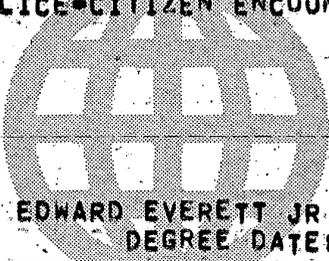


DECISION-MAKING MODELS OF
SOCIAL INTERACTION; THE CASE
OF POLICE-CITIZEN ENCOUNTERS.



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DECISION MAKING MODELS OF SOCIAL INTERACTION:
THE CASE OF POLICE-CITIZEN ENCOUNTERS

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA

By
Edward Everett Brent, Jr.

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

March, 1976

DECISION MAKING MODELS OF SOCIAL INTERACTION:

THE CASE OF POLICE-CITIZEN ENCOUNTERS

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ABSTRACT

DECISION MAKING MODELS OF SOCIAL INTERACTION:
THE CASE OF POLICE-CITIZEN ENCOUNTERS

By

Edward Everett Brent, Jr.

Chairman: Theodore R. Anderson

Interactions among police and citizens are examined from the perspective of a decision making framework utilizing mathematical models. The data consist of ratings in several dimensions of interactions as they occur in police-citizen encounters obtained from systematic observations of randomly selected police patrols in St. Paul, Minnesota.

A theoretical framework for social interaction as a decision making process is proposed. This framework views social interaction as a process which unfolds over time as two or more participants, in the context of some task, choose and carry out actions which have consequences for both participants. The focus of the study is on the response of each participant in the encounter to the actions of the other participant. Fundamental elements of this social interaction include the contingency of present events on past events, the character of the task, the underlying decision processes, the nature of the participants,

and the interaction sequence which results. The framework draws heavily from game theory, social exchange theory, learning theory, and a number of economic theories.

A series of Markov models are examined empirically with the use of log linear models. A second-order Markov model with heterogeneous roles is found to provide the best fit to the data, both with regard to the assumptions of stability, homogeneity, and order; and with regard to predictions of equilibrium distributions, multi-step transition probabilities, and distributions of runs.

The dynamic character of that model is explored mathematically by examining its eigenvalues and eigenvectors. The results are then interpreted substantively. The results support the view of this phenomenon as a process of interaction taking place over time among participants enacting complementary roles where citizens are compliant and submissive and officers are directive and controlling. The two roles differ primarily in the different situations which they encounter and secondarily in their different contingencies on past responses. The implications of these findings for the enforcement process are explored. It is suggested that future research explore social interaction in a variety of settings and further examine possible decision rules which may account for the observed behaviors.

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Chapter 1

INTRODUCTION

The objective of this research is to provide an integrated framework of social interaction as decision making and to examine that framework empirically for the specific case of police-citizen interactions. In this first chapter the approach to this topic and the assumptions to be used will be described. In addition, the scope of this framework will be specified. A brief review of past work in this area will set the context for the focus of this research and the methods which will be employed. Finally, the organization of the remainder of the report will be outlined.

The Phenomenon

An area of concern for sociologists has long been that of microsociology. Perhaps the best example of this concern is found in the work of Simmel. He viewed society as a patterned web of social interactions. His focus was on the fundamental patterns of social interaction among individuals which form the basis for macrosociological patterns. It is these patterns of interaction which are also the concern here. Specifically, the phenomena of interest here are observable sequences of social interaction in which two or more participants, in the context of some task, alternately choose and carry out actions having consequences for

both participants. The objective is to examine those interaction sequences in a way which will shed some light on the process of social interaction which generates them.

Social interaction encompasses a wide variety of phenomena, including doctor-patient interactions, teacher-student interactions, the interaction between two people on a date, police-citizen encounters, and so on. The primary concern in this study is with a specific case of social interaction: police-citizen encounters. The data to be analyzed here are from an extensive study in which interactions in about 5,000 police-citizen encounters were observed and coded.*

The nature of social interaction can be classified by considering some of the scope conditions for this phenomenon.

Scope Conditions

Sequence of related events. There must be some basis for considering these decisions as a sequence. For example, an interaction in which one person first talked with a passerby, then was interrupted by the telephone and talked with someone on the telephone for a minute, and finally returned to his discussion with his companion would not be considered to be a sequence of related events unless some identifiable criterion which was common to all of them could be found (e.g., the guy is a bookie placing bets on horses for people). An example of a phenomenon which would clearly be included in this class of phenomena would be a police-citizen

*"Comparative Quantitative Analysis of Police Encounters," Center for Studies of Crime and Delinquency, NIMH, U. S. Public Health Service. Richard E. Sykes, Principal Investigator; John P. Clark, Co-Principal Investigator.

interaction which occurs when a policeman stops a person for speeding, discusses the issue with the person for a period of time, writes a ticket, and sends the person on his way. For all series of events considered in this theory there must be some basis for considering them to be related to one another.

One or more social actors must be able to affect the outcomes.

These phenomena include only those series of events in which at least one person or some social organization (e.g., a group, a family, a formal organization, and so on) is able to at least in part affect what events occur. The social actor may not be the sole determinant of what occurs. Nor is it necessary the other participants (who may also affect the outcomes) also be social actors.

Value relevant interactions. The consequences of the events which occur must have some value to the social actors involved. E.g., the social actor must prefer some events over others. There must be some important distinguishing factor among these possible events which might serve to encourage the social actor to prefer that some events occur rather than others.

Examples of phenomena which would not be included in this class of phenomena are as follows: situations in which the social actor had no control at all over the possible events which would occur; situations in which the social actor could not distinguish which event occurred or did not care which event occurred; events which occur simultaneously in time; and situations in which events are unrelated to one another so that the outcomes of past events would have no relevance to future events.

Examples of phenomena which would be included in this class of phenomena are the following: a single event for which the individual has some degree of control over the outcomes and for which he prefers some of the outcomes over others; events for which only one social actor had any control over the outcome (e.g., a person playing a game of solitaire); events for which the outcomes not only have value relevance but also informational content (for example, in the situation where a policeman is interacting with a citizen and trying to find out information about a burglary which occurred, the statements made by the citizen have informational consequences for the policeman as well as value consequences).

Assumptions

There are a number of assumptions about these specific situations and about this type of situation in general which are implicit in these examples. These assumptions are important because they determine the approach which will be taken, the variables which will be considered, and the ways in which the phenomenon will be examined. Because of the important role these assumptions play in this theory development and testing it is critical that they be made explicit so that they may be evaluated and changed if demonstrated to be wrong. Stating assumptions sometimes has the unintended effect of making them seem more unchangeable than they should be. These assumptions are not irrevocably etched in stone. They are empirically testable propositions with some basis in empirical work and in logic. They serve now as important building blocks for a theory but they remain hypotheses beyond the scope of this work subject to further empirical verification.

These assumptions are as follows:

- 1) A key aspect of these phenomena is that they are processes which occur over time. Because of this, what happens early in the sequence of events may affect what happens later in the sequence. The choice of a college early in a career can drastically affect the alternatives available later in that career. The use of a racial slur by a policeman early in an interaction with a citizen of a different race can radically change the character of all subsequent interactions. An approach which does not recognize the tremendous impact the historical development of events can have on the outcome is simply not doing justice to the phenomenon.
- 2) The same underlying processes in different environments or different historical circumstances can produce quite different results--a policeman in one condition in which he is verbally insulted and physically abused by a citizen can be violent, while the same policeman interacting with a calmer, less hostile citizen may not be at all violent. Similarly, the same two participants, a policeman and a citizen may interact in very different ways depending on whether or not the incident takes place in a very emotionally charged atmosphere of racial strife.
- 3) Many of these actions within the context of the interaction appear to represent goal directed behavior. A policeman asks the citizen questions designed to determine how he should respond to the citizen's violation of the law. The participant answers questions in ways designed to encourage the policeman to be lenient. The son chooses a college with the intention

of achieving a certain career goal.

- 4) There is some degree of consistency in behavior. What appears to be inconsistency at one level may be consistency at another level. For example, a policeman might utilize a decision rule by which he gives traffic tickets to all of those citizens who do not respond submissively to him when he stops them for a violation. Suppose an observer notes that he fails to give a ticket to someone who does act submissively, while he gives a ticket to someone who commits the same offense but fails to respond submissively to his intervention. Although these actions appear inconsistent in terms of the offense, they are clearly consistent in terms of the decision rule which the officer applies.

Approach/Perspective

The approach taken here is based primarily on two considerations. First there is some desire to contribute to knowledge which meets the goals of theory (i.e., which makes possible predictions, explanation, a sense of understanding, and so on). Secondly, there is also a desire to adopt methods, techniques, and approaches which are compatible with the phenomenon being studied. The desire is to be sensitive to the nature of the phenomenon itself and to avoid as much as possible imposing arbitrary criteria upon the phenomenon being studied. With regard to the goals of this theory of the general phenomena occurring in a variety of settings rather than of the specific instances of those phenomena. These goals have led to the adoption of a descriptive approach, a generalizing approach, and the use of mathematical models. With regard to the

nature of the phenomenon, the assumptions discussed earlier have led to the adoption of an approach which is based on a systems perspective, is aimed at the micro-level, is first analytic and then synthetic, takes a purposive approach, and views the phenomenon as a process of decision making.

The objective of this research is to accurately reflect real behavior to allow its prediction and explanation rather than to prescribe what behavior should take place. This research is designed to explicate the general characteristics which all social interaction situations share and is not concerned with the situation-specific characteristics of particular examples of social interaction (a generalizing approach as defined by Berger, et al., 1972).

Two important ways theories or models may be developed are through analysis or synthesis (Coleman, 1960). An analytic theory or model is one which takes a "number of known facts about the real world--that is, empirical generalizations or laws-- and develops some explanatory scheme from which these generalizations may be deduced" (Coleman, 1964:36). A synthetic theory or model, on the other hand, is one which takes "a number of empirical generalizations or logical relations and instead of working 'inward' to explain them, work(s) 'outward' by using them as postulates of a theory and studying their joint implications" (Coleman, 1964:36-37). The approach chosen here will begin by first analyzing the phenomenon to develop a conceptually tight model of the process of decision making which takes place in these phenomena, and then working synthetically to generate from that model a variety of predictions about behavior in various situations.

This theory is developed on a micro-level. That is, it attempts to develop a theory of social interaction which considers the actions of individuals. From this micro-level where it is possible to consider relatively precisely the processes which go on, the theory will be synthetically developed toward a macro-level where the aggregate effects of such processes may be explored.

Coleman (1973:1-5) distinguishes two fundamentally different conceptions of man which are the basis for schools of thought: The first considers behavior as a response to the environment, the second considers it to be pursuit of a goal. The first is associated with a purposive approach, which is the basis of some theoretical work in sociology, is typified by the work of Weber and Parsons, and characterizes the entire field of economics. This work is most closely associated with the assumptions of the purposive approach and the mathematics and empirical techniques used will be derived primarily from that tradition. However, there is a deliberate attempt in this work to recognize and examine the impact of environmental factors on the processes as they occur. It has been shown elsewhere (Brent, 1975) that these approaches, while having fundamental differences, are not necessarily incompatible.

Closely related to this emphasis on the purposive element in social interaction, is the adoption of a decision making perspective of these processes. Theories of decision-making in their many forms and scattered about as they may be throughout the literature, offer mathematical models, empirical techniques, and theoretical concepts and frameworks which are critical for the understanding of the process of social interaction. This work draws heavily from

a variety of theories which in a loose sense may all be defined as decision theories. These include theories of social exchange, microeconomic theory, utility theories, statistical decision theory, game theory, theories of coalition formation, equity theory, linear programming, and the experimental game literature.

Perhaps the most critical characteristic of the approach taken in this work is that it seeks to consider the processual character of these phenomena. The process approach taken here involves viewing these phenomena as examples of dynamic decisions--sequences of decisions each of which are contingent upon past behavior (Rapaport & Wallsten, 1972:166). The importance of the processual character of events should not be underestimated. One word, for example, said early in the course of an interaction sequence (e.g., a racial slur in a very tense atmosphere) can change the character of the entire sequence of events which follows. Even though sociologists quite frequently emphasize the processual character of social phenomena (e.g., Olsen, 1968), they have only rarely been successful in addressing that processual character in empirical research.

Part of the reason for this avoidance of process on the part of sociologists might be that it is a very difficult phenomenon to deal with effectively. Rapaport & Wallsten (1972) have pointed out that such an approach frequently involves the need for dynamic models, and most social science techniques known by sociologists in general simply don't apply. In addition, sociologists are not that familiar with the use of mathematical models and either choose not to address the issue or, if they do address it, they frequently fail

to adequately develop and test the models. Qualitative techniques, as Rosenberg (1968) points out, are not adequate for such phenomena. Qualitative analysis simply cannot adequately handle the complexity of many repetitive events, the many possible actions which may occur, and so on. There are a number of problems associated with empirical examinations of decision processes. Slight errors in predicted behavior, over time can result in great differences in predictions.

Mathematical models are used for a number of reasons. The character of such phenomena as processes has meant that most of the currently in vogue techniques for sociology are not really appropriate and useful. So for that reason there is a need to turn to mathematical models as the most promising technique for handling process. One of the greatest strengths of mathematical models is their ability to allow the consequences of assumptions and propositions to be ferreted out.

A systems approach is also taken in this work. There have been many treatments of systems describing the perspective (e.g., many of the articles in Buckley, 1968 including the one by Hall and Fagen, 1968), or advocating the approach (e.g., see Buckley, 1967). The characteristics of this approach are too numerous to mention here. A number of the major characteristics which will be most critical for this work include the notion of boundaries for systems, inputs and outputs, the notions of feedback and purposeful systems, and the notions of structure and process associated with state-determined systems.

Existing Conceptualizations of Social Interaction

There have been a number of major theoretical efforts which are of relevance to social interaction. These include social exchange theory, game theory (and the related experimental games literature), equity theory, theories of coalition formation, praxiology, psychological learning theory, and a number of theories closely related to various areas of economics, including statistical decision theory, utility theory, and economic exchange theory. These theories differ markedly in their scope and specific substantive emphasis. They also take very different perspectives and approaches, make radically different assumptions, and in some cases have fundamentally different objectives. They are spread throughout a wide range of literature including sociology, psychology, and economics. As a result they have developed largely independently of one another and have different weaknesses and different strengths. There is no overarching conceptual framework which combines all of these or even which clearly states how they are alike and how they are different. But they all have one thing in common. They all may be viewed as special cases of the same phenomenon-- decision making.

These points may be illustrated by a detailed consideration of three of these theories which offer the most promise for developing improved conceptualization of the process of social interaction. The three which will be considered here are social exchange theory, psychological learning theory, and game theory. The others have been discussed in detail elsewhere (Brent, 1975).

Game Theory and Experimental Games Research

Game theory assumes the participants act with a high degree

of information search and processing ability, positing a type of rationality probably more restrictive than that assumed for any other segment of the literature. It assumes that each participant in the game has full knowledge of the utilities for each outcome which may result, is aware that the other participant is a strategic social actor, and will act to maximize his own future outcomes by influencing the external environment in a way which assumes that the other participant is also trying to maximize his own benefits. Game theory is best applied to the special case of decision making in the presence of a strategic other where there are only two social actors who are the participants, the outcomes of value for the participants are contingent upon the actions of both participants, communication is not possible, and they have completely conflicting interests.

This is an especially important area of research because, of all social science theories, it is probably the most formalized theory which does not rely heavily on formally oriented mathematical techniques. The initial work on game theory by Von Neumann and Morgenstern (1947) is widely heralded as a classic and a major innovation. Many works, have followed which describe the theory and are perhaps more readable, including Rapoport's *FIGHTS, GAMES AND DEBATES* (1960), and Luce and Raiffa's *GAMES AND DECISIONS* (1957). The mathematical element of this theory derives from the substantively oriented theory developed by Von Neuman and Morgenstern and does not emphasize formally oriented mathematical techniques. The theory is developed at the level of the symbolic or postulational style. The logical rigor of mathematics is used to deduce

relationships. The emphasis is divided between the substantive meaning of the propositions and their logical proof.

The theoretical framework of game theory is limited primarily by its scope. It is designed most clearly for situations in which there are only two actors present and they have competitive interests. Although there have been efforts to extend it to other phenomena those efforts have been criticized both on theoretical and empirical grounds. Theoretically, the concept of rationality becomes ambiguous for other situations (e.g., see Rapaport and Chamah, 1965; Coleman, 1973). Empirically, there seems to be little support for game theory beyond two-person zero-sum games (e.g., see Becker and McClintock, 1967). Game theory is important however, for its early development in a field where explicit theories and scientific investigation had not been considered before. It has made an important contribution by encouraging the scientific investigation of major aspects of social interaction which were once not considered amenable to scientific investigation.

The empirical research on game theory has been monumental. There have been literally hundreds of studies examining different game situations. There have been a number of reviews of this literature in the past which serve as a starting point for this review. These include reviews by Vinacke (1969), Becker and McClintock (1967), Gallo and McClintock (1965), and Rapaport and Orwant (1962).

There are two major dimensions along which past empirical research may be classified: First, there have been a number of

different experimental games which have been studied. Secondly, there are a number of classes of independent variables which have been examined. Together these two dimensions distinguish the major contributions which have been made in the empirical research on experimental games.

All of these games which have been examined have been tasks for which there are two or more participants who make decisions affecting the valued outcomes for both themselves and the other participants. The three major classes of such games are two-person games without communication, two-person bargaining games, and coalition formation games. These are distinguished by 1) the number of participants and 2) whether or not explicit communication is possible.

Past reviews of this literature have identified a number of categories of independent variables which have been examined in experimental settings (Becker and McClintock, 1967; Gallo and McClintock, 1965; and Vinacke, 1969). Here categories will be distinguished based on the conceptual framework presented earlier. Those categories overlap considerably with those of the earlier reviews. These categories of variables are as follows:

- 1) Characteristics of the participants--these correspond to the personality variables used in previous reviews but they are not limited to individual participants; they may also characterize social organizations which act as participants or participants other than social actors. These characteristics are inferred characteristics of the subjects and include general differences between participants, family background, psychopathology,

attitudes and traits, motives, and the systemic properties of the actors (e.g., their tendency to respond to external change).

- 2) Characteristics of the interaction--these variables include possibilities for communication and feedback and generally are measures of the possible modes of interaction for the interactive situation.
- 3) Task Characteristics--these variables include manipulations of the payoff matrices, the mode of presentation, the length of the interaction, power relationships, and so on. These are properties of the task itself which may be specified independently of the participant, the developing pattern of interaction, and the possible modes of interaction.
- 4) Situational variables--These are characteristics of the process of interaction as it develops over time. These include the strategy of the other participant, past outcome for the participant, and so on.

Generally, the major types of games have served as experimental paradigms within which a number of different issues have been explored. As Vinacke (1969) has pointed out, studies rarely examine interactions in more than one type of game. The dominant character of these empirical studies has, in fact, been one of examining the effects of one of many possible independent variables upon interaction within one of these contexts. As Becker and McClintock (1967) have observed, the utility of game theory "has not seen its success in predicting human choice behavior, but as a framework against which to examine how people act in these interpersonal situations" (256-7). The myriad of independent

variables which characterize this research bear testament to this view. Many of these independent variables have not been central to game theory. In fact, even with all of these empirical studies, it remains as true today as it was in 1967 that the great majority of studies of game theory "have not rigorously examined the formal model" (Becker and McClintock, 1967).

Vinacke (1969) has made a number of suggestions for the further development of research in this area. He suggests that since so many different independent variables appear to affect the interaction it is important that theories be developed which include many of those variables and that experimental studies be conducted which examine more than one of them at a time to assess interaction effects and to begin to develop a comprehensive structure of how these variables combine to affect social interaction.

Another suggestion of Vinacke (1969) is to examine more than one experimental game situation in each experiment. It should be recognized that game situations are actually values of a number of independent variables (many of the task variables) and varying them in the context of an experiment can lead to new insight into their effects. It is important too that the studies for these different game situations be recognized as part of a comprehensive set of data and not be seen as separable sections of the literature irrelevant to each other. It is important too that the generalizability of findings from one experimental game setting to another be further explored.

Vinacke (1969) also suggests that games involving more

participants be investigated. The great majority of studies involve two participants, only a few additional ones involve three, and virtually none involve four or more. The fundamental discontinuity between dyads and larger groups has been pointed out by Simmel (1971). This arises from the possibility of coalitions forming in groups of three or more while dyads cannot have coalitions without the dissolution of the group. In addition, Vinacke (1969) suggests that personality variables and the importance of social interaction itself may vary with the number of people who participate.

Becker and McClintock (1967) identify a number of problems with the development of the empirical literature for experimental games. First, the great majority of these studies which examine experimental situations for which there are repetitive choices may not be appropriate for testing the theory. They suggest that a strict interpretation of the theory would require that a series of plays be regarded as one game rather than a series of separate games. But no one has done this and hence "the conclusions drawn from experimental results should not be interpreted as necessarily valid conclusions about the formal model" (254-5).

There are a number of problems with the assumptions game theory makes about the utilities for the participants (Becker and McClintock, 1967). The assumptions of transitivity and continuity made by utility theories have not been supported empirically. Two additional assumptions of game theory appear to be contradictory; the assumptions that the participant will at the same time maximize his own utility and minimize the maximum harm that could

befall him. Efforts to resolve these dilemmas have led to yet another assumption that "each decision maker in a game situation projects himself into the position of his opponent(s), and views the game from his (their) standpoint" (Becker and McClintock, 1967: 256). Secondly, under some conditions when the opponent is assumed to take a minimax strategy then all the choices for the participant have the same expected value and the game theory would have no single prediction for his behavior. Finally, such an assumption presumes that the participant can effectively compare his utilities with those of the other participants. Such interpersonal comparison of utilities has never been demonstrated to be possible.

For situations other than zero-sum two-person games the notion of rationality does not have a unique form (Becker and McClintock, 1967; Rapaport and Chamnah, 1965; and Coleman, 1973). For such situations there are a number of possible strategies based on different motives which would be consistent with rational behavior. For instance, McClintock and McNeal (1966) suggest that at least three values or motives may underlie a person's choice in the prisoner's dilemma game. Those include "(a) attempting to maximize the joint outcomes across own and other's payoffs, (b) attempting to maximize own outcome regardless of the outcome to the other player, and (c) attempting to maximize the difference between own and other's outcome" (Becker and McClintock, 1967:269). Other possible decision rules are suggested earlier in their paper in the conceptual framework chapter.

There has been only limited empirical support for game theory

by this research. Becker and McClintock (1967) have pointed out, for example, that empirical studies of games without saddle points have found that subjects' choices "do not approximate the asymptotic proportions predicted by the minimax rule" (259). Yet, eight years later, it is still as true as it was then that these studies have not rigorously examined the formal model.

A number of apparently stable empirical relations have been found but they have not been systematically followed up in subsequent research. For example, Becker and McClintock noted that in most studies the strategies of participants seem to vary as a function of the other player's choices. Yet this has not been systematically explored in many studies. Similarly, they noted that the participants' choice proportions change through time. That too has been generally ignored in subsequent research. (A major exception to this criticism is the work of Rapaport and Chammah, 1965, which will be reviewed in a later section).

This literature has emphasized the static equilibrium behavior rather than focusing on the process which occurs over time. Such an emphasis overlooks a major component of this phenomenon: process. A lack of concern for the processual character of this phenomenon may result in misinterpretations of those aspects which are examined. For example, very few if any of these studies even test the assumption that the "equilibrium proportions" they are examining are indeed at some equilibrium. This could have serious consequences for the conclusions of such studies.

There has been a consistent failure to rigorously test game theory. The studies have generally failed to consider hypotheses of

game theory and have concentrated instead on the impact of independent variables not even considered in game theory. Even were the predictions of game theory to have been unequivocally supported by these studies, which they were not, it would still not be possible to infer that the theory was correct without also testing the assumptions of the theory (e.g., the participants could be asked to discuss their motives and the variables they consider in making their choices and those could be compared with those hypothesized to be considered by game theory). To my knowledge there have been few such tests of these assumptions.

To briefly summarize, the major problems with this literature are the failure to consider process, the limited scope of game theory theoretically, its lack of empirical support (and its apparent lack of empirical validity due to its emphasis on normative as opposed to descriptive theory), the empirical literature which has emphasized the use of the experimental paradigm for the study of almost everything but game theory, the application to situations for which the theory was not intended, and the major failure to test basic aspects of the theory and to meet the minimal requirements of testing models. Strengths have been the development of a formal framework for dealing with a phenomenon (which was not always considered to be subject to effective treatment by scientific means), the extensive exploration of an experimental paradigm, and the identification of a number of key independent variables which appear to affect social interaction in these settings.

Social Exchange Theory

The scope conditions of social exchange most closely resemble

the scope conditions of the general theory of decision making proposed here. Social exchange is concerned with the exchange of social commodities among two or more participants. There is generally at least some partial coincidence of interests which allows for their mutually beneficial exchange. Each participant is aware that the other participant is a social actor and may be aware of certain of the characteristics of that participant, but generally does not know the utilities for that other participant. The decreasing marginal utility of the commodities exchanged may sometimes be an issue which affects the exchange. Transactions are the dominant mode of interaction, although communications may be possible; and certainly the valued outcome for each participant is contingent upon both his own and the others' actions.

The participants are viewed as purposeful social actors with a primary external orientation preferring to effect changes in their environment as ways of coping with changes in that environment. The behavior may be of symbolic significance and the participants are assumed to have a relatively high level of information search and processing capability much greater than that assumed by learning theory, but less restrictive than that assumed by game theory.

The aspect of social exchange which most limits its scope and which makes it a special case of the general theory is the assumption that some sort of social commodity is exchanged. Social exchange is best considered as an example of decision making in the presence of a strategic other with the additional constraint that some social commodity is involved. This concept of some exchangeable commodity should not be overemphasized, however. By

considering abstract exchange properties and the great variety of such commodities which may occur, it can be shown that this concept may be extended to cover almost any type of choice task for which the participants may make decisions having valued consequences. (e. g., see Foa, 1971; Blau, 1964; and Turner, et al. 1971). From this perspective, the general theory proposed here may, with only slight over-simplification, be viewed as a generalized, formalized theory of social exchange.

Apart from its substantive focus the major characteristics of the social exchange literature are 1) a very low level of formalization, 2) sophisticated theoretical development, and 3) very few empirical studies of the phenomenon. The low level of formalization of exchange theory is a dominant aspect of its classic works (e.g., see Blau, 1964; Homans, 1961; Thibaut and Kelley, 1959); and major reviews and critiques which have followed (e.g., Ekeh, 1974; Mulkay, 1971; Shaw and Costanzo, 1970). These presentations are uniformly verbal theories written in an academic style (in terms of Kaplan's levels of formalization).

One notable attempt to formalize social exchange has been that of Emerson (1972). But this work has a number of problems. Emerson makes the mistake which Coleman (1960) noted of taking qualitative propositions and trying to generate a precise mathematical theory from them (such propositions are not generally precise enough and they would require extensive analysis and clarification and only then would they provide an effective basis for a mathematical system). Even at that point he does not clearly connect the theory to a powerful mathematical system but instead

makes a relatively unsuccessful attempt to develop an axiomatic theory. There are additional problems of inadequately formalized concepts which do not allow precise formal deductions from the theory.

This low level of formalization is a particularly important deficit for theories of process as these theories are. As Rosenberg (1968) has pointed out, verbal theories suffer from being too clumsy and too bulky for handling process over a period of time. What is generally required for handling the notions of change and process is to take the system of events at one point in time and follow them through several additional periods of time. This becomes very awkward verbally, but is easily done with mathematical equations.

Because of this weakness these theories have generally been unable to make precise testable predictions based on processes which occur over time and have been forced, instead to concentrate on the properties of those processes at specific points in time. That concentration has been the basis for the major contributions made by the social exchange theory literature. Major insights have been offered in the literature on social exchange regarding many of the important decision processes which take place in social interaction and some of the analytic properties of social interaction settings. For example, the analytic properties of exchange commodities have been nicely explored by Blau (1964) and Foa and his colleagues (Foa, 1971; Turner, et al., 1971). A wide variety of possible exchange rules and their analytic properties have been discussed by Meeker (1971). Comparisons which are possible and

which are made have been examined by Ekeh (1974), Thibaut and Kelley, (1959), and Blau (1964). The notions of distributive justice (one possible category of exchange rules) have been extensively examined by Ekeh (1974), and are a major emphasis of equity theory research (Walster, Berscheid, and Walster, 1973). These contributions have been of major importance in the formulation of the conceptual framework presented earlier in this paper.

There have been few empirical tests of this theory. Those which have been made have generally been peripheral to exchange theory and have not tested central propositions. The empirical literature can be divided into two major sections: First there are a number of relatively rigorous studies utilizing the experimental paradigm of the minimal social situation. Such articles are discussed in Gergen (1969) and have provided some information of relevance to exchange theory but have not in general rigorously tested the theory. In addition, a number of the studies from the experimental games literature may be of relevance in this category of the empirical literature. The second major group of empirical studies have been experimental studies in which exchange theory has been applied in social settings including social conflict (Nord, 1969), organizations (Levine and White, 1961, 1963), work groups (Homans, 1953), interactional relations (Dillon, 1968), family choice behavior (Bahr, 1972; Edwards, 1969), interpersonal relations (Blau, 1963). These studies generally utilize exchange theory in its entirety. Both of these sets of empirical studies, as with the research on experimental games cited earlier, are not of central importance for this paper because they ignore process. For that

reason they will not be discussed in depth here.

Learning Theory

Learning theory also may be considered to be a special case of the general decision model proposed here. As with most of these special cases perhaps they are distinguished from other special cases most by their scope conditions. Learning theories generally place their emphasis on the process of interaction between the participant and his environment. They generally posit a relatively low level of intellectual functioning on the part of the participant (often such theories are based on experiments with lower animals) in marked contrast to most other theories discussed here. Because of this they do not posit a great many of the possible decision processes which the participants might go through but rather view it as a rather simple reactive process (qua operant learning) where the participant changes his internal dispositions as a direct result of the impact of past interactions with the environment. These theories tend to view the process as backward past--that is, decisions are made on the basis of outcomes from past events rather than on expectations for future events (this distinction is made clearly by Coleman, 1973). In the terminology of Ekeh, they tend to view behavior as operant or conditioned as opposed to symbolic. Generally such theories tend to be applied primarily to situations involving only one social actor and his environment, however, they have been applied to situations where two or more social actors are interacting with each other. It is these latter applications which are of most interest here.

As with many of these special cases, the scope conditions of

learning theory do not clearly put it in a central position for this general theory which is proposed here. Nevertheless, there are many reasons why it is reasonable to think of this set of theories and studies as special cases of the more general theory of social interaction which is being proposed. Rosenberg (1968), for example, argues that learning theory shares both a formal and a substantive similarity with social interaction.

Learning models would appear to be appropriate for the analysis of social interaction for two reasons. The first is that there is a formal similarity between learning as a probabilistic phenomenon and social interaction. Contemporary mathematical learning theories are particularly well suited to handling the variability inherent in social stimuli, that is, stimuli that are produced by another individual; the models are stochastic and deal explicitly with stimulus and response variability. The second is that there is a substantive similarity. In principle at least, learning theory is intended to deal with certain systematic changes in individual behavior and with the development of certain kinds of behavioral stability. These aspects of individual behavior are also of chief concern in the analysis of social interaction (1968:209).

The issue of the substantive similarity between learning theory and some of the decision theories has also been addressed by Coombs, Dawes, and Tversky (1970). They point out that one of the differences between the data with which mathematical learning theory is concerned and the data of concern for decision processes theories is that

The former data are generated by an essentially 'mindless' guessing process that is presumed to be shaped by reinforcement, as in the theories of probability learning and concept formation; the data of decision processes are presumed to involve high-level cognition with a minimum of randomness (303).

But they point out that efforts to distinguish between these areas of the literature sometimes tend to caricature the different fields.

They appear to see the differences as differences in emphasis within a general concern for the same general phenomenon, and they see a number of areas in which the two areas are becoming connected.

Decision processes theory is largely concerned with preferential choice under steady-state or equilibrium conditions. In other words the theories deal largely with a static process. The conceptualization behind mathematical learning theory is that of a dynamic process. The former is concerned with the effect of the current state on choice behavior and the latter with the effect of choice on the current state. In still other terms the former may be regarded as the study of motivation and the cognitive nature of choice; the latter is the study of learning and the adaptive nature of choice. The recognition that behavior is both cognitive and adaptive is evident in recent developments in both fields. There are, for example, the current application of Bayesian statistical theory to decision making (see Sec. 5.2 and Edwards, Lindman, and Phillips, 1965), which incorporates the effect of new information on the decision variables, and the recent development by Bower (1966) of a multi-component theory of memory trace that recognizes the deeper cognitive nature of the learning process. These examples are evidence of diffusion into common problems and possibly even common data. (Coombs, Dawes, and Tversky, 1970:303)

If the scope conditions usually applied to these theories are not such as to make them central to this theory of decision making then it is nevertheless true that their methodological and formal emphases are such as to make them very critical for the development of this general theory. Unlike game theory, learning theory has always had a very strong empirical emphasis providing clearly testable mathematical models of behavior. Learning theories have also been in the forefront in providing mathematically sophisticated models of social interaction and they have provided the closest approximations yet to adequate tests of such models. The degree of formalization of this area far exceeds that of comparable theories such as exchange theory. Those models have been most concerned with the

processual character of the phenomena, which is the central focus of the decision theory proposed here. The contributions have been central to the development of this theory and provide the methodological base from which this work has begun. As Rosenberg (1968:227) points out,

Learning theorists have made important methodological contributions to the analysis of social interaction. As already noted, the application of verbally stated principles to the analysis of a social interaction sequence is a formidable task. Learning theorists have demonstrated the feasibility of using a quantitative language to track the changes in individual behavior that take place during interaction. They have also called attention to detailed aspects of behavior that must be included in a psychological account of social interaction. Moreover, although multi-person interaction models have appeared in the literature from time to time, it is primarily the learning theorists who have attempted to provide precise tests of their models using situations in which genuine interdependencies exist. Thus, their efforts have pointed up ways to construct a model that is at once psychologically detailed, mathematically tractable for dynamic social interaction, and empirically testable.

A great many different mathematical techniques are utilized in these learning models and a great variety are available, including Markov models, linear models, one-element models, n-element models, models including different amounts of reward, models considering only differences in probabilities, models for continuous response outcomes, models for finite responses, and finite-state models. Some of the better books available which deal with such learning theory models are Coombs, Dawes, and Tversky, 1970 and Atkinson, Bower, and Crothers, 1965. Rosenberg, 1968 also provides the best discussion to date of many mathematical learning theory models of social interaction. Many authors have applied these models to social interaction, including major works by Rapaport and Chaznah, 1965; Ofshe and Ofshe, 1970; and Suppes and Atkinson, 1960. These

works and others will be discussed in great detail in the next chapter. Learning theory is the basis for the great majority of work examining process for these phenomena and clearly provides a major contribution to this theory both in models proposed and the methodological techniques established for testing these models.

To summarize, learning theory, while generally developed for phenomena different from those of interest here, is sufficiently similar both formally and substantively to be of relevance. The major contributions consist of the models proposed and the techniques provided for testing them (although they certainly have their problems as I shall later try to demonstrate). The major weaknesses arise from the concern with participants who have relatively unsophisticated levels of information processing capability and the consequent lack of emphasis on fundamental processes of decisions which are likely to take place. This area would profit from consideration of some of the processes posited in social exchange theories. In addition, variables affecting these processes could be explored more carefully. Thus, this area could also profit from a merger with the experimental games literature in which such variables are studied in great detail.

In the next chapter an attempt will be made to develop a conceptual framework which draws heavily from the work in all of these areas. It is clear that these diverse areas of research are actually theories of restricted scope dealing with special cases of the general phenomenon of social interaction. The creation of a conceptual framework which makes it possible to link these areas offers promise for making a major contribution in a number of ways.

The deficiencies and strengths of one area to be applied to other areas, improving them significantly. In addition, these areas compliment one another with respect to the facets of the phenomena which they examine so that their synthesis will lead to an integrated theory which excludes fewer aspects and hence offers a more comprehensive view of the phenomenon.

