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An Identification Algorithm for Dynamic Intervention Modeling with Application to Gun Control

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Abstract

An identification algorithm for dynamic intervention models is developed. Using this algorithm the form of the dynamic component from which policy inference is drawn is determined from data structure and is free of a modelers a priori biases. A re-analysis of the gun related crimes for a pre and post history surrounding the implementation of the Massachusetts Gun Control Law is conducted using this algorithm and a dynamic modeling approach.

Introduction

In a previous paper (Deutsch and Alt (1977a)), a preliminary evaluation of the Massachusett's gun control law was conducted. Here, intervention analysis was conducted by formulation of a method of shift detection (Box and Tiao (1965)), in a traditional quality control framework in which the process was measured by the reported number of gun related crimes. The objective here was to identify the in-control from out-of-control state of the process as determined by new system observations with respect to historical values. Further associated with the determination of an out-of-control state was the assessment of the transition point or time frame in which the process went from a state of in-control to out-of-control. It should be noted that this approach identified from data alone, the transition point for which an assignable cause for the transition was sought.

In our previous analysis of the three gun related crime series, we detected that both the armed robbery and assault with a gun series did go out-of-control at an identical time point. The assignable cause that we suggested as an explanation of this transition was the activities of planned publicity that occurred immediately prior to and continued after the transition point. We concluded the paper with a suggestion that further description of the detected transition, in particular with regard to its dynamics should prove useful. That is, given that a transition or shift did occur in the reported levels of gun related crimes, was this a shift to a new more desirable steady state level or simply a transient phenomena in which the system returned to its original level of behavior. This question required models of dynamic intervention Box and Tiao (1975).

Since the data series are ordered by time in dynamic intervention modeling, replication is impossible. Thus, lack-of-fit tests to check adequacy of the fitted forms do not exist. For given data sets several different forms of dynamic models can be adequately fit in a statistical sense. However, each form may typically result in different policy implications for the analyst. To eliminate biases of a modeler's postulation of a specified form of dynamic component, an identification procedure was developed, which uses only information or structure contained in the data alone for the model form specification. This identification procedure was used to re-analyze the Boston Gun related crimes.

Evaluation of Change by Dynamic Models

Box and Tiao (1975) have suggested a class of general dynamic models capable of representing intervention effects. Since interventions are likely to have some effect on a system other than to change the level, a model representation which takes into account dynamic or delayed effects is desirable in order to describe the form of the intervention accurately. As suggested previously, the evaluation of change via shifts is identical to a subset of the dynamic models.

An overview of the dynamic model class and their descriptive capabilities is contained in the following sections, along with the specific relationships between the model specification of the shift approach and the dynamic models. Using these relationships, an identification algorithm for dynamic intervention modeling is described. Lastly the dynamic intervention analysis is conducted for the Boston Gun Control data.

Overview of Dynamic Intervention Models

Linear dynamic systems may be represented by relating an output Y to an input X. If the input is varied and x_t and y_t represent deviations at time t from equilibrium, the system can be represented by a linear filter of the form,

$$y_t = \omega_0 x_t + \omega_1 x_{t-1} + \omega_2 x_{t-2} + \ldots$$

$$= \omega(B) x_t,$$

where,

$$\omega(B) = \omega_0 + \omega_1 B + \omega_2 B^2 + \ldots$$

The operator $\omega(B)$ is called the transfer function. If we wish to allow not only for a simple change in level of the output y_t, but also the rate of change, to affect the output, we may represent the model by an equation of the form,

$$\delta(B) y_t = \omega(B) x_{t-b} \quad ,$$

where

$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \ldots \quad .$$

In this case the transfer function is a ratio of two polynomials in B, $\delta^{-1}(B)\omega(B)$, with $\delta(B)$ determining the rate of change.

In a physical system the input variable x_t is typically different than the output variable y_t and is uniquely identifiable. In social systems, the variable is not necessarily identifiable nor is it necessarily different in label than the output specification. For example in intervention analysis the x_t might represent the presence of a number of specific activities that are collectively acting to affect change. In fact these activities might be known or unknown. That is, a given program being implemented will typically have specific activities each contributing to a change in the operating environment which in turn will affect the output y_t. However, these known activities might occur simultaneously with other documented activities outside of the formal program but related to the programs existence. This is the posture that we adopt in intervention modeling. Thus we will in general denote the x_t, to be ξ_t which represents variations in the environment, either known or unknown, that may affect the single output measure, y_t of the environment. Beyond the changes in the output measure that are associated with environmental changes, we allow for random variations in the form of an additive noise structure, N_t. Thus

$$\delta(B)y_t = \omega(B)\xi_t + N_t,$$

where N_t may be modeled by any of the multiplicative autoregressive moving-average processes (see Box and Jenkins 1970).

Although the ξ_t could conceivably take on any form, we will consider only the case where they are indicator variables taking on the values zero or one. The presence of an intervention effect or essentially a change in the environment will thus be indicated by $\xi_t = 1$, otherwise $\xi_t = 0$ when there are no modifications.

Using the transfer function framework, we can model various types of intervention effects. For example, if we let the variable ξ_t denote a step change in the input,

$$\xi_t = S_t^{(T)} = \begin{cases} 0, & t < T \\ 1, & t \geq T, \end{cases}$$

where T is the time of intervention, we can model the output as a response to this step change. Three typical responses might be those pictured in Figure 1 parts (a), (b), and (c). On the other hand, it may be more convenient to think of the input as a pulse occurring once at time T, rather than as a step input as in the previous case. We can then let

$$\xi_t = P_t^{(T)} = \begin{cases} 0, & t \neq T \\ 1, & t = T, \end{cases}$$

and we can model output such as that shown in Figure 1 parts (d), (e), and (f).

In Figure 1(a) the output is represented by

$$y_t = \omega B S_t^{(T)}.$$

Relating this to the general form of a transfer function model, we can see that for this model, which we will refer to as the Constant Step Response Model, where only ω_1 is non-zero ($\omega = \omega_1$). In this model, ω measures the constant magnitude of the output response.

In Figure 1(b), we note that the output is a first-order response to the step input. We must thus incorporate a rate of change parameter

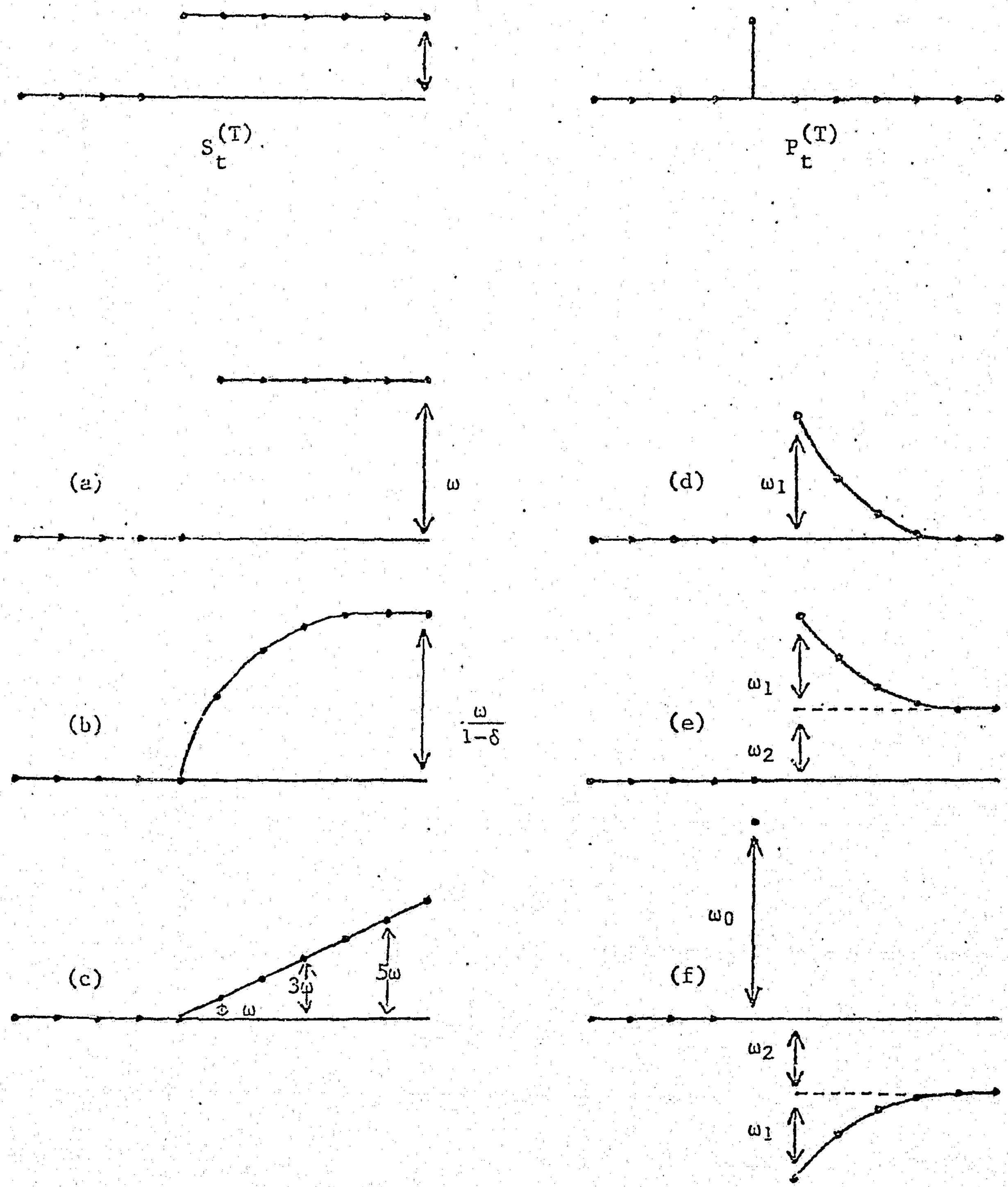

Figure 1. Six Basic Transfer Function Models

into this First-Order Step Response Model. This output is therefore represented by,

$$y_t = \frac{\omega B}{1- B} S_t^{(T)} \quad .$$

As in the Constant Step Response Model, only one ω is non-zero. However, in the present model all δ's are equal to zero except for δ_1. Here, $\omega/(1-\delta)$ represents the steady state gain or change in the output.

The transfer function output represented in Figure 1(c) is,

$$y_t = \frac{\omega B}{1-B} \quad S_t^{(T)} \quad .$$

We can easily see the resemblance between this Ramp Response Model and the First-Order Response Model. In the case of the Ramp Response Model, $\delta_1 = 1$, which indicates that the rate of change is constant over time. Thus, there is a constant response to the input for the output for each time interval (T, T+1) and ω measures this response.

