THE SUPPLY OF LEGAL AND ILLEGAL ACTIVITY: AN ECONOMETRIC MODEL

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The Supply of Legal and Illegal Activity: An Econometric Model

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Empirical investigations in recent years have amassed considerable evidence that increasing expected costs or decreasing expected benefits in a given illegal activity results in diminished participation in the affected activity. But an important question remains unanswered before policy recommendations can be drawn from such findings: to what extent do individuals respond to changes in expected returns by moving from one source of income to another? In this paper we derive an econometric model that is consistent with individual maximizing behavior and that can be used to estimate 1) the degree of substitutability or complementarity that exists between these alternative sources of income, and 2) the "net" or system-wide response of participation rates in the several income-generating activities as expected returns and costs vary.

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Introduction

One of the fundamental questions of interest to researchers studying criminal behavior concerns the extent of any deterrent effects which may be associated with different policy changes and sanctions. As one would expect, and as Block and Heineke [1975] have recently shown, it is not possible to establish the existence of deterrent effects from theoretical considerations alone unless preferences are strongly restricted. The situation is a familiar one: "uncertainty substitution effects" are consistent with the deterrence hypothesis while "income uncertainty effects" are qualitatively ambiguous.[1] Hence, as is usually the case in models of household decision making, determination of both the magnitude and the direction of supply and demand responses to parameter shifts is an empirical proposition.

To this end a number of econometric investigations have been undertaken in recent years, most of which have been supportive of the deterrence hypothesis.[2] These studies have been of two general types: 1) studies utilizing indices of overall criminal activity to measure the response of

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[1]The terms "uncertainty substitution effect" and "income uncertainty effect" were introduced by Block and Heineke [1973] to denote the stochastic analogs to the terms in the traditional Slutsky decomposition.

[2]There has been some discussion concerning the validity of several of the estimated "supply of offenses" equations. (See Nagin [1976], Fisher and Nagin [1976] and Passell and Taylor [1977], for example.) Comments have been essentially of two types: disagreement with identifying restrictions, and comments revolving about the rather poor quality of available data and the consequent difficulty of drawing valid inferences from such data. See Nagin [1976] for a bibliography.

offense levels to changes in policy parameters (see for example Orsagh [1973], Sjoquist [1973], Carr-Hill and Stern [1973, 1976], and Phillips, Votey and Maxwell [1972]); and 2) studies which have focused attention on particular crimes and used activity levels in those crimes to measure the effects of changes in policy parameters. See, for example, Ehrlich [1970, 1973, 1975], Vandaele [1973], and Avio and Clark [1976].

Models of the former type have rather obvious advantages and disadvantages. On the plus side one has the fact that since all criminal acts are grouped into a single index, one automatically has a measure of the system-wide response to any parameter shift in the model. So to some extent the effects of a change in the sanction for, say, burglary, on activity levels in other crimes have been accounted for. Of course, the negative side of the "index approach" lies in the question of just how much information is contained in movements of such an index. That is, just how meaningful are changes in a broad index of criminal activity as a measure of changes in a society's well being? The problem is the familiar one of weighting the components in an index, and in the case of criminal acts this problem is manifestly exacerbated. Crimes against persons and crimes against property must be assigned weights to obtain a single number which serves to represent the total number of murders, burglaries, rapes, robberies, etc.[3]

Of course models which rely on activity levels in single, relatively well-defined crimes to measure the response of policy changes do not have

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[3]The weighting problem is usually solved in reporting agencies by assigning an equal weight to all crimes included in the index. For example, the Uniform Crime Report's Index of Crime, prepared annually by the Federal Bureau of Investigation, is calculated in this manner.

this problem and should be used as the basis for tests designed to assess system-wide responses to policy changes. The point is that payoffs and sanctions in one crime may affect the level of activity in other crimes, and if so, changes in these payoffs and sanctions will have spillover effects. This is especially true of property crimes where economic theory leads one to suspect that the effort devoted to any one income-generating activity depends upon the distribution of returns to that activity and in general on the distribution of returns to all other competing sources of income. Clearly, even if it can be established that increasing sanctions and enforcement levels for a particular type of crime will decrease the incidence of that crime, one must also be able to account for changes in offense rates in other criminal activities which may be induced by the original policy change, before general statements concerning system-wide deterrence can be made. For example, will policy changes which decrease mean returns to burglars result in fewer burglaries, but increases in larceny, robbery and auto theft as individuals allocate more of their time to these now relatively more favorable opportunities? Hence before general conclusions concerning the overall deterrent effects of various policies can be reached, researchers must come to grips with the question of substitution among crimes as distributions of relative sanctions and returns change. This problem is addressed most satisfactorily by estimating a system of joint supply equations and assessing the response of the system as a whole to the policy changes of interest. None of the studies mentioned above or any other published or unpublished study with which we are familiar has attacked this problem.[4] The obstacle has certainly not been methodological, as the recent work on estimating demand systems is for the most part directly applicable to systems of activity supply equations. Instead the primary obstacle appears to be one of insufficient data and in particular insufficient data on returns by type of crime. However such information is now available to researchers at the Center.