Past Empirical Research

The great majority of research which has been carried out in these various areas has not examined the processual character of social interaction. The position taken here is that to ignore the processual character of such a phenomenon is a serious flaw and subjects such research to great risks of misinterpreting findings.

Those studies which have dealt explicitly with social interaction as a process have been much fewer in number than those which have not. The treatment of process is a very difficult topic (e.g., there are the combined problems of dealing with process and with mathematical models, neither of which are specialities of most social scientists). In general, the substantive contributions of particular studies tend to be rather sparse and repetitive, they offer little beyond what was found in other studies carried out decades earlier, and they are subject to serious reservations due to the many serious methodological flaws which pervade research in this area. It may be said, in fact, that this research is characterized more by the methodological problems than by the substantive findings.

The great majority of research in this area has come from learning theory (Rosenberg, 1968) and includes work from three different areas: linear models (e.g., Burke, 1959; 1960; 1962; Hall,

1962), finite state models (e.g., Suppes and Atkinson, 1960; Binder, Wolin, Terebinski, 1965a;1965b;1966a;1966b; Cohen, 1958; 1962; Suppes and Krasne, 1961; Suppes and Schlag-Rey, 1962), and continuous response models (Suppes, 1959; Suppes and Atkinson,1960; Rosenberg, 1962;1963a; Anderson, 1961;1964; Rouanet and Rosenberg, 1964). Of these, the work by Suppes and Atkinson (1960) probably marks the most extensive example of investigation of many different conditions under which social interaction occurs. Two other notable works in this vein which don't rely so heavily on learning theory are the work of Ofshe and Ofshe (1969) and Rapaport and Chamah (1965). These last three works have been extensively reviewed elsewhere (Brent, 1975). They provide some of the better examples of the work in this area and are indicative of the general problem.

There have been a number of general methodological problems with this research including a failure to seriously explore the mathematical properties of mathematical models or to interpret substantively the properties when they are found, and a tendency to examine the validity of the models to the exclusion of examining empirically the characteristics of the phenomenon. In short, this work generally suffers from a misuse of models in which the purpose of models (to help provide insight into the phenomenon) is overlooked, the model is studied for its own sake. Although these studies have suffered from methodological problems which have severely limited their usefulness, there have been a number of findings which may help to guide the current research. There is some evidence of the relatively minor role individual variables play in social interaction which is provided by the high correlations

Rapaport and Chammah (1965) found between the actions of the participants who interact with each other in the PD game. This suggests that the social character of the interaction is so strong that individual differences are quickly washed out in these social interactions. Additional evidence of the secondary importance of individual differences is provided by Rapaport and Chammah's (1965) finding of male/female differences which appear only as a result of the sequence of interaction over a period of time. They found that there were no differences in the noncontingent propensities of males and females. However their conditional propensities did differ and over a series of interactions those result in substantive differences. Thus, in this, the soundest study of individual differences methodologically the differences only show up as a joint result of individual differences and the sequential process of interaction.

There is some evidence that task variables play an important role in social interaction. Here we may include among task variables not only the conditions in which the participants interact as defined in the conceptual framework, but also the possible modes of interaction among participants and differences which result from different roles (i.e., differences in the structural positions of different participants in the same task). Rapaport and Chammah (1965), for example, found that the different reward structures in their PD games did affect the behaviors of the participants. Conceptually, too, one would expect task variables to affect the social interaction in important ways. It is hard to imagine that the social interaction in zero sum games with very high negative punishments could possibly be the same as nonzero-sum games where the

participants have complete coincidence of interests. The analysis of Burke (1959) is a very good example of the radically different types of behavior which could be expected within tasks having the same general structure but differing values of the parameters.

The substantive finding which has received the most support from these past studies is the primary importance of the processual character of social interaction. It appears that the most important characteristic of social interaction is that it is a process which occurs over time. The importance of this aspect empirically is illustrated by the finding of Rapaport and Chammah (1965) of the high correlations between the actions by the different participants in social interaction settings. These results appear to indicate that much of the behavior in social interaction is affected by the character of the social interaction itself, is social in nature, and is the result of a process. From a conceptual point of view there is also good reason to pursue this view of the phenomenon as process. A view of social interaction as a sequence of events has great face validity as an explanation of what occurs. The notion that decisions and outcomes of those decisions at one point in time are likely to affect similar decisions at future points in time is particularly appealing. The common sense notions of social interaction, which we all as individuals have, also appear to be consistent with the view that people respond to the actions of others in ways which are affected by past actions and which may later affect future interactions.

In this research a deliberate attempt will be made to avoid the methodological problems of past research by extensively

exploring the mathematical properties of the models, interpreting the results, and examining the models extensively for substantive implications. In addition the substantive insights of these past studies suggest primary consideration should be directed at the processual character of social interaction and the task conditions in which it takes place.

Organization Of The Report

In the second chapter an integrated framework conceptualizing social interaction will be presented. The third chapter describes the data which will be analyzed, establishes key links between those data and the conceptual framework, and describes the analysis strategy which will be pursued. Chapter four presents the results of the analysis. A number of Markov models are explored by testing key assumptions until one model is found which appears to fit the data best. Predictions of that model are then examined to provide further validation of it. In Chapter five are presented extensive interpretations of the findings and exploration of the logical implications of the model. It addresses the problem so often left unaddressed, namely the implications of the model for the phenomenon itself. Chapter six is a summary and discussion chapter in which avenues for future exploration are discussed and some of the broader consequences of these findings are considered.

Chapter 2

CONCEPTUAL FRAMEWORK

Observable sequences of social interaction are the result of a process of social interaction which takes place over time. It is these interaction sequences which are the subject of this research. The objective is to examine interaction sequences in a way which will shed some light on the process of social interaction which generates them. In this chapter a theoretical framework is proposed which suggests the processes which might account for such interaction sequences. This conceptual framework is based on a view of social interaction as a decision-making process. Different aspects of this interaction process are examined and related to relevant past work. Finally, this general framework is interpreted for the case of police-citizen interaction from which data for this study were drawn.

Overview

Observable sequences of social interaction are here viewed as the result of a process of social interaction in which two or more participants, in the context of some task, alternately choose and carry out actions having consequences for both participants. Fundamental elements of this social interaction include the nature of the participants, the character of the task, the underlying decision processes, and the interaction sequence which results.

For example, consider an encounter between a police officer and a citizen. The policeman has just pulled over a young male driver for a traffic violation. They talk for a while, the policeman gives the man a ticket and they both drive away. The officer and the citizen are participants in the interaction. The task is determined by the nature of the offense, the normative and legal restrictions on the behavior of the officer and the citizen after the occurrence of a traffic violation witnessed by the officer, and it consists of the possible actions they each may take and the potential consequences of those actions for the two participants. The interaction sequence consists of the series of communicative acts between the officer and citizen beginning with the officer pulling the man over and ending when they make their last comments and depart. This interaction sequence is illustrated in Figure 2.1 where a hypothetical series of interactions for such an encounter are presented.

Here this interaction sequence is viewed as the result of a process which unfolds over time where first one participant takes some action which has consequences recognized by the other participant; the other participant responds to that action, and so on, until the interaction sequence is complete and they no longer interact with each other. In Figure 2.2 is a diagram which illustrates conceptually how this process of social interaction between two participants in the context of a specific decision task might take place.

In this diagram the two participants are represented by different systems (the two rectangles in the upper part of the

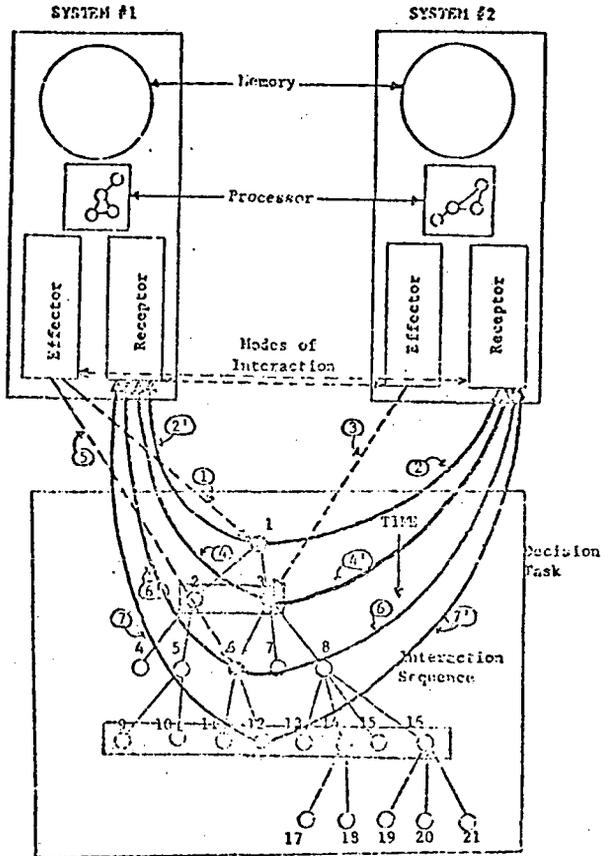
FIGURE 2.1

HYPOTHETICAL INTERACTION SEQUENCE*

- Situation: A young man in a new car has just run through a red light and is speeding along a side street. A policeman in a patrol car pulls him over and stops. The officer has just approached the man's car and speaks to him through the window.
- Officer: "Let's see your license, buddy."
- Citizen: Looking nervous, he fumbles with his wallet and finally hands his license to the officer.
- Officer: Reads the license. "Bill Smith..... What's your address, Bill?"
- Citizen: "1412 Rosemary. What's wrong, officer?"
- Officer: "What's wrong! You're in a heap of trouble, boy. You just ran that red light back there and I clocked you at 55 in a 35 zone. Just what are you trying to prove?"
- Citizen: "Nothing, officer. I guess I just wasn't paying attention. My wife just had a baby boy and I'm so nervous I just can't think about anything else. I'm real sorry. I'm usually a very careful driver."
- Officer: "Really? Have you ever had a ticket before, Bill?"
- Citizen: "Yes, but it was just a parking ticket, sir."
- Officer: "That's all, you're sure?"
- Citizen: "Oh, yes sir!"
- Officer: Hesitating and then writing a ticket. "O.K. I'm going to have to give you a ticket for that red light. I'll let the speeding go this time, but you'd better watch it in the future."
- Citizen: "Oh I will, sir. You'd better believe I will. Thank you."

*Adapted from an example in Wallen and Sykes (1974).

FIGURE 2.2
PROCESS OF SOCIAL INTERACTION



decision node numbers--1,2,....

process sequence numbers--①,②,....

information feedback--curved lines

actions taken by systems--dotted lines

states passed through during the interaction--⊗

states not passed through during the interaction--○

diagram) having properties which will be explored in more detail shortly. The decision task is represented (in the lower part of that diagram) by a series of possible states which may occur. The specific properties of this decision task also will be explored in more detail in sections to come. A particular sequence of events which occurs (e.g., the interaction sequence from Figure 2.1) is represented in this diagram as the series of connected black dots in the decision task diagram.

The interaction sequence occurring here and the underlying processes which generate it are illustrated by this diagram. The first action occurred in this example when participant 1 took some action initiating the encounter and placing the system in state 1 (this is represented by the dotted line connecting the effector of this participant to state 1). That action has consequences for participant 2, and that action is perceived by that participant (as represented by the solid curved line connecting the first state to the receptor of the second participant). Participant 2 then takes some action causing the system to move to state 3. That action is, in turn perceived by the first participant, who then takes some action resulting in the system moving to state 6. This process continues until the encounter is completed. Each participant takes some action which has consequences for the other participant. The other participant perceives that action and takes some other action based on his objectives and the alternatives which are available to him at that point in time. As these occur the system moves from one state to the next creating the resulting observable sequences of interactions such as those in Figure 2.1.

In the sections which follow specific components of this process will be considered in greater detail. After those components have been more clearly defined and their characteristics explicated, this framework will be utilized to develop a detailed theory of social interaction occurring in police-citizen encounters.

An Interaction Sequence

An interaction sequence is here defined as a series of states occurring over time which characterize the process of interaction between two or more participants. A state is an explicit description of a possible set of circumstances which may occur. The concern here is primarily with empirically observable interaction sequences--i.e., sequences of observable states. For example, the series of actions by the police officer and the citizen portrayed in Figure 2.1 represent an interaction sequence. In this case each state is described by the actions and statements which each participant makes at each point in time. States are morphological categories imposed upon the phenomenon by the researcher. The specific description of a state may vary. The usefulness of any particular description is measured by the extent to which that particular definition of the states provides insight into the phenomenon which occurs and is a good representation of major facets of the interaction which takes place.

The states of interest here are those which appear as sequences of comparable, related states which occur over a period of time and for which states may influence future states. It is necessary that the states have something in common so that they may be considered

to belong to some unified sequence of states. For example, a series of states which take place during the course of an encounter between a citizen and a police officer, which was initiated when some violation was observed and which is completed when the officer finishes writing out a ticket to the offender, may be considered to constitute a sequence of events because they all are part of some larger process. On the other hand, a series of states in which one person first briefly interacts with a passerby then returns to talking with his companion and finally answers the telephone to interact with still another person would not be considered a sequence of events unless some unifying theme could be found which tied them together and made it useful to consider them as a sequence.

Particular interaction processes may be characterized in a number of ways. These characteristics may be either formal characteristics which derive from the definition of social interaction as the occurrence of a sequence of states over time, or substantive characteristics which derive from the particular type of states which occur and some of their interesting characteristics. Formal characteristics center around the properties of either states or time. The time in which these states occur, for example, may be either discrete or continuous, there may be regular intervals or irregular ones, the time frame may be very long (e.g., a period of years) or relatively short (only minutes or seconds), events may or may not occur simultaneously, and events may have varying durations. The states themselves also may be discrete or continuous, they may or may not be mutually exclusive or exhaustive, and so on. Substantive characteristics include such considerations as the basis

for treating the states in a similar fashion (e.g., they are all decisions focused on solving a particular problem); the character of the outcomes (e.g., it is generally an approach-approach situation--see Lewin, 1951--in which the participants must decide between two incompatible but desirable events); and so on.

From conception of the interaction sequence as a series of states which occur over time it is possible to link up the observed interaction sequence to the notion of a decision task.* This is important because there have been many studies of decisions in the past and examining social interaction as a decision process makes it possible to utilize what is known about the general processes of decision making to explore the observable social interaction sequence. Social interaction is then viewed as a special case of the general phenomena of decision making processes.

One of the interesting characteristics of interaction sequences is that they consist of a series of events which actually occur. Those events are only a few of the many possible events which might have occurred. For example, the interaction described in Figure 2.2 is only one of the many possible sequences of events which might have occurred. At any point in that interaction sequence either of the participants might have acted differently resulting in a

*These states also provide a link with empirical observations and data analysis facilitating both the conceptualization of this process in terms of some available mathematical models and its empirical examination. That line will be explored later.

different series of events (e.g., the citizen might have tried to run away, or refused to answer a question of the officer; the officer might have probed the citizen more on specific issues or acted with greater hostility). It is possible to conceptualize an entire network of possible events for which the observed sequence is only one of many which might occur. At each point in that network, one of the participants or the other chooses what his next action will be and in enacting that moves the system to its next state. But this is simply a decision task. Decision tasks have been considered extensively in a number of areas of the literature. Social interaction may be viewed as a special case of a decision task for which two or more participants make decisions and the decision of one participant has consequences for both his own outcomes and those of the other participant.

The Decision Task

A decision task is a description of the circumstances in which decisions take place. Much of the social science literature has dealt with such decisions. Decisions are of central concern in much of the literature pertaining to game theory, exchange theory, learning theory, equity theory, experimental games, coalition games, economic decision theory, and systems theory. The general structure for decision making which follows represents an attempt to synthesize these various approaches. This discussion is based primarily on past efforts by Luce and Raiffa (1957), Singleton & Tyndall (1974), Newell and Simon (1972), Thibaut and Kelley (1959), and Ackoff and Emery (1972).

A decision was earlier defined as the purposeful act of

making up one's mind or the act of choosing between two or more available alternatives. This concept may be clarified by a formal description of decision tasks in terms of possible states and transitions among states.

A state is an explicit description of a possible set of circumstances which may occur. A decision state is a state which is defined in reference to possible or actual decisions. Important aspects of decision states may include characteristics of a particular participant (potential decision maker) or characteristics of the environment in which that participant is found.* For example, one possible decision state would be a policeman who wants to break up a bar fight. This state is defined in terms of variables of relevance to possible decisions by the actor (e.g., the decision as to what action to take). It is defined in terms of both characteristics of the decision maker (a policeman who wants to break up a fight) and characteristics of the environment for that decision maker (e.g., there is a bar fight occurring and many people are involved).

Available alternatives are those states which the decision maker may choose in a particular decision.

A terminal state is a state which has no further states from which the decision maker may choose.

*A major issue in developing and testing decision models is the difficulty in identifying states. States are not immediately evident for the phenomenon but are morphological categories imposed upon the phenomenon by the researcher. Procedures for determining precisely the best way to conceptualize states are not available. This remains a critical step in the research process (e.g., see Coombs, Dawes, and Tversky, 1970; Fararo, 1973; Cortes, et al., 1974; Ackoff and Emery, 1972).

A decision node is a state for which there are two or more alternative states which may be chosen by the decision maker.

Choosing to perform an action in the available repertoire of the actor and carrying out that action moves the actor from one state to another. This movement from one state to another is the result of input from some decision maker. A decision maker is some system which allows the choice of alternative states to be made.

The decisions by an actor in a series of decisions made in the context of one decision task may be summarized as strategies. Strategies are explicit descriptions of which alternative the decision maker will choose at each possible decision node which would be encountered in one pass through the decision tree for some task.

In this context, the interaction sequence would be simply one possible sequence of states through which the interacting participants passed over time. Interaction sequences may be contrasted with strategies. The strategy of one participant identifies a range of possible states which the system may go through in the course of the decision task. The strategy of a single player, however, cannot, by itself, specify entirely each precise state which will be encountered because other participants may have some independent effect upon those states. An interaction sequence, on the other hand, specifies the precise states which are encountered in the course of the decision task. An interaction sequence specifies the results of the actions of all of the participants.

A decision task is a fixed set of circumstances in which there are a number of alternative courses of action, decision nodes,

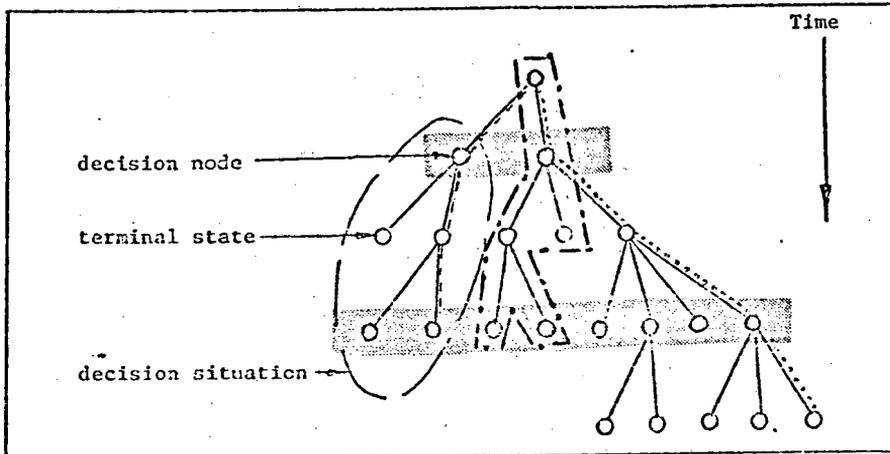
available alternatives, and terminal states available for a decision maker such that the available alternatives for each decision node lead ultimately to some terminal state. This may be represented by a decision space which specifies all states for that particular decision task and the possible transitions among them.

Decision tasks are defined relative to specific actors. A decision task exists when there is some actor of interest whose actions affect the states which occur. A decision task cannot exist when there is no actor who can affect these states. Decision tasks may include circumstances in which more than one actor affects the future states which will occur (e.g., in a police-citizen interaction whether or not the citizen will be arrested may be affected by both the citizen and the policeman). In fact, tasks where more than one actor affect the states which occur are more common than those in which only one actor is sole determinant of what states occur.

A decision situation is a subsection of a decision task characterized not only by the fixed set of circumstances which characterize the task but also by particular historical sequences of events and values of variables which restrict the state space (e.g., after one decision has been made the range of alternatives is limited and these limitations are a function of the particular alternative chosen).

These concepts are illustrated by the diagram in Figure 2.3 in this diagram states are represented as circles. Possible transitions between states are represented by lines connecting the

FIGURE 2.3

DECISION TASK REPRESENTED BY
DECISION TREE OR STATE SPACE

○	States
—————	Possible transition
- - - - -	Sample interact sequence #1
.....	Sample interact sequence #2
- . - . -	Strategy*
—————	Situation

* Notice that a strategy only specifies the choices to be made by the decision maker who adopts that strategy. Other decisions which are not under the control of that decision maker cannot be specified.

states. An arrow in the diagram represents the flow of time and makes it clear that it is not possible in this diagram to return to a state after going to another state. The decision task itself is identified by the boundary surrounding the set of possible states and the transitions among them. Only those states which occur within that boundary are part of the decision task. Because the decision task is defined relative to a specific actor (who may be called the primary decision maker), in those cases where more than one participant may have some impact on the states which occur in the decision task it is sometimes helpful to distinguish decisions under the control of the primary decision maker from those under the control of the other participant(s). Decision nodes enclosed by shaded regions in this diagram represent decisions under the control of participants other than the primary decision maker. Those decision nodes not in such shaded regions are under the control of the primary decision maker.

This representation of the decision task by a decision tree or state space helps to identify a number of important characteristics of decision spaces in general and social interaction in particular. The resulting interaction sequence is a series of events which occur over a period of time. An action at one point in time might have been viewed as appropriate and reasonable behavior, while that same action at some other point in time might seem neither appropriate nor reasonable. The actions in the past combine with the task itself to form the situation within which current decisions must be made. In some cases, the alternatives available may change depending on the past response of the other participant. This illustrates one of the major ways in which current decisions may be

dependent on past decisions. Some of the decisions in a decision space may be under the control of one participant, the others under the control of the other participant. When those decisions have outcomes for both participants then the dependency of the outcomes for each participant on the actions of the other is made clear.

This last characteristic is a fundamental characteristic of social interaction--the nature of the interdependence of the participants on each other. Different types of interdependence have been identified by both Jones and Gerard (1967) and Coombs, Dawes, and Tversky (1970).

Four types of interdependence between actors which are theoretically possible have been suggested by Jones and Gerard (1967). A pseudocontingency interaction is one in which the individual carries out a preestablished plan independent of the actions of the other participants or where the individual responds primarily to his own past actions rather than those of the other actors. The actions of neither actor influence the other to deviate from a fixed objective. The individual behaves as if he were the only person there or at most uses the actions of the other actor to time his actions only (as in plays). Reactive contingency interaction occurs when a player reacts almost exclusively to the immediately preceding action of the other actor. Examples of such behavior would probably include driving and chess games by beginners. Asymmetrical contingency is a mixture of the first two where one person is guided almost exclusively by his own preestablished plans and doesn't react to the actions of the other who is guided almost exclusively by the immediately preceding actions of the first actor. Examples of such

behavior occur when one individual has had a better opportunity than the other to work out the details of the interaction beforehand, such as with salesmen. And the final type of interaction is mutual contingency where each individual's responses are determined partly by the responses of the other participants and partly by internal factors.

Coombs, Dawes, and Tversky (1970) identify three types of experimental events which are also of relevance in discussing the issue of interdependence. They identify experimenter-controlled events, subject-controlled events, and experimenter-subject-controlled events. In experimenter-controlled events the response of the subject on the next trial is dependent only on the experimenter's action in the previous trial and not on the subject's own behavior. In the subject-controlled events it is only the subject's past behavior which affects behavior on the next trial and not the experimenter's action. And in experimenter-subject-controlled events both the subjects' and the experimenters' responses determine the future action of the subject.

The only types of interaction of concern here are those in which the actions of one system are affected by the actions of another system. It may also be true that the past actions of a system affect that system's own future actions. But those situations in which a system is totally unaffected by the actions of the other system are not of concern for this research.

When social interaction occurs there must be some form of interdependence between the different participants. The precise form of that interdependence, its magnitude, and the magnitude of that interdependence relative to the participant's dependence on his

own past responses or to other variables is an important source of variation in types of interaction.

A More Parsimonious Representation of Decision Tasks

All of this is very interesting, but how useful is this representation for understanding social interaction? For example, is it really feasible to create a decision tree describing the interaction which might occur in a police-citizen encounter such as the one described in Figure 2.1. If each of the 12 interactions in this example is considered to be a state, then each of them is also a decision node at which there are a number of possible alternative courses of action which the participant may take. To completely specify the decision task for this encounter it would be necessary to include all possible alternatives and all of the decision nodes which would be involved for the entire task. It is likely that if other alternatives had been chosen the interaction could have proceeded in a quite different way and many additional decision nodes would be made possible by the different alternatives. Even very short encounters between two or more participants are very complicated and offer an infinite number of possible states. In fact, most realistic decision tasks are clearly too complex to be analyzed using this decision tree representation. It is necessary to greatly simplify these processes before they can be analyzed.

One technique for doing this is to collapse the decision tree into a single matrix which can represent the entire interaction in a much simpler form without losing its essence. This matrix is called the normal form or outcome matrix representation of the

decision task--as opposed to the extensive form or decision tree representation which we have been using (Singleton & Tyndall, 1974; Thibaut and Kelley, 1959; Luce and Raiffa, 1957).

The key to the normal form representation of decisions is the concept of strategy. If all the strategies of the primary decision maker are listed as rows and all those of the other decision maker are listed as columns of a matrix, then the terminal states associated with the combination of alternatives from the two actors can be listed in the cells of the matrix. This process, in effect, takes multiple decision tasks and finds an equivalent task which involves only one decision for each actor (e.g., the choice of strategy) which is some logically possible combination of the multiple decisions that actor could make in the original decision task.

Such an outcome matrix is presented in Figure 2.4. In this figure there are A_n strategies for the primary decision maker (represented by the rows) and C_m strategies for the other decision maker (represented by the columns). Each cell of the matrix contains the outcome, O_{ij} , associated with that terminal state which results from the primary decision maker's choice of strategy "i" and the other decision maker's choice of strategy "j".

The extent to which such a matrix is a simplification of the decision task depends on the set of decision nodes in that task. When there are a series of very different decisions with different alternatives for the actor then this matrix becomes very complicated and offers little simplification of the model. The number of distinctive states required to describe such a decision task would

FIGURE 2.4

THE NORMAL FORM DECISION MODEL

ALTERNATIVE STRATEGIES FOR OTHER PARTICIPANT

ALTERNATIVE
STRATEGIES

THE DECISION MAKER

	C_1	C_2	...	C_j	...	C_m
A_1	O_{11}^*	O_{12}		O_{1j}		O_{1m}
A_2	O_{21}	O_{22}		O_{2j}		O_{2m}
\vdots						
A_i	O_{i1}	O_{i2}		O_{ij}		O_{im}
\vdots						
A_n	O_{n1}	O_{n2}		O_{nj}		O_{nm}

* In some cases there may be additional outcomes for the other participant. Cases in which this would be true will be pointed out in later sections.

produce a huge matrix. On the other hand, when the same alternatives are available for many decision nodes, or when alternatives which are available may be classified in similar ways for all decision nodes, then the matrix is considerably simplified. The matrix is sufficiently parsimonious to be useful for very simple events or complex events which are equivalent conceptually to many simple events. The latter may occur in two ways: either alternatives for the actor may be equivalent and their outcomes are also equivalent, or the alternatives may be conceptually different but lead to equivalent outcomes and hence obviate the need for distinguishing among those different alternatives (Camilleri, et al., 1972) To summarize, such a matrix is clearly useful for decision tasks with repetitive decision nodes, for those tasks containing a series of decisions having underlying similarities, for tasks having situations which people define as similar, and for tasks in which only a few distinct outcomes occur.

The critical factor affecting the parsimony of this approach is the analytical classification of the possible strategies. Unfortunately, in most cases there is no routine procedure for conceptualizing such phenomena (Camilleri et al., 1972). Although any decision task in extensive decision tree form may be reduced to normal matrix form, that reduction may not always contribute to greater simplicity.

Fortunately, in the area of social interaction there have been extensive efforts in the past to develop schemes for categorizing social interaction which may be used in observational research to code the interaction which occurs. Early work in this

area was conducted by Bales and Stradtbeck (1951), Flanders (1969), and many others. More recent work has resulted in very extensive coding schemes such as that by Wallen and Sykes (1974). A typology of nodes of influence presented by Tedeschi, et al. (1973) is also a set of categories of this type which could be used in observations of social interaction. Once a set of categories are established which are mutually exclusive and exhaustive and which offer promise of measuring meaningful components of the interaction process which occurs, then it is possible to conceptualize social interaction in terms of a normal form representation of the decision task.

For example, in the interaction code developed by Wallen and Sykes (1974) (which, incidentally, was developed specifically for police citizen encounters) there are a number of categories which may be approximately characterized as follows:

- 1) redirective response,
- 2) compliant response, and
- 3) negative response.

For the sake of argument, let us suppose that these categories could be used to describe all of the interactions which occur in a particular police-citizen interaction such as the example cited earlier. Of course these categories would not capture all aspects of that interaction, but a reasonable argument might be made that they capture interesting aspects worthy of study and amenable to study by themselves in this form. Then,

this would make it possible to greatly simplify the conceptualizations of this task.*

Types of Decisions

The decision task plays a very important role in the processes underlying social interaction. It consists of the many constraints which act upon the behaviors there. It is likely that the character of the social interaction which takes place is very much affected by the task. As a result, in past efforts to develop theories of decision making or to examine empirically particular decisions it has been necessary to distinguish between decisions and interaction occurring in different tasks.

There are many characteristics of tasks which would clearly be important here. However, it would be going too far afield from the main objective of this research to develop an extensive typology of tasks. Rather a number of important factors will be

*This also makes it possible to examine change in the interactions over time. Unless the events are comparable, at least in some respects, there will be no basis for determining whether the decisions change or stay the same. Change can only be defined with respect to some standard or reference point which is constant. If there is nothing about the series of decisions which is comparable across time then there is no basis for assessing change or lack of change. The consideration of the same categories at each point thus makes it possible to employ notions of change and stability in analyzing social interaction. The impetus of this for the examination of process should not be discounted. The real strength of a decision theoretical perspective appears to lie in its application to processual phenomena. Sociologists, although they frequently point out their concern with process (e.g., see Olsen 1968), have been notably negligent in developing adequate theories of processes and have generally not come to grips either empirically or theoretically with the processual characteristics of phenomena. It is hoped that this study will make some progress in that area.

mentioned, then four which have received the greatest attention in the past will be discussed in greater detail.

Some of the characteristics of tasks which surely would have important effects on the nature of the social interaction which occurs would include the following: the number of participants (i. e., this distinguishes learning theory from 2-person game theory, coalition theory, and so on--very different processes are possible with varying numbers of people); the complexity of the task (i. e., the number and variety of alternatives available, the length of the task, and so on); roles (i. e., in many common social interactions the participants are interacting not as individuals but are enacting particular roles such as teacher, student, police officer, judge, and so on); the nature of the outcomes for the participants (e. g., Foa and his colleagues and Blau (1964) have pointed out very different properties of exchanged commodities such as their concreteness, their symbolic character and so on which might affect the nature of the interaction; and a fundamental distinction in Lewin's field theory is the distinction between approach and avoidance situations involving desired or dreaded potential outcomes); and an entire family of measures of the power the participants have over each other, their relative dependence, and so on (e. g., Tedeschi and others (1973) have pointed out many of these; the power literature is replete with different notions of the dependence and power among participants; Thibaut and Kelley (1959) in exchange theory have distinguished fate control and behavior control; and so on).

Three aspects of tasks used most commonly in the literature

on decision makers to distinguish fundamentally different decision tasks are as follows: 1) the amount of information available, 2) the nature of the other participants, and 3) the degree of coincidence of interests of the participants in the decision process. Of critical importance is the amount of information available about the probability of specific choices by the other participant, p_j . Three possible levels of information--uncertainty, risk, and certainty--are commonly distinguished. These are illustrated in Figure 2.5.

In decision making under uncertainty the decision maker has no information regarding the probabilities. In decision making under risk the decision maker knows the probability distribution of the other participant's choices. In decision making under certainty the decision maker knows with virtual certainty the exact choice of the other participant. This distinction is important because if there is no information about the probabilities of the other participant's decisions then those cannot be used as a basis for the decision process. When there is knowledge of the distribution of choices then it can be used to determine the decision strategy of the decision maker. And when the other choices are known with virtual certainty then the decision situation and the behavior of the decision maker becomes greatly simplified.

The degree of coincidence of interests of the participants is important for circumstances in which the other participant is also a purposeful system. This may vary from pure coincidence of interests (both participants prefer the same outcomes); to partial conflict of interest (some outcomes are preferred over others by both

FIGURE 2.5
SPECIAL CASES OF THE DECISION MODEL

CHARACTERISTICS OF THE OTHER PARTICIPANT	PRECISE CHOICES KNOWN WITH VIRTUAL CERTAINTY	PROBABILITY DISTRIBUTION OF CHOICES KNOWN WITH VIRTUAL CERTAINTY	NO INFORMATION AVAILABLE
NATURE	(1) Decision Making Under Certainty	(2) Decision Making Under Risk	(3) Decision Making Under Uncertainty
SOCIAL ACTOR	(4)	(5)	(6) Decision Making in Presence of Strategic Other

participants, but in other instances outcomes preferred by one are preferred by the other), to pure competition (the most preferred outcome for one participant is the least preferred for the other participant). Clearly, the ways in which the other participant is considered by the decision maker would vary considerably depending on the coincidence of interests. In the case of pure coincidence of interests, for example, we might expect the primary behavior exhibited to be efforts to communicate and to coordinate actions. In pure conflict of interest, on the other hand, there would be efforts to prevent the other participant from anticipating one's own behavior in order to prevent that participant maximizing his outcome at the decision maker's expense.

The critical aspect of the nature of the other participant is whether the other participant is another purposeful system or not. The key notion here is whether the other participant can change his behavior with some freedom of choice based on the actions of the decision maker. When this is the case the decision maker must take into account the other participants' reactions to his behavior in the decision process. This opens the possibility for a number of much more complicated behavioral processes including taking the role of the other, symbolic interaction and so on. The use of these three characteristics to distinguish fundamentally different decision tasks for investigation involving substantially different processes is very common in the literature. Moore (1954), for example, in work which is on the fringes of literature relevant to decisions identified three folk models characterizing how people tend to relate to their environment; puzzles, games of chance, and

strategies. These correspond in large part to cells 1, 2, and 4 respectively. Authors more centrally oriented to decision theories such as Luce and Raiffa (1957) and Coleman (1973), and Ewart, et al., (1974) have also identified these same four sets of circumstances. The first three cells are uniformly labeled decision making under certainty, decision making under risk, and decision making under uncertainty respectively. The fourth cell is variously labelled a game (Luce and Raiffa, 1957), conflict (Ewart et al, 1974), or decision making in the presence of a strategic other (Coleman, 1973).

Decision Processes

The connection of the interaction sequence to the decision task in which it occurs has proved to lend some new insight into at least some of the aspects of that interaction by making salient some of the possible constraints which may operate and by making the decision making character of the process generating that social interaction more clear. But this alone is not enough. If the examination of the interaction sequence is to go beyond the level of simply describing an observable phenomena then it is absolutely essential that the underlying processes which produce that interaction be investigated. And before they can be investigated they must first be conceptualized.

Fortunately, there has been a great deal of work conceptualizing decision processes in a wide variety of social science literature (i.e., social exchange theory, game theory, learning theory, equity theory, coalition formation theories, and a wide variety of economic theories). Unfortunately that theorizing has been

carried out in very many widely separated areas of the literature with little sharing of ideas, major differences in perspective, and the absence of any overarching theoretical framework providing order and continuity in the development of this area.*

Here an effort will be made to present in one integrated framework a comprehensive conceptualization of possible decision processes which may underlie the observed social interaction and the nature of the connection of those processes to the observed states.