In Figure 1(d) and (e), we note that we have a Decaying Pulse Response Model and a Partially Decaying Response Model, respectively. In each case, the term,

$$\frac{\omega_1 B}{1-\delta B} \quad ,$$

represents the decaying portion of the model. The parameter ω_1 measures the initial response to the pulse input and δ measures the rate of decay of these first-order models. For the Partially Decaying Response Model,

$$y_t = \left\{ \frac{\omega_1 B}{1- B} + \frac{\omega_2 B}{1-B} \right\} P_t^{(T)}, \quad \text{where}$$

ω_2 measures the residual response after the initial effect. We can also relate these models to the general transfer function model form. For the Decaying Pulse Response Model, only ω_1 is non-zero and as in the case of the First-Order Step Response Model all subsequent δ's are zero except for δ_1. For the Partially Decaying Pulse Response Model, the first term with ω_1 is as above and the second term has δ_1 non-zero being equal to one, and all other δ's zero.

In Figure 1(f), we see the most complex of the six transfer function models depicted, the Inverse Decaying Pulse Response Model. The model is represented by:

$$y_t = \omega_0 + \frac{\omega_1 B}{1-\delta B} + \frac{\omega_2 B}{1-B} \quad P_t^{(T)} \quad .$$

In this case, ω_0 represents the immediate impact of the pulse input. The parameter ω_1 represents the delayed response which recedes gradually according to the rate of change parameter, δ. Beyond these components, is the permanent effect, measured by the quantity ω_2. The second and third terms in this model are exactly the same as the two terms in the Partially Decaying Pulse Response Model. The first term in the transfer function implies that only ω_0 is non-zero and all δ's are zero. A summary of the interrelationships of these six types of dynamic intervention models are contained in Figure 2.

<u>Relationship Between Evaluation by Shifts and Dynamic Models</u>

Whereas it has been noted that the underlying model of change in shift detection represents a subset of the models in the Dynamic Model class, we proceed now to describe the relationship between this shift parameter and all the dynamic models. These relationships are developed

Key: A = Constant Step Response Model
B = First-Order Step Response Model
C = Ramp Response Model
D = Decaying Pulse Response Model
E = Partially Decaying Pulse Response Model
F = Inverse Decaying Pulse Response Model

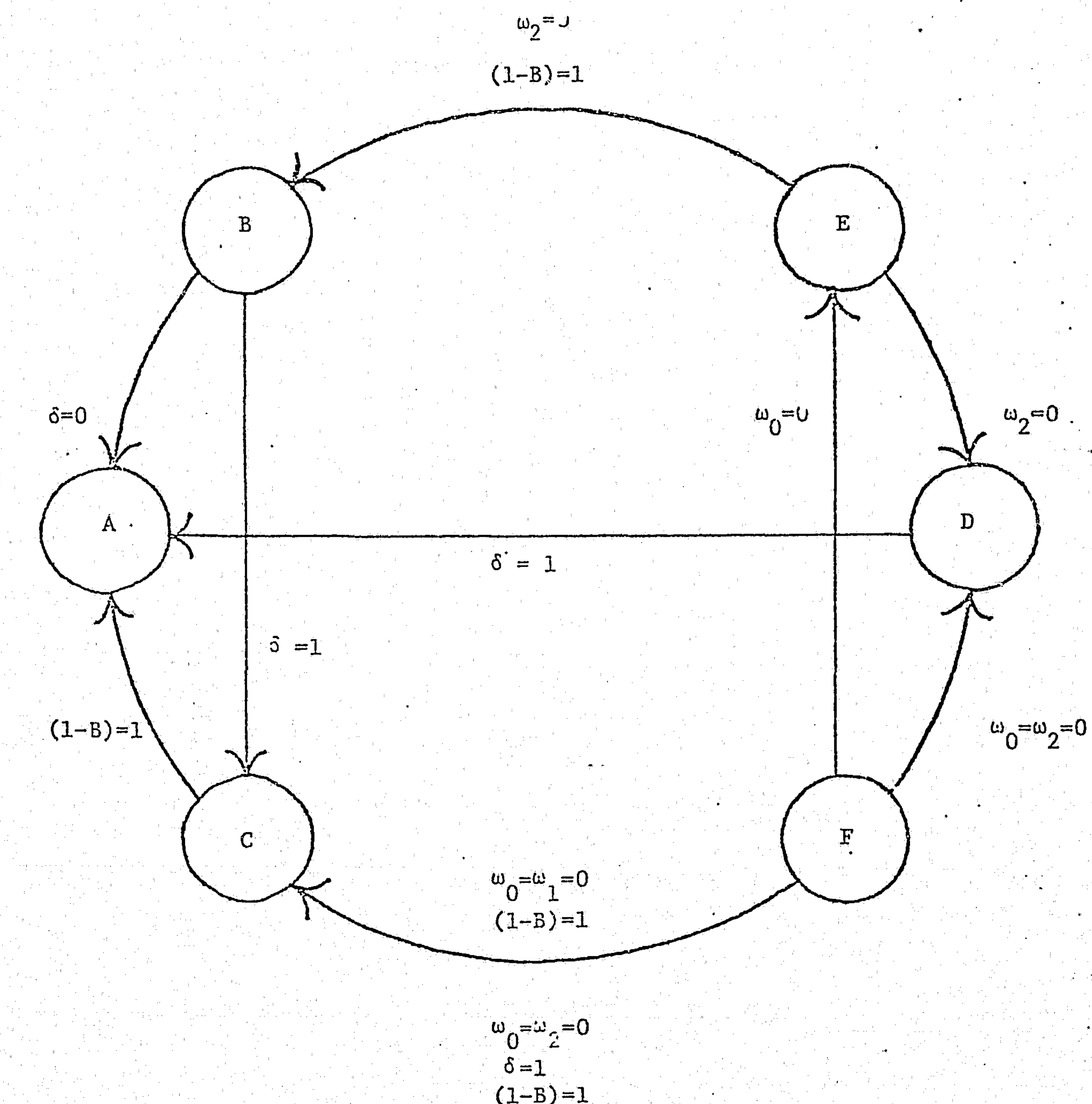

Figure 2. Relationships Among Six Basic Transfer Function Models

by considering the shift parameter to be sequentially estimated, that is, a function of time, δ_t.

For example, consider the first order step response model, in which the constant shift ω is equal to zero for all time points less than T+1 and equal to a constant for all time points $\geq$ T+1. Direct comparison with the shift model shows the shift parameter to be zero prior to T+1 and equal to a constant for all times $\geq$ T+1. This is seen from direct comparison of the model forms. For the shift parameter model the level at time t is;

$$Z_t = \beta_t \qquad t = 1,2,\ldots,T$$
$$Z_t = \beta_t + \delta \qquad t = T+1,T+2,\ldots$$

where β_t is the time series representation of the $t^{\underline{th}}$ observation for any underlying process of the multiplicative moving average class. Thus if the underlying model was a first order moving average process, $\beta_t = (1-\theta B)a_t$. Similarly the constant step response model is

$$Z_t = \omega B S_t^{(T)} + \beta_t \quad ,$$

$$S_t = \begin{cases} 0 \text{ for } t = 1,2,\ldots,T \\ 1 \text{ otherwise.} \end{cases}$$

From direct comparison of these models we see for all values of t less than T+1 that ω is zero. For time T+1 when S_t is one, δ_{T+1} is equal to ω. Table 1 summarizes the relationships between the shift parameter and the parameters of the dynamic models for the six basic models illustrated. Here the symbol delta is used to denote the shift parameter and the delta primes and omegas are used to denote the parameters of the dynamic models. From the table we see that a sequential plot of the magnitudes of the

Table 1. Relationship of the Shift Parameter to the Dynamic Model Parameters

		$t<T$	$t=T$	$t=T+1$	$t=T+2$	$t=T+3$	$t=T+4$	$t=T+5$	$t=T+6$	...	$t=T+k$
Constant Step Response Model	δ=	$\omega=0$	$\omega=0$	ω	ω	ω	ω	ω	ω	...	ω
First-Order Step Response Model	δ=	$\omega=0$	$\omega=0$	ω	$(1+\delta'\omega)$	$(1+\delta'+\delta'^{2})\omega$	...	...	...	...	$(1+\delta'+\ldots+\delta'^{k-1})\omega$
Ramp Response Model	δ=	$\omega=0$	$\omega=0$	ω	2ω	3ω	4ω	5ω	6ω	...	$k\omega$
Decaying Pulse Response Model	δ=	$\omega=0$	$\omega=0$	ω	$\delta'\omega$	$\delta'^{2}\omega$	$\delta'^{3}\omega$	$\delta'^{4}\omega$	$\delta'^{5}\omega$	...	$\delta'^{(k-1)}\omega$
Partially Decaying Pulse Response Model	δ=	$\omega=0$	$\omega_1+\omega_2$	$\delta'\omega_1+\omega_2$	$\delta'^{2}\omega_1+\omega_2$	...	...	...	...	...	$\delta'^{(k-1)}\omega_1+\omega_2$
Inverse Decaying Pulse Response Model	δ=	$\omega=0$	ω_0	$\omega_1+\omega_2$	$\delta'\omega_1+\omega_2$	$\delta'^{2}\omega_1+\omega_2$	...	...	...	...	$\delta'^{(k-1)}\omega_1+\omega_2$

shift parameter versus time gives rise to the shapes of the dynamic model configuration in Figure 1.

It should be noted however that the estimation procedure for delta described in [1] does not yield unbiased estimates of the specific dynamic model parameters shown in the table. Rather the estimates of delta obtained at time t are coupled or confounded with information associated with earlier estimates made prior to time t.

General Methodology for the Identification Algorithm

We have just seen that the shift parameter has the ability to express dynamics within the time series framework. However, we can achieve this dynamic property of δ by estimating it sequentially, if and only if we incorporate previous estimates of the parameter into the estimation procedure. This means that we must think of delta not strictly as a parameter, but as a time-dependent variable in the model, denoted by δ_t. In the following paragraphs, an algorithm for identifying forms of intervention effects utilizing successive estimates of δ_t over time is suggested.

