o/\partial W^o = -(pV''(W_u)(-\overline{L}_1 + \overline{G}_2 - \overline{F}_2) + (1-p)V''(W_s)(-\overline{L}_1 + \overline{G}_2))/J_3^o$$

$$(21') \quad \partial t_2^o/\partial W^o > 0$$

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[18] In Ehrlich (1973), $C^1 \equiv W_\ell(t_1)$, $C^2 \equiv W_i(t_2)$, $C^3 \equiv F_i(t_2)$; in Sjoquist (1973) $C^1 \equiv \overline{g}_w t_1$, $C^2 \equiv \overline{g}_c t_2$, $C^3 \equiv \overline{p} t_2$ and in Becker (1968), $C^1 + C^2 = Y_j$, and $C^3 \equiv f_j$. There is a problem in analyzing Becker's model since it is only partially specified and contains no explicit decision variable. The implicit decision variable seems to be the number of offenses, O_j, since Becker states that his approach implies existence of a function relating O_j to the probability of conviction and the punishment among other things (see p. 177). Writing $O_j(t_2)$ transforms the model into the time allocation framework. The Becker model does not include legal alternatives and hence monetary equivalents will be functions of only t_2, S and W.

[19] The Becker model deals only with the market for illegal activities.

(22) $\partial t_2^o/\partial\alpha = -\overline{G}_{2\alpha}(\partial EV/\partial W^o)J_3^o + \overline{G}_\alpha(\partial t_2^o/\partial W^o)$

(22') $\partial t_2^o/\partial\alpha > 0$

(23) $\partial t_2^o/\partial\beta = p(V''(W_u)F_\beta(-\overline{L}_1 + \overline{G}_2 - \overline{F}_2) + V'(W_u)\overline{F}_{2\beta})/J_3^o$

(23') $\partial t_2^o/\partial\beta < 0$

and as before

(24) $\partial t_2^o/\partial\gamma \gtrless 0$ iff $V'' \gtrless 0$

where J_3^o is the Jacobian associated with equation (19) evaluated at equilibrium.[20] Comparison of these expressions to equations (12'), (13'), (14'), (15') and (16") above indicates that fixing the allocation to leisure leaves the predictive consequences of model II unchanged with respect to illegal behavior. Clearly, this will not be the case for the participation rate in legal endeavors. Since if ε is an arbitrary parameter, then $\partial t_1^o/\partial\varepsilon = -\partial t_2^o/\partial\varepsilon$. Therefore

(25) $\partial t_1^o/\partial p > 0$

(26) $\partial t_1^o/\partial W^o < 0$

(27) $\partial t_1^o/\partial\alpha < 0$

(28) $\partial t_1^o/\partial\beta > 0$

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[20] Again, the proofs of these propositions are virtually identical to those above. Inequality (24) requires $\overline{F}$ be seperable as before.

(29) $\partial t_1^o/\partial\gamma \gtrless 0$ iff $V'' \lessgtr 0$

Finally note that

(30) $\partial t_1^o/\partial\delta = -\overline{L}_{1\delta}(\partial EV/\partial W^o)/J_3^o - \overline{L}_\delta(\partial t_1^o/\partial W^o)$

(30') $\partial t_1^o/\partial\delta > 0$

and therefore

(31) $\partial t_2^o/\partial\delta < 0$

Once the leisure margin is fixed, legal and illegal activities become gross substitutes and the model collapses into the simple portfolio model of equations (1) - (7) above.

To summarize the results appearing in this section, notice that if t_3 denotes the time allocated to leisure and θ is any parameter which affects only the distribution of returns and costs to illegal activity, then $\partial t_2^o/\partial\theta = -\partial t_3^o/\partial\theta$ whenever t_3 is free to vary. (Contrast the pairs (12),(12") - (16),(16") with the pair (17'),(18').) Once the leisure margin is fixed, then $\partial t_2^o/\partial\varepsilon = -\partial t_1^o/\partial\varepsilon$, where ε is any parameter in the model. (Contrast expressions (20') - (23') with expressions (25) - (28) and (30') with (31).)[21] So these models are not time allocation models in any usual sense of the word. But the more important question is whether either model describes criminal behavior. Since each model provides a number of unambiguous predictions, testing should be relatively straightforward. For example, one could begin by attempting to discriminate between the fixed and variable leisure margin versions of the model. To test the fixed leisure mar-