The connection between the observable interaction sequence and the decision processes which underlie it is not an automatic one by any means and that connection needs to be very deliberately and directly cultivated. The key to connecting the empirically observed sequence of interactions to underlying decision processes lies in that elusive notion of economists--utility.

Utility

The concept of utility forms the key link between observable interaction sequences and underlying decision processes because decision processes have been conceptualized in terms of goal-directed behavior. Utility may be defined as the preference for or the value placed upon different outcomes by the participants.** Such a concept is critical because it is on the basis of this that

*The closest approaches to such a framework recently completed by Ackoff and Emery (1973) or Kuhn (1974) have failed to consider most of the major contributions from these divergent areas and have failed to tie the theoretical framework to empirical research.

**There is the potential for circular logic in the definition of utility and the concept of rationality (a concept which will be developed in a later section on decision rules) when care is not taken to insure that they have independent definitions. This issue will be discussed in detail in that later section.

the participants compare and select alternatives. Without some concept of utility it would not be possible to predict from a theory which alternatives the participants might choose. Without some empirical measure of utility it would not be possible to test those predictions.

There have been many extensive theoretical developments of the concept of utility including many axiomatic treatments of utility (e.g., see Von Neumann and Morgenstern, 1947; Luce and Raiffa, 1957; Arrow, 1951; Fishburn, 1973; Newman, 1965). These treatments have many differences and points of contention which remain. Elaboration of these axiomatic treatments is far beyond the scope of this paper. However, there are a number of characteristics of the concept of utility upon which authors seem to agree. Clearly utilities are subjective. The utility of one outcome for one person may be very different from that for some other person (e.g., the utility of a house for one person who lives nearby may be quite different from the utility of that same house for the person who was born in it and has lived there for his entire life). In addition, very serious problems arise when one tries to compare the utilities of commodities between persons (e.g., it is not possible to say that person A has a greater utility for something than person B). The basis for such comparisons has never been demonstrated and such comparisons should always be avoided.

Unfortunately, the concept of utility at this time, although extensively developed theoretically in a variety of treatments (e.g., see Becker and McClintock, 1967, for discussion of a range of such treatments), has not been nearly so well developed empirically. The empirical determination of utilities for

experimental situations, to say nothing of other less rigidly constrained situations, requires extensive experimentation and is generally not available. For this reason empirical analysis of decision tasks must either include extensive empirical determinations of utilities or be designed in such a way as to minimize the importance of precise determinants of such utilities.

For some types of situations it may be possible to ignore in large part many of the characteristics of utilities and simply treat the objectively defined outcomes as crude indicators of their utility for the participants. For example, it can generally be assumed that more of some desirable commodity is preferred over less of it (e.g., more social praise is better than less; more positive affective comments are preferred over more negative affective ones). Such a procedure is difficult to avoid in many situations in which accurate measures of utility would require extensive experimentation and would so sensitize the subjects that effective analysis beyond the determination of those utilities would be precluded. In such situations, however, it must be constantly kept in mind that these are indeed extremely crude measures of utilities and should be treated with great caution. This procedure is particularly suspect in situations in which the interaction takes place over a sufficiently long period of time so that the utilities for particular outcomes may change (e.g., a police-citizen interaction which is carried out to completion may be an example of a situation in which the marginal utility of continued interaction has decreased to a point where the two participants no longer desire to maintain the relationship and it is broken off). In such cases the

researcher must be alert for the possible effects of changing utilities and those effects may be treated as possible explanatory variables.

Foa and his colleagues, Blau, and others have argued in effect that this concept needs to be extended and generalized to encompass social phenomena in general and not be limited to economic decisions. To some extent there has been a relatively successful extension of this concept. However, there is much work yet to be done. It is apparent that utility should be thought of as a very complicated multidimensional variable reflecting the relative value of a number of different aspects of outcomes not on one dimension but in a multidimensional space where each dimension represents an independent facet to be maximized. One way to help extend this concept is to consider the great variety of outcomes which result from social interaction and the ways they may combine to produce some overall decision.

Assessment of Outcomes

Recall that one of the major types of variables identified in the diagram of the normal form of the decision process in Figure 2.4 is the outcome of the decision task. For each participant consisting of at least one social actor a critical aspect of the theory hinges on the identification of that participant's assessment of this outcome. This assessment is a step in the process of decision making which that participant goes through. There are at least two important aspects of this assessment procedure: 1) the types of variables which are considered and 2) how they are combined by the decision makers to produce some overall assessment.

Variables may be considered for at least two reasons; for their value content or for their information content. In a communication, for example, a slight nod of one participant's head may have little consequence for the valued outcomes for some second participant, but it may have tremendous consequences for the amount of information possessed about the situation by the second participant and may drastically affect the processes which occur. The concern here is only with variables considered for their value consequences. The focus of this research is on transactions. But the presence of other types of interaction and the relationship of that to transactions should not be overlooked.

Another issue which must be addressed here, but which is not central to this section is the issue of marginal utility and decision making. Essentially, some authors posit that reasonable people (or other social actors) would make decisions based solely on future and present outcomes, ignoring past outcomes. They argue that past outcomes are already beyond changing and the most logical process would be to consider only new outcomes.

In this research, however, there is a need not to assume too much about behavior, but rather to treat issues such as this as issues to be resolved empirically. If this were true, then past outcomes would not be considered at all. This would greatly simplify the model of how these variables combine to produce an overall assessment of outcomes. But surely there are several instances of situations in which past outcomes are considered and do affect decisions. Interactions between the Palestine Liberation Organisation and Israel are a good example where one act of

violence is followed by retributions and other acts of violence. Other acts of revenge based on past outcomes are plentiful in many different settings. The paying back of favors done in the past are also very important aspects of human behavior and may be one of the strongest bases for machine politics in cities such as Chicago (where votes are often cast in return for past favors). There also are norms which encourage such behavior such as the expectations for helping friends in times of need and so on. Whether these specific actions are based on considerations of past outcomes is not entirely clear. But surely that issue may be resolved empirically.

A number of types of variables may be considered in assessing outcomes. Certainly both benefits and costs accruing to an actor for a certain choice of action would be considered in assessments of outcomes. By reward is meant here some desirable outcome. Costs are desirable outcomes denied or undesirable outcomes. Two types of costs may be distinguished: opportunity costs, "the dissatisfaction of having to avoid, give up, or do without what we would like to approach, keep, or acquire," and disutility costs, "the dissatisfactions of having to approach or accept what we want to avoid" (Kuhn, 1974:107).

Benefits and costs may also be distinguished by whether or not they are contingent upon actions of the decision maker or upon other conditions beyond the control of the decision maker. In most cases there are probably a number of costs and benefits which are known to occur for a specific alternative course of action in addition to a number of costs and benefits which are contingent upon either the

conditions beyond the control of the decision maker or the actions of the decision maker. For example, a family might know that in choosing to buy a second car as a way of solving their transportation problems there will be a number of costs and benefits which they will have regardless of what the conditions are. But in addition, there are a number of costs and benefits which are contingent upon the conditions which prevail (e.g., the car is a lemon or the son who will be driving more with two cars in the family is a reckless driver and more likely to have accidents now, and so on), or contingent upon the actions of the family (e.g., the choice of a station wagon has certain costs and benefits associated with it as opposed to the choice of a sedan).

Kuhn (1974:107-8,111) discusses many ways costs and benefits vary in the time relative to the decision in which they come into play. Some costs and benefits occurred prior to the decision, in which case they may be thought of as past costs and benefits or sunk costs and benefits. Such costs and benefits are not contingent upon the decision at hand but may have been contingent upon earlier decisions which have already been made. They are given and inalterable regardless of which alternative is chosen in the present decision. This aspect of the outcomes may not be changed, although the decision maker's evaluation of it may change (e.g., internal changes in perceptions and evaluations may result in decreased perceived costs). There may also be present costs and benefits. These would be costs and benefits experienced at or very near the time of the decision (e.g., the immediate costs involved in purchasing a house). Present costs may be broken down into

fixed and variable costs. Fixed costs are those which are incurred to prevent deterioration of or to maintain the cost/benefit ratio (e.g., to maintain the status quo) at its present level. Variable costs are those incurred to produce current benefits. There also may be future costs and benefits. Investments are defined as costs currently incurred for future benefit.

Costs and benefits also may differ in the extent to which they have long-term or short-term consequences. Some costs and benefits may have only temporary consequences such as the price of admission to a theatre. Others may have consequences lasting over long periods of time, such as the costs from a handicapping injury suffered in an auto wreck.

These variables are summarized in Table 2.6. The ways in which the utilities for these different outcomes might combine to produce an overall assessment of outcomes remains to be explicated. But they clearly illustrate the complex problem of evaluating utilities in order to come to some decision.

Comparisons

Once some notion of value is general enough to encompass reasonable ways in which outcomes might be assessed for social interaction rather than requiring that all social interaction be redefined in terms of some unidimensional criterion it then becomes possible to discuss intelligently some of the ways in which participants might reasonably make decisions about how to interact. Two of the major aspects of these decision processes which have been given some attention in the past are 1) comparisons and 2) decision rules. Making comparisons is a general process which is likely

TABLE 2.6

CHARACTERISTICS OF VARIABLES CONSIDERED IN EVALUATING OUTCOMES

Contingent - Known

Longterm - Shortterm

Benefits - Costs

Opportunity Costs - Disutility Costs

Variable Costs - Fixed Costs

Past - Present - Future Benefits and Costs

to occur in all circumstances in which there are alternative courses of action from which to choose. A number of particular types of comparisons have been discussed in the literature. Those types of comparisons and this aspect of the decision process will be discussed here.

There are two types of comparisons which are commonly noted in the literature on exchange theory (e.g., see Thibaut & Kelley, 1959; Blau, 1964). These are the comparison of outcomes for a particular activity with the general expectations for such activities, and comparison of outcomes for that activity with those for available alternatives. Blau refers to these comparisons as comparisons of particular expectations (expectations for the particular chosen activity and the conditions surrounding it), general expectations (expectations an individual has of the total benefits he will achieve in various aspects of his social life), and comparative expectations (expectations for the profits individuals expect in other possible activities). Thibaut and Kelley label the criteria for these comparisons the comparison level (CL) and the comparison level for alternatives (CL_{alt}) respectively. Blau labels them fair rates of exchange and going rates of exchange respectively.

In addition, Ekeh (1974) points out two other types of comparisons: interpersonal and intrapersonal. These are associated with two types of exchange: profitable exchange and fair exchange. In profitable exchange the "individual is comparing his own rewards with his own costs." In fair exchange "he is comparing his rewards, costs, or profits--either jointly or severally--with those

of his exchange partner in arriving at his own assessment of his position." The former involves intrapersonal comparisons, the latter, interpersonal ones.

Three major categories of exchange comparisons may thus be identified:

- 1) Viability of alternatives--comparisons by either the primary decision maker or the other participant which determine viability of the interaction relative to specific available alternatives. (This corresponds to CL_{alt} for Thibaut & Kelley, to comparative expectations according to Blau, and it is an example of an intrapersonal comparison according to Ekeh.)
- 2) Satisfaction--comparisons by either the primary decision maker or the other participant which determine whether the interaction is satisfying relative to what the primary decision maker or other participant would normally expect out of such an interaction. (This corresponds to CL for Thibaut and Kelley, it corresponds to general expectations for Blau, and it is an example of an intrapersonal comparison according to Ekeh.)
- 3) Social justice--comparisons of outcomes of the primary decision maker with outcomes for the other participant which determine whether the primary decision maker (or the other participant) is receiving his fair share of the benefit from the interaction. (This is an example of an interpersonal comparison identified by Ekeh.)

An important issue with regard to comparisons with expectations is that those expectations must be based on something. Typically they are based on similar experiences of the social actor

in past situations. The key issue here is "similar experiences." How does the actor determine which experiences are similar and which are not? This is a very important issue and one which has been addressed in other contexts (e.g., the problem of the definition of the situation in symbolic interactionism). A similar issue arises with regard to the social justice comparisons. What other participants are chosen by the actor for purposes of comparison? Are they other actors with whom he/she is interacting (e.g., the neighbors with whom they share a driveway), other actors interacting with the same general environment (e.g., comparing treatment of other families by the same police force), or simply other actors who are similar to them in some way (e.g., other middle class white families in suburbs)? This issue also has been addressed perhaps more directly in other areas (see the literature on reference groups) and will not be dealt with in detail here. This issue is particularly important because it reflects on the situations for which social justice comparisons may be made. Clearly, they may be made even when there is only one social actor in the immediate interaction (e.g., comparison of outcomes for this family with that of the Smiths next door in their purchase of a new automobile). They also have implications for the occurrence of generalized exchange relations (as discussed by Ekeh, 1974). In a generalized exchange situation there are bases for comparison of outcomes with those of other participants somewhat remote in the exchange network. In dyadic exchange the two participants interact in isolation with no relevant connections to others.

Another important aspect of social justice comparisons is

that they require a great deal more information than required by comparisons with expectations or with alternatives. For such comparisons it is necessary to know the outcomes and the utilities for the other participants. True, limited comparisons may be made with less than complete information, but clearly more information is required for these comparisons than for the others.

For social justice comparisons the notion of simple rationality or a maximization of outcome also becomes more suspect as attempts to make it desirable for the other actor to continue with the interaction (particularly in voluntary exchanges) become salient.

Such comparisons may be represented in terms of the symbols used here as follows:

comparison with alternatives: $O_{ij} - O_{alt}$

comparison with expectations: $O_{ij} - O_{exp}$

social justice comparisons: $O_{ij} - O'_{ij}$

$$(O_{ij} - O_{alt}) - (O'_{ij} - O'_{alt})$$

$$(O_{ij} - O_{exp}) - (O'_{ij} - O'_{exp})$$

Notice that there are three possible social justice comparisons: one comparing outcomes for each participant, a second comparing the divergence from alternatives for each participant, and a third comparing the divergence from expectations for each participant. (E.g., the outcome for one participant might be far greater than his expectations while that for the other might be far less than his expectations.)

Although several basically different types of comparisons have been identified, no work has considered how they might act simultaneously in decision making processes. Homans (1961), according to Ekeh (1974), has tended to confuse them. And Ekeh (1974) has argued that social justice comparisons (he calls them interpersonal comparisons) may occur in one context and intrapersonal comparisons in other contexts (e.g., "In two-person groups whose members are engaged in restricted exchanges interpersonal comparisons and fair exchange prevail. In multi-person groups, with an emphasis on generalized exchange, intrapersonal comparisons and profitable exchange prevail." p. 131). On the other hand, Blau and Thibaut and Kelley have both argued that the satisfaction comparison is the basis for the satisfaction of the actor, and the viability of alternatives comparison is the determinant of whether or not the actor will continue the action (e. g., they each contribute to different aspects of the decision process).

It is more likely that each of these types of comparisons may play a major role in particular decisions, and each may contribute to a number of aspects of that decision process. The extent of that role and the way they combine with each other is a function of a number of factors including the particular type of problem, the character of the participants, and other variables. These would be likely to interact rather than to have additive effects--the impact of one comparison on the decision varies with the results of other comparisons. The extent to which each of these played a major role in any one decision would depend upon the divergence of the result of that comparison from common results

(e.g., the greater the social injustice the more impact that comparison would have on a specific decision). In some cases one may be more important than the others.

Decision Rules

Perhaps the most interesting aspects of the decision process are decision rules. Decision rules are summarizations of the behavior taking place in decisions which connect each set of conditions and each actor to the alternative which is chosen. Mathematically, a decision rule may be conceptualized as a function which maps the space defined by task conditions and participant characteristics into the action space associated with that decision task.

The notion of decision rule can be made clearer by comparing decision rules to other similar notions presented earlier such as the decision nodes and strategies in the description of the decision task. Any particular decision rule may be a choice, a strategy, or some combination of these. A decision rule is some combination of these which predicts correctly the decision which will occur for a specific decision maker in all possible situations for the task at hand. It is less restrictive than strategies in that it may include some combination of them. It need not involve some coherent, rational, or constant strategy, but may involve chains of different strategies or choices. Decision rules describe the behavior throughout an entire interaction involving a series of decisions. A decision rule may be thought of as a "super strategy," being the descriptive counterpart to strategies which are defined on the basis of logical categories of behavior (e.g.,

strategies include logical categories such as the minimax strategy, the maximin strategy, equity, and so on, while decision rules may include combinations of these). A key assumption in this notion of decision rules is that there is some sort of behavior which is stable enough over interactions to accurately describe behavior while at the same time lending insight into the nature of that behavior in terms of logical categories such as strategies or their combinations.

Decision rules may be assessed in two ways: prescriptively and descriptively. Prescriptively, decision rules are useful when they correspond to meaningful, coherent strategies. This is useful because those strategies tell us something about the underlying nature of the individual participant who employs this decision rule. It appears to have identified something important about that participant. And it can be applied in other situations deductively to create hypotheses about the behavior of that individual in a wide variety of settings.

Descriptively, decision rules are useful when they accurately describe the actual behavior of decision makers. A decision rule which is correct 100% of the time and precisely predicts choices is preferred over rules which are less precise (e.g., they only predict vague categories of choice leaving precise predictions to other techniques) or less often correct (e.g., they are only correct 90% of the time).

The goal of course is to obtain decision rules which are both substantially accurate and precise in predictions and at the same time appear to identify consistent underlying characteristics

of the decision makers.

These decision rules constitute the critical link between the conditions of the situation and the characteristics of the participants and the behavior which results. These constitute the necessary element which makes it possible to create a very powerful theory allowing prediction of behaviors on the basis of decisions which take place in certain contexts by certain participants. Without some notion of these decision rules the connection between behavior and these conditions is not made and the theory cannot predict behavior, and it is a very disappointing theory indeed. Consequently it is of utmost importance that some way be found for specifying these decision rules which is both empirically accurate and conceptually insightful into the type of behaviors which occur.

Unfortunately, it is in general extremely difficult to empirically determine what decision rules are operating in decisions (e.g., see Wilcox, 1972). One difficulty is that many decision rules often are compatible (i.e., see Meeker, 1971) and predict the same behaviors in certain situations, so it may not be possible to distinguish which occurs. Another difficulty is that even if there is only a little error in our knowledge of which decision rules occur (e.g., it is known what decision rule occurs 90% of the time), the results of a series of decisions are affected drastically by the precise order of decisions which occur and the conditions which result from those decisions. Slight inaccuracies in the decision rules over a series of decisions may lead to very inaccurate predictions of the dynamic behavior over time. Because of these and other problems in determining the decision rules,

there is a critical weakness in most theories of decisions.

There is a strong tendency for many of these theories to address this problem by positing some decision rules without testing them. This makes the theory deductively viable and allows many predictions. But serious problems arise in testing the theory adequately and in understanding the actual processes which take place when this is done.

The issue of rationality

The most common strategy for past developments of theories involving decisions has been the assumption that a particular type of decision rule characterizes the behavior of the participants: rationality. Precise definitions of rationality vary. But a representative definition is that of Kuhn (1974:124) in which rationality is defined as "the process of selecting the preferred alternative." Rational behavior is simply behavior which is consistent with the decision maker's preferences. Kuhn (1974: 124-5) points out that this definition does not make clear precisely what types of behavior would not be included. It "comes perilously close to including all behavior."

There is in this definition, in addition to the lack of clarity and precision, a potential for logical circularity in the definition of rationality and the definition of preference or utility functions. Indeed, Edwards (1954) argues that the notion of subjective utility was introduced with the aim and the effect of accounting for what would otherwise have been considered irrational behavior based on objective utility functions. Homans (1961) has been repeatedly criticized for a tendency toward circularity in

connection with his development of exchange theory in his treatment of rewards and rationality notions. Coleman (1973:35-37) also discusses the problems of tautology in the definition of rationality and utility, arguing that either may be measured empirically and used to predict the other and those predictions may be tested empirically, but it is necessary that independent empirical measures of each be obtained in order to avoid creating a tautological fallacy.

An additional problem with rationality is that, while it may be possible to identify particular behavior which is rational for some types of decision situations, for other situations there are a number of alternative types of behavior each of which may be viewed as rational, but these are not always consistent with each other. Rapaport and Chazrah (1965) argue that game theory which is based on a prescriptive notion of rationality is not useful beyond zero sum two person games. Becker and McClintock (1967:268-9) address the same general issue in pointing out that for two person nonzero sum games there are a number of possible strategies which may be chosen beyond pure competition. For example, three such strategies they point out would be maximizing joint outcome for both self and other, maximizing own outcome regardless of the outcome for the other participant, and maximizing the difference between one's own outcomes and those of the other participant. The particular form rational behavior might take may vary drastically for different decision situations. Some of these forms for many situations have been discussed in Coleman (1973).

Normative models prescribing rational behavior have also failed

to adequately describe empirical results. For the development of a descriptive theory of decision making such as is the task of this paper, this is a serious problem. As Becker and McClintock (1967: 269) suggest, such efforts to construct prescriptive models for non-zero sum games have generally been unsuccessful. Efforts for other types of games also have been somewhat unsuccessful. And even for zero sum games the results have not been entirely in favor of such prescriptive models. The utility of prescriptive notions such as rationality appears to be much more associated with normative theories and normative uses than with empirical theory.

Notions of rationality imply certain information processing and search capabilities, and a willingness to invest a certain amount of effort in the decision making process. Some effort has been made by many authors to develop notions of rationality which explicitly take into account the constraints of different decision tasks (e.g., the amount of information available, the likely motivation level, the degree of information processing skill required, and so on) (Simon, 1955; Tversky, 1972). Others quite rightly have argued that explicating notions of rationality within different constraints blurs the distinction between normative and descriptive behavior (Becker and McClintock, 1967:241). Two such attempts are the notion of satisficing (Simon, 1955) which posits a limited form of rationality based on lesser capabilities or less willingness to process and search for information. Essentially what this rule claims is that the decision maker will search only until he finds an alternative which is acceptable, and will not continue to search for some optimal alternative. Another example

of this sort of decision rule which is essentially a concept of rationality modified to account for limitations in information processing and search efforts and capabilities is the notion of "elimination by aspects" of Tversky (1972). His thesis is that different aspects will be considered in turn and alternatives eliminated according to particular aspects until one alternative is chosen. A dissenting note with respect to this issue has been provided by Kuhn (1974) who argues that the types of decisions made (e.g., decisions based on marginal utility) actually may involve much less effort and ability than it appears, and are very likely to fall within the information processing capabilities of most people.

One might argue, as Kuhn does, that the notion of rationality specifically, and decision theory in general is oriented more around organizations than individuals. The amount of effort, the time, the number of calculations, the enormity of the information search process, the clearly defined limited goals, and the measurable outcomes in terms of money which are possible for organizations are all compatible with the lines of development most theories of decisions have taken (e.g., linear programming, decision making under uncertainty, game theory, etc.). Most of these theories (and particularly the notion of rationality) are based on analyses of critical decisions rather than day-to-day decisions. Such decisions are more commonly the province of firms with the resources to carry out extensive decision making processes.

At this point a clear approach to the issue of rationality is not precisely indicated. However, there are certainly some elements which approaches should have. First, it should be

recognized that notions of rationality must be limited by the information processing and search capabilities and the amount of effort decision makers might reasonably spend on particular decisions (although the extent of that limitation should not be overestimated). Secondly, it should be recognized that rational behavior may take a number of forms in many decision tasks and any notion of rationality must take into account that variety. At the same time the notion of irrationality should not be used as a dumping ground for as yet unexplained phenomena. A basic proposition of this work is that the great bulk of decision behavior is consistent, understandable behavior which awaits comprehension. Some flexible notion of maximization of benefits and minimization of costs (which may take a number of forms in different decision tasks) does appear to be useful and in order. But the precise form of that process is not yet clear. Kuhn (1974:125-8) has pointed out problems with a number of such general notions. Further consideration in this area is clearly desirable. Clearly, an approach aimed at providing empirically accurate descriptions of actual behavior must not casually assume a particular form of rationality but must instead be concerned with developing a method for empirically testing the actual decision rules which are used.

Examples of Decision Rules

There are two primary sources of decision rules in the literature: those suggested in statistical decision theory and those suggested in the exchange theory literature. Those rules from statistical decision theory (e.g., see Ewart, et al, 1974) are characterized by their use of "viability of alternatives" or

"satisfaction" comparisons for the decision maker. These are comparisons by the decision maker of his actual outcomes with those expected in general or those expected for some particular alternatives. These rules do not concern themselves with the utilities of the other participant except in so far as those utilities might help predict probabilities of the incidence of different conditions. These rules are perhaps more likely to be used when there are not repeated interactions with another social actor because they maximize the outcomes for the primary decision maker without regard for the consequences for the other participant, and make no attempt to insure that the other participant would receive enough net benefit from the exchange to continue interaction.

A number of decision rules commonly discussed in these two areas of the literature are listed in Table 2.7. Notice that the decision rules suggested by statistical decision theory are separated into three categories: those which consider utilities only, those considering probabilities only, and those considering both.

These decision rules may be expressed as follows:

- 1) Maximin criterion--This decision rule seeks the best payoff that a decision maker can be assured of. The decision rule is to determine the minimum possible payoff for each act and then select that act for which this minimum possible payoff is the maximum. The decision criterion for this rule expressed in the terminology developed in Figure 2.7 for the normal form outcome matrix would be as follows:

$$\max [\min_i (O_{ij})]$$

TABLE 2.7
DECISION RULES

DECISION RULES FROM STATISTICAL DECISION THEORY

1. Maximin criterion
2. Maximax criterion
3. Hurewicz criterion
4. Minimax regret criterion
5. Maximum likelihood criterion
6. Expected state of nature
7. Expected monetary value criterion
8. Bernoulli criterion

DECISION RULES FROM EXCHANGE THEORY

9. Reciprocity
10. Equity
11. Distributive justice
12. Status consistency or rank equilibration
13. Competition or rivalry
14. Altruism or social responsibility
15. Group gain

where \min_1 is the minimum value in a particular row, "i", (i.e., the minimum outcome associated with a specific action on the part of the decision maker, "i"), and O_{ij} is the outcome resulting from action "i" by the primary decision maker, and action "j" by the other participant.

- 2) Maximax criterion--This criterion seeks the optimum payoff that can possibly be obtained by the decision maker. The decision rule here determines the maximum possible payoff for each act and then selects that act for which this maximum possible payoff is the greatest. The decision criterion is as follows:

$$\max [\max_1 (O_{ij})]$$

- 3) Hurewicz criterion (pessimism--optimism coefficient)--This decision rule is based on a pessimism-optimism coefficient that emphasizes a weighted combination of the optimal and minimal payoff for each act. The exchange rule here selects that activity which has the highest score on the criterion--that is, the highest weighted combination of the pure maximax and the pure maximin criteria. This decision rule is expressed as follows:

$$(c) \cdot \{ \max [\max_1 (O_{ij})] \} + (1-c) \cdot \{ \max [\min_1 (O_{ij})] \}$$

- 4) Minimax regret criterion--Opportunity loss (regret) is the difference between the actual outcome and the optimal outcome which could have resulted given the choices by the other participants. The minimax regret decision rule selects the act which minimizes the maximum regret. The decision rule is

expressed as follows:

$$\min \left[\max_i (O_{ij}) - \min_i (O_{ij}) \right]$$

- 5) Maximum likelihood criterion--This decision rule identifies the state of nature that has the maximum likelihood of occurring, and the selects the act which has the most desirable value consequences for that state of nature. This decision criterion is expressed as follows:

$$\max(O_{i_{\max}, j}), \text{ where } i_{\max} \text{ is the } i \text{ for which } p_{i_{\max}} = \max(p_i).$$

- 6) Expected state of nature--This decision rule first computes the mean state of nature and then that act is selected which will have the most desirable value consequence if the actual state of nature is close to the mean. The criterion for this decision rule is expressed as follows:

$$\max(O_{i^*, j}), \text{ where } i^* \text{ is the } i \text{ for which } p_{i^*} = \min \left[\frac{\sum (p_i \cdot c_i)}{n} - p_i \right].$$

- 7) Expected monetary value criterion--This criterion calculates the expected monetary value for each act and then selects that act with the maximum expected monetary value. (This, of course, can also be done for value in general, and is not limited to monetary values.) This computation explicitly utilizes both values and probabilities. The criterion for this decision rule is as follows:

$$\max \sum_j [(O_{ij}) \cdot (P_i)].$$

This criterion maximizes the average payoff over the long run.

- 8) Bernoulli criterion--This criterion assumes that all conditions are equally likely and chooses the alternative course of action for which the average outcome is greatest. The criterion for this decision rule is expressed as follows;

$$\max_j \sum (O_{ij}).$$

The decision rules suggested in discussions of exchange theory are generally less formalized than those of statistical decision theory. Precise equations for these rules are not offered, and a number of alternative formal definitions are frequently possible. Here the verbal definitions of these decision rules taken from the literature will be presented.

- 9) Reciprocity--This decision rule requires that a decision maker help someone who has helped him.
- 10) Equity--This decision rule states that a participant tries to get out of an exchange outcomes commensurate with his inputs.
- 11) Distributive justice--This rule is much like the equity rule and states that the decision maker which has higher investments deserves more favorable outcomes. Investments include the values of acts, costs to the actor, and external status characteristics such as age and sex.
- 12) Status consistency or rank equilibration--This rule states that the decision maker will try to distribute rewards on the basis of status on an external dimension.
- 13) Competition or rivalry--This decision rule states that the decision maker will try to achieve more favorable outcomes than the other participant even at an absolute cost to the decision

maker.

- 14) Altruism or social responsibility--This decision rule maintains that the decision maker will try to maximize the outcomes of the other participant even at a cost to the decision maker.
- 15) Group gain--In this decision rule the decision maker tries to maximize the total outcome for both itself and the other participant.

One strategy in the past has been for the decision rules to be analyzed in terms of logical criteria they should meet. In this way a number of criteria may be identified and particular decision rules may be ruled out on the basis of them. This has been done frequently. Perhaps the best example of this is the work of Luce and Raiffa (1957) in which they summarize their analysis as well as those of Savage, Milnor, and others. This is one way the problem might be approached. On the other hand, this research is primarily empirically oriented. Perhaps a better approach for this type of study might be to examine empirical data for evidence of different possible decision rules and not rule out any on the basis of logical criteria which may appear reasonable and logically consistent but which may not be accurate descriptions of actual behavior.

Another distinction which bears consideration is between decisions which are made as a response to past actions versus decisions made in an effort to induce future actions (Coleman, 1973). For example, the decision maker just mentioned in the prisoner's dilemma game may elect to make cooperative choices, even though he earlier had unfavorable experiences, because of the potential

rewards he may receive in future decisions if the other participant is persuaded by his cooperative behavior to also be cooperative. Conceptually, one might argue that decisions based on future events and those based on past events appear to be very different processes. There is undoubtedly some merit to that argument. This distinction even appears to parallel many very significant distinctions made earlier such as that between symbolic behavior and operant behavior (Ekeh, 1974). However, Coleman (1973) argues that while they may appear quite different conceptually, in terms of the empirical analysis they are quite often equivalent. For this reason, while the distinction should be maintained and the different expected behaviors from the two perspectives should be looked for, empirically the analysis will probably be much the same regardless of which process is examined.*

The decision processes which occur have been conceptualized in a variety of contexts as the result of a series of processes including comparisons, decision rules, utility estimations, and assessments based on a combination of outcomes of relevance. In the past those processes have been considered primarily in an economic context or in a social context with largely economic characteristics. But these processes can be effectively applied in other, less economically oriented contexts. Consider, for example, the earlier example of the interaction between a police

*This is probably, in part, a reflection of the general problem that many alternative decision rules may account for the same behavior and determination of the decision rules which actually are used must therefore be very difficult.

officer and a young man stopped for a traffic violation (see Figure 2.1). It is possible to illustrate in that example a possible set of decision processes which might describe the behavior of either participant.

Suppose, for example, that the citizen's actions were guided by the objective of avoiding being given a traffic ticket. His preference might be for almost any combination of outcomes which result in him not receiving a traffic ticket over any combination which result in him receiving a ticket (i.e., calmly taking and acknowledging numerous insults and threats to his self-esteem so long as he did not in the end receive the ticket, would be preferred over even the most cordial and pleasant encounter when he received a ticket). In this case his concern is primarily with the future, long-term, disutility costs of a traffic ticket to the virtual exclusion of every other related outcome of the encounter. In terms of specific categories of action such as those three categories (redirective, compliant, and negative) suggested earlier, he would be expected to respond almost uniformly in the encounter with compliant behavior (at least until he finds out he will certainly receive a ticket). His decision rule then would be to respond uniformly with compliant behavior regardless of the nature of the actions by the officer.

Characteristics Of Participants In The Decision Process

An important aspect of the decision model is the character of the participants in the decision process. There are many different types of participants and those different types have implications for the processes which will occur. Participants are here viewed

from a general systems perspective in which they are conceptualized as information processing systems. There are a number of important characteristics of such information processing systems. These include 1) the types of systems, distinguished by their structure and function; 2) the character of the relationship between the system and its environment; and 3) the process by which specific systems transform the conditions of the decision situation into outcomes and value functions. These system properties constitute the major analytic properties of participants of interest here. In addition, there are a number of other properties of participants which also have implications for the character of the processes which occur. These include many of what have been identified as "individual variables" in the literature (e.g., see Vinacke, 1969). For example, participants consisting of more than two people may be distinguished by the type of social organization they represent (e.g., see Olsen, 1968).

Decision tasks are viewed from the perspective of one participant (the primary decision maker) but may include any number of additional participants. A participant is defined as any system which can or does affect the outcomes of the decision task for the primary participant. For example, if the primary participant is a particular individual, a victim of a robbery, and one of the other participants in the decision task is another individual, a policeman called to the scene of the crime; then the second person is a participant in this specific decision task as viewed from the perspective of the primary decision maker (the victim) only to the extent that he affects the outcomes of the decision task for that

primary decision maker. The policeman is a participant only to the extent to which he can affect the outcome for the victim. Those aspects of the policeman which do not affect the outcome are not relevant to this specific decision task--i.e., the fact that the policeman is a good poker player is not relevant to the victim (except possibly in mystery novels).

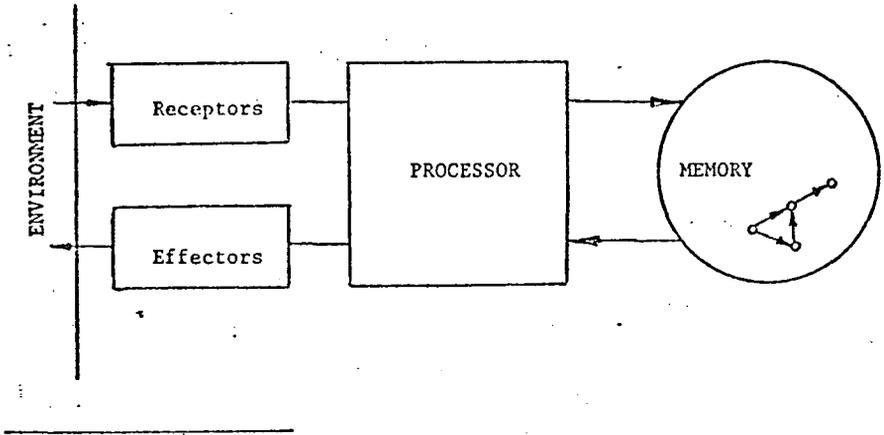
There are a variety of different types of participants which are possible, ranging from natural phenomena (e.g., a machine may break down) to human individuals. The participants may be defined as individuals, groups, organizations, and so on. In many respects their definition is somewhat arbitrary and varies with the purpose of the analysis. For one purpose a group may be defined as a number of individual decision makers interacting with each other; for other purposes they may be considered as one decision maker which interacts as a unit with other decision makers. And the propositions here to be posited for actors are the same whether the actor is a group, an individual, or whatever (Kuhn 1974:105).