To estimate delta at any point in time, we must first specify that time, which will be denoted by T. This is equivalent to specifying n_1, the number of observations before T and n_2, the number of observations after time T, where $n_1 + n_2 = N$ and N is the total number of observations used in the estimator. The quantities n_1 and n_2 enter directly into the estimator as do the observations associated with n_1 and n_2. Thus, by specifying n_1 and n_2, we effectively specify which observations will enter into the estimator as historical (before T) data and those which will be used as "future" (after T) data. Of course, the estimate, $\hat{\delta}_t$, is also functionally dependent on the form of the ARIMA model noise term and the associated moving-average and/or autoregressive parameter values (Deutsch and Alt, 1976). For our purposes, we will denote the ARIMA model form as M and the parameters from this model collectively as Λ. Thus, we can write,

$$\hat{\delta}_t = f(n_1, n_2, Y, M, \Lambda).$$

In sequential estimation, we must monitor for significant shifts if the sequential plot of $\hat{\delta}$ is to mimic the dynamics of intervention. To translate what it means to use past significant values of delta in the actual estimation procedure without correction, we refer to the functional description of $\hat{\delta}$ given previously. We know that if we estimate δ at time T, $\hat{\delta}_T$, with $n_2 = 1$, we are using the n_1 observations before time T and an additional observation, y_{n_1+1}. If we let Y_{n_1} denote the observations occurring before time T, we can write:

$$\hat{\delta}_T = f(n_1, n_2, Y_{n_1}, y_{n_1+1}, M, \Lambda).$$

Suppose we estimate δ_T and find that it is significant, i.e., not statistically equal to zero. Then, in effect we are saying that there has been an intervention effect at time T which is evidenced by the change from the previous n_1 observations to the observation Y_{n_1+1}. Thus, if we move ahead and stand at time T+1, we again let n_2=1 and this time $n_1^* = n_1+1$, where n_1 was the previous value of n_1 used to estimate δ_t at time T. Now, an additional observation y_{n_1+2} or $y_{n_1^*+1}$ will be used, and y_{n_1+1} will be grouped with the n_1 observations (Y_{n_1}) to form the set of n_1^* observations. However, since we have already concluded that an intervention effect of significant magnitude occurred at time T, the set of n_1^* observations which we are comparing the $y_{n_1^*}$ observation to is not internally consistent. That is, the n_1^* set of observations does not represent a single population. To form an historically consistent population we subtract the previous significant value of δ, $\hat{\delta}_T$, from the y_{n_1+1} observation to account for the difference in level with the previous observations. We can then proceed to estimate δ_{T+1}. At this point in time, we can write,

$$\hat{\delta}_{T+1} = f(n_1, N_2, Y_{n_1^*}, y_{n_1+2}, M, \Lambda)$$

where $Y_{n_1^*}$ includes the original n_1 observations and the corrected $n_1+1^{\underline{st}}$ observation. Of course, if $\hat{\delta}$ was not statistically significant, then no adjustment is needed since δ_T is effectively zero and there is already consistency between the n_1 observation and the $n_1+1^{\underline{st}}$ observation. For subsequent estimates of δ_t, we use the same procedure as we move ahead to time T+2, T+3, T+4, etc. The estimation procedure is depicted in flow chart form in Figure 3.

With this identification algorithm, the sample pattern produced will not match the theoretical patterns associated with the dynamic intervention model exactly but rather will deviate from it. The distortion from theoretical patterns are due to the stochastic additive noise component, N_t, in particular the form of the noise component and the specific associated parameter values. A computer simulation experiment was run to assess the nature and magnitude of the descrepancies associated with the stochastic components effect in the dynamic intervention model identification. Figures 4 through 9 graphically illustrate these variations for each of the six dynamic intervention models in the presence of additive noise whose form is first order autoregressive, first order moving average or first order integrated moving average.

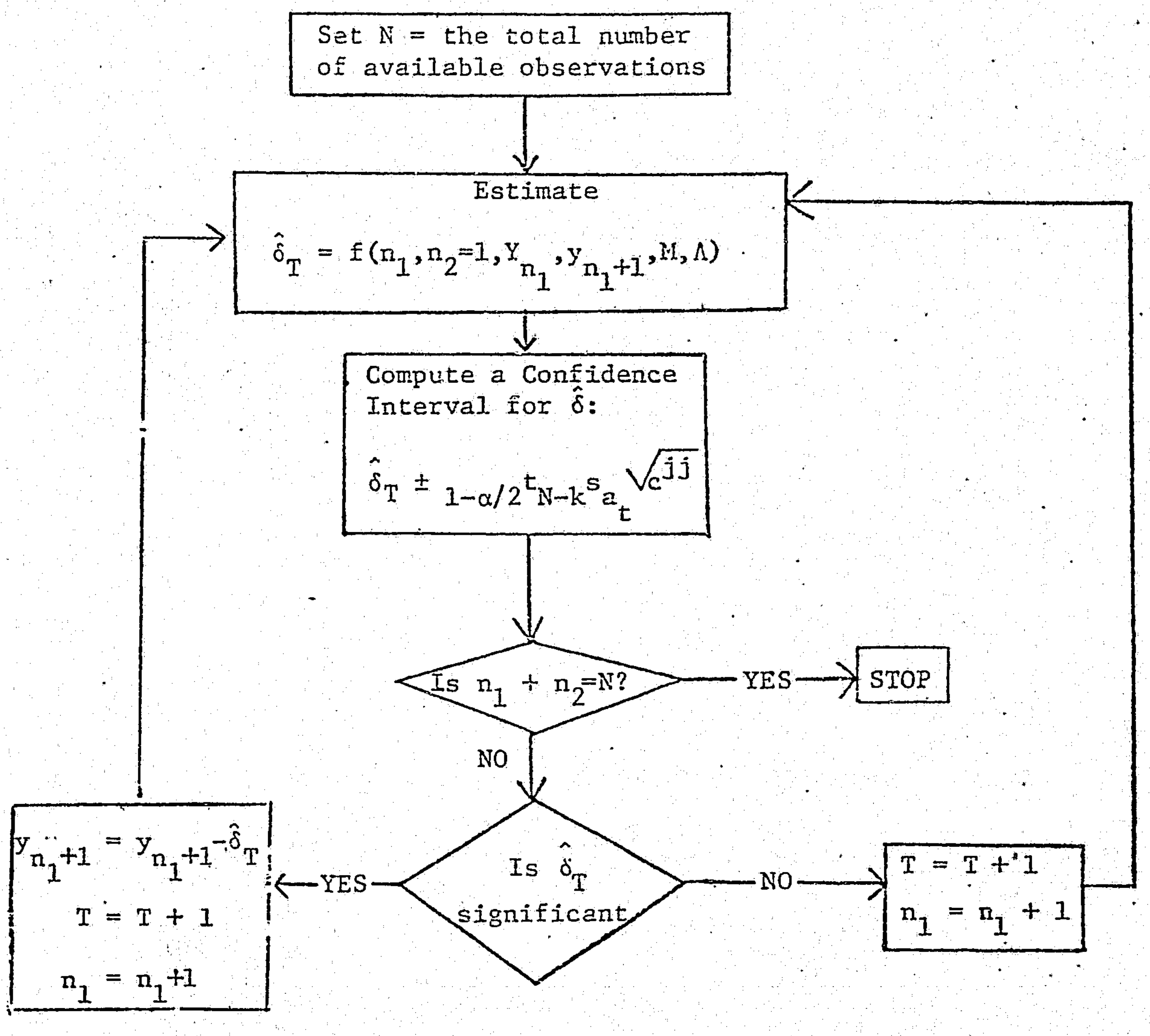

Figure 3. Flowchart for Identification Algorithm

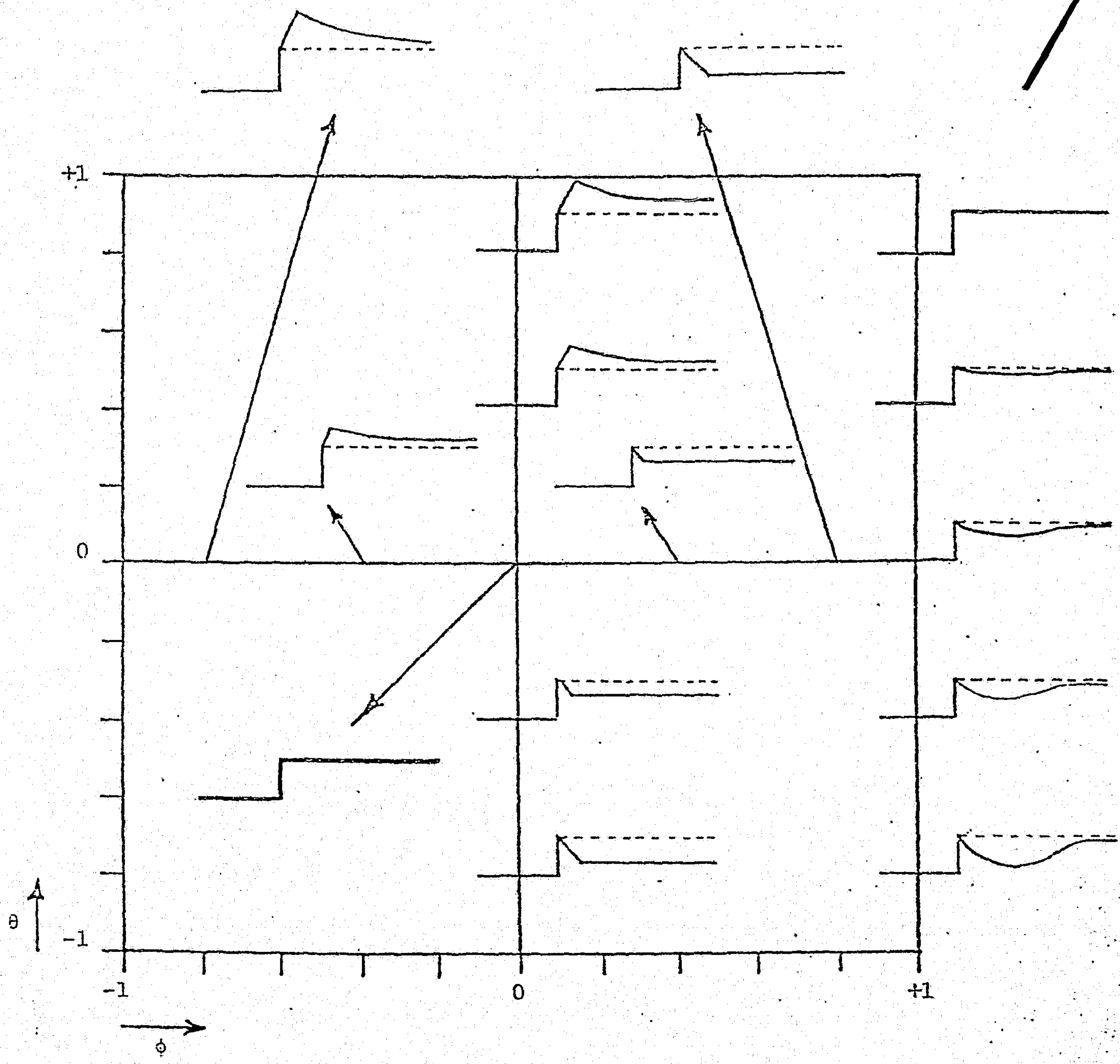

Figure 4. Effect of Variables M and Λ (ϕ and θ) on the Constant Step Response Model