The discussion of the previous paragraphs indicates the desirability of building and estimating a model which does not rely on broad indices of criminal activity and at the same time treats the "supply" decision of criminal agents as a choice over competing sources of income and/or satisfaction. In what follows we model the joint activity supply decision of an individual confronted with a set of legal and illegal income-generating prospects and derive the implied set of activity supply equations for the case of four income-generating prospects -- a generic legal activity and three illegal activities: larceny, burglary, and robbery. Attention is focused on constructing an econometric model capable of measuring the degree of substitutability between the legal and various illegal activities. Obviously it is the extent of substitutability between activities which determines the information loss incurred when policy prescriptions are based upon a system representing fewer than the full range of income-generating prospects confronting individuals.[5]

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[4]We should point out that Ehrlich (1970) has made a limited effort to estimate cross policy effects for several property crimes. He found all cross effects to be insignificant. We report the Ehrlich estimates below.

[5]This point is hardly novel, but merely a restatement of the fact that "partial" analyses become less applicable as the degree of interdependence between commodities or activities increases.

Outline of the Paper

We begin our investigation with a model of a single economic agent confronting the problem of allocating his time and income among n legal and illegal activities and m consumption possibilities, and derive the implied system of activity supply and commodity equations. To maintain the closest possible degree of contact between the underlying economic model and the resulting econometric model, we exploit several results from modern duality theory.[6] For our purposes the principal advantage of adopting these duality results is that they permit straightforward derivation of a system of activity supply and commodity demand equations which are consistent with utility maximizing behavior, simply by differentiating the indirect utility function as opposed to explicitly solving the utility maximization problem.[7] Among other advantages of estimating an econometric model which is consistent with an underlying utility maximization model is the substantial reduction in the number of parameters which need to be estimated when utility maximization is the maintained hypothesis and the restrictions implied by this hypothesis are imposed.

We proceed by approximating the agent's indirect utility function with a function which is quadratic in the logarithms of its arguments --

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[6] The literature on duality theory is quite large and growing rapidly. For a rigorous overview with an emphasis on applications see Diewert's [1974] survey article and the follow-up paper by Lau [1974].

[7] We should note that for reasonably general functional specifications for the direct utility function, obtaining explicit solutions to the utility maximization problem is very complicated if it is possible at all.

the transcendental logarithmic function.[8] This function provides a second order approximation to an arbitrary direct or indirect utility function and places no *a priori* restrictions on patterns of substitution between activities. The agent's commodity demand functions and activity supply functions for each legal and illegal income-generating activity are then derived and integrated over the wealth distribution to obtain aggregate demand and activity supply functions.

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[8] See Christensen, Jorgensen, and Lau (1971, 1973, 1975).

The Model

In this section we derive the system of activity supply and commodity demand equations implied by the hypothesis that legal and illegal "labor supply" decisions are made as if the individual's utility were being maximized. In each period the agent decides which of the n income-generating opportunities and m consumption possibilities confronting him are to be undertaken and how intensively each is to be pursued. The problem we address here differs from a traditional labor supply problem in that returns to most criminal activities are fundamentally stochastic. Seldom does an offender know the size of the gain to be realized from a crime. Furthermore, there is always the possibility that the individual will be arrested, convicted, and sentenced to jail or prison thereby incurring the cost of a severely restricted opportunity set in addition to any explicit costs incurred in his defense. From an empirical point of view a major difficulty with building a model in which returns are random lies in the apparent absence of a stochastic analog of Roy's Identity [1947] in many decision making contexts. To a large extent the existence of a stochastic analog to this identity depends upon how the income-expenditure constraint is treated. More specifically, the fact that returns and sanctions in each state of the world are uncertain means that the decision maker's plans will often not be realized. In some periods, surpluses will be generated, while in others deficits will be incurred. It is therefore necessary to adopt some convention regarding the relation between income and expenditures in the model. In what follows we require expenditures to equal income only "on average." Not only does this appear to be a reasonable condition to impose upon the income-generating relation as long as bankruptcy is disallowed, but in addition it permits straightforward extension of Roy's Identity to a world with stochastic "prices."[9]

We proceed as if consumption levels and time allocations to the several legal and illegal activities were determined by an agent maximizing utility subject to the requirements that expenditures equal income "on the average" and that the total time allocated to all activities, including leisure, be equal to total time available in the period. The following definitions and notation will be used:

W:	The agent's wealth at the beginning of the period.
t_i:	The time allocated to activity i. For convenience we denote t_n as legal activity.
r_i:	The unit return from activity i.
$S_i(t_i, W)$:	The monetary equivalent of the sentence if the agent is arrested for engaging in activity i and convicted and sentenced to jail or prison.[10] (Notice that S_i depends upon both the agent's wealth and his activity levels.) For convenience we assume S_i is proportional to t_i.
p_a^i:	The agent's subjective probability of being arrested for engaging in activity i. We have designated t_n as the time allocation to legal activity and assume $p_a^n = 0$. That is, we assume the probability of type one error is zero for individuals engaged ex-

[9] The non-existence of bankruptcy has a long precedent. For example, see virtually any of the portfolio models which have appeared in the literature in recent years.