[21] The condition $\partial t_2^o/\partial\varepsilon = \partial(\overline{t} - t_1^o)/\partial\varepsilon$ is precisely analogous to $\partial x^o/\partial\varepsilon = \partial(1 - x^o)/\partial\varepsilon$ in the simple portfolio model.

gin assumption one could test whether $\partial t_1/\partial\varepsilon + \partial t_2/\partial\varepsilon = 0$, where ε represents any of the parameters entering the model.[22] If this assumption is rejected, one could then proceed to test the twelve restrictions given as (12), (12") - (16), (16") and (17') and (18') above. As we noted previously, special interest lies in testing the independence restrictions, inequalities (12) - (16), and the gross complementarity of legal and illegal activity, (18'), since these properties of model II are associated with a much smaller class of models than are the remaining properties. If both versions of the model are rejected, one has evidence that the preference restrictions utilized in Model II are inappropriate. A more general model should be considered.

MODEL III - THE ALLOCATION OF TIME TO ILLEGAL ACTIVITY: THE CASE OF BERNOULLI CONSEQUENCES

In this section we present a model which fully accounts for non-monetary aspects of both the time allocation problem and the penalty. As the title of the section indicates, the model is concerned (as have been the other models in this paper) with the special case where the consequences of illegal activity are Bernoulli

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[22] Notice that this means the rate of substitution between t_1 and t_2 will always be constant and equal to unity. Or alternatively if t_1 and t_2 are interpreted as the time spent by an individual in each of two occupations and $\eta_{i\varepsilon} \equiv (\partial t_i/\partial\varepsilon)(\varepsilon/t_i)$, $i = 1, 2$, then the fixed leisure margin version of model II predicts that $t_1^o/t_2^o = -\eta_{2\varepsilon}/\eta_{1\varepsilon}$, where ε is any parameter in the model. In words, the relative sensitivity of the time allocation to occupation two to changes in any parameter is given by the observed proportion of time allocated to the other occupation, t_1^o/t_2^o.

distributed. Using the notation developed above, $Z(t_1,t_2,S,W)$ represents the agent's utility indicator with S being a vector of attributes of the penalty. For interpretive convenience we assume here that S is a scalar, the length of the sentence if convicted. It is natural to specify $S = S^o + S^1(t_2;\sigma)$, $S^1(0;\sigma) \equiv 0$ and $S_2 > 0$.[23] The term S^o is a constant and represents the minimal prison sentence for the class of activities in question. Analogous to above, we define $S_\sigma > 0$ and $S_{2\sigma} > 0$. The individual's problem is then to

$$(9) \quad \max_{t_1,t_2} \{(1-p)Z(t_1,t_2,0,W_s) + pZ(t_1,t_2,S,W_u)\}$$

subject to the condition $t_1 + t_2 + t_3 = \bar{t}$; where $W_s = L(t_1;\delta) + G(t_2;\alpha)$ and $W_u = W_s - F(t_2;\beta)$. Recall that the functions L, G and F contain only monetary aspects of the return to legal and illegal activity and monetary aspects of the penalty, respectively, since here non-monetary aspects of the offense decision enter $Z(\cdot)$ directly.

First order conditions for an internal maximum are

$$(32) \quad \begin{aligned} &(1-p)(Z_1^s + Z_W^s L_1) + p(Z_1^u + Z_W^u L_1) = 0 \\ &(1-p)(Z_2^s + Z_W^s G_2) + p(Z_2^u + Z_S^u S_2 + Z_W^u (G_2 - F_2)) = 0 \end{aligned}$$

where $Z^s \equiv Z(t_1,t_2,0,W_s)$, $Z^u \equiv Z(t_1,t_2,S,W_u)$, $Z_1^s \equiv \partial Z^s/\partial t_1$ etc.

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[23] Of course it is possible that $S^1(0;\sigma) > 0$, since people do occasionally receive prison sentences for crimes they do not commit. Note that the specification $S^2(0;\sigma) = 0$ also implies the existence of type 1 error.