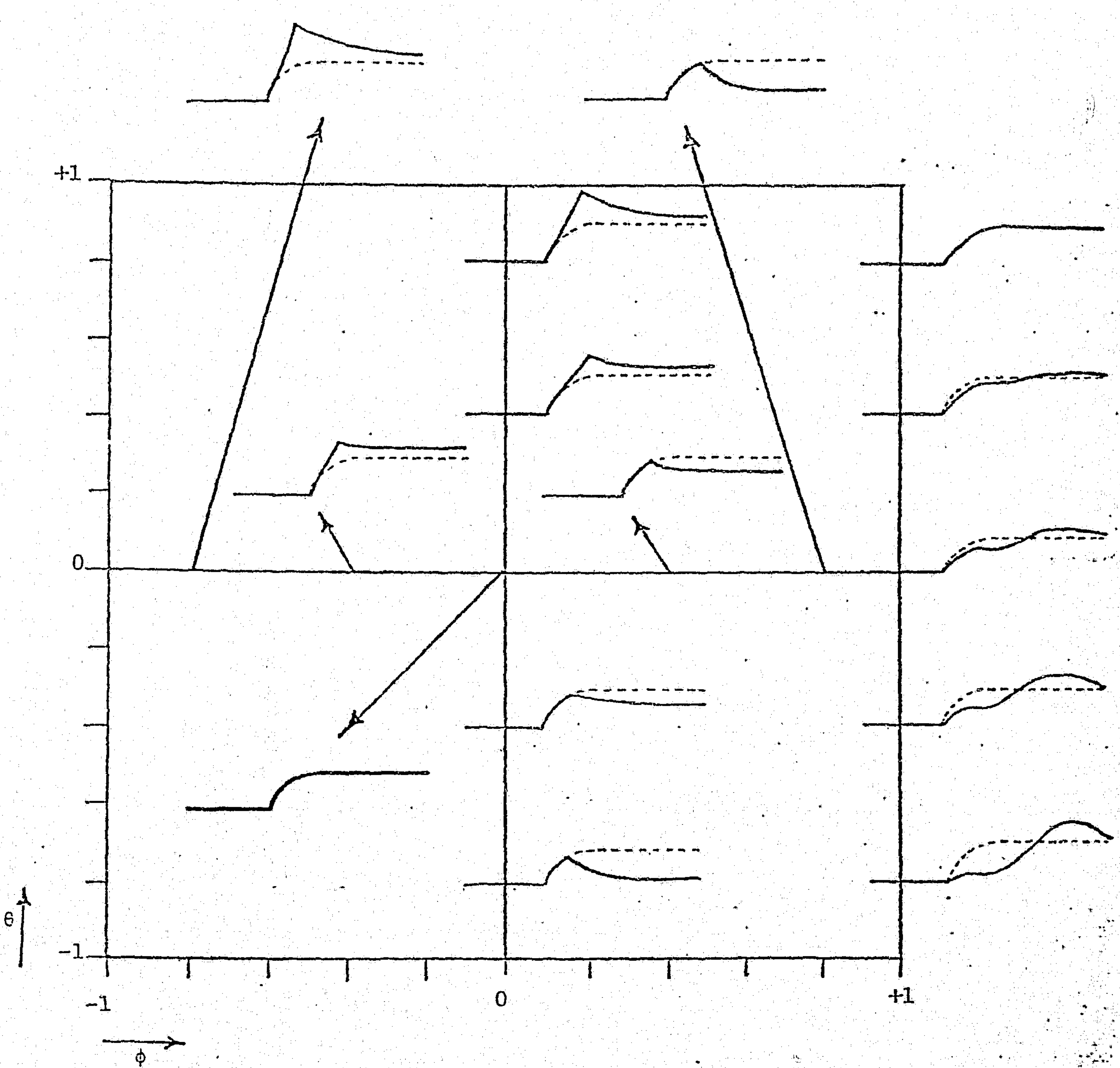

Figure 5. Effect of Variables M and Λ (ϕ and θ) on the First-Order Step Response Model

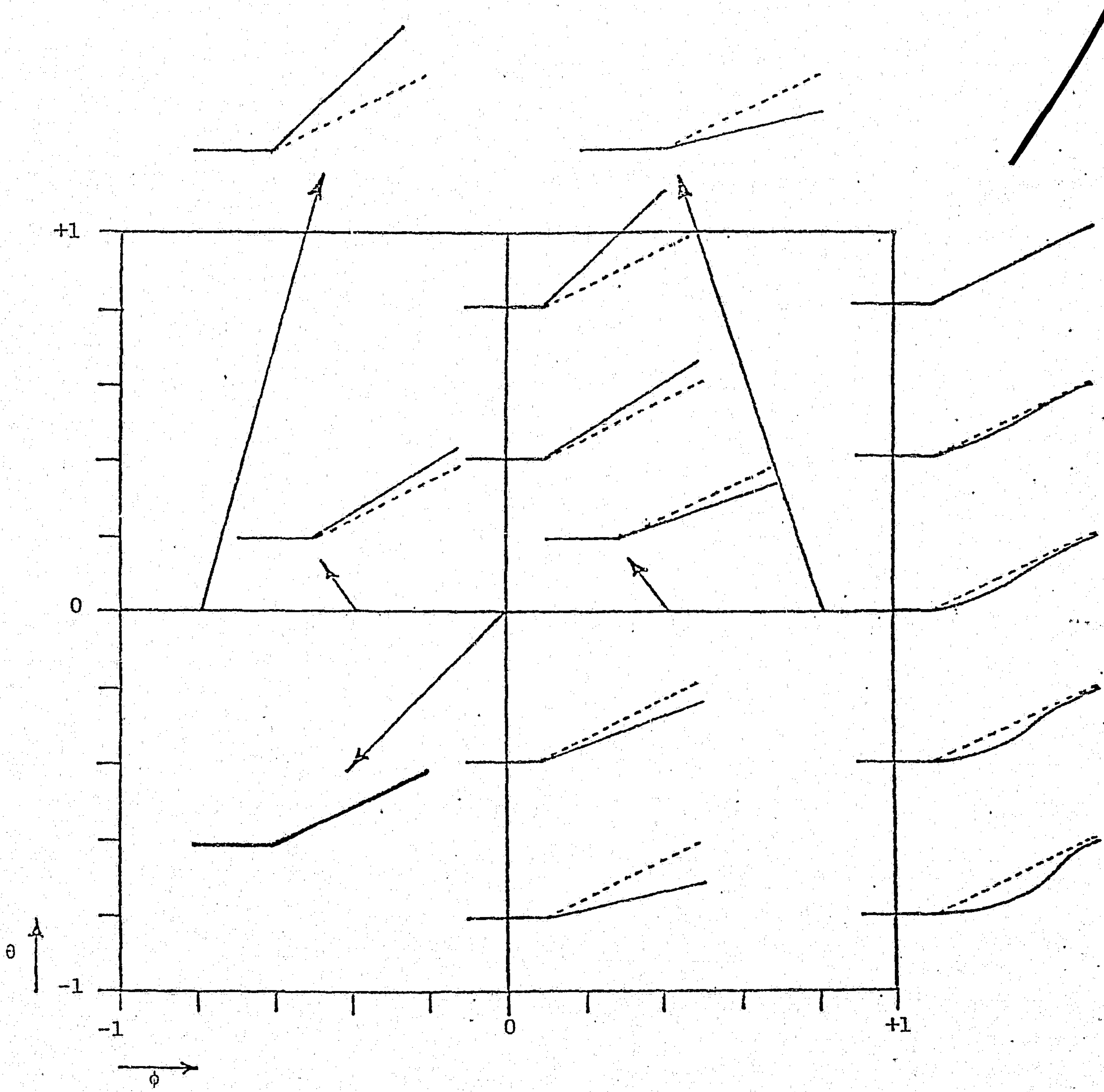

Figure 6. Effect of Variables M and Λ (φ and θ) on the Ramp Response Model

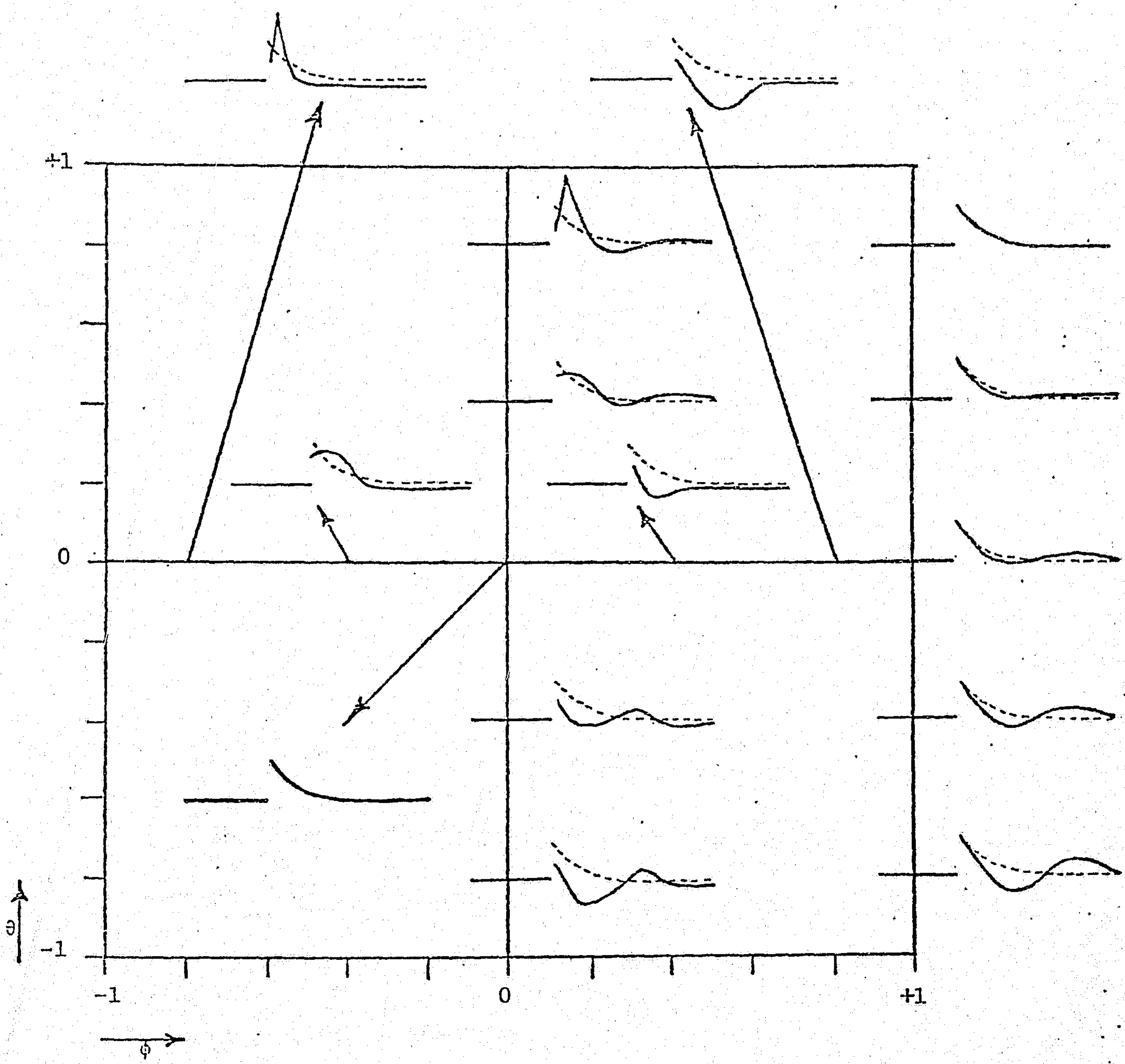

Figure 7. Effect of Variables M and Λ (φ and θ) on the Decaying Pulse Response Model