The participants in the decision processes are here viewed as information processing systems. The following diagram represents one such view proposed by Newell and Simon (1972:20-21). Such a system includes a mechanism for receiving information from the environment (a receptor), a mechanism for processing such information (a processor), a mechanism for storing results from such processes and allowing past actions and outcomes to affect present ones (a memory), and a mechanism for affecting the environment to produce particular responses (an effector).

Kuhn (1974) offers a similar model of the participants in

FIGURE 2.8

GENERAL STRUCTURE OF AN INFORMATION PROCESSING SYSTEM*



* (Newell and Simon: 1972:20-21)

which he distinguishes three components: a detector, a selector, and an effector. In Kuhn's model the detector is the mechanism by which the participant receives and processes information from the environment; the selector is the mechanism which makes decisions or "selects" behaviors for the system; and the effector is the mechanism which carries out actions by the system.

There are several basic processes which characterize these systems where each process is associated with one of the major components of those systems (Kuhn, 1974; Newell and Simon, 1972). These processes are information search, information processing, decision making, and action taken to effect decisions. It is through these processes that systems affect one another. These constitute the basic processes which occur in the general context of problem solving which occurs for such systems. In social interaction typically most of these processes are relatively straightforward and the interesting aspects of the process lie in the decisions--e.g., participants interacting face-to-face have little problems determining what each other says, in most cases, hence the interesting processes are information processing and decision making. Here the emphasis is on decision making. For other phenomena it might be wiser to concentrate on one or more of the other processes.

This general view of participants as systems is found in many works. But it should not be overlooked that there are many different types of systems which may occur and which have somewhat different processes. It is important to identify clearly which type of system is being considered here.

This issue has been explicitly addressed by Ackoff and Emery (1972:28-31). They identify a number of systems distinguished by their range of possible outcome functions and their range of possible structures. The functions of outcomes may be uni-uni (which signifies that there is one function in all environments), uni-multi (where one function occurs in any one environment but different functions may occur in different environments), or multi-multi (where different functions may occur in any of the possible environments). The structure of actions may vary in a similar fashion, being either uni-uni (where there is one structure in all environments), uni-multi (where there is only one structure in any specific environment but different environments may have different structures), or multi-multi (where different structures may occur in the same or different environments). Together these criteria distinguish a number of different systems.

The primary decision maker in the decision tasks considered in this research must be multi-multi both in function of outcomes and in structure. Such systems are called purposeful systems. These systems "can produce (1) the same functional type of outcome in different structural ways in the same structural environment and (2) can produce functionally different outcomes in the same and different structural environments " (Ackoff and Emery, 1972:31). Such a system can change its goals in constant environments--that is, it can select the goals as well as the means for pursuing them. All purposeful systems considered in this research consist of individuals enacting roles. Any participant including at least one individual is called a social actor. The concern here is only

with phenomena in which two social actors interact with each other.

The type of system must be considered because systems differing in their outcome functions or their structures may be capable of different processes. The conceptualization developed here is only for systems which may take on multiple structures and have multiple outcome functions. These processes do not necessarily apply to the operation of other types of systems.

Another way in which systems may differ importantly is in their relationships with their environment. Two ways in which systems relate to their environments are the effects of the environment on the system and the effect of the system on the environment (Ackoff and Emery, 1972:117-8). Specific systems may be relatively high or low in their responsiveness to the environment (objectiversion or subjectiversion respectively) and relatively high or low in their tendency to change the environment to suit their needs (externalizer or internalizer respectively) (Ackoff & Emery, 1972). Together these two dimensions identify four different types of systems distinguished on the basis of how they relate to their environments: objective externalizers, objective internalizers, subjective internalizers, and subjective externalizers.

These distinctions are important to keep in mind when analyzing the interactions of systems because the observed behaviors may not be the only processes which occur. It is also possible for the systems to effect internal changes as ways of adapting to changing conditions in their environment. Analyses which look only at overt

behavior would not allow such subtle changes to be detected.*

Yet another important characteristic of participants is the connection between their characteristics and the decision situation. This link is absolutely critical in any theory of decision making. Such a connection is provided by Ackoff and Emery (1972). According to them, the normal form or outcome matrix representation of decisions may be summarized in terms of a number of components: the primary decision maker, A, the other participant, B, the available strategies for the two participants, A_1 through A_n and C_1 through C_n respectively, and the possible outcomes from the decision task, O_{ij} for the primary decision maker and O'_{ij} for the other participant. In addition, there are three parameters which they identify for this decision task which summarize the relationships among these variables. These include the probability of a specific choice (familiarity), the efficiency of a choice (knowledge), and the relative value of an outcome (utility) (Ackoff and Emery, 1972:34).

Familiarity, or the probability of a particular choice is defined simply as the probability that a participant, A, will choose a specific strategy, A_1 , given the decision task characteristics (e.g., given the other actor, that other actor's possible strategies, the likelihood of selecting each of those strategies, and the possible outcomes of them). The efficiency of a choice

*Examples of such behavior are plentiful in the attitude change literature and in theories of cognitive balance, dissonance, equity theory, congruity, and so on. Socialization is another example of a phenomenon in which one of the major outcomes is the internalization of different values as a result of interactions with the environment of the individual system.

(knowledge), is defined as the probability that a given choice will result in a particular outcome in a specified decision task given that it is chosen by the actor, A. The relative value of some outcome of the decision situation is defined simply as the preference for each outcome relative to other outcomes for the actor (e.g., the most preferred outcome is that with the greatest relative value) (Ackoff and Emery, 1972:34-35). These three parameters together describe the process of decisions which can occur in such decision tasks.

To summarize, Ackoff and Emery (1972:82) suggest an individual's model of his choice situation consists of what that individual believes to be:

- 1) the courses of action available to the participant,
- 2) the possible outcomes of the available courses of action,
- 3) the possible courses of action of the other participant(s)
(possible values of the uncontrolled variables that can affect the outcomes of available courses of action for this participant),
- 4) the probability that each of the possible states of the choice environment is the true one (familiarity),
- 5) the efficiency of each available course of action for each possible outcome in each possible state of the choice environment,
and
- 6) the relative value of each possible outcome.

Together the three parameters, the probability of choice, the efficiency of choice, and the relative value, describe in general mathematical terms the process whereby a particular decision maker translates the characteristics of the choice situation

into some outcome. Each of these parameters is a function of the choice situation characteristics, and they may be represented as in the following table (Table 2.9).

These parameters describe the outcome which will result. That is, the probability of a particular outcome equals the sum of the probability of each choice times the probability of the outcome given that choice.

$$P(O_1) = \sum_k (P_k E_{k1})$$

Both the probability of choice and the efficiency depend on the properties of the decision task (the available strategies, the possible outcomes, and the choices of the other participant) therefore the outcome is also a function of those variables:

$$P(O_1) = F[(C_k), (O_1), S_a]$$

The particular decision maker then provides a transformation function which transforms the decision task characteristics into some outcome. And the particular characteristics of the individual are displayed in the nature of that transformation. One might expect different systems or different decision makers to respond differently in similar situations and to have different parameter values.

The decision maker also converts these decision situation characteristics into the expected relative value for that decision maker

$$EV = \sum_k \sum_l [P_k E_{k1} V_l]$$

TABLE 2.9

COMPONENTS OF THE CHOICE SITUATION

PARAMETERS OF THE CHOICE SITUATION	AVAILABLE COURSES OF ACTION (STRATEGIES)	POSSIBLE OUTCOMES	ENVIRONMENT (CHOICES OF OTHER PARTICIPANTS)
Probability of choice	$P_i = f$	$\{(C_{k1}),$	$(O_1), S_m\}$
Efficiency	$E_{ij} = g$	$\{(C_{k1}),$	$(O_1), S_m\}$
Relative value	$V_j = h$	$\{(C_{k1}),$	$(O_1), S_m\}$

and hence,

$$EV = P[(C_k), (O_1), S_k]$$

Unfortunately, as Ackoff and Emery admit, these functions are not as operational as they are programmatic. They don't describe the specific functions, but they do suggest a program for research which would determine such a function. There are three types of research, one for each of the parameters. Each examines what choices are made in controlled circumstances. These are as follows:

- " 1) Measures of familiarity derive from the effect that different properties of courses of action have on probabilities of choice displayed in situations in which the course of action chosen has no effect on the outcome. Hence they are measures of means preferences.
- " 2) Measures of knowledge derive from the effect of different efficiencies of choice on probabilities of choice for whose outcomes the relative values remain constant. Hence they are measures of sensitivity to efficiency.
- " 3) Measures of intention derive from the effect that differences in outcome have on probabilities of choice, where each available course of action can produce only one possible outcome and each possible outcome can be obtained. Hence they are measures of ends preferences" (Ackoff & Emery, 41-42).

Basic Modes Of Interaction Between Systems

Closely related to the characteristics of participants are the modes of interaction which are possible among participants.

There is a major distinction which is sometimes made to characterize fundamentally different types of interaction: the distinction between transactions and communications. These are two fundamentally different ways systems may interact. The first is interaction in which one system affects the outcomes of the other system in ways which may induce that other system to change its behavior. The second is interaction in which one system affects the other by an exchange of information rather than a change in outcomes. The first type of interaction may be labelled a transaction; the second, communication.

A transaction may be defined as "any interaction between parties analyzed with reference to its value content to the parties" (Kuhn, 1974:174). Roughly speaking, a transaction is a transfer of anything of value between the two parties. Valued things in transactions may include information, praise, affection, or permission, and are not limited to matter-energy quantities such as money, material goods, and so on. Transactions may be contrasted with communications. Transactions are concerned only with the value consequences of some interaction, not with the informational consequences (Kuhn, 1974:1-5).

Communication, on the other hand, is a process by which one system can affect another system without changing "either its environment or the components of its choice situation." Communication, as defined by Ackoff and Emery (1972:142), occurs when a message produced by one system produces a change in one or more of the parameters identified earlier which describe the relation of that second system to the decision situation (these

parameters are the probability of choice, the efficiency of choice, and the utility of the outcomes). An important aspect of this definition is that both systems must be purposeful for communication to take place--the choice parameters of a system cannot be changed if only one type of action or only one type of structure is possible. In addition, communication may be unintentional, it may take place between systems widely separated in time and/or space, and a system may communicate with itself (Ackoff and Emery, 1972:142).

As Kuhn (1974) is quick to point out, transactions and communications may be related to each other. Any particular interaction between systems may include elements of both value content and information content. The information transfer can affect the value transfer and vice versa. In most situations both of these processes occur simultaneously.

Ackoff and Emery (1972) identify a number of types of communication which are possible based upon the parameters of the choice situation they identified earlier. These include information, instruction, and motivation. Instruction is "a communication that produces a change in the efficiencies of any of a receiver's courses of action." Information is "a communication that produces a change in any of the receiver's probabilities of choice." And motivation is "a communication that produces a change in any of the relative values the receiver places on possible outcomes of his choice" (Ackoff and Emery, 1972:144).

The Case of Police-Citizen Encounters*

Now that the components of this conceptual framework have been elaborated it is possible to return to a consideration of the entire process as it was presented in the overview section. The way in which these factors combine to produce social interaction between two participants can be restated with greater detail.

The phenomenon of concern here is a police-citizen encounter such as commonly occurs in the course of the day-to-day activities of police officers as they are called to the scene of a crime to investigate or as they intervene in a crime in progress, and so on. In these circumstances the officers commonly find themselves interacting with some individual already on the scene--i.e., a victim, or a supposed criminal, or a witness.

Such phenomena are appropriate for investigation utilizing this framework because they involve relatively clearly defined series of interactions which begin when the officers arrive at the scene and which end when they depart. The interaction which takes place, as much or more than other interactions, tends to be focused around a specific issue. At least two individuals are interacting. Both of the individuals may have some impact on the course of the interaction--i.e., they affect the outcomes. And the events which occur have some valued consequences for the individuals (i.e., they have some preference for some outcomes over others; for example, the citizen typically does not want to be

*This conceptualization, when applied to police-citizen encounters, is similar to a conceptualization of such encounters by Sykes and Clark (1975).

arrested).

From the previous empirical studies it would appear that the most important characteristics of police-citizen interaction would be first its processual character, then the task conditions in which it takes place, and only secondarily the characteristics of the individuals. These characteristics will be considered in turn.

Process

Police-Citizen interaction may be viewed, as illustrated in Figure 2.1, as a series of interactions between a police officer and a citizen. Suppose, the police officer, first takes some action which is perceived by the citizen. The citizen then decides what action to take in response, based upon the informational and value consequences of that first action, and the objectives of the citizen. That decision involves a number of subprocesses including an assessment of the net utility of the outcomes for different alternative courses of action; comparisons of alternatives with each other, with expectations, and with the outcomes for the other participant; and any of a number of particular decision rules depending on the specific objectives of the citizen.

The actions of these participants tend to be purposeful. The citizen accused of a crime tends to present information in such a way as to prevent the officer arresting him. The citizen who has been victimized tends to present information and to cooperate with the policemen in order to lead to the punishment of the offender or the relief of the victim's condition. The policeman tends to act in such a way as to meet the requirements of his job-- to determine the necessary information, to mediate between

antagonistic citizens, and so on. The precise goals of the participants may be of a number of types. The policeman may be primarily concerned with avoiding violence, a hassle, or red tape while still fulfilling his obligations. The citizen accused of a crime might be attempting to avoid further trouble with the police, to remain out of jail, to avoid any penalties, and/or to end the interaction favorably as soon as possible.

That decision involves a number of subprocesses including an assessment of the net utility of the outcomes for different alternative courses of action; comparisons of alternatives with each other, with expectations, and with the outcomes for the other participant; and any of a number of particular decision rules depending on the specific objectives of the citizen.

Once the citizen responds this provides feedback to the officer. The officer then must make some decision as to the course of action to take given both his own and the past action of the other participant and the conditions of the task in which they are interacting. The character of this process is very much like a mutually adaptive process where each participant acts in part as a consequence of the other participant. It is a cyclical process of action, reaction, action, and so on.

One of the factors which most accounts for current action in such a process is the past actions which have occurred. Hence, the situation--consisting of the task constraints plus the past actions of the participants--is a very important aspect of this process. For example, one would expect a quite different response on the part of the officer if the citizen had just accused him of police

brutality and called him something obscene as opposed to when the citizen had just responded politely and cooperatively to a question.

This processual character of the phenomenon is not only an interesting factor which may account for some of the variation which occurs--i.e., an encounter is described by more than just aggregate measures such as the average number of cooperative responses. Encounters are also described by the development of the interaction process over time. The interaction sequence is a very important part of the encounter and the consideration of the encounter apart from that leaves out a very important facet of the interaction.

Task Conditions

The most important aspect of differences in police-citizen encounters which are likely to appear are the very different roles which the citizen and officer enact.

Role. Police and citizens generally have rather clearly defined roles which produce a very asymmetric relationship between the two participants. The policeman is obligated to perform certain functions in a variety of such situations, and the citizen is generally expected (those expectations are enforced with a combination of information and formal legal expectations for behavior) to behave in certain ways.

For both participants there are a number of very restrictive legal and normative prescriptions as to their behavior, possible outcomes, and alternative courses of action which are available. Police-citizen interaction, of all types of social interaction,

may be the most rigidly prescribed behavior. Clearly there are very strong legal prescriptive components to this interaction (e.g., the miranda ruling, standard ways of dealing with citizens resisting arrest, the standardized training of policemen, and so on). Great care will probably be taken both by the policeman and the citizen not to violate these prescriptions.

A second characteristic which appears important is that the policeman typically has been extensively trained and has experienced many such encounters before. For this reason the policeman has had a long time to prepare for such encounters and has been able to think out beforehand many of the types of actions which he might carry out in such a situation. For the citizen, on the other hand, such situations may usually be somewhat unique and new. There has been no previous premeditation of possible actions and the citizen may not be aware of all of the expectations for his role, may not have had time in advance to plan behaviors, and may be under shock or under stress from the traumatic event which precipitated this encounter. The policeman is also more familiar with the possible alternatives available in such situations.

A third characterization of many such encounters may be that one of the primary reasons the policeman is there is to gather information. Information search is a basic component of his role. Such information search may not be expected or allowed for the citizen.

Given all of this (the policeman's familiarity with the situation, his information search activity, and his legally defined role, as mediator, representative of the interests of the larger

society, and so on) it is likely that he would be much more likely to take control of the situation and to tend to affect his external environment, and to be less affected by the environment. The citizen (due to his lack of familiarity with the situation and the role expectations, the legal and normative restrictions on his role, and the requirements that he provide information at the request of the policemen) would tend to be more passive, internal in his changes and more affected by the external environment.

The interaction tends to be asymmetric contingent in terms of the contingency of actions for each participant on the past actions of others and of himself. That is, the policeman, due to his training and past experience is much more aware of the alternative courses of action available and the decision rules which may be effective. By right of his legally and normatively prescribed role of authority he also is much more in control of the situation than the citizen. One would thus expect the action of the policeman to be contingent primarily upon his own past actions, being affected relatively little by the actions of the citizen. The citizen, on the other hand, has not had the opportunity (usually) to plan his behavior in advance, is not familiar in some cases with all the alternative courses of action available, may well be traumatized by the events precipitating the encounter, and hence is forced into a more submissive role. The citizen is thus likely to have his behaviors contingent very heavily on the past behavior of the policeman and very little on his own past behavior. The policeman then, for a variety of reasons,

tends to dominate the interaction.*

In addition, in most police-citizen encounters the potential outcomes of the policemen differ considerably from those for the citizens. The police are simply carrying out their job and the outcomes of the interaction will not normally have lasting long-term consequences for them. For the citizen on the other hand, the consequences may be quite dire. If the citizen is a suspected criminal the outcomes may be extremely long-term, extremely negative, and even irreversible. Certainly these differences in potential outcomes will lead the citizens to be somewhat careful in their actions and one might expect them to take fewer risks in the interaction than the policemen.**

Police and citizens are also differentiated in the extent to which each controls both their own outcomes and outcomes for the other participant. In most cases the policeman has greater control over the outcomes of the situations for all participants than the citizen. The outcomes for the police officer, as mentioned before, tend to be of minor importance relative to other outcomes for most such interactions. And those outcomes are primarily under

*The finer structure of such interactions as explicated by the taxonomy of modes of influence of Tedeschi et al (1973) could also be applied here, and probably with some insight. However, the data available for this analysis does not include many of those distinctions and instead includes others. Therefore this discussion will not be carried out here. The interested reader is encouraged to consider some of the insights which may be gained from considering these modes of interaction and which would be likely to occur in this setting.

**Others (i.e., Sykes, Fox, and Clark, 1975) have also suggested a number of decision criteria officers might use in making decisions.

his control. He also has a great deal of control over the relatively important potential consequences for the citizen (i.e., he may arrest the citizen and charge him with a crime).

Other task characteristics. In addition to role there are other characteristics of tasks which may affect the interaction. These include a number of characteristics which affect the character of the role itself. The nature of the role the citizen is expected to play can vary considerably depending upon a number of factors. The citizen might, for example, be accused of violating a law or might be a victim seeking police help. The actions of citizens would be expected to vary considerably with the role they play. There is a need to differentiate among the different roles citizens may be called upon to play in different circumstances. Similarly, the police themselves, have a number of complimentary roles which they enact depending upon the nature of the circumstances (e.g., a service role for a car accident versus an enforcement role for violation of some law).

One of the more complex aspects of police-citizen interaction is the nature of the "commodities" exchanged.* In the course of such interactions verbal and nonverbal behaviors take place which have value consequences for the participants (e.g., positive affective statements and negative affective statements). These value consequences are frequently not easily compatible or comparable. They do not have clear relationships to easily observable characteristics (for example, a slight nod of the head may

*E.g., see Foa (1971), Turner, et al. (1971).

convey more meaning than the entire verbal behavior which goes on in a conversation). Nor do the acts have the same or even comparable consequences for each participant (an activity which is considered highly rewarding and complimentary for one participant may have insignificant consequences for others).

It should not be overlooked that these interactions take place among people who moments before may have been perfect strangers. For such a regulated, standardized, interaction to occur requires some very clear and powerful social interaction processes. It is for this reason that policemen typically wear distinctive uniforms, badges of authority, and so on. The beginning of such interactions is critical for the establishment of the definition of the situation. The police have only limited information as to the situation--information which may frequently be quite wrong--and they are not entirely sure how the citizens involved will interact--e.g., whether they will be hostile and noncooperative, helpful, incoherent, or what. The interaction can progress rather rapidly, hence it is important how fast the participants can process information and cope with the events which develop. One might expect them to make use of stereotypes or socially determined categories of people to help them process information faster. Because of the limited information one might also expect the first few interactions in such a situation to be rather clear actions designed to establish control over the situation and to impose a definition of the situation on the other participants (Fox, 1975). One might expect such a situation to be very volatile at first in the possible types of interaction which may

occur, but to settle down very rapidly into stable interaction.

One of the most critical aspects of police-citizen interaction is that it is an interaction between two or more social actors. Among other things, this means that simple maximization of own benefits is less likely; assessments of the outcomes for the other participants as well as for one's self is likely to occur, and social justice comparisons may be made and may be the basis for much of the interaction which takes place.

In addition, there are many other ways tasks may differ from each other. Many of these differences are very important and have strong effects on the nature of the interaction which occurs. Such tasks may differ in the number of participants present, the level of coincidence of interests, the roles of the actors present, the complexity and duration of the interaction, the available alternatives, the extent to which particular decisions affect future possibilities, and many characteristics which describe the matrix of possible interactions for the tasks in ways which may be important (e.g., measures of fate or behavior control, the concept of power, and so on).

There may be more than one policeman present or more than one citizen. The importance of the precise numbers of people present is mediated by the roles these people play. For example, two policemen may be fulfilling virtually the same role in an interaction so that it is not necessary to distinguish them as separate participants but they may instead be treated as one participant (e.g., "the police") which includes more than one social actor. This is also true for the citizens present. If they play roles which are

very similar or indistinguishable then it may be useful to consider them simply as "citizens", "victims", or "offenders" and not distinguish them as different participants.

The degree of coincidence of interests for the participants may also vary. There may be complete conflict between the citizen (s) and the police (e.g., the situation in which a policeman is trying to apprehend a suspect who is resisting arrest), there may be partial conflict (e.g., a person being beaten up in a fight which he started would share the policeman's desire to stop the fight but would not share the policeman's desire to punish the persons responsible), or there may be complete cooperation (e.g., the victim of a crime would share the goals of the policeman of capturing and prosecuting the offending party).

Encounters also may differ in their complexity and duration. A mass riot in which many police and hundreds of civilians are involved is clearly much more complex than a domestic squabble between a husband and wife with one policeman present. The number of available alternatives and possible courses of actions for participants, and the complexity of the overall interactions in terms of all participants is clearly much greater. Some encounters last only a very few seconds (e.g., a policeman telling a vagrant to move on), while others may last hours (e.g., the questioning of a suspect about a crime).

Modes of Interaction. The primary mode of interaction for police-citizen encounters is generally verbal, symbolic communication. The informational content of such communications is certainly important as is evidenced by its use as evidence in

court trials. It is through this symbolic communication that participants find out about the possible outcomes for them from the interaction, possible consequences they will face as a result of their action, and so on. The informational impact of such statements as "you're under arrest" is very great.

But there is also a transactional component to this interaction which should not be underestimated. Each action by the participants may have either positive, negative, or neutral affect; it may be compatible with the actions of the other participant (e.g., answering a question) or incompatible (e.g., refusing to obey a command); and it may be either a verbal interaction or some physical pressure, or so on). Each of these aspects of the interaction has value consequences for the participants as well as informational consequences. For example, a person might be expected to prefer positive affective comments to negative affective ones, behavior compatible with his own behavior to incompatible behavior, and so on.

This transactional component of the interaction may be important and worthy of study for a number of reasons. First of all, that component is important in its own right without regard for its connections with the communicative component. For example, the character of the police-citizen encounters in Chicago during the 1968 Democratic convention in terms of their transactional components (e.g., the degree of cooperation and conflict, the degree of physical violence, and the negative affect) clearly is an important phenomenon. The issue of police brutality and violence by both parties in such interactions is one which is worth addressing in

and of itself apart from any insight it might also lend into other aspects of the interaction.

Secondly, this transactional component is likely to have a close connection with the communication component of the interaction and to reflect differences which occur on that level. For example, the informational consequences of the statement "you're under arrest" are in many respects similar to the value consequences for that same statement. The verbal or physical response of the person to which such a statement is made will probably reflect both of these components and be expressed in a parallel fashion for both the transactional and the communicative component of the interaction. Much as verbal and nonverbal behavior are thought to reflect parallel channels of communication conveying similar information (Knapp, 1972; Birdwhistell, 1952, 1970; Kendon, 1972) the transactional and communicative component of interactions also might be expected to reflect similar concerns and convey valued consequences and information which paralleled each other (i.e., they represent corresponding states).

Another reason why it might be useful to examine the transactional component of interactions rather than the communicative component is because of the tremendous legal and normative restrictions on the behaviors of police and citizen in police-citizen encounters. The severe regulation of such behavior (e.g., the legal prescriptions against offering bribes, the requirement of reading a person his Miranda warning, and so on) may be so restrictive that the communicative component is ritualized and regulated so much that the real interesting behavior which explains

the character of the interaction is the transactional aspect.

Another reason for emphasizing the transactional component might be its greater parsimony both in terms of the complexity of the different states which are conceptualized and in the ways participants might be expected to respond to those states. Certainly there are almost infinite variations in types of information which may be exchanged in interactions. The types of valued consequences, however may be rather crudely categorized in a few basic categories such as positive or negative affect, cooperative behavior and so on. Such general categories for information are not to my knowledge available in the literature. The symbolic interactionists have repeatedly pointed out the great complexity of such communicative interaction. The complexity of the states which are possible is far overshadowed by the even greater complexity of the possible ways people may respond to such states. The complicated processes enjoined by the symbolic interactionists to explain interaction must surely be feared as much as they are admired. The subtlety of such symbolic interaction, its nuances, the complex decision, and so on are clearly far beyond current formalization capabilities.

Perhaps by examining only that simple aspect of this type of interaction which may reflect this more complicated behavior is a good strategy. Certainly, in terms of a formal theory and precise empirically testable predictions something must be done. Perhaps it is the case that interaction can be conceptualized as taking place on a number of levels at once. There is a physical level, a biological level, a psychological level, a social level, and so

on. Of the two levels which approach the greatest level of abstractions, the transactional and the communicative (symbolic), perhaps we are better disposed to concentrate first on the simplest in hopes of finding out something. This may prove inadequate and the issue may have to be joined on the level of completely symbolic communicative behavior. But it is a reasonable research judgement to begin with the simplest approach and hope it proves successful.

Individual Characteristics. There are a number of additional ways in which particular participants may differ. They may differ with regard to a large number of personality characteristics such as those pointed out by Vinacke (1969) or Becker and McClintock (1967). These include authoritarianism, Michaevelianism, inner-directedness, and so on. Participants may also differ in their ability to search for and process information (i.e., developmental differences due to differences in aging, IQ differences, and so on). Participants may differ with respect to a number of external characteristics such as status, age, sex, race, social class, and so on.* Many of these differences may be expected to influence the character of the interaction which occurs.

Specific hypotheses

This discussion illustrates some of the insights this

*Sex roles, for example may be compatible or incompatible with the perceived character of police-citizen interaction (i.e., when a policeman feels he should be dominant and citizens submissive, if the policeman is a man and the citizen a woman, the the sex role expectations of dominant-submissive relations may be compatable with the police-citizen roles.)

framework can provide for understanding a particular case of social interaction. Since this particular case of social interaction will be examined in some detail here, it may be helpful to formalize a number of these insights as specific hypotheses to be tested.

There is one assumption in particular which is implicit in the effort to model this phenomenon with a Markov process:

- 1) Police-citizen interaction is a process which takes place over time and specific actions at any point in time are, at least in part, a function of actions which preceded them-- i.e., they are contingent on past actions.

In addition, there are several assumptions pertaining to the role differences between officers and citizens:

- 2) Police respond more to their own past actions than to those of citizens (i.e., their current responses are more closely related to their own past actions than to the past actions of citizens),
- 3) Citizens respond more to the past actions of police than to their own (i.e., their current responses are more closely related to the past actions of police than to their own past actions), and
- 4) Police officers will be more likely to respond with more negative or redirective communicative acts than the response of the citizen which immediately precedes their action; and citizens will be more likely to respond with acts which are less negative and more cooperative than the response of the officer which immediately precedes them.

Two additional hypotheses pertain to the contingency on past events and the possible decision rules which guide citizen and officer behavior.

- 5) The model which provides the best fit to the data will be the model which assumes that current responses are contingent upon both the most recent response of the other participant and one's own most recent response; and
- 6) Both citizen and officers tend to utilize a status-equilibration decision rule to guide their responses.

The analysis which follows will seek to provide tests of these hypotheses as well as to explore other interesting properties of police-citizen interaction.

Chapter 3

METHODOLOGY

The overarching objective of this research is to develop a coherent conceptual framework of social interaction which admits of its processual character and to examine a specific case of such social interaction--police-citizen encounters--in an effort to test the utility of that conceptual framework. Hopefully in the process some greater understanding of interaction will result. Now that the conceptual framework has been presented it is necessary to somehow connect that framework to the empirical analysis. This is a critical step which should not be taken lightly. Many past works have been notably deficient in relating theoretical developments to the empirical phenomenon and have suffered as a consequence.

The objective of this chapter is to provide a number of essential connections: connections between the concepts of that conceptual framework and specific variables measured in the observations of police-citizen encounters to be analyzed here, and connections between the still rather formalized and conceptually-oriented Markov models and the more data-oriented log linear models which will be utilized to analyze the data. In the course of making these connections it is also necessary to describe briefly the character of the observations of police-citizen encounters and the methods of data collection employed in obtaining them. It is also

necessary to point out that there are subtle differences between mathematical models and other approaches to empirical investigation, and to warn of some of the problems which arise as a consequence of these differences. And finally, the specific strategy of empirical analysis to be employed will be outlined.

The extensive use of mathematical models in this research requires an understanding of the subtle differences between research utilizing such models and other more common types of research. There are many problems which arise from the use of such models which appear to derive in large part from their mix of logical and empirical bases for truth. The most effective use of such models must include extensive tests of both their assumptions and their predictions, an exploration of the logical implications of the models, and explicit interpretation of the models in terms of the phenomenon of interest. The nature of these problems and the implication they have for the analysis are very complicated and will not be discussed in detail here. In appendix 1 is presented a relatively brief discussion of some of these major points. In addition, the interested reader is encouraged to explore a number of past works dealing with some of these issues, including the following works in sociology: Lave and March, 1975; Leik and Meeker, 1975; Kaplan, 1964; and Willer, 1973.

A Markov Chain Model Of Social Interaction

Having earlier discussed some of the processes underlying social interaction and perspectives others have taken regarding various aspects of social interaction, it is now the task to develop a way of conceptualizing this process so that it may be examined

empirically. Here this will be done by working with the general states spoken of earlier in connection with the normalized decision form representation of the decision task. Those states, recall, are general categories which describe the system at some point in time. Those states should be meaningful representations of some substantive aspect of the social interaction. They should be mutually exclusive and exhaustive, and they should characterize all social interaction in question at all points in time. If one can characterize a particular form of social interaction in this fashion, it has already been shown how this may be used to conceptualize the contingency between the two participants and their underlying decision behavior. Here we will use this same concept to form a bridge between these conceptual discussions of the underlying process and the empirical examination of social interaction.

We may define a vector of all possible states which characterize social interaction:

$$S = (s_1, s_2, s_3, \dots, s_n)$$

These n states are mutually exclusive and exhaustive.

$$P(s_i, s_j) = 0 \text{ for all } i \neq j \quad (\text{mutually exclusive})$$

$$\sum_1 P(s_i) = 1 \quad (\text{exhaustive})$$

Any particular social interaction process observed over time could thus be represented and described as a sequence of states $s_{1(t)}$ which characterize the system at any time, t . For example, a particular interaction sequence might look as follows:

$$s_{1_t}, s_{2_{t+1}}, s_{1_{t+2}}, s_{3_{t+3}}, s_{2_{t+4}}, s_{2_{t+5}}, \dots, s_{2_{t+n}}$$

Up to this point the system has simply been described. In doing so assumptions of discrete space and discrete time have been implicitly made. These assumptions are not without their consequences, but they correspond to those already made earlier in conceptualizing the underlying processes and they are consistent with the method of data collection employed to obtain the data which will be analyzed here (see the section to follow). The states of the system at different points in time might be related to each other in any of a number of ways. If that relationship is one of a number of specific types given considerable attention by statisticians in the past then it will be possible to use some of the insights generated by those previous statisticians and researchers to derive a number of predictions about the characteristics of these interactions as they unfold over time. In particular, one of the simplest models which has been used to describe such processual data is a Markov model. A Markov process is one which has a number of important properties. A Markov process may be said to describe a particular type of social interaction if the following assumptions are valid:

Assumption 1: The Markov Assumption.

The probability of occurrence of a particular state, s_j , at time $t+1$ depends solely on the state, s_i , which occurred at the immediately preceding time t . In other words, a Markov process may be described by a transition matrix, T . The rows of this transition matrix represent the possible states occurring at time t , and the columns represent the possible states at time $t+1$. In each cell of the matrix is the conditional probability, p_{ij} , of state j occurring

at time $t+1$ given that state i occurred at the previous time, t .

$$T = \{P_{ij}\} = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & \dots & P_{ij} & \dots & P_{2n} \\ \vdots & & & & \vdots \\ P_{n1} & \dots & & & P_{nn} \end{bmatrix}$$

where $P_{ij} = p(s_{j,t+1} | s_{i,t})$ where $i = 1, j = n$

P_{ij} satisfies the constraints of probabilities. $0 \leq P_{ij} \leq 1$ where $i = 1, j = n$. Since the process must always occupy one of the states,

$$\sum_{j=1}^n P_{ij} = 1 \quad i = 1, 2, \dots, n$$

and the system is described by n possible states which may occur at any time and which are mutually exclusive and exhaustive. At any one time the current state of the system may be characterized by a probability vector S consisting of the probabilities that the state is in any one of the n possible states

$$S(t) = p(s_{1,t}), p(s_{2,t}), \dots, p(s_{n,t})$$

From this assumption,

$$S(t+1) = T \cdot S(t)$$

If this assumption is not true when states are defined a particular way, it might be true when they are defined in some other fashion (e.g., when a new state is defined which is the joint occurrence of two of the states as previously defined). Exploration of this assumption with a number of possible states defined by successively combining old states until the assumption is true is

referred to as examining the order of the Markov model. For example, the assumption might be found to be true for the second order. A Markov model of second order is conceptually equivalent to a nonMarkovian process where present actions are contingent on the past two actions.

Assumption 2: Stability.

The transition probabilities in the transition matrix remain constant over time. That is,

$$P_{ij} = p(s_{j,t+1} : s_{i,t}) \text{ where } 1 \leq i, j \leq n \text{ and } t=0,1,2,\dots$$

Assumption 3: Homogeneity:

The transition probabilities in the transition matrix are the same for all classes of individuals, types of interactions, and so on. That is, the Markov model which is described by the transition matrix is assumed to be the same for all interactions under consideration.

$$P_{ij} = P_{ijk} \quad 1 \leq k \leq m$$

where k is the category of some third variable for which the process is homogeneous. This is not a very restrictive assumption, since if there is some reason for believing there are substantial differences in the Markov process parameters for different sets of interactions, for different people, or whatever, then different parameters may be estimated for each of those subsets of the interactions considered.

Three standard ways of representing Markov models are as tree diagrams, transition diagrams (graph theoretic devices), and transition matrices. The tree representation of a task is closely

related to normal form decision task representation. Only transition diagrams will be used here. Such a representation is presented in Figure 3.1 This has all the properties of transition diagrams as discussed under Assumption 1 above.

The advantages of such models is that if they hold, it is possible to generate a number of interesting predictions about the phenomenon which should hold from the mathematical properties of Markov chains. Those properties have been extensively explored by mathematicians. In addition, the interaction can be summarized very efficiently with only a very few parameters. It is also possible to infer some of the underlying processes related to decisions of interest by interpreting the parameters and components of the model. On the other hand, if it doesn't hold, then by comparing the model to the data some insight into the nature of the data may be gained and some idea of the type of model which might better fit the data may be gained.