It is of considerable interest to calculate the effects on the time allocation to criminal activity of changes in the various parameters and to contrast these with the analogous calculations in models I and II. Straightforward but tedious computations reveal

$$(33)\quad \frac{\partial t_2}{\partial W^o} = \frac{H_{21}\{(1-p)\ (Z^s_{1W}+Z^s_{WW}L_1)+ p(Z^u_{1W}+Z^u_{WW}L_1)\}-H_{11}\{(1-p)(Z^s_{2W}+Z^s_{WW}G_2) +}{J^o_4}$$

$$\frac{p(Z^u_{2W} + Z^u_{SW}S_2 + Z^u_{WW}(G_2 - F_2))\ \}}{J^o_4}$$

$$(34)\quad \frac{\partial t_2}{\partial \alpha} = \frac{-H_{11}G_{2\alpha}(\partial EZ/\partial W^o)}{J^o_4} + G_\alpha(\partial t_2/\partial W^o)$$

$$(35)\quad \frac{\partial t_2}{\partial \beta} = \frac{pH_{11}Z^u_W F_{2\beta}}{J^o_4} + \frac{F_\beta\{pH_{11}(Z^u_{2W}S_2+Z^u_{WW}(G_2-F_2)) - (1-p)H_{21}(Z^u_{1W}+Z^u_{WW}L_1)\}}{J^o_4}$$

$$(36)\quad \frac{\partial t_2}{\partial p} = \frac{H_{11}(Z^s_2 + Z^s_W G_2 - Z^u_2 - Z^u_S S_2 + Z^u_W(G_2-F_2)) + H_{21}(-Z^s_1 - Z^s_W L_1 + Z^u_1 + Z^u_W L_1)}{J^o_4}$$

$$(37)\quad \frac{\partial t_2}{\partial \delta} = \frac{H_{21}L_{1\delta}(\partial EZ/\partial W^o)}{J^o_4} + L_\delta(\partial t_2/\partial W^o)$$

And finally the effect on the time spent in criminal activity due to changes in the severity of punishment (as measured by the length of the sentence) is given by

$$(38) \qquad \frac{\partial t_2}{\partial \sigma} = \frac{-pH_{11}Z_S''S_{2\sigma}}{J_4^o} + S_\sigma(\partial t_2/\partial S^o)$$

In expressions (33) - (38), $H \equiv EZ$ and J_4^o is the Jacobian associated with (32) evaluated at equilibrium. As would be expected, it is not possible to establish the sign of any one of these comparative static derivatives unless one is willing to make much stronger assumptions about the preferences of offenders.

The response of illegal activity to increases in illegal opportunities, $\partial t_2/\partial \alpha$, and legal opportunities, $\partial t_2/\partial \delta$, are composed of stochastic counterparts to neoclassical substitution and income effects. (See Block and Heineke (1973, 1975b)). Even if one is willing to assume that illegal endeavors are inferior activities, it is not possible to sign these terms, although as usual the direct substitution effect is signed. It is also interesting to note that the response of criminal activity to changes in sentence length, $\partial t_2/\partial \sigma$, may be written as in (38) as the sum of two components: the first measures the response of t_2 to a compensated change in σ, and is always negative; the latter measures the response of t_2 to a change in the minimal sentence.

The reader will recall that in both models I and II it was shown that if the penalty function was separable, increases in the dispersion of returns to illegal endeavors led to decreases in such activity if and only if the agent was risk averse and vice versa; i.e., $\partial t_2/\partial\gamma \lesseqgtr 0$ iff. $U'' \lesseqgtr 0$ in those models. It can be shown that in model III sign[U''] is neither necessary nor sufficient for determining the allocative effects of changes in the dispersion of returns.

In other words, if the utility function is left unrestricted vis a vis specialized assumptions about monetary equivalents, then no conclusions may be drawn concerning behavior toward risk by observing sign[$\partial t_2/\partial\gamma$]. This point is of interest due to the fact that sign[$\partial t_2/\partial\gamma$] is equivalent to determination of the responsiveness of offenses due to simultaneous and offsetting changes in the probability of conviction and in the severity of punishment. (See the discussion following inequality (7') above.) Therefore Becker's contention that the "common generalization" that a change in the probability of conviction has a greater effect on the number of offenses than a change in punishment implies offenders are, on average, risk takers, is not forthcoming in a more general time allocation model in which non-monetary aspects the offense decision are left unrestricted. In fact this "common generalization" is consistent with $U_{WW} \gtrless 0$.