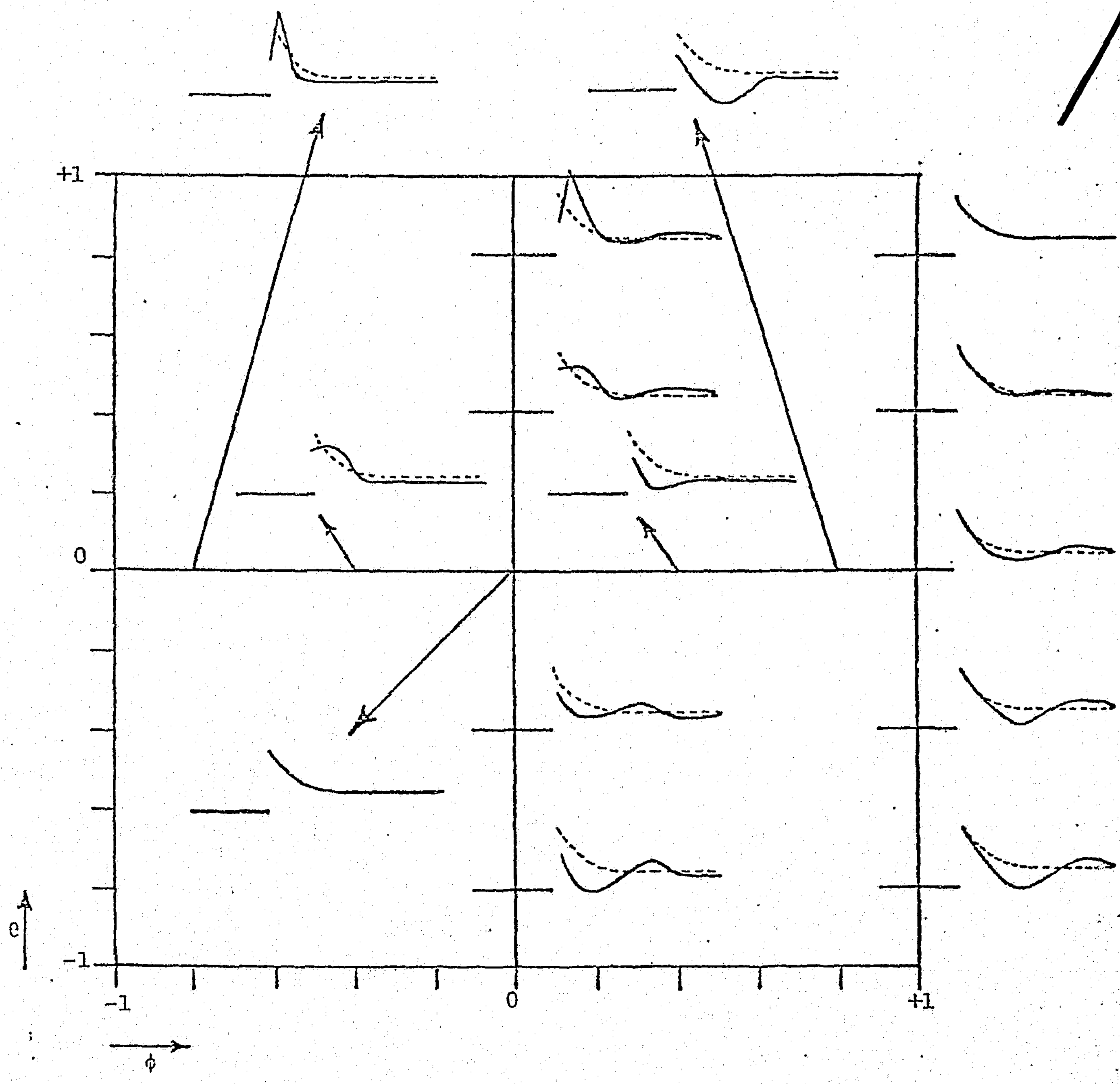

Figure 8. Effect of Variables M and Λ (φ and θ) on the Partially Decaying Pulse Response Model

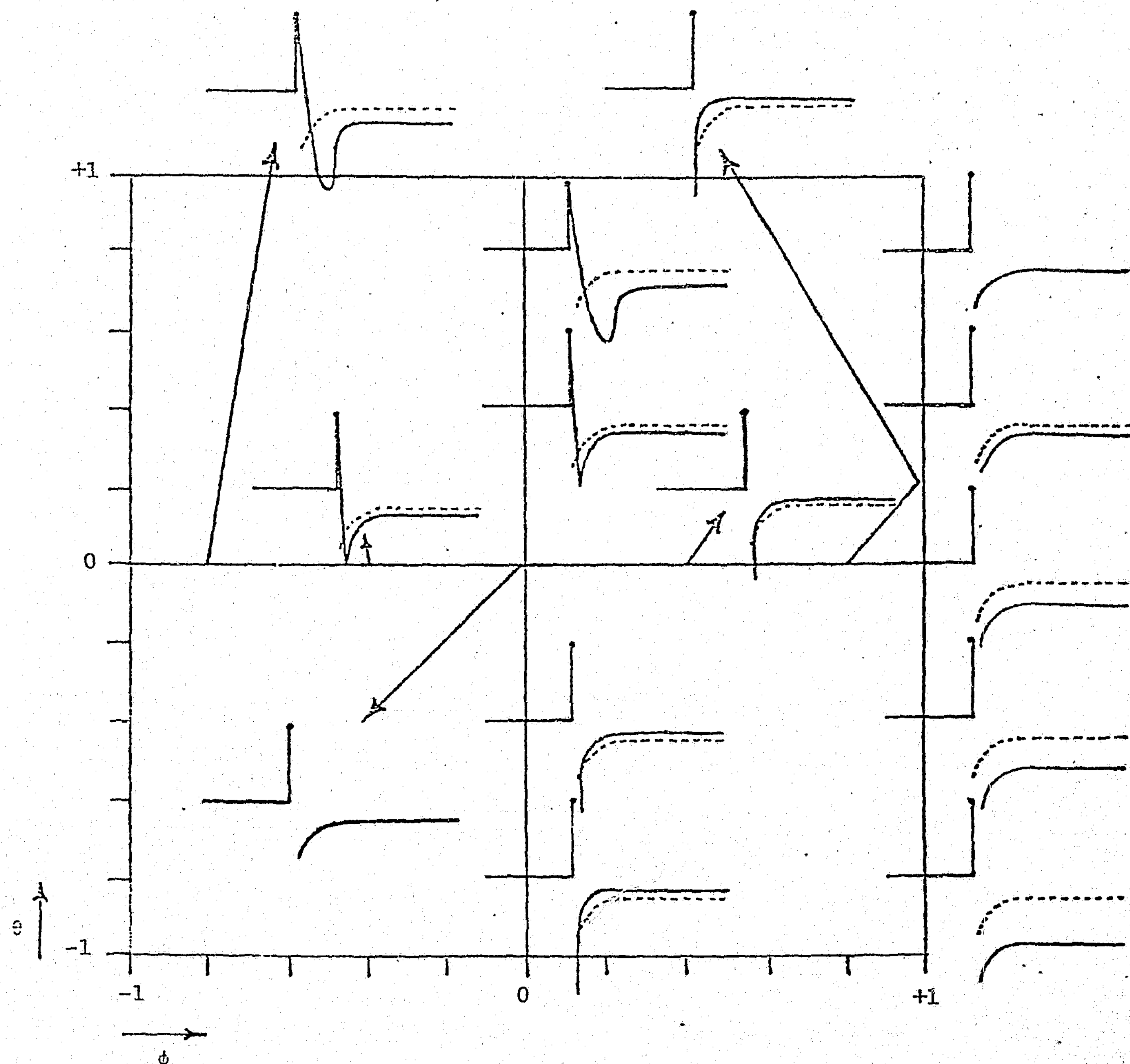

Figure 9. Effect of Variables M and Λ (ϕ and θ) on the Inverse Decaying Pulse Response Model

Consider Figure 4 which contains the chart for the Constant Step Response Model. Here we see on a domain of theta, the moving average parameter and phi, the autoregressive parameter overlays of the theoretical pattern and the observed patterns. For example, when $\theta = \phi = 0$ and the additive noise process is white noise, the theoretical pattern is identical to the sample pattern produced by the identification algorithm. In all other combinations of the parameters of the additive stochastic component, the solid line indicates the pattern produced by the algorithm and the dashed line the theoretical pattern. These charts can be used to assist in pattern matching the estimated sample pattern produced from the algorithm to the theoretical underlying form of the dynamic intervention.

Analysis of Massachusett's Gun Control Law Via Dynamic Intervention Models

The previous intervention analysis of the Gun Control Law via shift detection addressed the question, did a change take place? In this re-analysis we employ dynamic intervention modeling to address the nature of these changes, specifically focusing upon whether the changes was permanent or transient. In this re-analysis additional data through September 1977 was available. Thus the post intervention period contained almost two and one half years of monthly data.

In the following, a brief review of some pertinent aspects of the Gun Control Law are reviewed and the full data set exhibited. After which the step by step model building procedures for dynamic intervention analysis discussed, starting with the time series data for gun related crimes is presented.

The Updated Boston Gun Control Data

Gun-related offenses have become an increasing problem in major cities. In an effort to deter gun-related crimes, the State of Massachusetts, in April 1975, put into effect a law which mandates a one year minimum sentence upon conviction of carrying a firearm without a license. To assess the impact of this gun control law, three types of gun-related offenses were examined: armed robbery, assault with a gun and homicide. Monthly data were available on the number of each of these offenses for the city of Boston.