[10] See Block and Heineke [1975] and Block and Lind [1975a, 1975b] for a discussion of monetary equivalence and its applicability to the criminal choice problem.

clusively in legal activity.[11]

$p^i_{c/a}$: The agent's subjective probability of being convicted, given he is arrested for offense i.

x_j: The level of consumption of commodity j in the period.

P_j: The price of commodity j.

$U(t, x)$: The agent's utility indicator.

Given that the loss from a prison sentence is measured as its monetary equivalent, the individual's unit prospects from engaging in activity i are:

r_i: $1 - p^i_a p^i_{c/a}$

$r_i - S_i$: $p^i_a p^i_{c/a}$, $i = 1, 2, \ldots, n-1$

r_n: 1

Hence returns to illegal activity i in our model depend upon whether the agent is arrested or escapes; and if arrested, whether he is convicted and sentenced or is acquitted. In more detail, returns are r_i if the individual engages in illegal activity i and is either not arrested or is arrested but subsequently acquitted. This state occurs with probability $1 - p^i_a p^i_{c/a}$. If the individual is arrested and convicted for engaging in illegal activity i, returns are $r_i - S_i$ with probability $p^i_a p^i_{c/a}$.[12] The

[11] See Block, Heineke and Nold [1977] for a model in which "mistakes" occur at all levels in the criminal justice system.

[12] There is no particular difficulty in expanding the model to include other contingencies. For example, the state "arrested, convicted and placed on probation" could be added or we could differentiate between the state "not arrested" and the state "arrested and acquitted." The only problem is an empirical one since data are not available on such outcomes.

quantity $r_i - S_i$ may be either positive or negative. Finally, since $p_a^n = 0$, unit prospects from engaging in legal activity are r_n with probability one.

Given the contingencies and probabilities we have outlined, the agent's expected wealth is given by:

$$W + \sum_1^n (r_i - p_a^i p_{c/a}^i S_i) t_i \tag{1}$$

For notational simplicity, we define:

$$\omega_i \equiv r_i - p_a^i p_{c/a}^i S_i \qquad i = 1, 2, \ldots, n \tag{2}$$

and

$$\omega \equiv (\omega_1, \omega_2, \ldots, \omega_n)$$

Since $p_a^n = 0$, $\omega_n = r_n$, the return to legal endeavors. Equation (1). may now be written as:

$$W + \sum_1^n \omega_i t_i . \tag{1'}$$

Following Becker [1965], the formal problem is then:

$$\max_{t,x} U(t, x) - \lambda[\sum_1^m P_h x_h - W - \sum_1^n \omega_i t_i] \tag{3}$$

subject to $\sum_1^n t_i \leq T$, where T is total time available in the period, $\sum_1^m P_h x_h$ are total consumption expenditures, and λ is a Lagrangean multiplier.

First order conditions for a maxima in t and x require:

$$(4) \qquad \partial U/\partial t_i + \lambda \omega_i \leqq 0 \qquad i = 1, 2, \ldots, n$$

$$\partial U/\partial x_j - \lambda P_j \leqq 0 \qquad j = 1, 2, \ldots, m$$

$$\sum_1^m P_h x_h - W - \sum_1^n \omega_i t_i = 0$$

$$\sum_1^n t_i - T \leq 0$$

We assume throughout that the last relation in (4) holds as a strict inequality. The solution to (4) is given by:

$$(5) \qquad t_k \equiv \phi_k(\omega, P, W) \qquad k = 1, 2, \ldots, n$$

$$x_i \equiv \psi_i(\omega, P, W) \qquad i = 1, 2, \ldots, m$$

$$L \equiv T - \sum_1^n \phi_k(\cdot)$$

where L represents the individual's demand for leisure and $\phi_k(\cdot)$ and $\psi_i(\cdot)$ represent the agent's supply function for activity k and demand function for commodity i. Although we are assured of the existence of supply and demand equations (5), it will generally not be possible to solve for these functions explicitly unless $U(\cdot)$ is of a particularly simple form. In other words, if one chooses a functional form for $U(\cdot)$ that places relatively few restrictions on equations (5), it will usually not be possible to solve first order conditions for the implied demand and supply equations.