MODEL IV - THE ALLOCATION OF TIME TO ILLEGAL ACTIVITY: GENERALIZATIONS AND PROBLEMS

Each of the models investigated in this paper have the common attribute that there are but two consequences in the decision problem confronting the offender. At first blush this seems to be an eminently reasonable characterization of the problem. But is it? If the decision problem is viewed as a general time allocation problem, then Bernoulli consequences imply the individual will either succeed on every offense undertaken or fail on every offense undertaken - a hopelessly unrealistic state of affairs.

One suggestion for salvaging the time allocation model was given in Block and Heineke (1975a) and amounts to replacing the Bernoulli density with a more general density function. Then, letting λ be a continuous random variable defined on $[0,1]$ with distribution function $K(\lambda)$, the choice problem posed as model III becomes[22]

$$(39) \quad \max_{t_1, t_2} \{\int_0^1 Z(t_1, t_2, S, W^o + L + G - \lambda F)\, dK(\lambda)\}$$

subject to $S = S^o + S^1$ and $t_1 + t_2 + t_3 = \bar{t}$. In (39) it is possible for the offender to "fail" on any portion of the total number of offences committed. Although such a formulation does incorporate "partial success," a ubiquitous feature of the real world, several generalizations are badly needed. First, in model (39) only monetary aspects of the penalty are stochastic. It is clear that in any realistic model of criminal behavior, gains and penalties must be more generally stochastic. But even in such a model, a second and more difficult problem remains if prison sentences are a possible penalty - a problem not usually addressed in labor supply models: The individual may be apprehended and

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22 If λ can assume but two values, say zero and one, and $dK(\lambda)/d\lambda \equiv k(\lambda)$, then $k(1) = p$, $k(0) = 1-p$ and the function $k(\cdot)$ reduces to the Bernoulli density.

hence be unable to supply the planned number of offenses.[23] This predicament arises not from an anomaly unique to models of criminal choice, but instead is an intrinsic shortcoming of static models that could be remedied by modeling the decision problem as a dynamic process in which realized consequences in period t are used to update the model and become the basis for decisions in period t + 1.[24]

Other than a dynamic programming model, an additional possibility for circumventing the complications introduced by prison sentences is to view the individual's decision problem as either (i) that of choosing whether or not to commit any one offense or (ii) that of choosing the time allocation to any one offense. In the first instance the decision variable is discrete, assuming the values zero and one, while in the latter, $0 \leq t_2 \leq \bar{t}$ as before. The distinction between these approaches is more than merely pedantic, since the qualitative implications of the two models differ substantially. If potential offenders view their decision problem as one of determining the amount of time to allocate to an offense on an offense by offense basis, than model III, expression (9), is appropriate and no qualitative implications are forthcoming without imposing strong restrictions on the pre-

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[23] This is not to say that involuntary exit from the labor market does not occur in markets for legal skills, e.g., when the individual becomes too ill to work, but instead that it is an insignificant aspect of the total problem in these markets.

[24] See Block, Heineke and Sweeney (1977) for a simple dynamic model of the criminal choice.

ferences of offenders.[25] On the other hand, if the decision problem is viewed as a special case of the time allocation problem in which the potential offender decides to either commit an offense or not on an offense by offense basis, then strong qualitative implications are forthcoming.[26] In any event, the discussion points up the fact that further progress in modeling criminal behavior requires more effort be allocated to understanding the structure of the underlying decision process and less to the generation of *ad hoc* models.

Summary and Conclusions

The purpose of this paper was to provide some perspective on the problem of modeling the decision problem of a potential offender. The eight years which have passed since the appearance of Becker's path-breaking paper have seen several generalizations of Becker's framework. The papers of Allingham and Sandmo, Kolm, and Singh have viewed the offense decision as essentially a portfolio decision. We saw that this specification leads to a number of testable implications. The papers of Ehrlich and Sjoquist have adopted Becker's notion of the monetary or wealth equivalent of the psychic costs of an offense, and if such equivalences exist, there is no formal objection to this procedure. But if monetary equivalent functions are generally specified, there seems to be no conceivable advantage to be gained by the procedure.

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[25] If this interpretation is adopted, it would be desireable to treat p as a function of t_2 with $p'(t_2) < 0$. Since for most individuals it seems likely that the more time spent planning any given offense, the smaller will be the likelihood of failure.