The Boston gun-control data have been previously analyzed for each of the three offenses from January 1966 to October 1975. As described in Deutsch and Alt (1977), this approach estimated the effect of the Gun Control law via the delta approach. In doing so, the analysis determined the magnitude of the intervention effect and ascertained its significance. However, the results did not address whether the intervention effect had a lasting impact on the three gun-related offenses. The purpose here is to discover if the intervention effected permanent change in the time series under study, and if so, what form the intervention effect took. Thus, the analysis in this section will extend and enhance the results of the previous analysis.

Additional data for the present analysis was collected up to and including September 1977. The data used in this analysis are presented in Figures 10, 11, and 12. The data in Figures 10 and 11, Boston Assault with a Gun (BAG), and Boston Homicide (BOH) respectively represents reported monthly Uniform Crime Report data. However, the data for Boston Armed Robbery (BAR) Figure 13, includes all armed robberies, whether the weapon used was a firearm or other dangerous weapon. Even

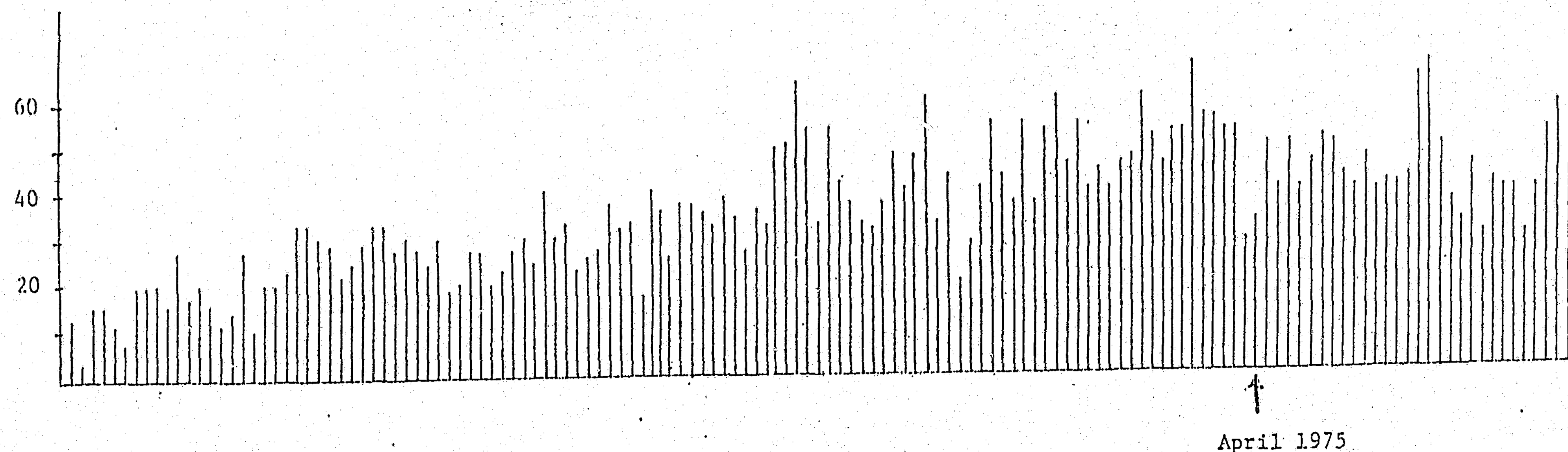

Figure 10. BAG Data

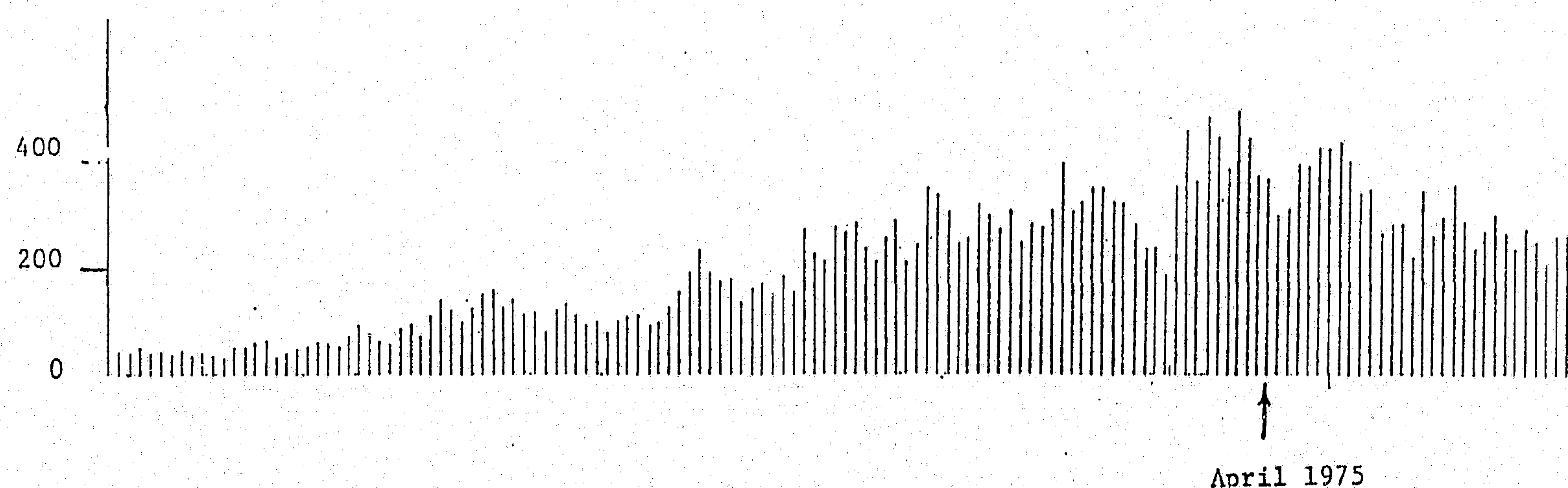

Figure 11. BAR Data

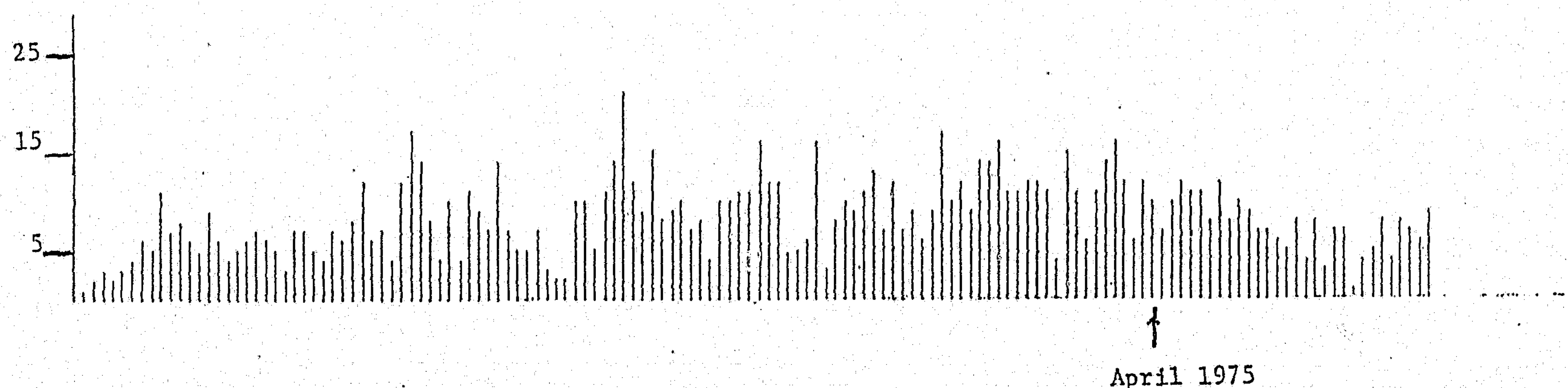

Figure 12. BOH Data

though separate reports were made for armed robbery with weapons other than firearms from January 1974, reports before this time were not delineated in this manner. Therefore, to achieve consistency, all armed robberies, regardless of weapon, were included in the data for the entire time period. This configuration of the armed robbery data is the same as that used in the previous analysis, so that the results are directly comparable.

Analysis of the Boston Gun Control Data

The first step in the present analysis was to identify the correct model form for the three sets of updated data using the ARIMA model forms proposed by Box and Jenkins (2,6). Plots of the data along with the sample autocorrelations and partial autocorrelations of the data were studied to postulate a model form for each of the three time series. Plots of the BAG, BAR, and BOH series indicated nonstationarity. Notice the change in mean throughout the series in Figures 10 to 12 respectively. Also, for the BAG and BAR data, there was strong indication of a seasonal correlation at lag twelve. When a seasonal difference is taken (D=1 for S=12) in addition to the removal of nonstationarity (d=1), plots of both the BAG and BAR data appeared stationary and nonseasonal. Since the BOH data did not appear to be seasonal, one difference (d=1) alone was imployed to reduce this series to a stationary series.

From the sample autocorrelation and partial autocorrelations computed the BAG, BAR, and BOH data for the partial autocorrelations were seen to tail off exponentially, while the autocorrelations cut off. In addition for the BAG and BAR data at lags 1, 12, and 24, the same behavior was noted. Thus, a $(0,1,1) \times (0,1,1)_{12}$ model form was tentatively identified for BAG and BAR while a (0,1,1) model form was selected for BOH. The parameters of these models were then estimated. For the BAG data, significant estimates (α = .05) of the nonseasonal and seasonal moving-average parameters were found to be $\theta_1 = 0.837$ and $\theta_{12} = 0.748$. For the BAR data, these estimates were $\theta_1 = 0.504$ and $\theta_{12} = 0.748$. For the BOH data, the nonseasonal moving average parameter was estimated as $\theta_1 = 0.757$. Diagnostic checks applied to the residuals of the fitted models revealed no inadequacies in these model forms. These models are consistent with the results from the previous analyses (6) of the three data sets which postulated the same forms of the models with the parameter estimates given below:

BAG:	$\theta_1 = 0.826$	$\theta_{12} = 0.775$
BAR:	$\theta_1 = 0.512$	$\theta_{12} = 0.790$
BOH:	$\theta_1 = 0.810$	

The difference in the parameter estimates results from using 141 observations (through September, 1977) in this analysis compared to the 118 used in the previous one (through October, 1975). These models represent the form for the noise component, N_t, of the dynamic intervention models.

The next step in the analysis is to obtain estimates of δ, the shift parameter while employing the identification algorithm. To do this, the raw data was first transformed to eliminate the seasonal component in the BAR and BAG case. The transformed observations (w_t's) were obtained recursively by

$$w_t = z_1, - z_{t-12} + \theta_{12} w_{t-12}$$

where $w_1 = z_1$, $w_2 = z_2, \ldots, w_{12} = z_{12}$. The rationale for this transformation is explained fully in Deutsch and Alt (1977). Because of the assumption

$w_1 = z_1$, etc. the resulting 129 w_t's were used to obtain an estimate of θ_1 for the integrated moving average model. For the BAG data, $\theta_1 = 0.836$ compared to the previous value for the raw data of $\theta_1 = 0.837$. For the BAR data, the parameter estimates for the raw data and the transformed data were 0.504 and 0.483, respectively.