To surmount this problem one need only calculate the indirect utility function and apply Roy's Identity [1947]. The indirect utility function, say $g(\cdot)$, gives the maximum utility the agent can attain when confronted with expected returns ω, commodity prices P and wealth level W. By definition:

$$(6) \qquad U\big(\phi(\omega, P, W), \psi(\omega, P, W)\big) \equiv g(\omega, P, W)$$

where ϕ and ψ are vectors of supply and demand functions. Activity supply and commodity demand equations (5) may then be written as[13]

$$(7) \qquad t_k = \frac{-W\partial g/\partial\omega_k}{\sum_1^m P_h \partial g/\partial P_h + \sum \omega_j \partial g/\partial \omega_j} \qquad k = 1, 2, \ldots, n$$

$$x_i = \frac{W\partial g/\partial P_i}{\sum_1^m P_h \partial g/\partial P_h + \sum_1^n \omega_j \partial g/\partial \omega_j} \qquad i = 1, 2, \ldots, m$$

which are analogs to Roy's Identity for problem (3).

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[13]See Heineke [1977] and accompanying references for more detail.

These equations are of course activity supply and commodity demand functions for but one individual in the population under study. Since only aggregated data are available for most studies, it will be convenient to sum individual offense and demand equations into aggregate offense and demand equations corresponding to the same aggregates as those on which data are available. Under appropriate assumptions we may integrate over the wealth distribution, which yields

$$(8) \qquad T_k(\omega, P; f) \equiv Q\int_0^\infty \phi_k(\omega, P, W)\, f(W)dW, \qquad k = 1, 2, \ldots, n$$

and

$$(9) \qquad \chi_i(\omega, P; f) \equiv Q\int_0^\infty \psi_i(\omega, P, W)\, f(W)dW, \qquad i = 1, 2, \ldots, m$$

as the aggregate supply of activity k and the aggregate demand for commodity i, respectively.[14] Here ϕ_k and ψ_i are the individual activity supply and commodity demand functions given in (5) and (7), Q is the number of individuals and f(W) represents the wealth distribution. Market wide supply elasticities are then:

$$(10) \qquad \frac{\partial T_k}{\partial \omega_i}\frac{\omega_i}{T_k} \equiv \frac{Q}{T_k}\int_0^\infty \frac{\partial \phi_k}{\partial \ln \omega_i} f(W)dW, \qquad i = 1, 2, \ldots, n$$

$$(11) \qquad \frac{\partial T_k}{\partial P_j}\frac{P_j}{T_k} \equiv \frac{Q}{T_k}\int_0^\infty \frac{\partial \phi_k}{\partial \ln P_j} f(W)dW, \qquad j = 1, 2, \ldots, m$$

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[14] See Heineke [1977] for details.

with analogous expressions for market demand elasticities. Notice that aggregate activity supply functions and aggregate demand functions depend not only on the usual return and price variables but also on the moments of the distribution of wealth. Hence if sample surveys of the population are available so that sample moments of $f(W)$ can be computed, one can estimate equation (8), thereby directly accounting for the effects of an unequal distribution of wealth on the level of criminal activity.

The Translog Model

From an econometric point of view the aggregate supply functions and commodity demand functions (8) and (9), and the implied direct and cross elasticities of supply and demand are only of limited interest until a specific functional form has been assigned to the indirect utility function $g(\cdot)$. The primary concern in choosing $g(\cdot)$ is that the chosen class of functions be capable of approximating the unknown indirect utility function to the desired degree of accuracy.[15] Because a central concern of the present work is to study the extent of substitutability between alternative legal and illegal sources of income, it is important to choose a functional form which does not *a priori* restrict substitution possibilities. Any of the so-called "flexible" functional forms which have appeared in the literature in recent years have this property.[16]

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[15] It is also desirable (less expensive) to choose functional forms that yield supply equations which are linear in the parameters.

[16] In general, "flexible" functional forms are second order approximations to the primal or dual objective functions in optimization problems. These functions include the generalized Cobb-Douglas function, Diewert [1973], the generalized Leontief function, Diewert [1971], the transcendental logarithmic function, Christensen, Jorgensen and Lau [1971, 1973, 1975] and a number of hybrids.

We have approximated the agent's indirect utility function with a transcendental logarithmic function and hence, via equations (8) and (9), (10) and (11), approximate the implied aggregate activity supply functions, commodity demand functions and corresponding elasticities. The translog indirect utility function is defined as:[17]

$$(12) \quad \ln g(\cdot) = \alpha_o + \sum_1^n \alpha_i \ln\omega_i + \sum_1^m \alpha_i' \ln P_i + \alpha_{m+1}' \ln W +$$

$$1/2 \sum_1^n\sum_1^n \beta_{ij} \ln\omega_i \ln\omega_j + 1/2 \sum_1^m\sum_1^m \beta_{ij}' \ln P_i \ln P_j +$$

$$\sum_1^n\sum_1^m \gamma_{ij} \ln\omega_i \ln P_j + \sum_1^n \Pi_i \ln\omega_i \ln W +$$

$$\sum_1^m \Pi_j' \ln P_j \ln W + \mu(\ln W)^2$$

Application of identities (7) to equation (12) yields the following system of individual demand and supply equations:

$$(13) \quad t_k = \frac{-W(\alpha_k + \sum_1^n \beta_{ik} \ln\omega_i + \sum_1^m \gamma_{kj} \ln P_j + \Pi_k \ln W)\omega_k^{-1}}{\alpha + \sum_1^n \beta_i \ln\omega_i + \sum_1^m \beta_j' \ln P_j + \Pi \ln W}, \quad k = 1, 2, \ldots, n$$

$$x_s = \frac{W(\alpha_s' + \sum_1^m \beta_{sj}' \ln P_j + \sum_1^n \gamma_{is} \ln\omega_i + \Pi_s' \ln W)P_s^{-1}}{\alpha + \sum_1^n \beta_i \ln\omega_i + \sum_1^m \beta_j' \ln P_j + \Pi \ln W}, \quad s = 1, 2, \ldots, m$$

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[17] Given that income generation is viewed as "work," a necessary condition for an internal solution to (4) is $\omega > 0$. Of course, our supply and demand functions are not defined for non-positive expected returns.

where we have simplified notation by defining:

$$(14)\qquad \alpha \equiv \sum_1^m \alpha_i' + \sum_1^n \alpha_j$$

$$\Pi \equiv \sum_1^m \Pi_i' + \sum_1^n \Pi_j$$

$$\beta_i \equiv \sum_1^n \beta_{ik} + \sum_1^m \gamma_{is} \qquad i = 1, 2, \ldots, n$$

$$\beta_j' \equiv \sum_1^m \beta_{sj}' + \sum_1^n \gamma_{kj} \qquad j = 1, 2, \ldots, m$$

Equations (13) are the empirical counterpart to equations (7) above. To arrive at the empirical counterpart to the aggregate activity supply and commodity equations given in (8) and (9) one need only substitute equations (13) into (8) and (9) and integrate. Integration of the resulting equations is significantly simplified if the restriction

$$(15)\qquad \Pi = 0$$

is used.[18] Transforming aggregate activity supply functions into per capita value transferred and per capita legal earnings functions we have:

$$(16)\qquad \frac{\omega_k T_k}{Q} = \frac{-\int_0^\infty Wf(W)dW(\alpha_k + \sum_1^n \beta_{ik}\ln\omega_i + \sum_1^m \gamma_{kj}\ln P_j) - \Pi_k \int_0^\infty W\ln W f(W)dW}{\alpha + \sum_1^n \beta_i \ln\omega_i + \sum_1^m \beta_j' \ln P_j}$$

$k = 1, 2, \ldots, n$

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[18]This restriction was suggested by Diewert [1974] in a slightly different context and has been used by Berndt, Darrough and Diewert [1976] to generate market demand functions.

while per capita demand for commodity s is given by:

$$(17) \qquad \frac{P_s X_s}{Q} = \frac{\int_0^\infty Wf(W)dW(\alpha_s' + \sum_1^m \beta_{sj}' \ln P_j + \sum_1^n \gamma_{is} \ln \omega_i) + \Pi_s' \int_0^\infty W \ln W f(W) dW}{\alpha + \sum_1^n \beta_i \ln \omega_i + \sum_1^m \beta_j' \ln P_j}$$

$s = 1, 2, \ldots, m$

If the wealth distribution $f(W)$ can be estimated, it will be possible to estimate the parameters of (16) and (17) by more or less straightforward regression techniques.

Using equations (16) and (17) we may calculate empirical counterparts to the aggregate supply elasticities displayed as (10) and (11) above. These are:

$$(10') \qquad \eta_{ki} \equiv \frac{\partial T_k}{\partial \omega_i} \frac{\omega_i}{T_k} = -\delta_{ik} + \frac{\lambda_1 \beta_{ik}}{\lambda_1(\alpha_k + \Sigma\beta_{sk} \ln \omega_s + \Sigma\gamma_{kj} \ln P_j) + \pi_k \lambda_2}$$

$$- \frac{\beta_i}{\alpha + \Sigma\beta_s \ln \omega_s + \Sigma\beta_j' \ln P_j}, \qquad i, k = 1, 2, \ldots, n.$$

and

$$(11') \qquad \rho_{kj} \equiv \frac{\partial T_k}{\partial P_j} \frac{P_j}{T_k} = \frac{\lambda_1 \gamma_{kj}}{\lambda_1(\alpha_k + \Sigma\beta_{ik} \ln \omega_i + \Sigma\gamma_{ks} \ln P_s) + \pi_k \lambda_2}$$

$$- \frac{\beta_j'}{\alpha + \Sigma\beta_i \ln \omega_i + \Sigma\beta_s' \ln P_s} \qquad j = 1, 2, \ldots, m$$

$$k = 1, 2, \ldots, n$$

where λ_1 and λ_2 are the mean and the higher moment $E(W \ln W)$ of the wealth distribution and δ_{ik} is the Kronecker symbol. Elasticities of supply with respect to mean wealth are given by

$$(18) \qquad \eta_k \equiv \frac{\partial T_k}{\partial \lambda_1} \frac{\lambda_1}{T_k} = 1 + \frac{-\Pi_k \lambda_2}{\lambda_1(\alpha_k + \Sigma \beta_{ik} \ln \omega_i + \Sigma \gamma_{kj} \ln P_j) + \Pi_k \lambda_2}$$

The Econometric Model

In this section we specialize the n + m equation model of per capita earnings and expenditures given above as equations (16) and (17) to the model which can be estimated and provide the stochastic specification needed for estimation.