The Data

The data analyzed in this study were collected in a recent study conducted by Richard Sykes and his research team.* During the second phase of the study one thousand six hundred and twenty two police citizen encounters were observed by trained observers over a period of approximately one year in St. Paul, Minnesota.

Four observers rode in randomly selected shifts in the squad

*"Comparative Quantitative Analysis of Police Encounters," Center for Studies of Crime and Delinquency, NIMH, U. S. Public Health Service. Richard E. Sykes, Principal Investigator; John P. Clavle, Co-Principal Investigator.

FIGURE 3.1
FIRST-ORDER TRANSITION MATRIX

Past Action at Time $t-1$ by Actor A	Action at time t by actor A			Row Total
	(1)	(2)	(3)	
(1)	P_{11}	P_{12}	P_{13}	$P_{1\cdot}$
(2)	P_{21}	P_{22}	P_{23}	$P_{2\cdot}$
(3)	P_{31}	P_{32}	P_{33}	$P_{3\cdot}$
Column Total	$P_{\cdot 1}$	$P_{\cdot 2}$	$P_{\cdot 3}$	$P_{\cdot \cdot}$

cars of police patrols. Each patrol normally consisted of two police officers and one observer. Those observers coded directly encounters as they took place. The sampling design involved the coding of critical events. Each encounter which occurred was coded; within each encounter interactions were coded as they took place. This design may be contrasted with a design where time intervals would be coded regardless of whether an interaction took place or not.

These data are the result of significant advances in measurement made by those researchers in developing an extensive interaction code, "Police IV," (Wallen & Sykes, 1974) and in developing sophisticated hardware and procedures for coding observations which make possible real-time encoding of data. (Sykes, 1973) The hardware system enabled the observers to code directly on magnetic tape as the interaction progressed. Those tapes were later converted directly into conventional computer data storage tapes utilizing machines developed in that line of inquiry by the investigative team. These data consist of systematic observations using that interaction coding scheme and hardware system to code behavior along several dimensions and create a record of the interaction as it takes place over time. Process data such as these for such a large number of cases collected in a field study are not at all common in the social sciences and constitute an unusual and significant opportunity to systematically explore the process of social interaction in a natural setting.

The quality of these data appears to be very high. The very sophisticated equipment is likely to have reduced much of the error

involved in collecting this data (to say nothing of making this data collection feasible). In addition, the training procedures for the coders were extensive and the checking and monitoring of the data collection process were vigorously pursued. Extensive cleaning of the data was carried out and a number of previous analyses have already been conducted which are likely to have exposed problems which might have existed in the data and have provided an opportunity for them to be corrected.

The Sample

Not all of the 1,622 police-citizen encounters observed in this data set were amenable to the relatively simple conceptual framework and empirical analysis procedures which are available for this analysis. In particular, the conceptual framework is focused on the social interaction between participants. It is clear that if there are more than two participants the character of the interaction process becomes exceedingly complex. For this reason it is useful to limit consideration in this study to only those encounters for which a minimal number of participants were present. Because, in almost all cases, there were two police officers present, this would include encounters with only one citizen present.

The conceptual framework also suggests that there are a number of other variables which might have strong impacts upon the character of the social interaction which occurs. Those include the characteristics of the individuals, the particular role expectations and the nature of the encounter situation. Controlling for very many of these at once immediately reduces the working number of encounters drastically. So it is only possible to control for a very few of

these variables in order to maintain sufficiently high numbers of encounters for analysis. It is not possible to know precisely which of these variables would be most important, but the decision here is to control for the role demands upon the participants and, in the same process to control one aspect of the encounter situation. Only those encounters in which one citizen was present and that citizen was an alleged offender were considered. It is considered likely that the interaction between police and victims would be quite different in character from that of police and alleged offenders.

This resulted in a sample of 159 encounters which could be examined. That sample was further reduced by consideration only of those encounters in which 20 or more interactions took place. It is reasoned that many very brief encounters would not make it possible to examine the dynamic character of the interaction process over time. It is quite possible, and even likely that this selection biased the sample in favor of a specific type of encounter-- i.e., perhaps one in which there was a more serious crime which occurred, or in which there was less cooperation on the part of the citizen, or perhaps it would be different from the shorter encounters in other ways. However, an examination of the entire sample of these 159 encounters when compared with that of the subsample for specific types of analysis (transition probabilities for a first-order model) found essentially no difference. This is not the best test of the differences between these two samples, but it is some indication that they may not be very different. Nevertheless, it should be recognized that the generalizability of this study may

not extend beyond relatively long police-citizen encounters with alleged offenders. It should also be recognized that the analysis is even further biased in favor of the long encounters because the unit of analysis for this study is an interaction which occurs during an encounter. The encounters with more interactions thus contribute more to the results than the shorter encounters. That bias is assessed in some respects by the later analysis of the stability of the encounter processes over time which indicate very little difference between early and late encounters. Again, this is not the best test of this bias, but it is an indication that the magnitude of the problem may not be great at all.

Operationalization of Concepts

There are two types of data which need to be considered in this study: data pertaining to the encounter as a whole and data pertaining to the specific interactions taking place during the encounter. The unit of analysis for this study is the particular interaction, not the entire encounter. Hence the primary concern here is with the data characterizing the interactions (the process codes), although a number of important variables characterizing the encounter as a whole will be examined.

Because the primary concern of this view of the interaction is its processual character, it is necessary to have data which reflect the sequence of interactions as they are observed over time. Those data must maintain the sequence of the interactions as they occurred and they must characterize important aspects of that interaction in terms of categories which apply at all points during the encounter in order to meet the prerequisites set down for the data in the

conceptual framework. The data examined here of police-citizen encounters meet all of these constraints. Each interaction as it occurred was coded along a number of dimensions. The sequence of the interactions as they occurred was preserved, and in addition, the elapsed time during the encounter was also preserved. Hence these data may be treated as discrete state data with either discrete or continuous time.

The procedures and codes utilized in this data collection procedure are based on the Police IV Interaction Code. This observational code is the fourth in a series of such codes developed by Duane Wallen and Richard E. Sykes (Wallen and Sykes, 1974). That code is very extensive and complex and is described in detail in a manual which was used for training observers in its use. Here only the specific parts of the code which were utilized extensively in this research are described. Those are the process codes.

The two process codes of interest here are the interact code and the operator code. These codes were used to describe the interaction which took place during the encounter. Each interaction (i. e., a statement, question, threat, or other verbal action) was coded using these codes. The interactions were coded in the sequence in which they occurred as they occurred by the observers utilizing electronic encoding equipment.

The interact code has the following structure:

$$(R + (R) + CD + IF + A)$$

That is, it consists of a minimum of four numbers ((R) is implied usually, more may be added when complex codes are required). The first two numbers of the code on the left represent the role of

(1) the participant initiating the action, and (2) the participant to which the action is directed. For the sample of encounters examined here there was only one citizen present and hence the role can only take on two effective values:*

Role: 1 - citizen
0 - officer

For example, the citizen may speak to the officer (this would be coded 10...), or the officer may speak to the citizen (01...), or one of the officers may speak to the other officer (00). The third number in the interact code specifies the content domain of the interaction. Communicative acts in the legal or encounter nexus content domain refer to the reason for the police-citizen encounter (e.g., for a traffic offense, statements referring to the alleged offense). The content domain may be one of these possible categories.

Content Domain:

- 3 - legal or encounter nexus; communicative acts dealing with the reason for the police-citizen encounter--e.g., for a traffic offense, communicative acts concerned with the alleged violation.
- 5 - behavior management; communicative acts concerned with the physical or informational management of the encounter--e.g., a direct order by the officer for the citizen to step aside.
- 7 - interpersonal; communicative acts dealing with the person to person contact, including interpersonal relations of the participants with one another or the seeking or giving of assistance.

*There were generally two officers present. However roles are examined here, not individuals and hence the communicative acts of citizen-officer are treated as coming from the same role. In addition intra-role communicative acts (e.g., officer to officer acts) were not considered. Only inter-role communicative acts are analyzed.

The fourth number in the interact code is the intensional form.

The intensional form may take any of four possible values:

Intensional Form:

- 4 - statement: provision of content domain relevant information
- 6 - question: effort to elicit domain relevant information or action
- 8 - mand: obligatory or coercive directive (e.g., a command or order)
- 0 - accusation: accusation relevant to the domain

These categories refer to the function of the communicative act, rather than its grammatical character, where those differ.

The final number in this code would be the affect code. The affect code may take on any of three possible values: This code rates the affective or emotive loading of the communicative act on a positive to negative scale.

- 3 - positive regard: the act enhances sociability and might include complimentary qualities of warmth, deference, understanding, and so on
- 5 - neutral regard: this act neither enhances sociability nor serves as a detriment to interpersonal relations
- 7 - negative regard: this act is normatively considered as detrimental to interpersonal relations and may include derogatory content such as hostility or sarcasm.

The affect coded is the affect directed at the recipient of the action (e.g., a person mad at the world would not be coded as negative in affect unless that anger was directed at the recipient of the communicative act).

There are many more qualities to this code which could be considered and this is a much simplified version of the code. There are many significant variations, exceptions, and special cases which were critical to the encoding of the data, but which do not bear substantially upon the interpretation of the output data because they were all processed to produce this outcome code

for the interaction. The interested reader is encouraged to explore the entire Police IV coding manual.

For the purposes of simplifying the coding scheme to make it possible to code interactions as they progress, an abbreviated version of this code was used. This involved the use of the operator code. The operator code takes the following form:

$$(R_2 + \text{operator})$$

When an operator of the form $(R_2 + \text{operator})$ follows an interact code of the form $(R_1 + (R_2) + CD + IF_1 + A_1)$ then the operator specifies a second segment of the same form of interaction with role R_2 , content domain, CD_1 , intensional form, IF_{1c} , and affect A_1 where IF_{1c} is a complement of IF_1 which may take one of two forms, which for our purposes here may be described as compliant or noncompliant. The operator takes on one of two values which indicate the form of this complement of IF as below:

- Operator: 4 - noncompliant, indicating a response which may be interpreted as noncompliant relative to the original interact code which this code follows (e.g., a refusal to answer a question, a denial of an accusation, and so on)
- 9 - compliant, indicating a response which may be interpreted as compliant relative to the original interact code (e.g., an answer to a question, an admission, and so on)

This code can only be used when the interaction occurs in a number of contiguous segments or strings which reflect interaction which is continuous in some fashion (i.e., pertaining to the same content). The operator is an optional form of coding. Even though it may be possible to code the event using an operator, the coder may elect to code it using the complete code. However, all coders exhibited extensive use of the operator code. Interaction is commonly relatively

fast so that it was surely simpler and easier. It is likely that the code was routinely used when appropriate.

These codes are quite complex and an extensive empirical analysis of the entire code category scheme would require huge masses of data exceeding even the very large data sets collected by Sykes and his colleagues. In addition to the problems in achieving the necessary sample size such an analysis would involve a tremendously complicated set of data which would require great skill and time to model. Hence, it is desirable to reduce these codes to a manageable level of complexity for the purposes of this analysis. The objectives in this simplification are to maintain useful and interesting distinctions provided by the code to insure that important facets of the police-citizen interaction are being represented by the codes, to maintain codes which could have reasonable connections with potential underlying decision processes (e.g., choose codes which might reflect states which could reasonably be differentially valued), and to insure that there will be sufficient variation in the distribution of codes so that they may be effectively analyzed without overwhelming problems of sampling error affecting the estimations of parameters for the models.

The content domain was immediately excluded because of the four substantive codes, it contained the categories for which some notion of differential preference was least reasonable to assert (e.g., could it reasonably be said that behavior management acts would be preferred more or less than those which were personal?). It is also possible to immediately eliminate the second role (the person to whom the interaction is directed) because in the sample

where only one citizen, an alleged offender, was present there could only be one possible direction for an interrole communicative act when the role of the person carrying out that act is known (e.g., the only interrole communicative act for an officer would be with the citizen as the object).

This leaves only the three codes; affect, intensional form, and operator. In Table 3.1 are presented the distribution of interactions among the possible categories created by the simultaneous consideration of these codes occurring for the encounters studied. From these data it is clear that there are problems in the distribution of these data. Positive and negative affect, for example, occur very infrequently relative to neutral affect. In addition, noncooperative responses are somewhat uncommon relative to cooperative or non-operator responses. Mandes and accusations also occur considerably less frequently than questions and statements. Thus, although there is an impressive number of interactions examined, the frequencies are rather unevenly distributed among these possible cells.

A number of possible recombinations of these cells were examined in an effort to produce fewer and hence simpler cells with reasonable distributions of events so that they could be analyzed. The simplified version which was finally chosen for analysis is that presented in Figure 3.2. These three categories of response are considerably simpler than the 36 categories possible in Table 3.1. They also have a reasonable frequency of occurrence. And it is possible to posit at least a rough order of preference among the categories where a compliant response would probably be preferred

TABLE 3.1
 DISTRIBUTION OF OBSERVED RESPONSES AMONG SELECTED
 CATEGORIES FOR SAMPLE OF POLICE-CITIZEN
 ENCOUNTERS WITH ONE ALLEGED VIOLATOR
 PRESENT

Intensional Form	Operator								
	None			Compliant			Noncompliant		
	Affect			Affect			Affect		
	+	0	-	+	0	-	+	0	-
Statement	20	325	18	8	1299	14	5	59	6
Question	16	493	3	14	1124	14	2	22	6
Hand	6	192	13	26	346	17	0	23	12
Accusation	21	214	31	21	358	24	17	114	15

FIGURE 3.2
REVISED CODING SCHEME

	Operator		
	None	Compliant	Noncompliant
Statement	1	2	3
Question			
Mand	3		
Accusation			

1 = Redirective Communicative Act (N = 875)

2 = Compliant Communicative Act (N = 3265)

3 = Negative Communicative Act (N = 769)

by the participant to whom the response is directed over a negative or redirective response, and a redirective response would probably be somewhat preferred over a negative response. These categories also offer promise of being relevant to a number of possible decision rules and strategies which might eventually be examined.

To illustrate these codes, the example first presented in Figure 2.1 is reproduced in Figure 3.3 with the revised codes to the left in the margin. The O or C indicates whether the actor initiating that response was the officer or the citizen, and the subscript (1, 2, or 3) indicates the code of the response as defined in Figure 3.2.

In addition to these process codes it is also necessary to employ a number of variables which characterize the entire encounter. Variables characterizing the characteristics of the participants or the nature of the decision task, for example, characterize all interactions within a particular encounter. There were a great many such variables measured for the police-citizen interaction data considered here. Only a few of these are considered in this study, however. In selecting a sample of encounters for examination, as mentioned earlier, only those encounters were selected which had similar decision tasks in the sense that they had only one citizen present and that citizen was cast in the role of a suspected offender. Another task characteristic is examined later in testing the assumption of homogeneity of the Markov process: the nature of the offense as measured by whether it was a traffic offense or some other type of offense. One characteristic of the participants also will be examined; the apparent social class of the citizen

FIGURE 3.3
HYPOTHETICAL INTERACTION SEQUENCE*

- Situation:** A young man in a new car has just run through a red light and is speeding along a side street. A policeman in a patrol car pulls him over and stops. The officer has just approached the man's car and speaks to him through the window.
- O₃ Officer: "Let's see your license, buddy."
- C₂ Citizen: Looking nervous, he fumbles with his wallet and finally hands his license to the officer.
- O₁ Officer: Reads the license. "Bill Smith.... What's your address, Bill?"
- C₁ Citizen: "1412 Rosemary. What's wrong, officer?"
- O₁ Officer: "What's wrong! You're in a heap of trouble, boy. You just ran that red light back there and I clocked you at 55 in a 35 zone. Just what are you trying to prove?"
- C₂ Citizen: "Nothing, officer. I guess I just wasn't paying attention. My wife just had a baby boy and I'm so nervous I just can't think about anything else. I'm real sorry. I'm usually a very careful driver."
- O₁ Officer: "Really? Have you ever had a ticket before, Bill?"
- C₂ Citizen: "Yes, but it was just a parking ticket, sir."
- O₂ Officer: "That's all, you're sure?"
- C₂ Citizen: "Oh, yes sir!"
- O₁ Officer: Hesitating and then writing a ticket. "O.K. I'm going to have to give you a ticket for that red light. I'll let the speeding go this time, but you'd better watch it in the future."
- C₂ Citizen: "Oh I will, sir. You'd better believe I will. Thank you."

Adapted from an example in Wallen and Sykes (1974).

(e.g., whether middle/upper or lower). Other variables could and should be examined in future research. These were chosen here because they appeared to be the most likely variables (as identified in the conceptual exploration of the case of police-citizen encounters) to have an impact upon the processes which occurred.

Analysis Procedures

The empirical data required for analysis of the assumptions and predictions of Markov models for social interaction consist primarily of contingency tables. For example, to estimate the transition probabilities for a system of interacting individuals from a state at time t to another state at time $t+1$ one could simply examine the contingency table consisting of rows representing the possible states at time t and columns representing the possible states at time $t+1$. An observation would be recorded for each cell x_{ij} when the system was in state i at time t and was in state j at time $t+1$.

Typical contingency tables which must be analyzed would involve a number of dimensions reflecting the choices at different related times during the course of the encounter and one or more other variables of interest which might affect choices. For example, consider the four-dimensional contingency table with dimensions

- 1) choice of participant A at time t ,
- 2) choice of participant B at time $t-1$,
- 3) choice of participant A at time $t-2$, and
- 4) role of participant A.

This would be a $3 \times 3 \times 3 \times 2$ matrix because there were three

possible choices at each point in time for each participant (re-directive, cooperative, and negative) and two possible roles (citizen or officer).

These contingency tables may be analyzed by comparing the fit to the data of a number of log-linear models with varying effects present. For every contingency table there is a general log linear model, analogous to an N-factor ANOVA model which imposes no restrictions upon the data and which provides perfect maximum likelihood estimates of the expected values for each cell. For example, in the $3 \times 3 \times 3 \times 2$ matrix under consideration each cell (i_1, i_2, i_3, i_4) would have some observed frequency, $x_{i_1 i_2 i_3 i_4}$, and some expected value, $m_{i_1 i_2 i_3 i_4}$. For such a data matrix the general log linear model would appear as follows:

$$\begin{aligned} \log m_{i_1 i_2 i_3 i_4} = & u + u_1(i_1) + u_2(i_2) + u_3(i_3) + u_4(i_4) \\ & + u_{12}(i_1 i_2) + u_{13}(i_1 i_3) + u_{23}(i_2 i_3) + u_{24}(i_2 i_4) \\ & + u_{14}(i_1 i_4) + u_{34}(i_3 i_4) + u_{123}(i_1 i_2 i_3) \\ & + u_{234}(i_2 i_3 i_4) + u_{124}(i_1 i_2 i_4) + u_{134}(i_1 i_3 i_4) \\ & + u_{1234}(i_1 i_2 i_3 i_4) \end{aligned}$$

Where the first subscripts of the u-terms refer to a set of dimensions and the second ones (in parentheses) refer to categories for those dimensions.*

The u-terms are analogous to the main and interaction effects

*In the remainder of this paper only the first subscript will be included in order to simplify the presentation.

represented in ANOVA (multifactor analysis of variance). A first-order u-term (one having only one subscript) corresponds to the effects of a dimension independent of the effects of other dimensions. A second-order u-term corresponds to the dependence of the two dimensions represented by the two subscripts (this is analogous to some contingent relationship among these variables or some association between them). A third-order u-term corresponds to an interaction effect where the relationship between two of the three dimensions represented is contingent upon the value of the third dimension. Additional rth-order u-terms are analogous to their corresponding (r-1)th-order interaction terms in the ANOVA model.

When attention is restricted to hierarchical log linear models (models where higher order u-terms only appear when all lower-order u-terms possible with those same dimensions also appear) a standard iterative proportional fitting procedure (Deming and Stephan, 1940) may be applied for computing the maximum likelihood estimates of the expected cell values for the models.

The fit of each model to the data may be assessed by one of a number of statistics distributed as chi square with degrees of freedom equal to the number of cells in the table minus the number of parameters which are estimated. The recommended statistic (Bishop, Fienberg, and Holland, 1975) is the likelihood ratio statistic, G^2 defined as follows:

$$G^2 = -2 \sum_i x_i \log \frac{\hat{n}_i}{x_i}$$

with degrees of freedom appropriate for the estimate of \hat{n}_i .

where \hat{m}_1 is the maximum likelihood estimate of the frequency in cell 1 and x_1 is the observed frequency in cell 1.

This statistic is recommended because it is the quantity minimized by the maximum likelihood estimates of m_1 , and hence is an appropriate summary measure of goodness of fit of the model. In addition, it may be partitioned in a way which makes it possible to conveniently test for the presence of specific effects in the data. It can be shown that "if $G^2(2)$ (the fit of model 2) and $G^2(1)$ (the fit of model 1) are asymptotically distributed as chi square with v_2 and v_1 degrees of freedom, respectively, then $G^2(2:1)$ is asymptotically distributed as chi square with $v_2 - v_1$ degrees of freedom" (Bishop, Fienberg, and Holland, 1975:127). When two direct models differ only by a single u-term, the difference in goodness of fit statistics G^2 of the two models applied to the entire data matrix is equivalent to an examination of the marginals for the u-term of interest.

Log linear models thus constitute a general procedure for testing a number of hypotheses about the presence or absence of specific effects in complex multidimensional contingency tables. These procedures may be utilized to explore multidimensional contingency tables for a wide variety of purposes. One specific task which can be performed utilizing these general techniques is the exploration of a number of assumptions of Markov models (e.g., see Bishop, Fienberg, and Holland, 1975, chapter 7).

The exploration of Markov models is greatly facilitated by the application of log linear models. This is the case because the general program for these models, CTAB, now available in SPSS is

a flexible program capable of making the varied tests of relevance to Markov models (in addition to many others). Other general programs for those tests were not previously available. As a consequence, these tests previously required great efforts in developing specific computer programs or calculating by hand goodness of fit statistics for extensive matrices. In addition, the general log linear approach provides additional insights which might not have been made so salient by Markov techniques alone.

The pursuit of Markov structures in multidimensional contingency tables also nicely supplements the general log linear modeling procedures. One of the characteristics of such a general procedure as log linear models, like ANOVA which preceded it, is that there are so many possible effects to investigate, and so many possible combinations of models that there is some need for direction in the search through the possible models. The specific hypotheses of Markov models offer one set of specific hypotheses which may be explored with a finite amount of time and which offer the promise of a great many interesting deductions and interesting properties if they prove to hold. If they don't hold, they also offer the very useful baseline of a simple Markov process against which to contrast the processes which actually occur.

The specific points at which Markov assumptions may be translated into log linear models have been sketched out by Bishop, Fienberg, and Holland (1975). Here those connections will be elaborated, hopefully clarified, and generalized.

These connections may be illustrated with the $3 \times 3 \times 3 \times 2$ contingency table discussed earlier. The first three dimensions

of that table represent choices of participants at different points in time relative to each other (e.g., choice at times t , $t+1$, and $t+2$). The assumption that a first-order Markov model fits the data would imply that the choice of a participant at any time would depend solely on the choice which immediately preceded it, (i.e., the choice at time $t+1$ would depend solely on the choice at time t , and the choice at time $t+2$ would depend solely on the choice at time $t+1$). Stability over time implies that whenever one r th-order u -term representing choices at different points in time appears, then all possible r th-order u -terms representing choices at comparable points in time must also appear. That is, if the choice at time $t+1$ depends solely on the choice at time t , and if that dependence is stable over time then the choice at time $t+2$ must depend solely on the choice at time $t+1$, and the effects must be the same. Thus, in general, for some n -dimensional contingency table containing k dimensions each representing choices (actions) at related periods of time (e.g., action at times t , $t+1$, $t+2$, ..., $t+(k-1)$), then a log linear model corresponding to a stable Markov process of r th order would include all u -terms of r th order or less representing only dimensions of choice.* For instance, in our

* Note, only hierarchical models are considered here so all u -terms of order less than r must also be included. Conceptually, this is not bothersome, since it is quite reasonable for main effects and lower order effects to also exist. Empirically, we can examine models in such a way as to identify the relative contributions to fit of each of these terms and provide some insight in that fashion as to the relative importance of the higher and lower order effects.

example matrix, a second order, stable Markov process would be represented by the model:

$$\log m_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4 + u_{12} + u_{13} + u_{23} + u_{123}$$

A first order, stable Markov process would be represented by the model:

$$\log m_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4 + u_{12} + u_{23}$$

A zero order, stable (Nonmarkovian) process would be represented by the model:

$$\log m_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4$$

One might also examine a self-contingent model as described earlier in the conceptual discussion of social interaction. In this model the response of a particular participant is contingent only upon his own previous actions. In the data under examination here since the data have been constrained so that the participants always respond to each other sequentially and one person never responds twice in a row, the person's most recent own past response would be the response which occurred two steps back (e.g., if the current response is t, it would be the response occurring at time t-2). Such a first-order self-contingent stable Markov process would be represented as follows:

$$\log m_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4 + u_{13}$$

From time to time it is also of interest to assess the relative contribution of other variables in estimating the expected

values. The strategy here will be to progressively explore various assumptions of the Markov process, identifying important variables which need to be considered in providing accurate estimates of the expected values and identifying others which may be safely ignored. As important variables are found further tests of additional variables will be made on the expanded model incorporating all relevant variables found to be important in past tests. So, for example, if a first-order Markov model is found to fit the data relatively well then other variables will be examined as they impact upon that first order model. Let us assume for the moment that it will be found that a second order Markov model is required to adequately fit data in a four-dimensional matrix containing only choices at times t , $t+1$, $t+2$, and $t+3$. Then to investigate the impact of role on that process we would require a four-dimensional matrix containing choices at time t , $t+1$, $t+2$, and role. In such a matrix we might consider the following models:

$$1) \log \pi_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4 + u_{12} + u_{23} + u_{13} \\ + u_{123} + u_{14} + u_{24} + u_{34}$$

$$2) \log \pi_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4 + u_{12} + u_{23} + u_{13} + u_{123} \\ + u_{14} + u_{24} + u_{34} + u_{124} + u_{134}$$

$$3) \log \pi_{i_1 i_2 i_3 i_4} = u + u_1 + u_2 + u_3 + u_4 + u_{12} + u_{23} + u_{13} + u_{123} \\ + u_{14} + u_{24} + u_{34} + u_{124} + u_{234} + u_{134} + u_{1234}$$

For model (1) role impacts only upon the first-order u -terms for choices. This is conceptually equivalent to the only difference between choices for different roles being some different distribution of choices (e.g., officers might be less cooperative than

than citizens in overall responses). These terms represent situational differences in the technical sense of situations as the combined product of the task characteristics and past events which affect the interactions for the individuals involved. In model (2) role impacts upon both the first order u-terms for choices and upon their second order terms. In this model the impact of role is both in terms of differences in the distribution of choices and in the different first-order contingencies (i.e., officers may differ from citizens not only in having fewer aggregate cooperative responses, but they may also tend to respond to the last response of the citizen in different ways than the citizen responds to the officers last response). Here role affects not only distributions of decisions but possible first-order decision rules and the contingent character of the interaction such as would be expected for a first-order Markov process. And in the final model, model (3), the effects of role extend even further to also include effects upon second-order contingency and the possible second-order decision rules which the different participants may be utilizing in a second-order Markov process.

By examining all of these models and the differences in the G^2 measure of goodness of fit between them one may assess the extent to which a particular effect is present in the data. It might be, for example, that officers and citizens have no noticeable difference in decision rules, but they do differ considerably in the proportion of choices they make of each type. This would be quite different from the case where they show marked differences in decision rules.

The general strategy of analysis which will be employed here will be to examine a series of multidimensional contingency tables similar to the one just discussed. Each of these will be conceptually equivalent to this example matrix collapsed along certain dimensions perhaps, and expanded along others to make it possible to test a variety of hypotheses with the greatest amount of clarity and simplicity. Families of hierarchical log linear models will be examined for each matrix in order to test specific assumptions of the Markov models for these interaction data. When all of the important effects which contribute to providing a fit for the data appear to have been found the permissible collapsed version of the data will be presented which will illustrate the variables which must be considered in adequate models of this process and their relevant effects. After an adequate model has been forced the substantive interpretation of these effects and the logical implications of the model will be explored with regard to how they relate to the hypotheses posed initially and to the insights into the phenomenon generated by this analysis.

One problem which should not be overlooked is the fact that independence in a model implies independence in the collapsed versions of that same table, but not vice versa. This is the old problem which has always existed in categorical analysis of interaction. It is never possible to conclude a variable has no effect without investigating that variable. Nor is it possible to conclude that there is no interaction with other variables. For this reason, there is always the possibility of confounding results by not considering key variables. For example, a comparison of a third and a second-order model may make it possible to choose a second-order model over a third-order model. But this does not necessarily imply that a fourth-order model might not be still better, or a fifth order model, and so on.

Chapter 4

RESULTS: SELECTION AND VALIDATION OF A MARKOV MODEL

It is generally necessary to test both the assumptions and the predictions of models because the fit of a model to the data is rarely perfect and slight discrepancies may lead to predictions which are incorrect even though the assumptions are substantially correct. In this chapter a number of models of police-citizen interactions will be examined empirically.* The model from among these which best fits the assumptions of Markov models will then be further examined empirically by testing several of its predictions. If the results indicate that the model is an adequate representation of the data, then in the chapter to follow the logical properties of that model will be examined and empirically interpreted in order to explore the substantive implications of the model for the process of social interaction which occurs among police and citizens.

Testing Assumptions and Selecting a Model

Three assumptions of Markov models will be tested here. These include 1) order, 2) stability over time, and 3) homogeneity.

*Sykes (1973) earlier explored some of the assumptions of Markov models using data from a related data set, and his results suggested Markov models would be a fruitful way of analyzing data of this sort.

Any model which is finally selected, if it is to be a Markov model must have current events depend only on past events, must have transition probabilities which are stable over time, and must have transition probabilities which are the same for different sub-samples. If any one of these assumptions does not hold for a particular model, however, it is often possible to create another model for the same set of data for which they will hold. For example, if the transition probabilities are a function of the two most recent past events instead of only the last, then events can simply be redefined to include both of those past events and the new model does meet the Markov assumption. Given these possibilities for modifying particular models until they meet the Markov assumptions, testing these assumptions becomes simultaneously both a validating procedure and a search procedure for finding an appropriate model for the data. In this section a number of models will be examined and the model which best fits the data will be selected for further analysis.

Tests of Order

Because it is hypothesized that the behavior of officers and citizens in police-citizen encounters is different due to the different roles the two are enacting, tests of the order of the Markov chain were run separately for officers and citizens. The data for these tests consist of two separate four-dimensional data matrices (one for officers and one for citizens). Those matrices are analogous to those considered earlier. Their dimensions are as follows: 1) the participant's own current response at time t ; 2) the immediately preceding response of the other

participant at time $t-1$; 3) the most recent previous response of the first participant at time $t-2$; and 4) the second most recent response of the other participant at time $t-3$. These combine to form a $3 \times 3 \times 3 \times 3$ matrix containing 81 cells. Those matrices are the top half and the bottom half respectively of Table 4.1.

The fit of five different log linear models to the data in each matrix were assessed. These models included models of 1) a zero-order Markov process, 2) a first-order other-contingent only Markov process, 3) a first-order both other-and-self-contingent only Markov process, 4) a second-order Markov process, and 5) a third-order Markov process. By comparing the relative fit of these models it is possible to make a judgement as to which Markovian process model would provide the best representation of these data (both in terms of statistically determined empirical fit and in terms of conceptual clarity and substantive meaning). The results of these tests are presented in Table 4.2.

TABLE 4.1
OBSERVED FREQUENCIES OF ACTS AND TRANSITION PROBABILITIES AT TIME t
BY THOSE AT TIMES t-1, t-2, t-3 AND BY ROLE

Dimensions 5: Role of Participant ₁	Dimension 4: Actions at Time t-3 by Participant ₂	Dimension 3: Actions at Time t-2 by Participant ₁	Dimension 2: Actions at Time t-1 by Participant ₂	Dimension 1: Action at Time t by Participant ₁							
				1		2		3			
				Freq	Prob	Freq	Prob	Freq	Prob		
Officers	1	1	1	3	.600	2	.400	0	.000		
			2	10	.400	12	.480	3	.120		
			3	0	.000	0	.000	2	1.000		
		2	1	1	4	.160	18	.720	3	.120	
				2	11	.250	25	.568	8	.182	
				3	0	.000	4	.667	2	.333	
			3	1	1	2	.500	2	.500	0	.000
					2	5	.385	3	.231	5	.385
					3	4	.667	1	.167	1	.167
	2			1	1	13	.464	13	.464	2	.071
					2	112	.291	207	.538	66	.171
					3	12	.345	4	.182	6	.273
		3		1	1	10	.303	18	.545	5	.152
					2	167	.187	595	.655	133	.149
					3	3	.214	6	.429	5	.357
			1	2	1	4	.211	9	.474	6	.316
					2	82	.649	53	.270	55	.221
					3	23	.315	21	.288	29	.397
	3			1	1	2	.333	3	.500	1	.167
					2	11	.274	26	.553	10	.213
					3	6	.429	3	.214	5	.357
		2		1	1	0	.000	3	.750	1	.250
					2	8	.305	14	.538	4	.154
					3	9	.200	23	.511	13	.289
			3	1	1	0	.000	5	.500	5	.500
					2	16	.400	9	.225	15	.375
					3	6	.167	7	.184	23	.577
Citizens	1			1	1	2	.118	14	.824	1	.059
					2	6	.353	11	.647	0	.000
					3	0	.000	1	.333	2	.667
		2		1	1	8	.056	129	.896	7	.049
					2	8	.031	248	.947	6	.023
					3	6	.665	68	.737	18	.196
			3	1	1	1	.650	12	.600	7	.350
					2	0	.000	7	.778	2	.222
					3	2	.143	4	.286	8	.571
	1			2	1	3	.158	9	.474	7	.358
					2	14	.341	22	.537	5	.122
					3	0	.000	9	1.000	0	.000
		2		1	1	12	.063	163	.662	14	.074
					2	21	.033	606	.959	5	.028
					3	4	.028	104	.717	37	.255
			3	1	1	0	.000	11	.917	1	.083
					2	2	.061	10	.303	21	.636
					3	1	.050	13	.650	6	.300
	1			2	1	0	.000	3	.750	1	.250
					2	3	.250	6	.667	1	.033
					3	2	.182	6	.745	3	.273
		2		1	1	9	.076	108	.508	2	.017
					2	4	.062	57	.891	3	.047
					3	9	.123	46	.630	18	.246
			3	1	1	5	.137	26	.703	6	.162
					2	2	.051	9	.223	27	.667
					3	6	.115	24	.462	22	.423

TABLE 4.2
 COMPARISON OF goodness of fit of different models to assess the order of a Markov chain model
 to fit the interaction data for OFFICERS + CITIZENS SEPARATELY

MODEL	CONFIGURATION	DF	χ^2	G^2	MODEL COMPARISON	G^2	DF	TENDS WHICH DIFFER
OFFICERS								
1)	Zero-Order Markov Model $\omega_{12}^1 \omega_{21}^1 \omega_{33}^1$	72	1278.60	787.14	1-2	270.96*	12	First Order Effects $\omega_{12}^1 \omega_{21}^1 \omega_{33}^1$
2)	First-Order Markov Model $\omega_{12}^1 \omega_{21}^1 \omega_{33}^1 \omega_{11}^1 \omega_{22}^1 \omega_{33}^1$	60	893.46	516.18	2-3	379.83*	8	Self-Contingent Effects $\omega_{11}^1 \omega_{22}^1$
3)	First Order + Self-Contingent Markov Model $\omega_{12}^1 \omega_{21}^1 \omega_{33}^1 \omega_{11}^1 \omega_{22}^1 \omega_{33}^1 \omega_{12}^1 \omega_{21}^1 \omega_{33}^1$	52	164.39	166.75	3-4	125.95*	16	Second-Order Effects $\omega_{12}^1 \omega_{21}^1$
4)	Second-Order Markov Model $\omega_{12}^1 \omega_{21}^1 \omega_{33}^1 \omega_{11}^1 \omega_{22}^1 \omega_{33}^1 \omega_{12}^1 \omega_{21}^1 \omega_{33}^1 \omega_{11}^1 \omega_{22}^1 \omega_{33}^1$	36	36.62	40.80	4-5	40.80	36	Third-Order $\omega_{11}^1 \omega_{22}^1 \omega_{33}^1$
5)	Third-Order Markov Model $\omega_{12}^1 \omega_{21}^1 \omega_{33}^1 \omega_{11}^1 \omega_{22}^1 \omega_{33}^1 \omega_{12}^1 \omega_{21}^1 \omega_{33}^1 \omega_{11}^1 \omega_{22}^1 \omega_{33}^1 \omega_{12}^1 \omega_{21}^1 \omega_{33}^1$	0	0	0				
CITIZENS								
1)	(same as above)	72	1431.28	895.13	1-2	360.28*	12	(same as above)
2)	(same as above)	60	951.78	550.85	2-3	379.36*	8	(same as above)
3)	(same as above)	52	210.64	205.49	3-4	137.69*	16	(same as above)
4)	(same as above)	36	62.01	65.80	4-5	65.80	36	(same as above)
5)	(same as above)	0	0	0				

*Significant at or beyond the .05 level.