The seasonally adjusted BAG and BAR data, henceforth referred to as TBAG and TBAR, now follow an integrated moving-average process of order one. The raw BOH data has also been shown to follow this model form. The three sets of data are now used to estimate δ values at $n_1 = 95, 96, 97, \ldots$ for $n_2 = 1$. Up to this point, the procedure for the data analysis is identical to the previous analysis. However, we now depart from the previous technique by applying the identification algorithm i.e., adjusting for significant values of the shift parameter so that the form of the dynamics can be unconfounded and identified.

First, we will consider the results for the BAG data. Beginning at $n_1 = 94$ or October 1974, we calculate estimates of δ. We noted a large significant shift at $n_1 = 98$ or February 1975 and the March 1975 data. A negative significant shift of -28 exists at this point in time. In the previous analysis, a shift of approximately the same magnitude was detected at the same point in time. By adjusting for this and subsequent significant delta's a pattern should begin to emerge. A plot of δ versus time for $\alpha = 0.1$ is shown in Figure 13. The plot suggests that there has been a fairly constant decrease with a slight bowing upward. This pattern closely matches the expected patterns for the constant step model as shown in Figure 4. From this figure we see that for $\phi = 1$ and $\theta > 0$, as in the case of our noise structure, estimated patterns for the constant step function will dip from the steady state gain level and then asymptote to the steady state value. The plot of delta's in Figure 13 corresponds to this behavior

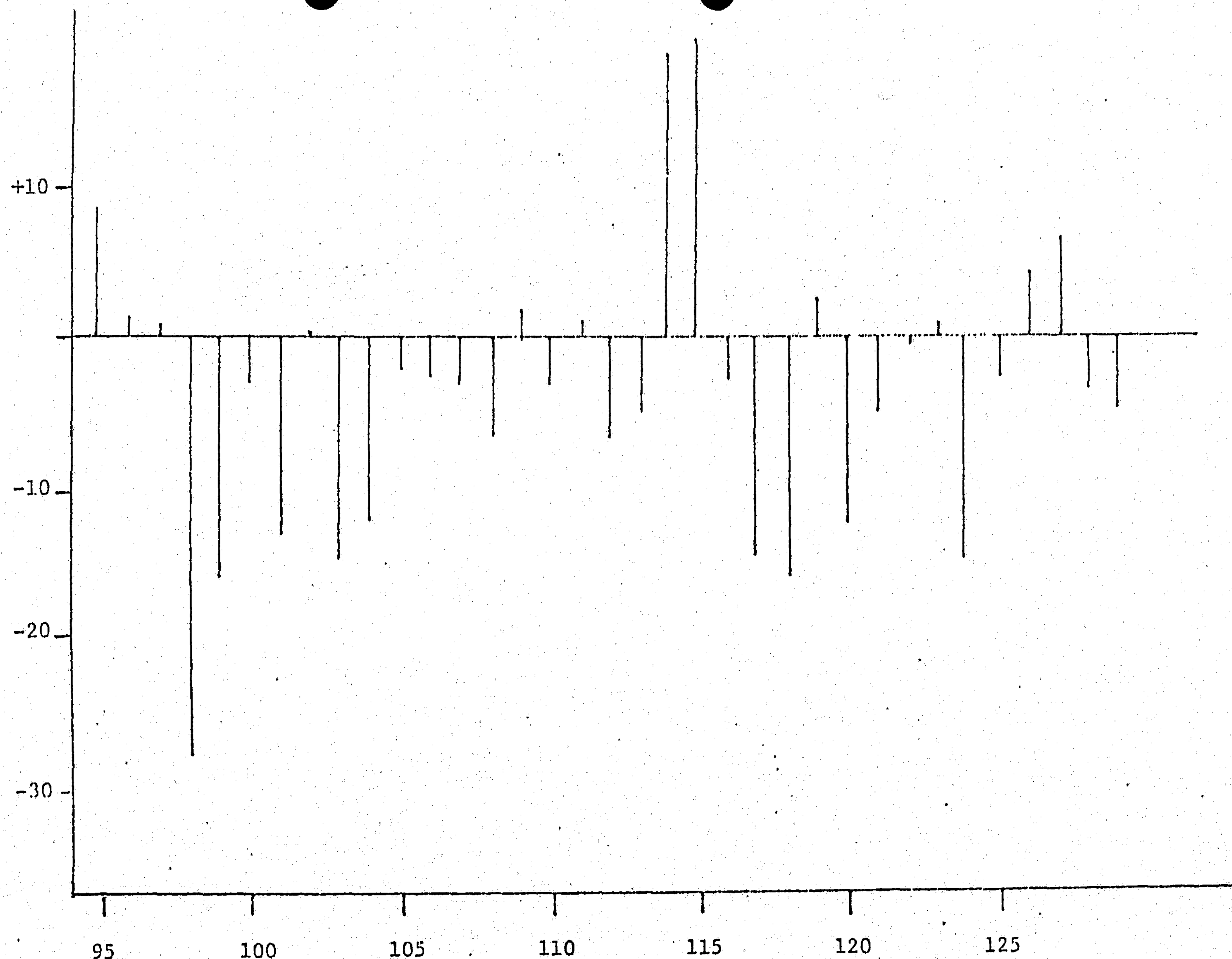

Figure 13. Output of Identification Algorithm for BAG Data

with the exception of having a negative steady state level instead of the positive level illustrated.

Thus, a constant change step function model is tentatively entertained to describe the dynamics. The estimated parameters and associated 90% confidence intervals were found to be;

$$\hat{\mu} = 0.1406, \qquad [-0.1256,\ 0.1537],$$
$$\hat{\omega} = -16.4509, \qquad [-24.6282, -8.2736].$$

Model 1 was overfit with the first order step response model which collapses to the constant step model when the rate of change parameter is zero. The estimated value of this parameter was 0.0676 with a corresponding 90% confidence interval of [-0.0877, 0.2229]. Clearly this interval contains zero, therefore the constant step response model is adopted. The appropriate dynamic intervention model for the entire Boston Assault with a Gun time series is:

$$(1-B)(1-B^{12})z_t = \hat{\omega}BS_t + (1-\hat{\theta}_1 B)(1-\hat{\theta}_{12}B^{12})a_t$$

where

$$S_t = \begin{cases} 0 & t \quad 98 \text{ (February 1975)} \\ 1 & t \quad 98, \end{cases}$$
$$\hat{\omega} = -16.4509$$
$$\hat{\theta} = 0.8267 \quad \text{and}$$
$$\hat{\theta}_{12} = 0.7751 \ .$$

Figure 14 contains the shift pattern produced by the identification algorithm for the BAR data for $\alpha = 0.1$ and 0.075. Regardless of the choice of alpha there is a pattern of temporally persistent shifts noted. Thus a step input form of a dynamic intervention model is chosen. As seen from this figure the two alpha choices do not yield identical patterns with the identification algorithm, the larger value of alpha results in larger magnitudes in the latter portion of the time plot. It should be noted that the larger the alpha value the greater the likelihood that a given estimated shift appears significant and a correction will be made for its level. For example, the patterns produced up to t = 109 is identical at which point a shift of -59.89 was estimated with a corresponding significance level of 0.0986. At the 0.1 level this estimate is significant whereas at the 0.075 level this estimate is not considered significant. Thus for the 0.1 run, observation 110 is corrected for $\hat{\delta}_{109}$ $(y_{110} - \hat{\delta}_{109})$ whereas in the 0.075 run, y_{110} remains as recorded (not corrected). Therefore in estimation of subsequent delta's in the 0.10 run a gain in estimated magnitude will be created above the corresponding value of delta estimated for the 0.075 run;

$$(\hat{\delta}_{t,\alpha_1} - \hat{\delta}_{t+1,\alpha_1}) - (\hat{\delta}_{t,\alpha_2} - \hat{\delta}_{t+1,\alpha_2}) = \text{gain}_t$$

When both alpha levels produce significant or insignificant delta estimates at time t the gain at time t is zero.

For the choices of alpha exhibited in Figure 14 shifts were noted for the .075 value but not the 0.10 value at t =109, 114, 117, 121, 122, 123, 124, 125 and 128. Thus the latter segment of the 0.10 run has a gain above the 0.075 run as exhibited by the larger magnitude estimates. It should be noted that the pivotal consideration is t = 109. Given that the 109th observation is determined to be consistent with past history as with the 0.075 run many more subsequent observations are also deemed consistent with the past history which also includes the magnitude of the