[26] This decision process is a time allocation problem in the sense that the decision to commit an offense is a decision to allocate a fixed amount of time to illegal activity. If an offense takes t_2^* "hours," then, under this interpretation either $t_2^o = 0$ or $t_2^o = t_2^*$, in contrast, to the models investigated in this paper in which $t_2^o \in [0, \bar{t}]$.

Next we found the models of Becker, Ehrlich and Sjoquist rested upon rather strong, implicit assumptions about the functional form of monetary equivalences and hence about the nature of the underlying utility function. In effect the assumptions used in these models transform the offense decision problem into a simple portfolio problem. This model provides the theoretical underpinnings for the qualitatively unambiguous theories of deterrence which have been reported in the literature. These results were reported above as model II and a special case of model II in which the time allocated to leisure is fixed. Both of these models support the traditional hypothesis concerning the deterrent effects of changes in the "gains" and "costs" of crime. Not so traditional results forthcoming from models II include the normality of illegal activities in each model; the independence of legal labor market decisions from all parameter shifts in illegal markets and the complementarity of legal and illegal activity, when the leisure margin is free to vary; and if the allocation to leisure is fixed, the prediction that changes in labor force participation rates, due to any parameter shift, will be identical in magnitude but of opposite sign, to changes in the amount of time allocated to illegal activity.

In the final section we discussed several problems which persist once psychic costs have been more generally accounted for. For one thing a time allocation model with Bernoulli distributed consequences implies the offender either succeeds or fails on every offense undertaken. More general distributions of consequences eliminate this difficulty. One fundamental problem remained: It may not be possible for the agent to carry out his plans if prison sentences constitute punishments. Several approaches to solving this problem were given.

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Appendix

Model I

By equation (3)

$$\text{(A-1)}\quad \partial x^o/\partial W^o = -((1-p)(U''(W_s)g_x + pU''(W_u)(g_x - f_x))/J_1^o$$

$$= (R(W_s)(1-p)U'(W_s)g_x + R(W_u)\,pU'(W_u)(g_x - f_x))/J_1^o$$

$$= (R(W_s)A + R(W_u)B)/J_1^o$$

where $R(W_s) = -U''(W_s)/U'(W_s)$, etc. Now $A > 0$, $B < 0$ and $A = -B$ by the first order condition for an internal maximum. Since decreasing absolute risk aversion implies $R(W_u) > R(W_s)$ the numerator of (A - 1) is negative and $\partial x^o/\partial W^o > 0$. It follows immediately that $\partial x^o/\partial \alpha > 0$. Also, since $(g_x - f_x) < 0$ by the first order conditions, risk aversion alone implies $\partial x^o/\partial \beta < 0$.

To show $\partial x^o/\partial p < 0$ rewrite the first order condition as $U'(W_s)g_x = p(U'(W_s)g_x - U'(W_u)(g_x - f_x))$ and compare with the numerator of equation (6).

Finally, from equation (7) we have

$$\text{(A-2)}\quad \partial x^o/\partial \gamma = \partial x^o/\partial \beta - (\partial x^o/\partial p)(pf_\beta/f)$$

Substituting for $\partial x^o/\partial\beta$ and $\partial x^o/\partial p$ and rearranging yields

$$\text{(A-3)}\quad \frac{\partial x^o}{\partial\gamma} = \{p(g_x - f_x)\, U''(W_u)f_\beta - pf_\beta\, g_x\, (U'(W_s) - U'(W_u))/f\ +$$

$$+\ pU'(W_u)\ (f_{x\beta}f - f_\beta f_x)/f\}/J_1^o$$

$$= \{pf_\beta\ (g_x - f_x)\ U''(W_u) - g_x(U'(W_s) - U'(W_u))/f\}/J_1^o$$

if $f(x;\beta) = f_1(x)f_2(\beta)$. The numerator of this expression is negative iff $U'' > 0$ and positive if $U'' < 0$. Therefore if the penalty function is separable, $\partial x^o/\partial\gamma \gtrless 0$ iff $U'' \gtrless 0$.

Procedures precisely analogous to those used thus far, will verify the results reported in footnote 6, for the case when $U'' > 0$ and $-\ \partial R/\partial W > 0$.