Information is available on values stolen for the four property crimes of robbery, burglary, larceny and motor vehicle theft. We decided not to include a motor vehicle theft equation in this model for two reasons. First, and foremost, there is the question as to whether values stolen adequately reflect the returns to many auto thieves due to the large portion of all auto thefts which are for "joy-riding." More precisely, available statistics indicate that approximately eighty-five percent of all auto thefts fall into the "joy-riding" category and hence are what Stigler has termed "consumption crimes" rather than "production crimes" which are the subject of this paper. Second, the UCR "value stolen" series are gross returns which have not been adjusted for recoveries. This presents a problem for each of the property crimes studied, but is especially acute for motor vehicle thefts, where dollar values per offense tend to be very large but where a large portion of all stolen vehicles are recovered, causing the value stolen series to seriously overestimate the return to the thief. To the extent that robberies, burglaries and larcenies result in cash transfers, gross and net returns will tend to be similar, since little cash is ever recovered. But since burglaries in particular result in transfers of durables along with cash, the returns to burglary will be overestimated by the value of recovered property. This would not seem to be a serious problem since only about fifteen percent of all burglaries are solved and of those solved, only a small percent of the stolen property is recovered.[19]

For the reasons outlined in the last paragraph we include only the crimes of burglary, robbery and larceny as possible sources of illegal income along with a generic legal activity to represent legitimate earnings. This gives a model with four activity supply equations and m commodity demand equations. In an effort to keep the size of the model within the realm of estimation possibilities, we aggregate all commodity demand equations into one, say x_1, and normalize the returns to each activity and wealth with respect to the price of this aggregate commodity, say P_1. Our model then becomes:

$$(19) \qquad \frac{\omega_k' T_k}{Q} = \frac{-\lambda_1'(\alpha_k + \sum_1^4 \beta_{ik} \ln \omega_i') - \Pi_k \lambda_2'}{\alpha + \sum_1^4 \beta_i \ln \omega_i'}, \qquad k = 1, 2, 3, 4$$

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[19]Another problem here is the fact that estimated market values of stolen merchandise overstate "fence" values.

$$(20) \qquad \frac{X_1}{Q} = \frac{\lambda_1'(\alpha_1' + \sum_1^4 \gamma_{i1}\ln\omega_i') + \Pi_1'\lambda_2'}{\alpha + \sum_1^4 \beta_i \ln\omega_i'}$$

where $\omega_i' \equiv \omega_i/P_1$, $\lambda_1' \equiv E(W/P_1)$ and $\lambda_2' \equiv E[(W/P_1)\ln(W/P_1)]$. Since equations (19) and (20) are homogeneous of degree zero in the parameters, a normalization of parameters will be necessary to permit estimation. It is convenient to set

$$(21) \qquad \alpha = -1$$

for this purpose.

The next step in implementing the econometric version of the model is to provide a stochastic framework for the earning and expenditure equations, (19) and (20). We do this by appending classical, additive disturbance terms to each of the five equations. These disturbances arise either as a result of random errors in the maximizing behavior of individual agents or as a result of the fact that the translog indirect utility function only approximates underlying preferences. We assume that noncontemporaneous disturbances are uncorrelated both within and across equations and that right hand side variables in equations (19) and (20) are uncorrelated with the disturbances in each equation. The latter assumption assures identification of the earnings and expenditure functions.

The appropriateness of our assumption of zero correlation between right-hand variables and disturbances hinges primarily upon whether probabilities of arrest which enter these calculations are exogenous. The usual argument to the contrary has been in terms of the "capacity" of police departments. Briefly, the argument goes that as the number of offenses increases police resources are stretched increasingly thin and arrests per total offenses fall, thereby yielding the ubiquitous negative partial correlation between offense rates and probabilities of capture -- but for the wrong reason. This argument requires that offense levels explicitly enter police agency production functions to account for agency capacity constraints. This hypothesis is tested and rejected in Darrough and Heineke [1977] utilizing results reported by Phillips and Votey [1975] and results reported by Ehrlich [1973]. Additional evidence supporting the exogeneity of expected returns is provided by Wilson and Boland [1977], who find that police "capacity" is not related to arrest rates for burglary, larceny, robbery and motor vehicle theft.