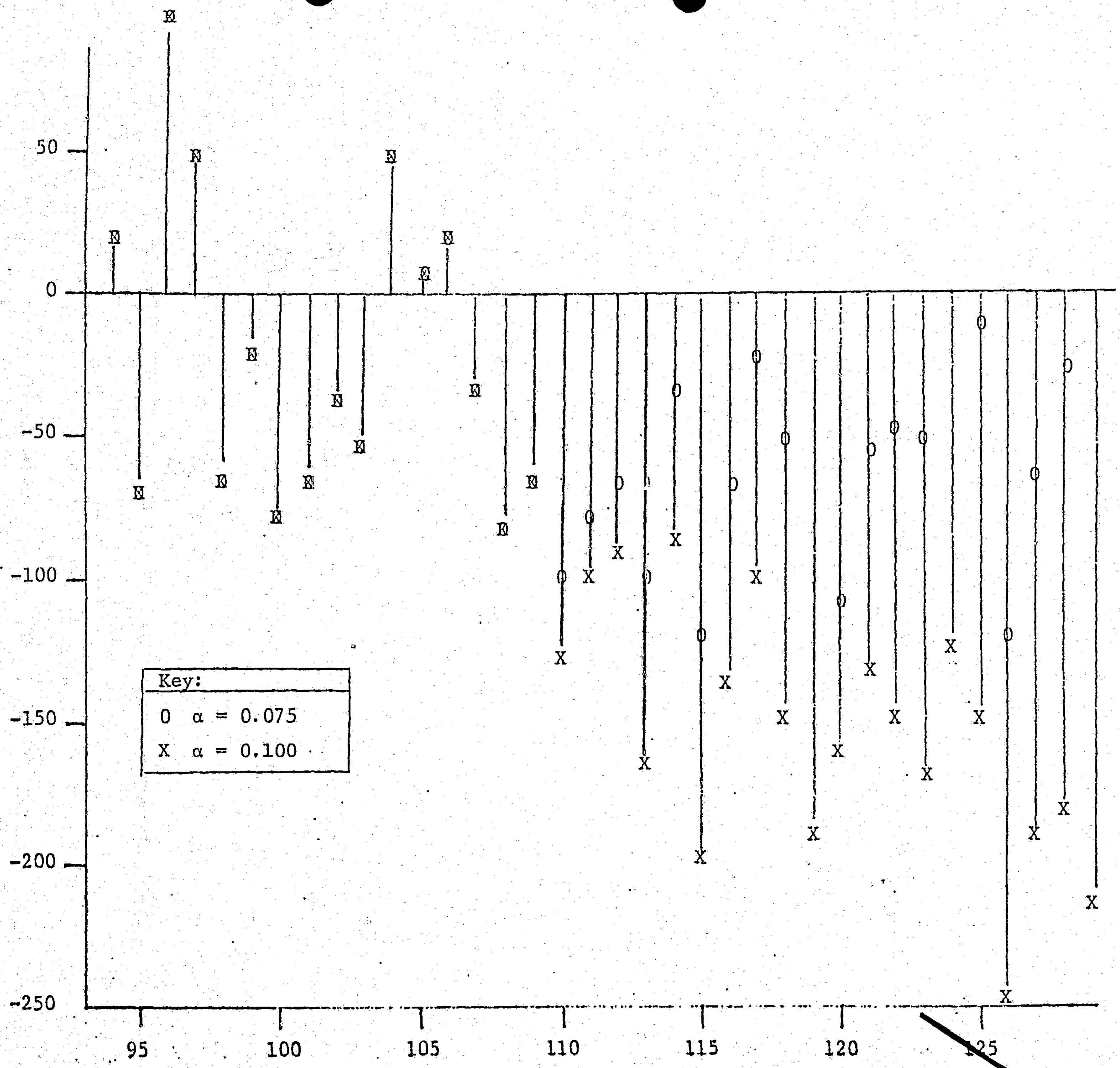

Figure 14. Output from Identification Algorithm, Bar Data

109th observation. On the other hand, if the 109th observation is viewed as inconsistent subsequent observations are also deemed inconsistent.

The identification of the specific pattern which suggests the tentative model type is obviously a function of alpha. It is recommended, as with any statistical inference procedures in which a type one error is user specified, that a range of alphas should be employed. For the most reliable estimate of initial dynamic model identification, patterns which stabilize for variation in choice of alpha are recommended. Thus we prefer to employ the pattern which is not unduly dominated by a single alternative decision of our sequential identification algorithm.

For $\alpha = 0.05$ a near identical pattern to that obtained for $\alpha = 0.075$ is obtained. Therefore Model 1, a constant step response model is selected for the BAR data. The estimated parameters and associated 90% confidence intervals are:

$$\hat{\mu} = 5.0560; \quad [-01.6328, \ 11.7452]$$

$$\hat{\omega} = -72.2838; \quad [-124.4209, \ -20.1467].$$

The mean for the observations prior to the point of intervention is seen to be insignificant. Overfitting with Model 2 produced an insignificant estimate of delta and Model 1 was accepted as the appropriate dynamic intervention component. The final dynamic intervention model for armed robbery is:

$$(1-B)(1-B^{12})z_t = \hat{\omega}BS_t + (1-\theta_1 B)(1-\theta_{12}B^{12})a_t$$

where

$$S_t = \begin{cases} 0 & t < 98 \text{ (February 1975)} \\ 1 & t \geq 98 \end{cases}$$

$$\hat{\omega} = -77.8398$$
$$\hat{\theta}_1 = 0.5128 \quad \text{and}$$
$$\hat{\theta}_{12} = 0.7905 \ .$$

Figure 15 pictorally represents the output of the identification algorithm for the BOH data. The pattern produced here is again typical of that expected for a constant step response model in the presence of an additive noise process which is represented by an integrated moving average process (see Figure 4). The fitted dynamic components parameters and associated 90% confidence intervals are:

$$\hat{\mu} = 0.540, \quad [-0.0038, \quad 1.0838]$$
$$\hat{\omega} = -3.2524, \quad [-4.3890, \quad -2.1158].$$

Again, the mean is seen to be insignificant and the final dynamic intervention model for the entire Boston Homicide series is,

$$(1-B)z_t = \hat{\omega}BS_t + (1-\theta_1 B)a_t$$

where

$$S_t = \begin{cases} 0 & t < 108 \text{ (December 1974)} \\ 1 & t \geq 108, \end{cases}$$

$$\hat{\omega} = -3.2524 \quad \text{and}$$
$$\hat{\theta}_1 = 0.8100 \ .$$

Summary and Discussion of Dynamic Modeling Results

Dynamic intervention models have been developed for time series data of gun related crimes in Boston. In developing these models a newly developed identification algorithm which does not require the modeler to arbitrarily postulate the dynamic component form was utilized. In

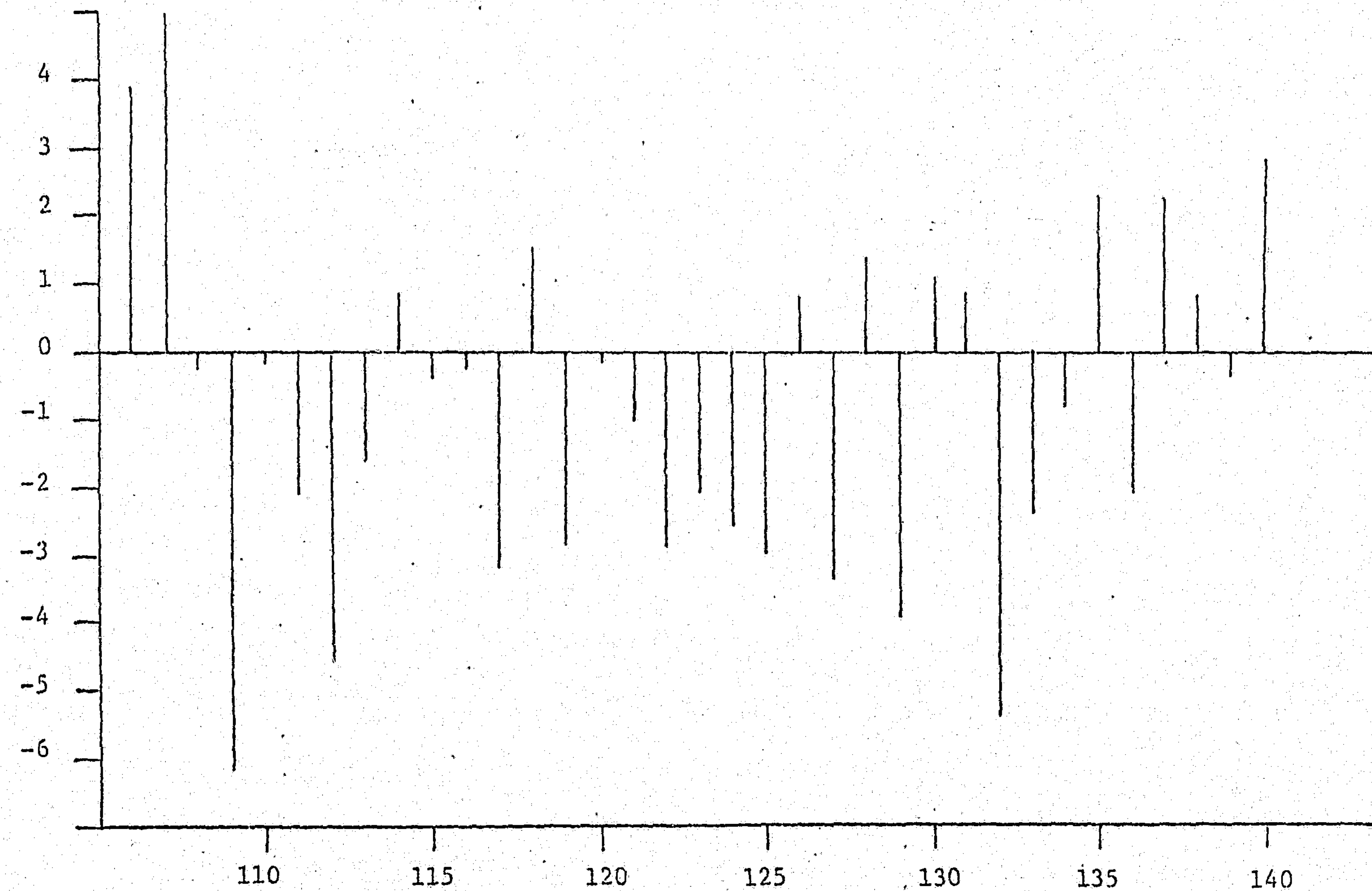

Figure 15. Output from Identification Algorithm, BOH Data ($\alpha = 0.1$)

time series experiments, replication is not possible. Therefore lack-of-fit procedures can't be employed in developing a statistical model that not only explains the variation in a statistical sense but is also of the appropriate form. In many ways the form of the model is most important, particularly since it's form and associated parameter estimates are the primary components of the dynamic intervention model that is related to policy inferences. The components of the dynamic intervention model that describe the system behavior after intervention can often be represented by several different forms that would be adequate in a regression sense alone. However in fitting these functions, structure is not only fit but the variation associated with the additive noise component is also described. Thus the identification algorithm which identifies the form of the intervention component from the information contained in the data avoids these pitfalls.

With the aid of this identification procedure the dynamic intervention component for each of the three gun related time series in Boston exhibited a constant reduction in the activity level of the gun related offenses. This constant reduction was seen to be a permanent change in level that perpetuated for the two and one half year post intervention period. The percentage reduction in each of these gun related crime series from the level without the intervention may be computed by taking the ratio of the constant gain at time t, to the sum of the $t^{\underline{th}}$ observed value and the steady state gain,

$$\%\ \text{change}_t = \frac{\omega}{z_t + \omega} \times 100.$$

The resulting percentage changes for each month of the series were computed. The low, high and average over all months in the post intervention period are:

	BAG	BAR	BOH
low	-19.47	-14.08	-21.31
average	-26.99	-18.58	-29.21
high	-36.00	-22.35	-76.47

As seen from the above summary, sizeable reductions in all three gun related crimes were noted throughout the two and one half year post intervention period.

It should be noted that in our earlier analysis, Deutsch and Alt (1977), we had found significant reductions for armed robbery and assault with a gun but not homicide. In our present analysis with dynamic intervention models, the models have a greater resolution capability to describe dynamic changes in that a separable component that resolves additively with a noise component is contained. Previously, with the shift detection method, Deutsch and Alt (1977), a negative shift in with a significance level of 0.125 was noted at t = 109 but sequential analysis conducted without the identification algorithm masked subsequent significant shift patterns.

The analysis conducted in this paper assumes a single consequence intervention process. That is only changes in mean level are addressed. In general, one may have a multi-consequence intervention condition in which not only the mean level of the process changes between the pre and post intervention periods but there is a simultaneous change in the covariance. The methods and considerations for multi-consequence intervention analysis are described in Alt, Deutsch and Goode (1977).

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