From the results of this analysis it is clear that for both officers and citizens the Markov process which provides the best fit of the data (in terms of both parsimony and the statistical standard of goodness of fit) is a second-order Markov process. The contributions of the first-order other-contingent effects, the first-order self-contingent effects, and the second-order effects all are highly significant for both citizens and officers, and the contributions in both cases of the third-order effects are not significant.

It is important to qualify this statement by pointing out that it is not possible to conclude categorically from these results that these decision processes are second-order processes. There may be higher-order processes which offer substantially better fits to the data. One can never assume that unexamined variables will not provide surprises. However, it is safe to conclude that a second-order model is to be preferred over the other models considered here. Substantively too, it may be reasoned that if the third most recent event does not have a major impact relative to more recent events then it is not likely that further removed events would be likely to have stronger impacts.

The next step is to empirically test the assumption that the behaviors of officers and citizens differ due to the different roles they are enacting.

Role Heterogeneity

In order to explore the effects of role on the second-order Markov process, a five-dimensional data matrix will be investigated. This matrix is simply the matrix resulting from a combination of

the four-dimensional data matrices from the previous section. The dimensions of this revised matrix are therefore 1) the participant's own current response at time t , 2) the immediately preceding response of the other participant at time $t-1$, 3) the most recent previous response by the first participant at time $t-2$, 4) the second most recent response of the other participant at time $t-3$, and 5) the role of the first participant. This matrix is the entire matrix presented in Table 4.1.

The log linear model of a second-order Markov process for this matrix is the same as the second-order Markov process log linear model obtained in the preceding section. Since the superiority of a second-order model over the other models has already been documented, only log linear models based on this second-order Markov process are considered here. Additional u -terms which are in the saturated model for this matrix would be first-order u -terms for the effects of role, and so on up to and including the fifth order term involving role. These effects of role on the second order processes are progressively added, beginning with the first order term including role and ending with the fifth order term including role which results finally in the saturated model for this data matrix. These models and their fit to the data are presented in Table 4.3.

From the results presented in Table 4.3 it may be seen that there are significant interactions of role with the distribution of responses, with the first-order contingency of responses on past responses (both self-contingent and other contingent), and with the second-order contingency of responses on past responses; but

TABLE 4.3
EFFECTS OF ROLE HETEROGENEITY IN SECOND ORDER MARKOV PROCESS

MODEL DESCRIPTION	DF	F ²	G ²	MODEL PAIR	G ²	DF	TERMS IN MODEL WHICH DIFFER
1) Second-Order Markov Model $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8} \omega_9^{\omega_{10} \omega_{11}} \omega_{12}^{\omega_{13} \omega_{14}} \omega_{15}^{\omega_{16} \omega_{17}}$ $\omega_{18}^{\omega_{19} \omega_{20}} \omega_{21}^{\omega_{22} \omega_{23}} \omega_{24}^{\omega_{25} \omega_{26}}$	117	1679.82	1902.96	1-2	0.87	1	Main Effect of Role ω_5
2) Model 1 + Main Effect of Role $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8} \omega_9^{\omega_{10} \omega_{11}} \omega_{12}^{\omega_{13} \omega_{14}} \omega_{15}^{\omega_{16} \omega_{17}}$ $\omega_{18}^{\omega_{19} \omega_{20}} \omega_{21}^{\omega_{22} \omega_{23}} \omega_{24}^{\omega_{25} \omega_{26}}$	116	1678.91	1902.09	2-3	1503.10 ^a	8	Effect of Role on Dis- tribution of Responses $\omega_5^{\omega_6 \omega_7 \omega_8 \omega_9}$ $\omega_5^{\omega_{10} \omega_{11} \omega_{12} \omega_{13}}$ $\omega_5^{\omega_{14} \omega_{15} \omega_{16} \omega_{17}}$
3) Model 2 + Effect of Role on Distribution of Responses $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8} \omega_9^{\omega_{10} \omega_{11}} \omega_{12}^{\omega_{13} \omega_{14}} \omega_{15}^{\omega_{16} \omega_{17}}$ $\omega_{18}^{\omega_{19} \omega_{20}} \omega_{21}^{\omega_{22} \omega_{23}} \omega_{24}^{\omega_{25} \omega_{26}}$	108	825.05	398.99	3-4	82.68 ^b	12	Effect of Role on First- Order Contingency on Fast Responses $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8}$
4) Model 3 + Effect of Role on First Order Contingency on Fast Responses $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8} \omega_9^{\omega_{10} \omega_{11}} \omega_{12}^{\omega_{13} \omega_{14}} \omega_{15}^{\omega_{16} \omega_{17}}$ $\omega_{18}^{\omega_{19} \omega_{20}} \omega_{21}^{\omega_{22} \omega_{23}} \omega_{24}^{\omega_{25} \omega_{26}}$	96	375.20	356.31	4-5	123.24 ^b	8	Effect of Role on Contingency on Fast Responses $\omega_5^{\omega_6 \omega_7 \omega_8}$
5) Model 4 + Effect of Role on Contingency on Own Fast Responses $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8} \omega_9^{\omega_{10} \omega_{11}} \omega_{12}^{\omega_{13} \omega_{14}} \omega_{15}^{\omega_{16} \omega_{17}}$ $\omega_{18}^{\omega_{19} \omega_{20}} \omega_{21}^{\omega_{22} \omega_{23}} \omega_{24}^{\omega_{25} \omega_{26}}$	88	219.12	233.07	5-6	126.47 ^b	16	Effect of Role on Second- Order Contingency on Fast Responses $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5}$
6) Model 5 + Effect of Role on Second-Order Contingency on Responses $\omega_{12}^{\omega_1 \omega_2} \omega_3^{\omega_4 \omega_5} \omega_6^{\omega_7 \omega_8} \omega_9^{\omega_{10} \omega_{11}} \omega_{12}^{\omega_{13} \omega_{14}} \omega_{15}^{\omega_{16} \omega_{17}}$ $\omega_{18}^{\omega_{19} \omega_{20}} \omega_{21}^{\omega_{22} \omega_{23}} \omega_{24}^{\omega_{25} \omega_{26}}$	72	93.64	106.60				

there is no significant interaction of role with the third-order contingency of responses on past responses. These results confirm the finding that role heterogeneity is an important factor and must be considered in models of these data. The lack of significant interaction of role with the third-order contingency of current responses on past responses again provides support for the view that a second-order Markov process is sufficient for modeling these data. There is no evidence that the third-order contingency of present responses on past responses is necessary for such a model. These results suggest that there is no need to consider further the responses preceding those made at time $t-2$. Hence further tests will include only responses at times t , $t-1$, and $t-2$.

Stability over Time

The next step is to test another assumption of Markov chains; stability. Specifically, the assumption is that the contingencies of current responses on past responses are stable over time. It is not an assumption of Markov chains that the distribution of responses is stable over time. The contingencies of current responses on past responses correspond to the second and higher order u -terms involving choices at different points in time in the log linear models; in Markov models they correspond to decision rules.

In order to assess this assumption of stability it is desirable to compare these contingencies of present responses on past responses at points which differ considerably with regard to the time when they occur. This is accomplished by examining data matrices related to the data matrices already considered. The

two halves of the matrix in Table 4.1 were again considered separately producing two matrices, one for citizens and one for officers. Each of those matrices was then collapsed across the r responses at time $t-3$ since it has been shown that the data may be adequately fitted with a second order model. Then those matrices were each expanded along a new dimension, the time during the encounter at which the response was made. The matrix of data ultimately considered here represent a sample from that matrix including only actions which took place during the first or the last ten interactions in each encounter. The dimensions of each of these matrices are thus 1) the participant's own current response at time t , 2) the immediately preceding response of the other participant at time $t-1$, 3) the most recent previous response of the first participant at time $t-2$, and 4) the time during the encounter in which the interaction took place.

The effects of time on the responses may be assessed in much the same way the effects of role were previously assessed. Five models are considered. These begin, as in the analysis for the effects of role, with the basic second-order Markov model and progressively include terms reflecting the instability over time of the second-order process, beginning with the first-order term involving time and concluding with the fourth-order term which includes time, resulting in a saturated model. These models, and their fit to the data are summarized in Table 4.5 for both citizens and officers separately.

As may be seen in the results in that table, the greatest instability over time for officers is a result of different

TABLE 4.4

DATA FOR TESTS OF STABILITY OVER TIME OF SECOND-ORDER MARKOV CHAIN: OBSERVED FREQUENCIES AND TRANSITION PROBABILITIES AT TIME t BY THOSE AT $t-1$, $t-2$, AND BY TIME OF THE INTERACTION SEPARATELY FOR CITIZENS AND OFFICERS

Dimension 4: Stability Over Time	Dimension 3: Participant ₁ at Time $t-2$	Dimension 2: Action by Participant ₂ at Time $t-1$	Officers						Citizens						
			Dimension 1: Action by Participant ₁ at Time t						Dimension 1: Action by Participant ₁ at Time t						
			1 Freq	Prob	2 Freq	Prob	3 Freq	Prob	1 Freq	Prob	2 Freq	Prob	3 Freq	Prob	
First 10 Interactions	1	1	4	.667	2	.333	0	.000	0	.000	8	.889	1	.111	
		2	2	.400	3	.600	0	.000	6	.059	92	.902	4	.039	
		3	3	.300	5	.500	2	.200	0	.000	16	.889	2	.111	
	2	1	45	.315	67	.468	31	.217	2	.143	11	.786	1	.071	
		2	25	.231	58	.537	25	.231	5	.039	127	.962	0	.000	
		3	31	.554	10	.178	15	.268	0	.000	7	.350	13	.650	
	3	1	5	.625	6	.000	3	.375	2	.400	1	.200	2	.400	
		2	2	.167	7	.583	3	.250	4	.060	40	.597	23	.343	
		3	11	.344	13	.406	8	.250	4	.308	5	.335	4	.308	
	Last 10 Interactions	1	1	4	.364	7	.636	0	.000	1	.167	4	.667	1	.167
			2	4	.222	11	.611	3	.167	9	.076	105	.882	5	.042
		2	3	1	.143	3	.428	3	.428	2	.167	10	.833	0	.000
1			31	.279	56	.504	24	.243	6	.300	13	.650	1	.050	
3		2	61	.268	124	.544	43	.188	8	.040	189	.940	4	.020	
		3	27	.415	21	.323	17	.262	2	.333	2	.333	2	.333	
3	1	3	.600	6	.000	2	.400	0	.000	6	.857	1	.143		
	2	3	.333	3	.333	3	.333	5	.060	69	.821	10	.120		
		3	4	.190	5	.143	14	.667	1	.050	10	.500	9	.450	

TABLE 4.5

COMPARISON OF GOODNESS OF FIT OF DIFFERENT MODELS TO ASSESS STABILITY OVER TIME
OF THE SECOND-ORDER MARKOV MODEL OF SOCIAL INTERACTION

MODEL	CONFIGURATION	DF	χ^2	G^2	MODEL PAIR	G^2	DF	TERMS WHICH DIFFER
<u>COEFFICIENTS</u>								
1)	Second-Order Markov Process + Main Effect of Time							
	$u_{11}^{11}+u_{22}^{11}+u_{33}^{11}+u_{12}^{11}+u_{23}^{11}+u_{13}^{11}+u_{123}^{11}$	26	69.19	71.86	1-2	46.9 [*]	6	Effect of Time on Dis- tribution of Response
2)	Model 1 + Effect of Time on Distribution of Response							
	$u_{11}^{11}+u_{22}^{11}+u_{33}^{11}+u_{12}^{11}+u_{23}^{11}+u_{13}^{11}+u_{123}^{11}$ $+u_{14}^{11}+u_{24}^{11}+u_{34}^{11}$	20	23.93	24.98	2-3	11.09	8	Effect of Time on First- Order Contingency on Past Responses
3)	Model 2 + Effect of Time on First- Order Contingency on Past Responses							
	$u_{11}^{11}+u_{22}^{11}+u_{33}^{11}+u_{12}^{11}+u_{23}^{11}+u_{13}^{11}+u_{123}^{11}+u_{14}^{11}$ $+u_{24}^{11}+u_{34}^{11}+u_{124}^{11}+u_{234}^{11}$	12	12.98	13.89	3-4	10.99 [*]	4	Effect of Time as a First- Order Contingency on Own Past Responses
4)	Model 3 + Effect of Time as First- Order Contingency on Own Past Responses							
	$u_{11}^{11}+u_{22}^{11}+u_{33}^{11}+u_{12}^{11}+u_{23}^{11}+u_{13}^{11}+u_{123}^{11}+u_{14}^{11}$ $+u_{24}^{11}+u_{34}^{11}+u_{124}^{11}+u_{234}^{11}+u_{134}^{11}$	8	2.75	2.90	4-5	2.90	8	Effect of Time on Second- Order Contingency on Past Responses
5)	Model 4 + Effect of Time on Second- Order Contingency on Past Responses							
	$u_{11}^{11}+u_{22}^{11}+u_{33}^{11}+u_{12}^{11}+u_{23}^{11}+u_{13}^{11}+u_{123}^{11}+u_{14}^{11}+u_{24}^{11}+u_{34}^{11}$ $+u_{124}^{11}+u_{234}^{11}+u_{134}^{11}+u_{1234}^{11}$	0	0	0				
<u>CONSTANTS</u>								
1)	(same as above)	26	55.53	61.32	1-2	16.98 [*]	6	(same as above)
2)	(same as above)	20	39.82	45.34	2-3	8.44	8	(same as above)
3)	(same as above)	12	32.79	35.50	3-4	11.43 [*]	4	(same as above)
4)	(same as above)	8	20.73	25.17 [*]	4-5	25.17 [*]	8	(same as above)
5)	(same as above)	0	0	0				(same as above)

distributions of responses the officers and citizens make over time. This effect is consistent with the Markov model assumption-- i.e., it is expected that there may be different distributions of responses in early stages of the interaction relative to those at equilibrium. For officers the only other statistically significant effect is the instability of the first-order self-contingent response behavior over time. This might reflect some slight tendency for the officer to change this aspect of his behavior over time. However, it is so slight relative to the huge effects found earlier for role and the contingency of present decisions on previous decisions that it is likely to be of little substantive significance. Similar effects are found for citizens, and, in addition, the interaction term expressing the relationship between both own and others' past actions on present actions is somewhat unstable for them. However, these effects are also orders of magnitude less than other effects and quite likely are not of substantive interest given their small effects relative to those of role and past responses.

The conclusion which appears warranted here is that the behavior of officers and citizens in police-citizen encounters, while slightly unstable over time in one or two aspects for each role, is generally stable; and the instability is so small relative to other effects that it may be ignored. These findings thus provide support for the earlier finding that a second-order Markov process with heterogeneous roles will adequately fit the data.

Possible Sources of Heterogeneity

Tests of the heterogeneity of roles in the police-citizen interaction process do not exhaust the possible sources of heterogeneity in this process. Other possible sources of different response patterns are clearly identified in the conceptual framework. These include different task characteristics and different individual characteristics of the participants. Here, an example of each of these will be considered for the possible impact it might have upon the police-citizen interaction process. The apparent social class of the citizen participant (as evidenced from the appearance and demeanor of the citizen based upon the judgment of the observer) is an example of an individual characteristic of the participants which may affect this interaction process. The general nature of the offense which precipitated the encounter (i.e., whether it was a traffic offense or some other type of offense) is one empirical indicator of the variety of "tasks" or situations in which the officers and citizens might find themselves. Without question, these two do not exhaust these variables. But they do represent two of the variables which, based upon this analysis of the police-citizen interaction situation utilizing the conceptual framework presented earlier, offer promises of affecting the quality of the interaction.

The effects of these variables on police-citizen interaction are assessed separately in the same way the effects of time were assessed in the preceding section. In each case a variant of the multidimensional data matrix used earlier was again used. The matrices were four dimensional with the first three dimensions

being the by now standard 1) current action of self at time t , 2) most recent past action of the other participant at time $t-1$, and 3) the most recent past action of self at time $t-2$. The fourth dimension in each case was the variable in question. In one case it was the apparent social class of the citizen, and in the other case it was the nature of the offense (whether it was a traffic offense or a nontraffic offense). For each of these cases two separate matrices were considered: one for officers and one for citizens. Those data matrices are presented in Tables 4.6 and 4.7.

Five models similar to those considered in previous sections are again considered. These begin with the basic second-order Markov model and progressively include terms reflecting the effects of either class or offense, beginning with the first order term involving that variable (class or offense) and concluding with the fourth order term involving that variable, resulting in a saturated model. These models and their fit to the data are reported in Tables 4.8 and 4.9.

From the results in Table 4.8 it is clear that the apparent social class of the citizens does not appear to affect the second order Markov process describing the interaction. In addition, from Table 4.9 it is clear that the nature of the offense has no significant impact upon this second order Markov process for citizens. There are, however, some effects which are marginally significant of the nature of the offense on the Markov process describing the interaction of the officers. There does appear to be some slight tendency for the officers to change their

TABLE 4.6

DATA FOR TESTS OF IMPACT OF SOCIAL CLASS OF CITIZEN IN SECOND-ORDER MARKOV CHAIN: OBSERVED FREQUENCIES OF ACTS AND TRANSITION PROBABILITIES AT TIME t BY THOSE AT TIME t-1, t-2, AND BY SOCIAL CLASS OF THE CITIZENS

Dimension 4: Social Class of Citizen	Dimension 3: Action by Participant ₁ at Time t-2	Dimension 2: Action by Participant ₂ at Time t-1	Officers						Citizens						
			Dimension 1: Action by Participant ₁ at Time t						Dimension 1: Action by Participant ₁ at Time t						
			1		2		3		1		2		3		
Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob		
Middle/Upper	1	1	2	.500	2	.500	0	.000	0	.000	3	.750	1	.250	
		2	3	.333	5	.556	1	.111	3	.042	63	.887	5	.070	
		3	1	.250	1	.250	2	.500	2	.154	10	.769	1	.077	
	2	1	22	.259	49	.576	14	.165	2	.222	7	.778	0	.000	
		2	35	.199	114	.648	27	.153	6	.034	170	.960	1	.006	
		3	14	.318	16	.364	14	.318	1	.083	5	.417	6	.500	
	3	1	6	.857	0	.000	1	.143	0	.000	2	.667	1	.333	
		2	1	.143	4	.571	2	.286	3	.057	37	.693	13	.245	
		3	6	.273	8	.364	8	.364	1	.091	4	.364	6	.545	
	Working	1	1	12	.444	12	.444	3	.111	4	.160	19	.760	2	.080
			2	9	.281	17	.531	6	.188	22	.079	249	.889	9	.032
			3	5	.294	6	.353	6	.353	2	.046	28	.651	13	.302
2		1	80	.280	165	.535	61	.194	13	.351	22	.594	2	.054	
		2	109	.184	393	.671	86	.145	17	.029	562	.954	10	.017	
		3	85	.503	35	.207	49	.290	2	.040	17	.340	31	.620	
3		1	11	.423	5	.182	10	.385	3	.167	10	.556	5	.278	
		2	9	.186	21	.488	14	.326	10	.052	137	.706	47	.242	
		3	24	.304	24	.304	31	.392	4	.073	30	.545	21	.382	

TABLE 4.7

DATA FOR TESTS OF IMPACT OF NATURE OF OFFENSE IN SECOND-ORDER MARKOV CHAIN: OBSERVED FREQUENCIES OF ACTS AND TRANSITION PROBABILITIES AT TIME t BY THOSE AT TIME $t-1$, $t-2$, AND BY NATURE OF OFFENSE

Dimension 4: Nature of Offense	Dimension 3: Participant, at Time $t-2$	Dimension 2: Action by Participant ₂ at Time $t-1$	Officers Dimension 1: Action by Participant ₁ at Time t						Citizens Dimension 1: Action by Participant ₁ at Time t						
			1		2		3		1		2		3		
			Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob	Freq	Prob	
Traffic	1	1	9	.409	11	.500	2	.090	4	.210	13	.694	2	.105	
		2	8	.223	13	.371	14	.400	16	.073	195	.890	8	.036	
		3	4	.333	6	.500	2	.167	3	.033	24	.667	9	.250	
	2	1	68	.262	133	.514	58	.224	10	.357	17	.607	1	.036	
		2	84	.195	274	.636	73	.169	13	.030	410	.956	6	.014	
		3	67	.508	29	.220	36	.273	2	.067	13	.433	15	.500	
	3	1	10	.476	5	.233	6	.286	0	.000	6	.750	2	.250	
		2	4	.182	7	.318	11	.500	10	.061	111	.681	42	.258	
		3	22	.333	18	.273	26	.374	1	.024	24	.571	17	.405	
	Nontraffic	1	1	9	.529	7	.412	1	.059	1	.062	14	.875	1	.062
			2	6	.162	26	.703	5	.135	12	.052	205	.894	15	.065
			3	2	.091	11	.500	9	.409	3	.091	25	.758	5	.152
2		1	77	.310	135	.544	36	.145	15	.312	28	.583	5	.104	
		2	100	.192	352	.674	70	.194	20	.028	491	.946	8	.015	
		3	54	.397	39	.287	43	.316	2	.046	12	.279	29	.674	
3	1	10	.500	2	.100	8	.400	4	.444	10	.526	5	.263		
	2	8	.190	25	.595	9	.214	9	.062	106	.731	30	.207		
		3	15	.250	16	.267	29	.483	9	.200	17	.373	19	.422	

TABLE 4.8

COMPARISON OF GOODNESS OF FIT OF DIFFERENT MODELS TO ASSESS EFFECTS OF SOCIAL CLASS
OF CITIZENS ON THE SECOND-ORDER MARKOV MODEL OF SOCIAL INTERACTION

MODEL	CONFIGURATION	DF	χ^2	G ²	MODEL PAIR	G ²	DF	TERMS WHICH DIFFER
<u>CITIZENS</u>								
1)	Second-Order Markov Process + Main Effect of Class $u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$	26	16.80	18.92	1-2	2.37	6	Effect of Class on Dis- tribution of Response $u_{11}u_{12}u_{21}u_{22}$
2)	Model 1 + Effect of Class on Distribution of Response $u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$ $+u_{11}u_{21}u_{22}u_{11}$	20	15.32	16.55	2-3	8.47	8	Effect of class on First- Order Contingency on Past Responses $u_{12}u_{21}u_{22}$
3)	Model 2 + Effect of Class on First- Order Contingency on Past Responses $u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$ $+u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$	12	6.67	8.08	3-4	2.49	4	Effect of Class on a First- Order Contingency on Own Past Responses $u_{11}u_{12}$
4)	Model 3 + Effect of Class on First- Order Contingency on Own Past Responses $u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$ $u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}$	8	4.38	5.59	4-5	5.59	8	Effect of Class on Second- Order Contingency on Past Responses $u_{12}u_{21}$
5)	Model 4 + Effect of Class on Second- Order Contingency on Past Responses $u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$ $u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}u_{11}u_{12}u_{21}u_{22}$	0	0	0				
<u>CITIZENS</u>								
1)	(same as above)	26	17.37	19.96	1-2	3.54	6	(same as above)
2)	(same as above)	20	15.62	16.42	2-3	4.74	8	(same as above)
3)	(same as above)	12	10.50	11.63	3-4	2.85	4	(same as above)
4)	(same as above)	8	7.90	8.83	4-5	8.83	8	(same as above)
5)	(same as above)	0	0	0				

TABLE 4.9

COMPARISON OF GOODNESS OF FIT OF DIFFERENT MODELS TO ASSESS EFFECTS OF NATURE OF OFFENSE OF CITIZENS ON THE SECOND-ORDER MARKOV MODEL OF SOCIAL INTERACTION

MODEL	CONFIGURATION	DF	χ^2	ϵ^2	MODEL PAIR	ϵ^2	DF	TERMS WHICH DIFFER
<u>CITIZENS</u>								
1)	Second-Order Markov Process + Main Effect of offense $u+u_1+u_2+u_3+u_4+u_{12}+u_{23}+u_{13}+u_{123}$	26	46.89	48.22*	1-2	13.86*	6	Effect of offense on Distribution of Response $u+u_{12}+u_{23}+u_{13}$
2)	Model 1 + Effect of offense on Distribution of Response $u+u_1+u_2+u_3+u_4+u_{12}+u_{23}+u_{13}+u_{123}$ $+u_{124}+u_{234}+u_{134}$	20	33.32	34.36*	2-3	19.51*	8	Effect of offense on First-Order Contingency on Past Responses $u_{124}+u_{234}$
3)	Model 2 + Effect of offense on First-Order Contingency on Past Responses $u+u_1+u_2+u_3+u_4+u_{12}+u_{23}+u_{13}+u_{123}+u_{124}$ $+u_{234}+u_{134}+u_{124}+u_{234}$	12	14.52	14.85	3-4	1.54	4	Effect of offense on a First-Order Contingency on Own Past Responses u_{134}
4)	Model 3 + Effect of offense on First-Order Contingency on Own Past Responses $u+u_1+u_2+u_3+u_4+u_{12}+u_{23}+u_{13}+u_{123}+u_{124}$ $+u_{234}+u_{134}+u_{124}+u_{234}+u_{134}$	8	13.01	13.31	4-5	13.31	8	Effect of offense on Second-Order Contingency on Past Responses u_{1234}
5)	Model 4 + Effect of offense on Second-Order Contingency on Past Responses $u+u_1+u_2+u_3+u_4+u_{12}+u_{23}+u_{13}+u_{124}+u_{234}$ $+u_{134}+u_{124}+u_{234}+u_{134}+u_{1234}$	0	0	0				
<u>CITIZENS</u>								
1)	(same as above)	26	34.82	37.83	1-2	9.96	6	(same as above)
2)	(same as above)	20	25.20	27.87	2-3	9.42	8	(same as above)
3)	(same as above)	12	16.79	18.45	3-4	6.29	6	(same as above)
4)	(same as above)	8	11.10	12.16	4-5	12.16	8	(same as above)
5)	(same as above)	0	0	0				

distribution of responses, and even their second-order decision rules with the different types of offense. These effects, however, as with those of instability over time, are considerably smaller than those observed earlier for both role and for past interactions, and for this reason they will be ignored in the further development and testing of the Markov model. It is likely that these effects are of little substantive importance relative to those much larger effects, and would contribute more to complicating the model than to gaining greater understanding or fitting the data more accurately.

The general conclusion from these tests would thus appear to be that the effects of these two sources of heterogeneity are very small, when present at all, and may be safely ignored in the development of a Markov model of this process.

Summary Results of Tests of Assumptions

Tests of the assumptions of stability, homogeneity, and order of the Markov process were made. These findings indicate that a second-order Markov model with heterogeneous roles provides an adequate fit of these data. The superiority of a model of this order at fitting the data has been clearly demonstrated over models of both first and third order. The heterogeneity of roles has been clearly established. And there is convincing evidence that these processes are relatively stable over time and do not vary substantially for different classes of citizens or for traffic versus nontraffic offenses.

These results indicate that the simplest data matrix which included all of the effects which were found would need to include

four dimensions: 1) the participant's own current response at time t , 2) the immediately preceding response of the other participant at time $t-1$, 3) the most recent previous response of the first participant at time $t-2$, 4) the role of the first participant. Such a matrix is presented in Table 4.10. This matrix should have all of the information which is required for the estimation of the parameters for the model of these data as a second-order Markov process with heterogeneous roles.

TABLE 4.10

PERMISSIBLE COLLAPSED TABLE FOR EFFECTS OF ROLE ON SECOND-
ORDER MARKOV MODEL OF SOCIAL INTERACTION

Dimension 5: Role of Participant ₁	Dimension 3: Action of Participant ₁ at Time t-2	Dimension 2: Action of Participant ₂ at Time t-1 ²	Dimension 1: Action of Participant ₁ at Time t					
			1		2		3	
			Freq	Prob	Freq	Prob	Freq	Prob
Officers	1	1	18	.462	18	.462	3	.077
		2	133	.291	245	.536	79	.173
		3	18	.474	7	.184	13	.342
		1	14	.226	39	.629	9	.145
		2	186	.193	634	.657	145	.150
		3	12	.155	33	.503	20	.308
	2	1	6	.182	16	.485	11	.333
		2	109	.433	65	.261	75	.301
		3	33	.257	29	.252	53	.461
		1	5	.125	26	.650	9	.225
		2	23	.329	41	.586	6	.086
		3	2	.037	16	.695	5	.217
Citizens	1	1	29	.064	650	.835	23	.051
		2	33	.074	911	.951	14	.015
		3	19	.061	218	.703	73	.235
	2	1	6	.087	49	.710	14	.203
		2	4	.053	26	.347	45	.600
		3	9	.105	41	.477	36	.419

The Second-Order Markov Model With Heterogeneous Roles

At this point the next task is to explicitly state the mathematical model of a second-order Markov process with heterogeneous roles called for by the results of the tests of assumptions. Such a statement is relatively straight-forward and is merely a mathematical restatement of the basic properties of this model. Next the parameters for this model will be estimated from the data in Table 4.10 using standard procedures. This model will then provide the basis for the remainder of the analysis. In the section immediately following this one a number of predictions of this model will be generated and tested empirically. It is only after all of this has been done and only if the tests of those predictions further support the model that the logical properties of the model will be explored and it will be empirically interpreted.*

A mathematical model of a second-order Markov process with heterogeneous roles constitutes a relatively simple extension of the simple first-order homogeneous Markov model discussed earlier in Chapter 3, and whose transition matrix is presented in Figure 3.1. This second-order property of the model may be stated mathematically as follows:

*it is generally a good idea to test as many assumptions and predictions of a model as possible before exploring its substantive implications and using the model to direct future research. In this case, since there were a few minor departures from the assumptions of stability and heterogeneity (apart from role heterogeneity which was incorporated into the model), this further testing of the model is particularly appropriate.

$$\begin{aligned}
 & P[O_k(t); C_j(t-1), O_1(t-2), \dots, C_n(t-r), O_n(t-r-1)] \\
 & = P[O_k(t); C_j(t-1), O_1(t-2)], \text{ for all } i, j, k, a, n, r \\
 \text{and} \quad & P[C_k(t); O_j(t-1), C_1(t-2), \dots, O_n(t-r), C_n(t-r-1)] \\
 & = P[C_k(t); O_j(t-1), C_1(t-2)], \text{ for all } i, k, j, a, n, r
 \end{aligned}$$

Because the process is second-order the states may simply be redefined to include two events occurring one after another in time (i.e., instead of states 1, 2, and 3, this model has states 11, 12, 13, 21, 22, 23, 31, 32, and 33). With the states so redefined the Markov assumption again holds that the current action is contingent only upon the past state and no other past states.

The second departure from the simplest version of a Markov model arises from the observed heterogeneity of roles. The property of role heterogeneity may be expressed mathematically as follows:

heterogeneous distributions of responses

$$P[O_1(t)] \neq P[C_1(t)]$$

heterogeneous first-order other-contingent transition probabilities

$$P[O_j(t); C_1(t-1)] \neq P[C_k(t); C_1(t-2)]$$

heterogeneous first-order self-contingent transition probabilities

$$P[O_k(t); O_1(t-2)] \neq P[C_k(t); C_1(t-2)]$$

heterogeneous second-order transition probabilities

$$P[O_k(t); C_j(t-1), O_1(t-2)] \neq P[C_k(t); O_j(t-1), C_1(t-2)]$$

Any one or more of these statements may be true.

It is clear that the contingency of the action of citizens upon past actions, and the distribution of responses of citizens both differ from those of officers. Hence it is always important in estimating transition probabilities for this model to distinguish officers from citizens. This simply means that in expressing the mathematical model it is necessary to always distinguish between officer's and citizen's states (i.e., instead of having 9 states: 11, 12, 13, 21, 22, 23, 31, 32, and 33, there are now 18 states: 01C1, 01C2, 01C3, 02C1, 02C2, 02C3, 03C1, 03C2, 03C3, C101, C102, C103, C201, C202, C203, C301, C302, and C303). The remaining properties of the model (e.g., homogeneity and stability) are the same as for the simple first-order model discussed earlier.

The transition matrix for the second-order Markov model with heterogeneous roles can thus be represented by an 18×18 matrix with the states defined by actions at time $t-1$ and t represented by columns and the states defined by actions at time $t-2$ and $t-1$ by rows. In each cell of that matrix would be the conditional probability of some state, $C_i O_j$ ($O_i C_j$), at times $t-1$ and t , given that some other state, $O_h C_i$ ($C_h O_i$), occurred at times $t-2$ and $t-1$. Such a matrix is illustrated in Figure 4.11. If this matrix were partitioned into four sections (I, II, III, and IV) as in Figure 4.11, then those quantities in the cells in section II would represent the transition probabilities which describe the response of officers to the past responses of themselves at time $t-2$ and the citizens at time $t-1$; those quantities in the cells in section III would represent the transition probabilities which describe the response of citizens to the past responses of

FIGURE 4.11

TRANSITION MATRIX FOR SECOND-ORDER MARKOV MODEL
WITH HETEROGENEOUS ROLES

STATES DEFINED BY ACTIONS AT TIMES t-2 and t-1 (t-2,t-1)	States Defined by Actions at Times t-1 and t (t-1,t)					
	o_1c_1	...	o_3c_3	c_1o_1	...	c_3o_3
o_1c_1 : o_3c_3	I $P(o_i c_j : o_h c_i) = 0$ where $h, i, j = 1, 2, 3$			II $P(c_i o_j : o_h c_i) = c^{(1)}$ where $h, i, j = 1, 2, 3$		
c_1o_1 : c_3o_3	III $P(o_i c_j : c_h o_i) = o^{(1)}$ where $h, i, j = 1, 2, 3$			IV $P(c_i o_j : c_h o_i) = 0$ where $h, i, j = 1, 2, 3$		

officers at time t-1 and to themselves at time t-2. The quantities in the cells in section I and IV would represent the transition probabilities describing the response of citizens to themselves at time t-1 and officers at t-2, and those describing the response of officers to themselves at time t-1 and citizens at time t-2. In this data, however, the responses were coded in such a way that the response of one participant never directly follows his own previous response and can only follow directly the response of the other participant.

$$P[O_i(t), O_j(t-1)] = [P C_j(t), C_i(t-1)] = 0, \text{ for all } i, j$$

Further, the events represented by the rows and those represented by the columns overlap in time one with the other. The second act of the two acts defining an event for the rows is the first act of the two acts defining an event for the columns (the act occurring at time t-1). Hence the transition probabilities in cells where the event at time t-1 represented by the row is not the same as the event at time t-1 represented by the column, must be zero.

$$P[O_i(t-1), C_j(t); O_h(t-2), O_k(t-1)] = \delta_{ik} = 0 \text{ for all } i \neq k \\ 1 \text{ for all } i = k$$

$$P[C_i(t-1), O_j(t); O_h(t-2), C_k(t-1)] = \delta_{ik} = 0 \text{ for all } i \neq k \\ 1 \text{ for all } i = k$$

Hence there are only three possible values other than zero for each row and the matrix is greatly simplified.