Parameter Restrictions

Notice that it will be necessary to estimate only four of the five equations in (19) and (20) since the budget constraint implies that the parameters of the remaining equation can be determined from definitions (14) above. We have chosen to estimate the four per capita earnings equations, in which case the parameters of the expenditure function may be obtained from:

$$(22)\qquad \alpha_1' = -1 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4$$

$$\beta_1' = \beta_{11}' + \gamma_{11} + \gamma_{21} + \gamma_{31} + \gamma_{41}$$

$$\beta_i = \beta_{i1} + \beta_{i2} + \beta_{i3} + \beta_{i4} + \gamma_{i1}, \qquad i = 1, 2, 3, 4$$

$$\Pi_1' = -\Pi_1 - \Pi_2 - \Pi_3 - \Pi_4$$

Earnings equations (19) comprise a complete econometric model of the time and income allocation problem confronting the individual.

Our maintained hypothesis of utility maximizing behavior imposes "equality," "homogeneity," and "symmetry" restrictions on the parameters of the system given by equations (19) and (20), and reduces the number of parameters to be estimated from sixty to twenty-two.[20] From an econometric point of view this dramatic reduction in the number of parameters to be

[20] These restrictions are given in Heineke [1977]. See Christensen, Jorgensen and Lau [1975] for further discussion.

estimated provides a powerful incentive for building econometric models which are consistent with utility maximization.

Estimating Policy Implications

One of the purposes of this study was to suggest methods of measuring the extent of any "system-wide" deterrent effects which may be associated with changes in sanctions and/or enforcement levels for a single crime. Such a measure must account for changes in offense rates in related illegal activities which are induced by a policy change in the activity in question. In what follows we suggest measuring the "system-wide" effects of policy changes with the response of the total value transferred in property crimes within the system, to the policy change of interest. By definition the total value transferred in the activities of burglary, robbery and larceny is given by:

$$(23) \qquad V = \sum_{1}^{3} r_j T_j$$

If θ_i represents any parameter associated with the expected return to illegal activity i then the elasticity of V with respect to policy parameter θ_i is given by

$$(24) \qquad \frac{\partial V}{\partial \theta_i}\frac{\theta_i}{V} = \left(\frac{\partial \omega_i}{\partial \theta_i}\frac{\theta_i}{\omega_i}\sum_{j=1}^{3} r_j T_j \eta_{ji} + r_i T_i \frac{\partial r_i}{\partial \theta_i}\frac{\theta_i}{r_i}\right)/V, \quad i = 1, 2, 3$$

Here we use θ_i to represent the "gross" return, r_i, the probability of arrest and conviction, p^i_{ac}, and the mean sentence, μ_i, associated with crime i.[21] Using Y to denote net income foregone per year of imprisonment and $\delta \equiv (1 + d)^{-1}$, where d is the annual discount rate, the elasticities of expected unit returns

[21] Since in our model elasticities of expected returns with respect to p_a, $p_{c/a}$ and p_{ac} are equal, we use p_{ac} as the probability measure in the remainder of the paper.

$(\partial\omega_i/\partial\theta_i)(\theta_i/\omega_i)$, with respect to each of these parameters are

$$\frac{\partial\omega_i}{\partial p^i_{ac}}\frac{p^i_{ac}}{\omega_i} = \left(1 - \frac{r_i}{\omega_i}\right) \quad 22$$

$$(25) \qquad \frac{\partial\omega_i}{\partial\mu_i}\frac{\mu_i}{\omega_i} = \left(1 - \frac{r_i}{\omega_i}\right)\left(\mu_i\delta^{\mu_i}\ln\delta/(\delta^{\mu_i} - 1)\right)$$

$$\frac{\partial\omega_i}{\partial r_i}\frac{r_i}{\omega_i} = \frac{r_i}{\omega_i}, \qquad i = 1, 2, 3$$

Due to the fact that the income-expenditure constraint in our model is in terms of expected values, expected returns must always be positive and elasticities of expected returns with respect to probabilities of arrest and conviction will always be less than the same measurements with respect to gross returns, r_i. The magnitude of this difference is, however, an empirical proposition of some interest.

In addition to system-wide value elasticities, it may be of interest to calculate simple market elasticities. This is the case, particularly for direct elasticities, because other studies present their results in terms of these elasticities. Because $(\partial T_k/\partial\theta_i)(\theta_i/T_k) = \eta_{ki}(\partial\omega_i/\partial\theta_i)(\theta_i/\omega_i)$, it is a simple matter to calculate market supply elasticities with respect to probabilities, mean sentence lengths and gross returns.