It has been shown (Anderson and Goodman, 1954; Birch, 1963) that the maximum likelihood estimate of the transition probability,

P_{ij} (the probability that event j will occur at time t given that event i occurred at time $t-1$) is simply

$$P_{ij} = \frac{x_{ij}}{x_{i1}} = \frac{x_{ij}(t)}{x_{i1}(t-1)}$$

Where x_{ij} is simply the number of occurrences of x_{kj} or the number of occurrences of event i at time $t-1$ and event j at time t . These estimates are readily obtained from the data presented in Table 4.10. The estimates of these parameters for the second-order Markov process with heterogeneous roles and the resulting model are presented in Table 4.12. This model provides the basis for all of the analysis which follows.

There are a number of properties of this model which have important substantive interpretations and which will be helpful to know for purposes of generating and testing predictions in the sections to come. Some of these properties will be established here for use in generating predictions. A more intensive discussion of their substantive interpretations will be presented in the later section in which the model is interpreted and explored.

This model is periodic with a period of 2. That is, the probability of the system going from state $O_i(t)C_j(t+1)$ to $O_i(+n)C_j(t+1+n)$ is zero unless n is a multiple of 2 (Feller, 1968). It has already been pointed out that the probability of a state where the officer's response precedes that of the citizen, being followed directly by another state in which the officer's response precedes that of the citizen is zero and the same is true of states where the citizen's response precedes that of the officer. In Table 4.13 is presented the two-step transition matrix

TABLE 4.12
TRANSITION MATRIX FOR THE SECOND-ORDER MARKOV MODEL WITH HETEROGENEOUS RULES

Actions of Officers and Citizens at Times t-2 and t-1 (t-2, t-1)	Actions of Officers and Citizens at Times t-1 and t (t-1, t)															N		
	O ₁ C ₁	O ₁ C ₂	O ₁ C ₃	O ₂ C ₁	O ₂ C ₂	O ₂ C ₃	O ₃ C ₁	O ₃ C ₂	O ₃ C ₃	C ₁ O ₁	C ₁ O ₂	C ₁ O ₃	C ₂ O ₁	C ₂ O ₂	C ₂ O ₃		C ₃ O ₁	C ₃ O ₂
O ₁ C ₁									.46 (18)	.16 (18)	.37 (37)							39
O ₁ C ₂													.23 (15)	.53 (27)	.18 (9)			59
O ₁ C ₃															.48 (20)	.17 (7)	.35 (18)	81
O ₂ C ₁									.22 (14)	.52 (29)	.14 (9)							68
O ₂ C ₂												.17 (18)	.67 (34)	.16 (15)				93
O ₂ C ₃															.18 (18)	.56 (32)	.26 (19)	63
O ₃ C ₁									.17 (6)	.50 (17)	.33 (11)							36
O ₃ C ₂												.43 (11)	.56 (49)	.27 (79)				149
O ₃ C ₃															.81 (37)	.27 (35)	.43 (35)	117
O ₁ O ₁	.13 (3)	.77 (27)	.06 (3)															33
O ₁ O ₂				.29 (8)	.39 (6)	.07 (6)												16
O ₁ O ₃							.14 (4)	.33 (16)	.29 (7)									27
O ₂ O ₁	.06 (29)	.88 (40)	.01 (2)															43
O ₂ O ₂				.03 (3)	.91 (12)	.01 (1)												159
O ₂ O ₃							.06 (19)	.70 (21)	.23 (7)									30
O ₃ O ₁	.08 (6)	.71 (49)	.21 (14)															69
O ₃ O ₂				.05 (4)	.37 (26)	.60 (4)												75
O ₃ O ₃							.11 (10)	.47 (41)	.42 (36)									87

containing the probability of a response $C_1 C_j$ (or $C_1 O_j$) at times t and $t+1$ respectively given the response $O_g C_h$ (or $C_g O_h$) at times $t-2$ and $t+1$ respectively. This matrix is obtained by simply multiplying the 1-step transition matrix by itself. In this matrix it may be seen that the probabilities of any one of the states following themselves two steps later are greater than 0.

It has been shown in general (Feller, 1968) that for Markov chains such as this which are periodic with period t and where all states have similar properties as these do, the states can be divided into t mutually exclusive classes G_0, \dots, G_{t-1} such that a one-step transition leads to a state in the neighboring class of states. Each of these submatrices corresponds to an irreducible closed set. For each of these submatrices there is an equilibrium distribution of responses which is totally independent of the initial distribution and is defined solely in terms of the transition matrix. In this case those submatrices are $G_0 = I + IV$ and $G_1 = II + III$ where I, II, III, and IV are the partitioned sections of the matrix presented in Figure 4.11.

It may also be shown that this second-order heterogeneous Markov process is equivalent to two second-order homogeneous Markov processes which alternate. If we think of the partitioned submatrices I and IV in Figure 4.11 as the one-step transition probabilities associated with the processes for citizens and officers respectively ($C^{(1)}$ and $O^{(1)}$), then our matrix simplifies to the following form:

TABLE 8.13

TWO-STEP TRANSITION MATRIX FOR SECOND-ORDER MARKOV MODEL
WITH HETEROGENEOUS ROLES

Actions of Officers and Citizens at Times t-2 and t-1 (t-2, t-1)	Actions of Officers and Citizens at Times t and t+1 (t, t+1)																								
	$O_1^t O_1^{t+1}$	$O_1^t O_2^{t+1}$	$O_1^t C_1^{t+1}$	$O_2^t O_1^{t+1}$	$O_2^t O_2^{t+1}$	$O_2^t C_1^{t+1}$	$O_2^t C_2^{t+1}$	$O_2^t C_3^{t+1}$	$O_3^t O_1^{t+1}$	$O_3^t O_2^{t+1}$	$O_3^t C_1^{t+1}$	$O_3^t C_2^{t+1}$	$O_3^t C_3^{t+1}$	$C_1^t O_1^{t+1}$	$C_1^t O_2^{t+1}$	$C_1^t C_1^{t+1}$	$C_1^t C_2^{t+1}$	$C_1^t C_3^{t+1}$	$C_2^t O_1^{t+1}$	$C_2^t O_2^{t+1}$	$C_2^t C_1^{t+1}$	$C_2^t C_2^{t+1}$	$C_2^t C_3^{t+1}$		
$O_1^t C_1^{t+1}$.066	.356	.040	.152	.278	.036	.011	.046	.020																
$O_1^t C_2^{t+1}$.018	.252	.015	.018	.504	.008	.011	.120	.043																
$O_1^t C_3^{t+1}$.042	.376	.029	.009	.059	.103	.029	.141	.141																
$O_2^t C_1^{t+1}$.032	.178	.019	.207	.373	.050	.021	.066	.033																
$O_2^t C_2^{t+1}$.012	.171	.010	.022	.625	.010	.009	.105	.035																
$O_2^t C_3^{t+1}$.016	.131	.030	.027	.176	.365	.035	.145	.128																
$O_3^t C_1^{t+1}$.025	.136	.015	.165	.296	.040	.040	.192	.064																
$O_3^t C_2^{t+1}$.029	.290	.023	.009	.244	.064	.018	.207	.069																
$O_3^t C_3^{t+1}$.025	.207	.039	.015	.096	.166	.030	.204	.179																
$C_1^t O_1^{t+1}$.066	.066	.011	.220	.407	.143	.042	.015	.029							
$C_1^t O_2^{t+1}$.074	.207	.040	.114	.309	.099	.015	.040	.024							
$C_1^t O_3^{t+1}$.084	.074	.040	.167	.132	.174	.073	.071	.112							
$C_2^t O_1^{t+1}$.030	.030	.005	.258	.649	.164	.005	.009	.017							
$C_2^t O_2^{t+1}$.060	.041	.005	.184	.615	.145	.005	.006	.005							
$C_2^t O_3^{t+1}$.011	.030	.020	.316	.181	.207	.048	.045	.102							
$C_3^t O_1^{t+1}$.040	.040	.007	.202	.376	.131	.029	.035	.049							
$C_3^t O_2^{t+1}$.018	.033	.040	.069	.228	.052	.111	.305	.185							
$C_3^t O_3^{t+1}$.020	.020	.027	.212	.121	.120	.120	.116	.179							

	t-1, t
	$o^{(1)}$
t-2, t-1	$c^{(1)}$

The two-step transition matrix presented in Table 4.13 is simply the product of the one-step transition matrix times itself. This is equivalent to the product of the two separate processes occurring over time as follows:

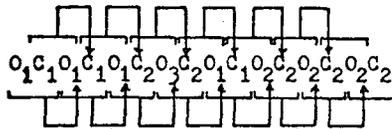
	t, t+1
	$o^{(1)}c^{(1)}$
t-2, t-1	$c^{(1)}o^{(1)}$

It may be shown that the n th step transition matrix is composed of the products of the two different processes: $OCOCOCO\dots$ or $COCOCO\dots$ respectively.

This may also be illustrated with the help of a hypothetical set of responses which may characterize the social interaction for a particular police-citizen encounter. Suppose the following sequence of interactions occurred:

$$O_1 C_1 O_1 C_1 O_1 C_2 O_3 C_2 O_1 C_1 O_2 C_2 O_2 C_2 O_2 C_2$$

Because it is a second-order process it is known that the response of the officer and citizen is a function of the past two events which occurred. The probability of the next response given the past two is expressed in the transition matrix. This transition probability may be illustrated with respect to this data as follows:



The first two responses, O_1 and C_1 lead to the choice by the officer of O_1 . The second pair of responses, C_1 and O_1 , in turn, lead to a new response by the citizen of C_1 , and so on. This might also be represented as the initial distribution of responses times the transition matrix for the officer, then that for the citizen, then that for the officer, and so on, until all transitions have been made and the encounter has ended.

$$X(t+n) = O_n C_{n-1} \dots O_2 C_1 X(t)$$

or $= C_n O_{n-2} \dots C_2 O_1 X(t)$, depending on the sequence which occurs.

Testing Predictions of the Model

Having tested a number of assumptions of a number of possible Markov models and selecting and specifying a specific Markov model (the second-order model with heterogeneous roles) which appears to be an appropriate fit to the data, the next step is to test some of the predictions of the model. One of the advantages of Markov models is that they have been relatively well examined in the past and there are a number of predictions which have been derived for such models which may be tested. In selecting aspects of the model to test it is important that they be selected not only for their adequacy as tests of the model but also for their substantive relevance. Here three specific tests of predictions will be made.* The observed and predicted values will be compared for

- 1) the equilibrium distribution of responses,
- 2) multi-step transition probabilities, and
- 3) the distribution of runs (sequences of responses all of the same type)

It is important not to lose sight of the primary goal of this research which is to gain a better understanding of the social interaction which takes place between police and citizens in these

*A fourth test was also examined. This was a test of the mean recurrence times (e.g., the mean time it takes for the system to return to some particular state after once having been in that state). However, it appears that this particular test is not an adequate test of this model because the mean and the variance of the recurrence times for these data are large relative to the length of encounters. As a result, only a very severely truncated distribution of recurrence times is observable. Hence it is not possible to estimate these empirically. This test and the problems with it are discussed in detail in Appendix 2.

brief encounters. One way of doing this is to develop a model which has assumptions which are compatible with the data and which has precise, testable, correct predictions. But the model itself remains secondary to the primary goal of gaining greater understanding.

The Equilibrium Distribution of Responses

One of the most convenient predictions of Markov models is that for aperiodic ergodic Markov chains the Markov process approaches an equilibrium distribution of responses which is totally independent of the initial distribution and is defined solely in terms of the transition matrix (Feller, 1968). As pointed out in the statement of the model, it is possible to partition the transition matrix for the second-order model with heterogeneous roles into two irreducible closed aperiodic Markov chains. Because the equilibrium distribution probabilities of each response is greater than zero these submatrices are also ergodic. Thus, these submatrices should exhibit equilibrium distributions defined solely in terms of the transition matrix.

Empirically, it is possible to estimate the equilibrium distribution of responses for these data by examining the responses which occur after equilibrium has been reached. For purposes of this test the distribution of responses in the last ten interactions in encounters were computed as estimates of the equilibrium distribution.* These empirical estimates are reported as the

*It will be shown in a later section that the rate at which the system approaches equilibrium is fast enough to make this a reasonable estimate of the equilibrium distribution.

"observed" distribution in Table 4.14.

It is also possible to predict the equilibrium distribution from the Markov model. There are a number of ways this may be done. Here the predicted equilibrium distribution is found simply by raising the transition matrix to a very high power (the 24th order). When this is done the probabilities in each column approach a stable value which is the same for each row. The equilibrium distribution probability of each response is then simply the probability in the column associated with that response.

Estimates of the predicted frequency distribution of these responses for the last ten interactions are then obtained by simply multiplying the predicted probability of each response occurring times the total number of responses which occurred during the last 10 interactions obtained from the observed estimates. The predicted equilibrium frequency distribution for each of these submatrices are also reported in Table 4.14.

These predicted and observed equilibrium distributions are quite close. Their goodness of fit was tested by fitting log linear models to these two sets of data and examining their fit. The results of those tests are presented in Table 4.15. It is clear from those results that the model provides very good predictions of the equilibrium distribution of responses.

Multi-step Transition Probabilities

From the transition matrix for a Markov chain it is possible to derive multi-step transition probabilities--i.e., transition matrices describing the probability a particular response will occur on the n th interaction given that some particular response

TABLE 4.14

EXPECTED AND OBSERVED EQUILIBRIUM DISTRIBUTION OF STATES FOR
SECOND-ORDER MARKOV MODEL WITH HETEROGENEOUS ROLES

STATE	EQUILIBRIUM	DISTRIBUTION
	OBSERVED FREQUENCY	PREDICTED FREQUENCY
O_1C_1	11	9
O_1C_2	111	108
O_1C_3	5	9
O_2C_1	18	14
O_2C_2	228	226
O_2C_3	9	14
O_3C_1	7	7
O_3C_2	65	63
O_3C_3	21	26
C_1O_1	6	9
C_1O_2	20	17
C_1O_3	7	5
C_2O_1	119	103
C_2O_2	201	221
C_2O_3	84	72
C_3O_1	12	15
C_3O_2	6	16
C_3O_3	20	19

TABLE 4.15

TEST OF GOODNESS OF FIT OF PREDICTED AND OBSERVED EQUILIBRIUM DISTRIBUTIONS

MODEL CONFIGURATION	DF	χ^2	G^2	MODEL PAIR	AG^2	ΔDF	TERMS WHICH DIFFER
Officers First							
1) Model Excluding Effects of Predicted vs. Observed				1-2	3.57	9	Effects of Predicted vs. Observed
$u+u_1$	9	3.54	3.57				u_2+u_{12}
2) Model Including All Effects							
$u+u_1+u_2+u_{12}$	0	0	0				
Citizens First							
1) Model Excluding Effects of Predicted vs. Observed				1-2	9.28	9	Effects of Predicted vs. Observed
$u+u_1$	9	9.11	9.28				u_2+u_{12}
2) Model Including All Effects							
$u+u_1+u_2+u_{12}$	0	0	0				

occurred on the first interaction. This particular prediction has been one which has generally not fit the data well in Markov models of mobility (Coleman, 1973; Spilerman, 1970; McFarland, 1969; Leik and Meeker, 1975). There has been a recurring problem with such models underestimating the proportion of people who do not move at all. This has been called the deficient diagonal problem by Coleman (1973), and has been the subject of a number of revisions of the basic Markov models, including the mover-stayer model by Elmen, Kogan, and McCarthy (1955) and the underlying states model of Leik and Meeker (1975). For this reason alone it is worthwhile to explore this prediction for this model to see if the same problem exists. Perhaps this would provide some evidence of the extent to which this is a problem endemic to Markov models or a problem of a particular set of data to which they have been applied. In addition, examination of the multi-step transition probabilities is a good measure of the extent to which the model is useful for predicting behavior over a long period of time. If the model is really appropriate, then it should be able to generate relatively accurate predictions over long time periods.

It is a relatively simple matter to estimate empirically the multi-step transition probabilities. It is simply necessary to create a contingency table where the rows represent the state the system is in at time t , and the columns represent the state it is in at time $t+20$, and in each cell is the frequency f_{ij} with which the system is observed to be in state S_j at time $t+20$ and in state S_i at time t . The resulting frequencies obtained in this way are displayed in Table 4.16.

TABLE 4.16

OBSERVED AND PREDICTED FREQUENCIES FOR 3-STEP TRANSITION MATRIX FOR SECOND-GRADE
RAPPOV MODEL WITH HETEROGENEOUS ROLES

Actions of Officers and Citizens at Times t-2 and t-1 (t-2, t-1)	Actions of Officers and Citizens at Times t+18 and t+19 (t+18, t+19)															N			
	0_1C_1	0_1C_2	0_1C_3	0_2C_1	0_2C_2	0_2C_3	0_3C_1	0_3C_2	0_3C_3	C_1O_1	C_1O_2	C_1O_3	C_2O_1	C_2O_2	C_2O_3		C_3O_1	C_3O_2	C_3O_3
0_1C_1	1 (0)	1 (5)	0 (0)	2 (1)	15 (11)	0 (1)	0 (0)	1 (3)	4 (1)										24
0_1C_2	5 (6)	5 (7)	5 (6)	13 (13)	15 (15)	15 (16)	2 (1)	3 (3)	13 (13)										33
0_1C_3	2 (1)	9 (7)	2 (1)	3 (1)	6 (16)	3 (1)	2 (1)	4 (0)	2 (2)										33
0_2C_1	0 (1)	6 (9)	2 (1)	7 (1)	14 (19)	2 (1)	1 (1)	4 (5)	3 (2)										39
0_2C_2	9 (11)	114 (132)	10 (11)	18 (17)	323 (277)	20 (17)	5 (9)	62 (77)	21 (32)										582
0_2C_3	2 (1)	9 (10)	1 (1)	1 (1)	19 (21)	3 (1)	0 (1)	5 (6)	5 (2)										45
0_3C_1	3 (0)	2 (4)	0 (0)	0 (0)	7 (8)	1 (1)	0 (0)	2 (2)	3 (1)										17
0_3C_2	4 (3)	28 (36)	3 (3)	3 (3)	83 (75)	2 (3)	3 (2)	22 (21)	10 (9)										158
0_3C_3	1 (2)	17 (19)	5 (2)	4 (2)	27 (40)	5 (3)	4 (1)	18 (11)	3 (3)										94
C_1O_1										2 (1)	2 (1)	0 (0)	4 (6)	11 (10)	6 (4)	1 (1)	3 (1)	1 (1)	29
C_1O_2										2 (1)	4 (1)	3 (0)	2 (9)	17 (20)	9 (6)	0 (1)	4 (1)	1 (2)	42
C_1O_3										0 (0)	0 (1)	0 (0)	2 (4)	7 (8)	3 (3)	3 (1)	0 (1)	3 (1)	18
C_2O_1										4 (5)	11 (9)	2 (3)	70 (58)	120 (124)	39 (41)	7 (8)	8 (9)	6 (10)	267
C_2O_2										9 (10)	26 (20)	4 (6)	117 (126)	316 (271)	61 (85)	16 (18)	19 (17)	14 (23)	582
C_2O_3										2 (2)	3 (6)	3 (2)	30 (36)	81 (78)	25 (25)	8 (5)	5 (6)	10 (7)	167
C_3O_1										0 (1)	4 (2)	1 (1)	6 (10)	22 (24)	5 (7)	1 (1)	3 (2)	4 (8)	46
C_3O_2										0 (1)	4 (2)	1 (1)	8 (12)	21 (25)	10 (8)	3 (2)	4 (2)	1 (2)	54
C_3O_3										2 (1)	3 (2)	1 (1)	9 (11)	13 (24)	13 (8)	1 (2)	3 (2)	6 (2)	51

The prediction of multi-step transition probabilities is one of the more straightforward of the predictions which may be made with Markov models. A fundamental property of Markov models which was pointed out earlier when they were first discussed (see Chapter 3) is that the distribution of responses at any time $t+n$ is simply the product of the distribution at some time t times the n th power of the transition matrix, T .

$$X(t+n) = T^n X(t)$$

The n th-step transition probability is then simply the n th power of the one-step transition probability

$$T^{(n)} = T^{(1)n}$$

For this model then the predicted 20th-step transition matrix is simply the one-step transition matrix from Table 4.12 raised to the 20th power. By multiplying each transition probability by the row totals from the observed frequencies, predictions of the frequencies for each cell which would be observed if the model fits the data can be made. These predicted frequencies are also presented in Table 4.16. In addition, the predicted and observed 4-step transition frequencies are presented in Table 4.17.

The predicted and observed transition frequencies for both the 4-step and the 20-step transition matrices are relatively close. The goodness of fit of these frequencies were tested by fitting log linear models to these two sets of data and examining their fit. The results of those tests are presented in Tables 4.18 and 4.19. In general the model provides a relatively good fit of these transition frequencies. There is, however, some evidence that

TABLE 6.17

OBSERVED AND PREDICTED PARADIGMS FOR 4-STEP TRANSITION MATRIX
FOR SECOND-ORDER MARKOV POLYMER WITH KINETICALLY INDEPENDENT SITES

Actions of Offshoots and Offshoots at Sites 1-2 and 3-4 (1-2, 1-1)																	N		
	$0_1^0 0_1^1$	$0_1^1 0_1^2$	$0_1^2 0_1^3$	$0_1^3 0_1^4$	$0_1^4 0_1^5$	$0_1^5 0_1^6$	$0_1^6 0_1^7$	$0_1^7 0_1^8$	$0_1^8 0_1^9$	$0_1^9 0_1^{10}$	$0_1^{10} 0_1^{11}$	$0_1^{11} 0_1^{12}$	$0_1^{12} 0_1^{13}$	$0_1^{13} 0_1^{14}$	$0_1^{14} 0_1^{15}$	$0_1^{15} 0_1^{16}$			
$0_1^0 0_1^1$	2 (1)	16 (8)	1 (1)	6 (2)	11 (17)	0 (1)	0 (1)	1 (4)	2 (2)								37		
$0_1^1 0_1^2$	8 (9)	129 (172)	7 (8)	19 (13)	265 (244)	6 (10)	5 (7)	65 (64)	19 (12)								493		
$0_1^2 0_1^3$	1 (1)	11 (11)	1 (1)	1 (1)	14 (14)	3 (3)	1 (1)	6 (7)	8 (8)								43		
$0_1^3 0_1^4$	1 (1)	9 (15)	1 (1)	15 (11)	27 (31)	6 (3)	1 (1)	6 (8)	4 (4)								69		
$0_1^4 0_1^5$	17 (16)	140 (206)	14 (11)	24 (26)	67 (57)	19 (13)	4 (13)	102 (120)	27 (46)								963		
$0_1^5 0_1^6$	0 (1)	12 (14)	1 (2)	1 (1)	11 (20)	21 (8)	1 (2)	17 (10)	6 (2)								49		
$0_1^6 0_1^7$	0 (1)	7 (6)	1 (1)	2 (2)	6 (11)	1 (1)	1 (1)	6 (4)	2 (2)								27		
$0_1^7 0_1^8$	4 (4)	176 (65)	7 (5)	6 (6)	167 (167)	4 (4)	6 (4)	82 (36)	17 (13)								250		
$0_1^8 0_1^9$	1 (1)	23 (20)	6 (6)	6 (6)	136 (11)	3 (3)	22 (26)	22 (11)	11 (11)								122		
$0_1^9 0_1^{10}$										3 (1)	2 (1)	0 (0)	10 (7)	12 (13)	3 (5)	0 (1)	1 (1)	2 (1)	33
$0_1^{10} 0_1^{11}$										4 (2)	20 (6)	3 (2)	11 (16)	20 (37)	8 (5)	0 (1)	5 (1)	2 (1)	83
$0_1^{11} 0_1^{12}$										2 (1)	1 (1)	0 (0)	5 (6)	3 (7)	3 (2)	4 (1)	0 (1)	1 (1)	20
$0_1^{12} 0_1^{13}$										17 (8)	17 (14)	2 (4)	114 (96)	206 (207)	61 (67)	12 (11)	5 (10)	4 (14)	432
$0_1^{13} 0_1^{14}$										10 (12)	24 (27)	4 (8)	168 (202)	576 (490)	227 (146)	3 (10)	7 (19)	5 (23)	746
$0_1^{14} 0_1^{15}$										3 (6)	7 (11)	5 (4)	26 (62)	51 (101)	47 (42)	23 (16)	21 (15)	28 (18)	271
$0_1^{15} 0_1^{16}$										2 (2)	3 (3)	1 (1)	12 (13)	27 (31)	11 (10)	6 (7)	3 (3)	5 (5)	70
$0_1^{16} 0_1^{17}$										0 (2)	5 (1)	1 (1)	8 (13)	15 (24)	4 (9)	10 (6)	23 (10)	12 (4)	78
$0_1^{17} 0_1^{18}$										0 (2)	4 (4)	5 (1)	13 (17)	8 (27)	14 (12)	8 (3)	7 (6)	21 (7)	82

TABLE 4.18

COMPARISON OF GOODNESS OF FIT OF DIFFERENT MODELS TO ASSESS GOODNESS OF FIT OF OBSERVED AND PREDICTED 20-STEP TRANSITION FREQUENCIES FOR SECOND-ORDER MARKOV MODEL WITH HETEROGENEOUS ROLES

MODEL CONFIGURATION	DF	χ^2	G^2	MODEL PAIR	G^2	DF	TERMS WHICH DIFFER
OFFICERS FIRST							
1) Model excluding predicted vs. observed effects $u+u_1+u_2+u_{12}$	81	59.62	64.15	1-2	64.15	81	Effects of Predicted vs. Observed $u_3+u_{13}+u_{23}+u_{123}$
2) Model including effects of predicted vs. observed $u+u_1+u_2+u_3+u_{12}+u_{13}+u_{23}+u_{123}$	0	0	0	1-2			
CITIZENS FIRST							
1) Model excluding predicted vs. observed effects $u+u_1+u_2+u_{12}$	81	55.70	59.96	1-2	59.96	81	Effects of Predicted vs. Observed $u_3+u_{13}+u_{23}+u_{123}$
2) Model including effects of predicted vs. observed $u+u_1+u_2+u_3+u_{12}+u_{13}+u_{23}+u_{123}$	0	0	0				

TABLE 4.19

COMPARISON OF GOODNESS OF FIT OF DIFFERENT MODELS TO ASSESS GOODNESS OF FIT OF OBSERVED AND PREDICTED
FOUR-STEP TRANSITION FREQUENCIES FOR SECOND-ORDER MARKOV MODEL WITH HETEROGENEOUS ROLES

MODEL CONFIGURATION	DF	χ^2	σ^2	MODEL PAIR	$\Delta\sigma^2$	ΔDF	TERMS WHICH DIFFER
OFFICERS FIRST							
1) Model excluding predicted vs. observed effects $u+u_1+u_2+u_{12}$	81	137.33	145.10	1 - 2	145.10*	81	Effects of predicted vs. observed $u_3+u_{13}+u_{23}+u_{123}$
2) Model including effects of predicted vs. observed $u+u_1+u_2+u_3+u_{12}+u_{23}+u_{13}+u_{123}$	0	0	0				
CITIZENS FIRST							
1) Model excluding predicted vs. observed effects $u+u_1+u_2+u_{12}$	81	150.19	160.17	1 - 2	160.17*	81	Effects of Predicted vs. Observed $u_3+u_{13}+u_{23}+u_{123}$
2) Model including effects of predicted vs. observed $u+u_1+u_2+u_3+u_{12}+u_{13}+u_{23}+u_{123}$	0	0	0				

*Significant at or beyond the .05 level.

TABLE 4.20
 EXPECTED AND OBSERVED DISTRIBUTION OF PENS FOR SECOND-ORDER
 MARKOV MODEL WITH HETEROGENEOUS RULES

	C101		C102		C103		C201		C202		C203		C301		C302		C303	
	Obs	Pre																
1	37	36	305	299	33	35	44	44	194	144	33	32	25	25	213	193	99	90
2	2	3	72	76	6	4	10	10	75	90	8	10	1	1	26	41	9	16
3			15	19			1	2	34	56	3	3	0	0	4	8	1	3
4			6	4			1		27	35	1	1			0	2	0	1
5			1	2					13	22	1	0			1	0		1
6			1						7	14								
7									10	9								
8									10	5								
9									3	3								
10									3	3								
11									2	1								
12									1	1								
13									1	1								
14									0	0								
15									4	1								
16									1	0								
17									0	1								
18									1	0								
Total	39		400		39		56		384		46		27		244		110	

	C101		C102		C103		C201		C202		C203		C301		C302		C303	
	Obs	Pre																
1	33	34	51	52	25	24	258	255	199	144	224	205	61	60	41	39	62	59
2	3	2	11	11	0	1	61	64	73	90	29	42	4	5	11	12	8	11
3			3	2			14	17	35	56	4	9	1	1	2	3	1	2
4			1	1			7	4	24	35	0	2			0	2	1	0
5							1	1	10	22	1	0			2	0		
6									10	14								
7									9	9								
8									9	5								
9									2	3								
10									3	3								
11									2	1								
12									1	1								
13									1	0								
14									1	0								
15									2	1								
16									2	0								
17									0	0								
18									1	0								
Total	36		66		25		341		384		258		66		56		72	

this information and knowledge of the number of times a particular event occurred when it was not preceded by its own occurrence, it is possible to predict the distribution of the length of runs for that event. Whenever an event occurs and is not preceded by itself this constitutes a new run (apart from how long that run may eventually be). Thus, the number of times an event occurs but is not preceded by itself is simply the number of runs observed. These numbers are available from the empirical estimation of the distribution of runs. So, given the number of runs which occur, and given the two-step transition matrix in Table 4.13, it is possible to predict the frequency with which runs of different length occur for each state. For any event, E_1 , the following equations hold:

$$P[E_1(t+1), E_1(t)] = P[E_1(t+1) | E_1(t)] \cdot P[E_1(t)]$$

$$P[E_1(t+2), E_1(t+1), E_1(t)] = P[E_1(t+2) | E_1(t+1), E_1(t)] \cdot P[E_1(t+1), E_1(t)]$$

and since the process is second order, this simplifies to

$$\begin{aligned} P[E_1(t+2), E_1(t+1), E_1(t)] &= P[E_1(t+2) | E_1(t+1)] \cdot P[E_1(t+1), E_1(t)] \\ &= P[E_1(t+2) | E_1(t+1)] \cdot P[E_1(t+1) | E_1(t)] \cdot P[E_1(t)] \end{aligned}$$

and because the process is stable, this simplifies to

$$= P[E_1(t+2) | E_1(t+1)]^2 \cdot P[E_1(t)]$$

and in general it can be shown that

$$P[E_1(t+k), \dots, E_1(t)] = P[E_1(t+k) | E_1(t)]^k \cdot P[E_1(t)]$$

The coefficient on the left of the above equation can be interpreted as simply the probability that the run will be of length $k+1$ or greater. Multiplying both sides of this equation by the frequency of runs for all events which occurred gives the

corresponding equation for computing the frequency of runs of length k or greater.

$$\text{freq}[E_1(t+k) \dots E_1(t)] = P[E_1(t+1) | E_1(t)]^k \cdot \text{freq}[E_1(t)]$$

From this equation we can calculate the frequency with which runs of length greater than or equal to $k+1$ will occur. For example, consider the event C_1O_1 . In this data 39 runs of this event were observed. From the two-step transition matrix we know that the probability of event C_1O_1 occurring in the next pair of interactions following its first occurrence is .066. From these data and the formula above we can predict the following:

$$\text{freq}(\text{runs for } C_1O_1 \text{ of length } \geq 1) = 39$$

$$\text{freq}(\text{runs for } C_1O_1 \text{ of length } \geq 2) = 3$$

$$\text{freq}(\text{runs for } C_1O_1 \text{ of length } \geq 3) = 0$$

The frequency with which runs of differing lengths occur is found by finding the difference between the relevant predicted frequencies.

$$\begin{aligned} \text{freq}(\text{runs of length}=1) &= \text{freq}(\text{run of length } \geq 1) - \text{freq}(\text{run of length } \geq 2) \\ &= 39 - 3 = 36 \end{aligned}$$

$$\text{freq}(\text{run of length}=2) = \text{freq}(\text{run} \geq 2) - \text{freq}(\text{run} \geq 3) = 3 - 0 = 3$$

Using these same procedures the predicted distribution of runs has been calculated for all states and those predicted distributions are also presented in Table 4.20.

It is readily apparent that the predicted and observed distribution of runs is quite close. This is confirmed when log linear models are fitted to those distributions to assess the goodness of fit between the predicted and observed frequencies. The results of those tests are presented in Table 4.21. There it is quite clear that the model provides an excellent prediction of the distribution

TABLE 4.21

COMPARISON OF GOODNESS OF FIT OF DIFFERENT MODELS TO ASSESS GOODNESS OF FIT OF PREDICTED AND OBSERVED DISTRIBUTIONS OF RUNS FOR SECOND-ORDER MARKOV MODEL WITH HETEROGENEOUS ROLES

MODEL CONFIGURATION	DF	χ^2	G^2	MODEL PAIR	G^2	DF	TERMS WHICH DIFFER
OFFICERS FIRST							
1) Model Excluding Effects of Predicted vs. Observed							Predicted vs Observed Effects
$u+u_1$	50	48.57	54.37		54.37	50	u_2+u_{12}
2) Model Including All Effects							
$u+u_1+u_2+u_{12}$	0	0	0				
CITIZENS FIRST							
1) (same as above)	48	46.75	52.49		52.49	50	Predicted vs Observed Effects
2) (same as above)	0	0	0				

of runs.

This test concludes the tests of predictions of the second-order Markov model with heterogeneous roles. To briefly summarize, these tests have found that the model provides excellent predictions of equilibrium distributions and distribution of runs, and good predictions of the multi-step transition probabilities. However, there is evidence that the predictions of transition probabilities for this model, as with other Markov models in quite different applications, suffer from the deficient diagonal problem. The model tends to overestimate the number of changes which occur across many steps. The model thus displays some of the weaknesses commonly found in Markov models. However, the model appears overall to provide a good fit and provides excellent predictions of other properties, including both properties which hold at equilibrium (equilibrium distributions) and properties which characterize the process (the distribution of runs). The model clearly is a good fit to the data and its interpretation and logical development is clearly warranted. However, because of the small weaknesses the model has shown in assumptions and predictions, some degree of caution in evaluating the findings is called for.

Chapter 5

CONCLUSIONS: LOGICAL EXPLORATION AND INTERPRETATION OF THE MARKOV MODEL

From the results in the previous chapter it is clear that, of the models examined, the one which best fits these data is a second-order Markov model with heterogeneous roles. That model has been shown to meet the assumptions of Markov models and to predict relatively accurately a number of characteristics of police-citizen interaction. However, fitting a model to the data, even one which is as successful as this one, does not, by itself, make a major contribution to understanding the phenomenon which is being modeled. The task of this chapter is to take the findings from the previous chapter and from them to determine important characteristics of the phenomenon of police-citizen interaction. Specifically, in this chapter the model will be explored mathematically in order to determine its implications for various aspects of the process, and the properties of the model will be interpreted in order to understand what they imply about the phenomenon itself. This effort will be guided by the predictions and insights available from the application of the theoretical framework to the case of police-citizen interaction in Chapter 2.

The Dynamic Character of the Markov Model

Exploring specific predictions of Markov models which may be of interest for the substantive area or which provide rigorous tests of the accuracy of the model is commonly done and does contribute to validating the model. However, often it is desirable to go beyond this to get a more comprehensive view of the dynamic process which is represented by the Markov model. One method for doing this is to examine the eigenvalues and eigenvectors of the transition matrices. Examination of these can provide information pertaining to the equilibrium points for the system, the stability of the system, the speed with which it approaches equilibrium, and other similar properties. For relatively simple systems it is even possible to construct phase diagrams which graphically display equilibrium points, the path the system will follow over time at all points, and characteristic patterns of change near and between equilibrium states. Unfortunately, when the number of states of the system is more than 2 or 3 those diagrams become very difficult to construct and interpret, and are not practical as an analysis technique. Fortunately, it happens that for this particular data set, the dynamic process is relatively straightforward and the process can be understood relatively well by simply examining the eigenvalues and eigenvectors.