Using equations (25) we have

$$\frac{\partial T_k}{\partial p^i_{ac}}\frac{p^i_{ac}}{T_k} = \eta_{ki}\left(1 - \frac{r_i}{\omega_i}\right), \qquad \begin{array}{l} i = 1, 2, 3 \\ k = 1, 2, 3, 4 \end{array}$$

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[22] In terms of earlier definitions $Y\int_0^{\mu_i}\delta^x dx \equiv S_i$, the monetary equivalent of the mean sentence, if imprisoned for μ_i years.

$$(26) \quad \frac{\partial T_k}{\partial \mu_i}\frac{\mu_i}{T_k} = \eta_{ki}(1 - \frac{r_i}{\omega_i}) \left(\mu_i \delta^{\mu_i} \ln\delta/(\delta^{\mu_i} - 1)\right), \quad i = 1, 2, 3 \quad k = 1, 2, 3, 4$$

$$\frac{\partial T_k}{\partial r_i}\frac{r_i}{T_k} = \eta_{ki}\frac{r_i}{\omega_i}, \quad i = 1, 2, 3, 4 \quad k = 1, 2, 3, 4$$ [23]

Equations (26) indicate that the smaller is the expected loss from participating in a criminal activity, the smaller will be the supply response from increased probabilities of capture and conviction and from increases in the mean prison sentence. Therefore policies designed to affect the supply of illegal acts by altering a component of the expected return, will have the smallest impact when expected losses are small. It follows that jurisdictions with relatively low enforcement and sanction levels not only experience higher crime rates *ceteris paribus* than do similar jurisdictions with relatively high enforcement and sanction levels, but these jurisdictions also find that policy changes which are undertaken to lower crime rates have a smaller impact than the same policy change would have if enacted elsewhere.

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[23] Since $r_4 = \omega_4$, for legal endeavors $(\partial T_k/\partial r_4)(r_4/T_k) = \eta_{k4}$, $k = 1, 2, 3, 4$.

A slightly different way of assessing the overall deterrent effects of selected policy changes would be to calculate the change in the total amount stolen in all property crimes due to one more arrest or one more conviction for crime i. (These computations would of course be especially useful if one had estimates of the marginal costs of arrests and convictions by type of crime.)[24] Our results use the fact that $\partial V/\partial a_i = (\partial V/\partial p^i_a)(\partial p^i_a/\partial a_i) = (\partial V/\partial p^i_a)(1/T_i)$[25] and $\partial V/\partial c_i = (\partial V/\partial p^i_{c/a})(\partial p^i_{c/a}/\partial c_i) = (\partial V/\partial p^i_{c/a})(1/a_i)$, where a_i and c_i are the number of arrests and convictions for property crime i, respectively. In which case

$$(27)\qquad \begin{aligned} \frac{\partial V}{\partial a_i} &= \frac{\partial \omega i}{\partial p^i_a}\frac{1}{\omega_i}\Big(\sum_j r_j T_j \eta_{ji}\Big)/T_i \\ \frac{\partial V}{\partial c_i} &= \frac{\partial \omega_i}{\partial p^i_{c/a}}\frac{1}{\omega_i}\Big(\sum_j r_j T_j \eta_{ji}\Big)/a_i \end{aligned} \qquad , i = 1, 2, 3$$

are the responses of V to an additional arrest and to an additional conviction.

Another question of interest concerns the response of total value stolen to changes in the distribution of wealth. We address two hypothetical situations: First, we calculate the response of value transferred

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[24] See Darrough and Heineke [1977] for estimates of marginal cost of "solution" functions, by type of crime.

[25] This calculation assumes that the probability of conviction given arrest is not affected by an additional arrest.

to a change in the mean of the wealth distribution, λ_1, income and gross returns, r_i, held constant. Second, we calculate the response of total value stolen to an equal percentage change in income, gross returns and wealth. The result might be interpreted as the response of property crime earnings to a secular increase in returns, income and wealth -- given a "passive" enforcement and sanctions policy which leaves enforcement levels and sanctions unchanged.

The response of total value stolen to equi-proportional changes in mean wealth, returns and income may be written as

$$\text{(28)} \qquad \frac{\partial V}{\partial \xi}\frac{\xi}{V} = \left[\sum_{j=1}^{3} r_j T_j \left(\sum_{i=1}^{4} \eta_{ji} + \eta_j + 1\right) d\xi/\xi\right] / V$$

where $d\xi/\xi$ represents an equal percentage change in mean income, wealth and returns. In the first case where mean wealth alone changes, equation (28) simplifies to

$$\text{(29)} \qquad \frac{V}{\partial \lambda_1}\frac{\lambda_1}{V} = \left[\sum_{k=1}^{3} r_k T_k \eta_k\right] / V.$$

Summary

In this paper we have derived an econometric model of legal and illegal labor supply. The closest possible degree of correspondence between the underlying economic model and the resulting econometric model was achieved by exploiting several results from modern duality theory.

The resulting econometric model is quite powerful and can be used to estimate: 1) the degree of substitutability or complementarity that exists between the income-generating activities of burglary, robbery, larceny and legitimate employment and 2) the "net" or system-wide response of participation rates in these several income-generating activities as expected returns and costs vary. Also explicitly derived were simple empirical measures that might be used to assess the system-wide effects of changes in such major criminal justice variables as arrest rates, conviction rates and sentencing practices.

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