As a first step in this analysis, consider what eigenvectors and eigenvalues represent. Essentially, an eigenvalue is a root of a polynomial equation and an eigenvector is one possible solution to that equation corresponding to a particular eigenvalue. This can easily be illustrated. For any transition matrix, say A , λ is an

eigenvalue of A and v is an eigenvector of A corresponding to λ if and only if

$$A \cdot v = \lambda \cdot v,$$

where λ is a single-valued parameter. But this is true if and only if

$$A \cdot v - \lambda \cdot v = 0$$

and this is true if and only if

$$(A - \lambda I) = 0 \text{ has a nontrivial solution,}$$

which, in turn, is true if and only if

$$\det (A - \lambda I) = 0.$$

The determinant of $(A - \lambda I)$ is a polynomial of degree n , and is called the characteristic polynomial of A . The roots of this polynomial are the eigenvalues of A . For particular eigenvalues, $\hat{\lambda}$, a nontrivial solution to the equation $(A - \hat{\lambda} I) = 0$ is an eigenvector corresponding to that eigenvalue.

The general solution for such a system can be expressed as follows:

$$A^k x(0) = \alpha_{11} \lambda_1^k v_1 + \alpha_{22} \lambda_2^k v_2 + \dots + \alpha_{nn} \lambda_n^k v_n$$

That is, the distribution of responses in this system after k interactions is the sum over all eigenvalue and eigenvector pairs of some constant times the k^{th} power of the eigenvalue times its associated eigenvector. The α 's here are constants which reflect the initial distribution for this process. Regardless of what the initial distribution of responses is, this general form may be used to describe the system.

The general solution is a very useful form for expressing the solution for this system because from it a number of properties of

the system can be easily derived. For probability matrices, for example, it can be shown (e.g., see Feller, 1968:432) that when the Markov chain is aperiodic one of the eigenvalues will be equal to 1.0 and the others will be less than 1.0.

Any quantity which is less than 1.0, when raised to the k th power, of course becomes smaller as k increases and approaches zero as k approaches infinity. Thus, if we allow k to go to infinity, the general solution simplifies to the following form:

$$\begin{aligned} \lim_{k \rightarrow \infty} A^k x(0) &= \lim_{k \rightarrow \infty} \alpha_1 \lambda_1^k v_1 + \alpha_2 \lambda_2^k v_2 + \dots \\ &+ \alpha_n \lambda_n^k v_n = \alpha_1 1^k v_1 = \alpha_1 v_1 \end{aligned}$$

where λ_1 and v_1 are respectively the eigenvalue and eigenvector pair for which the eigenvalue is 1.0. Thus, as the number of interactions becomes very large the actions of the participants approach the stable equilibrium distribution corresponding to the eigenvector associated with the eigenvalue of 1.0.

Another interesting property of this system is that the rate at which the system approaches this equilibrium distribution, u , is determined by the value of the eigenvalue with the second largest modulus, $|\lambda|_*$

$$\begin{aligned} u &= \max_{\lambda \neq 1} |\lambda| \end{aligned}$$

*The modulus is simply the absolute value of the product of an eigenvalue and its complex conjugate (i.e., the modulus, $|\lambda|$, of $a+bi$ is $|(a+bi) \cdot (a-bi)| = |a^2+b^2|$).

This can easily be demonstrated. As k is increased by increments for each interaction, the contribution of each component of this equation is multiplied by the eigenvalue associated with it. Since each of these other than the largest is less than one, this means that the contribution of each component of the equation to equilibrium solution decreases by the value of the associated eigenvalue for each trial. Without regard for their initial contributions due to the initial starting distribution, the limiting rate at which the system approaches the equilibrium solution is determined by the component which decreases slowest. This is the component associated with the second largest eigenvalue, and hence the rate of approach to the equilibrium solution is the value of the second largest eigenvalue.

If there are eigenvalues whose modulus is close to 1.0 then the behavior of the system can be described by the combinations of the large valued eigenvalues and their associated eigenvectors. When there are many of these, very complicated dynamic behavior can be observed. It is at this point that phase diagrams are so useful for helping to understand the processes which take place. For this particular set of data, there are no eigenvalues other than the largest which approach 1.0 or exceed it, so this type of behavior need not be of concern here. In such simple cases the equilibrium distribution and the rate at which the system approaches equilibrium fairly well characterize the system.

The analysis of eigenvalues and eigenvectors may take any of several specific forms for this particular data. There are a number of reasons for this. Recall, that this model proposed here is equivalent to two separate Markov processes which alternate.

Because of this alternating character, the actions later in the course of the encounter for either participant are the result of both his own and the other participant's past actions. This is illustrated below:

citizen acts first $x(1) = O.x(0)$ $x(2) = CO.x(0)$ $x(3) = COO.x(0)$	officer acts first $x(1) = C.x(0)$ $x(2) = CC.x(0)$ $x(3) = COC.x(0)$
--	--

where $x(0)$ is the initial distribution of responses,

$x(i)$ is the distribution of responses at the i th step, and

O and C are the transition matrices for officers and citizens respectively.

This dual Markov process may be represented as two separate processes or it may be represented, as in Figure 4.12, as a periodic Markov model with two irreducible closed sets in the matrix of transition probabilities. With each cycle the system moves from one of the irreducible closed sets to the other (i.e., first the officer speaks, then the citizen speaks, and so on). Because of this inherent periodic character built into this model, one of the eigenvalues found when this entire matrix was solved for the eigenvalues would have to be -1.0 . This -1.0 represents the periodic character of the process. The distribution at each point would then be the sum of the distribution due to the eigenvector associated with this eigenvalue. On odd trials they would be subtracted from one another, and on even trials they would be added to each other.

As an alternative to exploring the eigenvalues and eigenvectors for the entire matrix at once there are other

simpler ways of analyzing this matrix. Because, by the second transition the system has passed through the matrix completely in one cycle, at that point both of the processes have had an impact and the entire process which the system goes through is then represented in the resulting two-step transition matrix in each of the irreducible closed sets of the transition matrix, it is possible to explore these subsets separately rather than analyzing the entire matrix. This is a great simplification in terms of conceptualizing the process. This approach is taken here. In addition, the eigenvalues and eigenvectors for each of these subsets of the transition matrix must be related to each other in a very simple fashion. This relationship is as follows:

At equilibrium we know that

$OCv_1 = v_1$, where v_1 is the equilibrium distribution and O and C are transition matrices for the officers and citizens respectively.

During the next half-cycle of the process, however, the process returns to the other irreducible closed set in the matrix where, because it is equilibrium, the equilibrium distribution is v_2 . The transition is made by simply multiplying v_1 by the transition matrix C , giving v_2 , i.e.,

$$C \cdot v_1 = v_2$$

Similarly it may be shown that

$$O \cdot v_2 = v_1$$

Thus, the eigenvectors describing the equilibrium distribution of

these subparts of the transition matrix are related to each other; each eigenvector is the product of the other eigenvector times the appropriate 9×9 transition submatrix. Hence, the analysis of either one of these matrices alone will provide all of the information needed.

The two-step transition matrix which provides the data from which this analysis was conducted is presented in Table 4.13. In Table 5.1 are presented the 9 eigenvalues for the citizen to citizen subsection of the two-step transition matrix. As expected, the maximum eigenvalue is 1.0 and the other eigenvalues are less than 1.0 (i.e., in all cases the modulus of the eigenvalue is less than 1.0). The rate of approach to the equilibrium solution as defined previously proves to be .404. This is relatively fast. Within very few cycles the equilibrium distribution should predominate and be closely approximated by the data. The remaining eigenvalues are considerably less (i.e., the largest remaining eigenvector has a modulus which is roughly half that associated with the rate of approach to equilibrium). This indicates that the effects of the other eigenvectors are quickly diminished and the system is characterized as being relatively stable. The presence of other very large eigenvalues relative to the value of 1.0 would indicate interesting behavior related to divergences from equilibrium. The presence of other eigenvalues on the order of magnitude of the rate of approach to equilibrium would indicate perhaps some interesting processes in the approach, while the presence of others considerably smaller, indicate that the approach is basically the function of that one eigenvalue and its associated eigenvector (i.e.,

TABLE 5.1

EIGENVALUES FOR CITIZEN-CITIZEN TRANSITION PROBABILITIES

1.001

.404

.201 + .0151

.201 - .0151

.072 + .0071

.072 - .0071

.018

.010 + .0011

.010 - .0011

the system approaches the equilibrium distribution in a linear fashion from one direction with a relatively constant rate). Thus, these eigenvalues suggest a relatively simple sort of system, highly stable, and direct in its approach to equilibrium. The equilibrium distribution of this system is determined by the eigenvector corresponding to this largest eigenvalue. The equilibrium distribution for both officer-citizen and citizen-officer states are presented in Table 4.14.* This system is also periodic. That particular component has been deliberately left out of this method of analysis since that is already known and it simplifies the analysis. So we have then, a relatively simple system which is highly stable, periodic, and approaches equilibrium rather rapidly and directly.

Order

The fact that a second-order Markov model provides the best fit to these data of the interaction of police and citizens over time provides some insight into the nature of the decision processes those police and citizens might be using in this interaction. As pointed out earlier, the different components which contribute to fitting the data in the log linear models have interpretations both substantively and in terms of the Markov model. For both citizens and officers the effects which contribute most

*In this case these were computed from the transition matrix directly by raising it to a very high power (24, which is high enough for this system) and noting the distribution at that state after it had stabilized. These could just as easily have been computed by actually computing the eigenvectors for the matrix from the eigenvalues.)

to fitting the data are those corresponding to first-order self-contingent decision rules, next are those corresponding to first-order other-contingent decision rules, and finally are those corresponding to second-order decision rules.

Although the connection of these empirically observed patterns of behavior to possible theoretically-based decision rules remains somewhat tenuous at this point, these findings do suggest directions which future research concerned with understanding more about decision rules might take. These findings suggest that decision rules the participants might be employing in their interaction would probably involve effects of own past actions, the other participant's past actions, and some combination of the two.

The finding of the good fit of a second-order Markov model to this process of social interaction is a clear manifestation of the interactive character of that process. The empirically observed contingency of current actions on the past actions of one's self, the other participant, and their combination provides support for the view that this is indeed social interaction in which the actions of one participant affect those of the other participant, and the interactions of the second participant in turn affect those of the first. Conceptually this provides support for the view of this process as two different subsystems, the two participants, connected to each other by the actions which they perform, with feedback loops between them. Each action has consequences for both participants and their actions are a complex response to both their own and the other's past behavior. This mutually contingent character of social interaction is one of its most

fundamental characteristics.

Role differences

The results of the analysis of the impact of role differences upon the interaction between officers and citizens provide a number of insights into the character of that process. The relative impact of the role differences upon the different aspects of that interaction process as identified by the components of the log linear models lend insight into the way in which role differences affect the interaction and suggest that the primary differences are differences in the situation which the two participants face more than a difference in the strategies or decision rules which guide their response to the other participant. The directions of the role differences, in turn, provide an understanding of the different roles which are being enacted and suggest that police and citizens take on extremely different, but complementary roles.

The relationship of the role of the participants to the interaction process varies considerably for different components of that process. Those results were presented earlier in Table 4.3. The largest difference in the interaction process as a function of role is in the different distributions of responses for officers and citizens. The contribution of the relationship of role to the distribution of responses to fitting the data (a change in G^2 of 1503.10 with 8 degrees of freedom) is far greater than the contributions of the relationship of role to the different contingencies on past responses. The contingencies on past responses, recall, may be interpreted as empirical indicators of the decision rules which the participants may be using; and the distribution

of responses may be interpreted as differences in the situation, where situation is here used in the technical sense in which it is defined as the combined result of the task conditions and the actions which have occurred up to that point in time (see the conceptual framework). These results then indicate that the primary difference between officers and citizens in these encounters is they face very different situations. This difference in the situations the two participants confront is considerably greater than the differences they exhibit in decision rules which they may be employing.

A difference in situations faced by the two participants, however, is not the entire story. While those differences clearly are much stronger than the differences between the participants in decision rules, there remain significant differences in the decision rules for the two roles which must be taken into consideration to provide a model which adequately fits the data. The differences in first-order self-contingent, first order, other-contingent, and second order effects as a function of role all contribute significantly to providing an adequate fit of the data (these produce differences in G^2 of 123.24 with 8 degrees of freedom, 42.68 with 12 degrees of freedom, and 126.47 with 16 degrees of freedom, respectively). Officers and citizens thus differ from each other primarily in the situations they face but they also differ in various types of decision rules which they may employ.

A related question of interest is for each role what is the relative contribution of each of the effects: first-order self-contingent, first-order other-contingent, and second order. These

different effects reflect the nature of the contingency of present responses of each participant on past responses of both participants. In addition, they have implications for the types of decision rules which may be employed by those participants. A number of hypotheses were suggested earlier about these effects.

One hypothesis presented earlier is that police respond more to their own past actions than to those of citizens. One way to assess the magnitude of the contingency of current responses on past responses is to look at the relative fit of models to the data which do and do not include these components. Such an analysis was performed when the order of the Markov process was being examined. From the results in Table 4.2 it may be seen that the change in G^2 as a function of the first-order other contingent effects for officers was 270.96 with 12 degrees of freedom, while the change in fit as a function of the first-order self-contingent effects was 349.43 with 8 degrees of freedom. The contribution of the self-contingent effects to fitting the data were clearly greater than those of the other-contingent effects. These results suggest that the officer's own past behavior is a better indicator of his current behavior than is the past behavior of the citizen. The hypothesis is confirmed.

On the other hand, a second hypothesis was that citizens respond more to the past actions of police than to their own past actions. Data for the same analysis is relevant here. The contribution of first-order other-contingent effects to fitting the model for citizens was 340.28 with 12 degrees of freedom, while the contribution of their self-contingent effects was 349.36 with 8 degrees

of freedom. For citizens too it appears that current behavior is more closely related to their own past behavior than to the past behavior of the other participant. This effect, however, is less pronounced for citizens than for officers. This hypothesis is disconfirmed.

These results thus indicate that for both citizens and officers the encounter is characterized by a contingency upon the past actions of both participants, with the primary contingency being upon the individual's own past actions. The primary difference between the two roles lies in the different situations which they each are confronted with in police-citizen encounters. In addition, although both roles display contingencies or past events, the specific relationship of past to present events is not the same for the two roles--i.e, they employ different decision rules.

Having examined the relative contributions of role differences to different aspects of the Markov process, and inferring from them some of the properties of the phenomenon, the next step is to examine the directions of those differences to determine what insights those differences might provide into the phenomenon. Here the direction of these differences will be only briefly examined.*

*Detailed examination of these difference will not be made in this research. The objective here is to determine a mathematical model which describes the structure of this interaction in order to obtain some insights into the processes which take place. It remains for future studies to examine more closely the specific relationships which are present and to provide parameters describing the directions of such effects. This limited objective is necessary because extensive additional work would be required to determine specific detailed characteristics of the model such as this. This research addressed the prior questions of whether such a model exists and what is its basic structure?

These data indicate that the difference in the roles is that in general officers tend to be much less cooperative than citizens. Those findings are clearly seen in the equilibrium distributions as reported in Table 4.14. There it was seen that events in which the officer responds negatively or by redirecting the conversation and the citizen responds cooperatively (e.g., O_1C_2 , O_3C_2 , C_2O_1 , C_2O_3) occur much more frequently than those events where citizens respond negatively or by redirecting the conversation and officers respond cooperatively (e.g., O_2C_1 , O_2C_3 , C_1O_2 , C_3O_2). Clearly officers tend to respond less cooperatively and more often redirect the conversation or respond negatively.

An hypothesis from the theory which is appropriately considered here is the hypothesis that police officers will be more likely to respond more negatively than the response of the citizens which immediately precedes them, and citizens are more likely to respond more positively or cooperatively than the response of the officers which immediately precedes them. This hypothesis may be tested by examining the responses of both officers and citizens as a function of the previous response of the other participant and role. Those response frequencies and response probabilities may be derived from the data presented in Table 4.12. There it may be seen that citizens are more likely to respond cooperatively than officers regardless of the most recent response of the other participant, and officers are more likely than citizens to respond with either redirective comments, or with a negative communicative act, regardless of the most recent response of the other participant. Thus the hypothesis is confirmed.

Yet the most common event of all at equilibrium is the event where both respond cooperatively. And the eigen analysis

presented earlier clearly demonstrated that the system was *very* stable, rapidly approached equilibrium and had an equilibrium state in which cooperative responses played a major role. These results indicate that, while the roles the two participants play are quite different, they do appear to be somehow complementary (complementary in the sense that they together create a very stable system with relatively cooperative net behavior). The officer is directive and punishing when necessary to get the information he wants and to control the situation (as is his implicit and in some ways explicit performance expectations), while the citizen is passive, allows the officer to direct the interaction and responds rather cooperatively and submissively to accusations, commands, and noncooperative responses on the part of the officer. This is a rather interesting pattern which one would not expect to find routinely in all types of social interaction. The result is relatively stable interaction in which the officers play roles which appear to involve directive, controlling activities; and the citizens play roles which appear to be characterized primarily as submissive and compliant.

The complementary character of these roles may be demonstrated by considering what the result would have been if the participants had been interacting with other participants who responded in the same way in which they respond (e.g., what would have resulted if citizens responded to officers in the same way that officers respond to citizens?).

The dynamic behavior which would be predicted if both participants responded as the officers do may be obtained by algebraically analyzing the submatrix of transition probabilities for officers

in the one-step transition matrix presented in Table 4.12. This matrix represents only the effects of decisions the officers make upon the interaction and this analysis assumes that only that process occurs. The eigenvalues resulting from this analysis are presented in Table 5.2. The eigenvalues for the interaction as it would occur if both participants responded as the citizens respond is obtained in a similar fashion by analyzing only that submatrix, and these results are also presented in Table 5.2. The equilibrium distributions which would be predicted for these different circumstances are obtained by raising those same submatrices to very high powers until all the transition probabilities in each row are the same and then creating a vector of those row transition probabilities. The predicted equilibrium distributions for these hypothetical interaction processes are presented in Table 5.3.

From these results it may be seen that both of these hypothetical interaction processes would exhibit dynamic behavior quite different both from the interaction process which actually occurs and from each other. Citizen-citizen interaction, for example, would approach equilibrium almost twice as fast as it is approached in citizen-officer interaction (with a rate of approach of .78 for citizen-citizen interaction versus .404 for citizen-officer interaction). At equilibrium, citizen-citizen interaction would be characterized primarily by mutually cooperative responses (over 80% of the time compared to less than 50% of the time for such responses for officer-citizen interaction).

Officer-officer interaction, on the other hand, would result in a very different equilibrium. In this case the equilibrium

TABLE 5.2
EIGENVALUES FOR HYPOTHETICAL INTERACTIONS

Eigenvalues for Citizens Interacting with Citizens	Eigenvalues for Officers Interacting with Officers
1.000	1.000
.780	-.459
-.602	.452 + .0121
.520	.452 - .0121
-.502	-.301
.115	-.070
-.114	.241
.071	.163
.240	.072

TABLE 5.3
EQUILIBRIUM DISTRIBUTIONS FOR
HYPOTHETICAL INTERACTIONS

Citizens interacting with citizens		Officers interacting with officers	
O_1C_1	.004	C_1O_1	.097
O_1C_2	.046	C_1O_2	.191
O_1C_3	.004	C_1O_3	.114
O_2C_1	.044	C_2O_1	.160
O_2C_2	.806	C_2O_2	.152
O_2C_3	.038	C_2O_3	.042
O_3C_1	.005	C_3O_1	.090
O_3C_2	.036	C_3O_2	.066
O_3C_3	.017	C_3O_3	.092

responses would be almost evenly distributed across all of the 9 possible responses, resulting in a very fluid equilibrium in which the responses of particular individuals change with great regularity. This system would also approach equilibrium in a way very different from the citizen-officer or citizen-citizen interaction systems. There are three eigenvalues of approximately the same modulus: one real negative one and two complex ones. The approach of this system to the equilibrium distribution would be quite complicated and would include approaching the equilibrium distribution from both sides as the system differed from the equilibrium distribution first in one direction and then in another. The rate of approach, however, is very much like that of the officer-citizen interaction.

Departures From The Markov Process

A number of insights may be gained by interpreting the departures from a perfect fit of the Markov model for this data. As indicated earlier, one major use of mathematical models is as baselines against which to examine effects which occur as departures from the simple models. This baseline analysis or residual analysis can be quite effective in pointing out important aspects of a phenomenon which may be either small effects which need to be tacked onto the main model, or major differences in assumptions pointing to new models for consideration in the future. As Coleman (1960) pointed out, it is when the model fails that something is learned.

In some senses this analysis proved the data to be in surprisingly close fit with a Markov model. The discrepancies observed here were all relatively minor compared to the major

characteristics of the model, and do not appear to be sufficient to propose that a Markov model is clearly wrong and some other model is required. These discrepancies do, however, offer some leads for future study.

The slight departures from stability over time are interesting, for example, because they suggest that there may be changes in the decision rules employed by the participants in the interaction. Such changes might indicate that the participants were learning how to respond in ways which elicit desired responses from the other participant. Or they might constitute deliberate shifts in strategy over time.

One might have expected more change to occur for citizens since they are not usually participants in such encounters and have not been extensively trained for them (although the amount of socialization and preparation for such encounters which a citizen might normally receive in school, through his families, and through his peers, should not be discounted). There are slightly more changes for citizens--they show changes both in self-contingent responses and second second-order responses while officers change only in self-contingent responses.

Alternatively, these differences might reflect different phases of development of the encounter over time--e.g., perhaps an encounter goes through phases such as problem solving groups are said to go through and different decision rules apply at different phases (e.g., see Fox, 1975). However, one would expect more pronounced effects if there was really a phasing process at work here, hence that explanation may be at least somewhat discounted.

It is not clear at this juncture what the appropriate interpretation of these departures from perfect stability would be. But surely this is a phenomenon worthy of further consideration.

In another case it is the lack of a departure from the Markov process which proved unexpected. This is true for the lack of significant differences in the interaction process observed for citizens of different classes. The interaction process does not appear to vary significantly for these different citizens hence there is no need for creating different models of the process for the different characteristics of the citizens. This finding clearly disconfirms the hypothesis that the interaction between citizens and officers is governed by status equilibration decision rules.

The slight tendency for officers to exhibit different distributions of responses and different first-order contingencies for traffic and nontraffic offenses also suggests a number of possible explanations. These results suggest that officers may tend to respond differently depending on the nature of the offense which the citizen has allegedly committed. Such a finding is an indicator of the sensitivity of the model to the change in the task which might limit its generalizability. It would be interesting in the future to explore a variety of situations, including not only different offenses, but also entirely different encounters (i.e., doctor-patient encounters, or other encounters between professionals and clients). The effects of the decision task on these processes merits exploration.

The third discrepancy between the model and the data for this study was the occurrence here, as in so many cases before, of

the deficient diagonal problem. This tendency for Markov models to overestimate the amount of change which will take place in multi-step transitions has occurred with great regularity in studies differing greatly in substance (e.g., in studies of occupational mobility studies of the occurrence of violence in cities, and studies of social interaction). This suggests that the problem is one endemic to Markov models rather than one which is due to some peculiarity of a particular phenomenon or a particular process. The cause of this problem whether it lies in measurement error, or whatever, would surely be worth pursuing in some future study.

The question here, of course, is how serious a problem this poses for this study. Fortunately, as noted earlier, the dynamic system of this process is very stable and approaches equilibrium very rapidly. When that is the case, since the deficient diagonal problem disappears as the system approaches equilibrium, the impact of this problem is reduced. For systems which were very unstable or which only very slowly approached equilibrium, this would be a more serious problem. Thus, in spite of this problem, this model produces quite good predictions both of the equilibrium distribution and of other aspects of the process which occur (e.g., the distribution of runs).

Summary

To summarize, these findings indicate that the interaction which occurs between police and citizens in encounters of this sort is the result of a second-order Markov process with heterogeneous, and complementary roles. Current responses of the participants are

closely related to the participant's own most recent past action, the most recent past action of the other participant, and some combination of the two (in that order).

Although a second-order process with these characteristics describes the interactions of both participants, there are extreme differences in their specific actions as a function of role. The primary difference for these two roles is in the distribution of responses. The citizen is generally compliant and submissive and the officer, though he frequently responds cooperatively, is more inclined to either respond negatively or to redirect the conversation resulting in behavior which is directive and controlling. As a result, these two participants, when they interact with one another, find themselves confronted with very different situations; the officer finds himself in a situation in which he rarely is called upon to respond to a negative or redirective act; while the citizen is in a situation in which he frequently has to respond to negative or redirective actions. These participants also differ significantly in the decision rules which they employ in their interaction.

Although the actions of the participants playing the roles of officer and citizen are quite different, they are complementary in the sense that the interaction which takes place among these two participants begins with actions distributed very much like they are distributed at equilibrium (perhaps due to prior socialization and shared expectations) and rapidly moves toward a very stable equilibrium in which the different response distributions for the two roles are maintained.

The interactions observed, and the processes which generate them appear to be relatively stable over time, little affected by differences in the nature of the encounter, and even less affected by differences in the characteristics of the population of citizens involved.

These findings provide very good support for the approach which was taken in this study. The processual character and the mutual contingency between the participants which are fundamental characteristics of this approach are soundly supported by the excellent fit of the Markov process model to the data and the significant contingencies of present actions on the past actions of both participants. Many concepts thought to be important in this approach for the process, such as the situation and role differences prove to be critical in explaining the variance in the data. The examination of this process across a narrow range of variation in other variables of importance in the framework including task characteristics, individual differences, and different times during the encounter, provide evidence in support of the Markov model as a model which characterizes the process within a respectable range of variation in police-citizen encounters.

There is also respectable support for the "theory" of police-citizen interaction which was provided by interpreting this framework for viewing social interaction in the context of this specific type of encounter. The general ranking in importance of the concepts in the framework proved to be fairly close to the magnitude of their effects. Contingency on past actions, role differences, and situational differences appear to be major

factors affecting the interaction process, while relatively small variations in some of the other variables (individual characteristics and task characteristics) provide evidence that this process is a fairly general one holding for a restricted class of police-citizen encounters something like the ones examined here. The hypotheses of a second-order Markov model describing the data, the greater negative responses of officers, and the greater contingency of officers present response on their own past responses were all confirmed. While the hypothesis of greater contingency of citizens on the officers' past responses and the use of a status equilibration decision rule by the participants both were convincingly disconfirmed.

Chapter 6

DISCUSSION

At this point it is appropriate to put these results into perspective by considering first what implications might be drawn from the character of the interaction which takes place in police-citizen encounters for the broader issues of law enforcement in a democratic society, and secondly what avenues are suggested for future exploration.

In the last chapter it was shown that the roles of police and citizen are very asymmetric, and a change in the behavior of either of those participants would lead to interaction with different characteristics. In particular, it appears that officers take rather directive, controlling roles and citizens take passive, submissive roles in such encounters. If citizens were to take roles which were more aggressive--say behavior much like that of the officers themselves--the interaction would be much more dynamic and would have a much greater proportion of negative and redirective comments. In other words the interactions would probably be quite tense and involve more open conflict between the two participants.

Now imagine, for a moment, what sorts of people would be likely to act more firmly in these circumstances. It is likely that people who felt strongly that they were legitimate in what they were doing

and who felt the police were acting out of line would be likely to act more aggressively and less submissively. Such encounters would be likely to occur when there were very strong value differences in the population, particularly if those value differences led those people to do something which placed them in a position where they were confronting police officers carrying out their duties. Such encounters appear to have occurred rather commonly in the 1960's (e.g., peace demonstrations and civil rights demonstrations).

It appears that while this type of interaction on the part of the police might be appropriate and effective when they are dealing with citizens who are guilty & are generally from lower socioeconomic strata lacking in education and hence don't question the legitimacy of the officer's actions; it is not effective and possibly not appropriate when interacting with highly educated citizens who feel quite legitimate (even righteously indignant) in what they are doing and command a greater respect which they feel is due them. It would appear possible that a range of behavior on the part of the police officer which is adjusted to the task which confronts him would be helpful.

Another consideration is the impact of this behavior on the citizens. If a citizen is confronted with such encounters very often what sort of behavior does this encourage in him? How is this passive role consistent with notions of free speech, and principles such as innocent until proven guilty? Doesn't this pattern of observed interactions suggest that citizens would be reluctant to speak out for their rights and demand fair treatment?

And wouldn't that sort of action by them be more consistent with our notions of a free society?

Yet another issue is how effective an officer would be if he were to change his tactics. It is likely that if the officer changed his behavior the citizen's would also change to produce some new role behaviors which were compatible. Could officers utilizing different behaviors still handle citizens who were violent, abusive, or whatever? Would the interaction require greater time? Would he be more susceptible to violence and injury?

These are issues which cannot be resolved here. But they do appear to be raised by the very asymmetric character of interaction between police and citizens. It is not clear what changes in interaction patterns would do for encounters such as the ones studied here. But it does appear to be at least logically reasonable that police officers who were adaptive and who had a wide range of behaviors which they could apply strategically to cope with the variety of situations they deal with would be more effective officers, might avoid problems such as those encountered in the 1960's, and might provide a form of street justice which is more compatible with our ideals.

A second issue which bears addressing is the issue of what direction researchers should be counseled to go in the future? What would be the logical next step in research into social interaction, given the findings of this study?

Clearly future research might profitably explore other examples of social interaction and compare the social interaction which occurs in the police-citizen encounter with other instances

of interaction. This line of inquiry is consistent with the concern just expressed about the police interaction strategy in varied settings. But such explorations should not just examine other types of police-citizen encounters. They should also consider other instances of interaction. They could consider other instances of professional-client interaction (e.g., doctors and patients, students and teachers, clergy and laymen). Interaction among people who are not playing rigidly prescribed roles also might be interesting (e.g., interaction between patients in a waiting room, students in a line, and so on). Interactions in groups of more than two also would be interesting. In short, there are a wide variety of possible circumstances in which interactions may occur which would be interesting to explore in hopes of finding out more about other substantive phenomena and perhaps contributing to a better understanding of the effects of task characteristics on social interaction.

A second issue which clearly demands further attention in future studies is the issue of decision rules. One of the weakest points remaining in this empirical examination of the interaction is the difficulty in precisely determining what decision rules describe the interactions.

This is not by any stretch of the imagination an easy task. It is quite likely that a number of different decision rules might be employed by the participants, individuals may differ in decision rules, and decision rules might differ with the situations. In addition, the number of possible decision rules increases geometrically with the number of categories and the number of inter-

actions which take place.

The approach which is suggested in this effort is to analyze decision rules utilizing the notion of underlying states. Underlying states are simply states which are presumed to exist and to be the states which may more appropriately characterize the interaction, but which are not directly observable. For example, there has been extensive use of this notion in learning theory where underlying knowledge states are presumed to exist and to be measured only indirectly by the observable response states. Meeker (Leik and Meeker, 1975) also has utilized the notion of underlying states to represent underlying value states.

Underlying states are analyzed by specifying precisely their relationship to observable states and then using the observations to estimate parameters describing the behavior of the interaction system relative to those underlying states (e.g., see Leik and Meeker, 1975, or Suppes and Atkinson, 1960). Frequently those relationships are such that underlying states in effect selectively incorporate information from a number of past actions to estimate the underlying state. Formally this procedure is equivalent to selectively considering information from several past states rather than just the most recent ones. This is analogous to changing the order of the Markov model; and depending on the precise nature of the assumptions made, it may be precisely the same as changing order in some cases.*

*I am indebted to Richard Sykes for pointing this out to me.

Unfortunately a straightforward generalization of the approach to underlying states (making assumptions similar to those by Suppes and Atkinson and Meeker) for the present data involves a very large number of parameters and becomes very difficult (if not impossible) to solve. In addition, the analysis of underlying states in the past has been justified primarily in an effort to improve the fit of the model to the data. However, in this study the Markov model based on observable states has proven a very good fit. Hence, the need for this further exploration is not as great with respect to this criterion of empirical fit as it has been for those other studies in the past. For these reasons, and because of time constraints, this further analysis was not pursued in this study.

There nevertheless remain very good reasons for eventually pursuing this approach in some future study. This approach appears very amenable to the conceptual framework developed here which places a high reliance on decision rules. It would provide a great improvement in substantive understanding and explanation if these decision rules could be more effectively explored empirically.

Appendix 1

ISSUES ARISING FROM THE USE OF MATHEMATICAL MODELS

There are many serious problems which arise in the application of mathematical models. These have been dealt with in detail elsewhere (Brent, 1974b) and hence there is no need to discuss them at length here. Here an effort is made to identify the source of the problem and to point out some of the major aspects of that problem. This is important because a great deal of the literature considered here suffers from serious flaws in the application of mathematical models.

Mathematical models differ fundamentally from other forms of knowledge. Those differences are the primary source of many serious methodological problems occurring in the literature. All forms of scientific knowledge (including models) have a similar structure which includes a number of different components (a logical system, an empirical system, and the phenomenon itself) and relationships among those components (operational definitions connecting the empirical and logical systems, and some isomorphism between each of these and the phenomenon of interest). The character of the relationships among those components for mathematical models is quite different from other forms of scientific knowledge more commonly employed in the social sciences (e.g., common social science theories). These differences have important implications

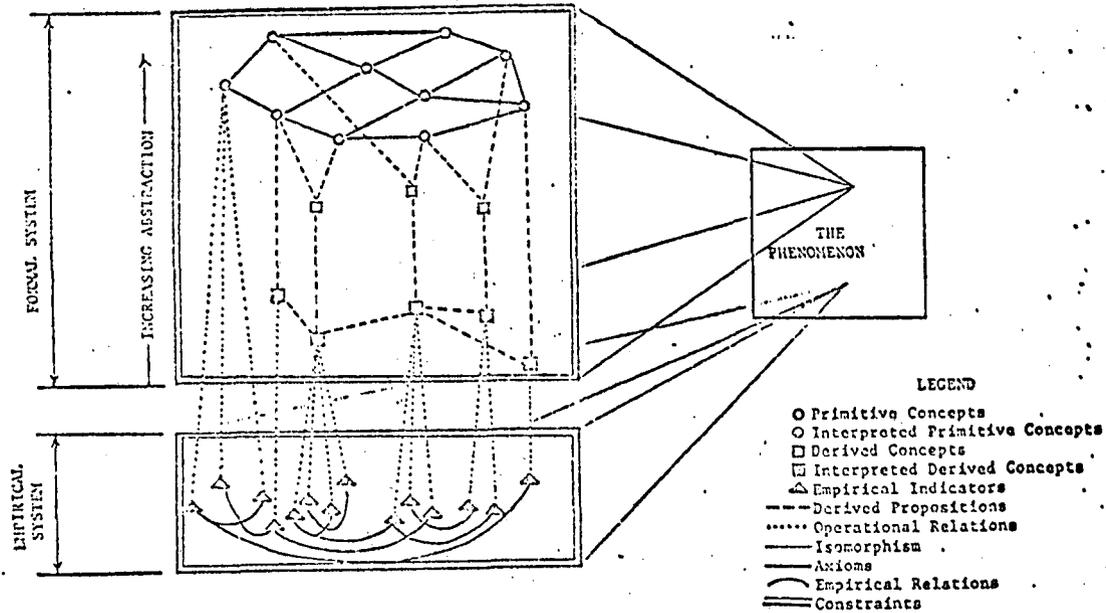
for the interpretation of such models, the ways such models are empirically tested, the pattern of growth of knowledge for models relative to other forms of knowledge, and for other aspects of these systems of knowledge. Serious problems arise when the researcher is not aware of the need for different approaches in the treatment of mathematical models and fails to adequately cope with these differences.

The Structure of Scientific Knowledge

Although there is no shortage of controversy and disagreement among philosophers of science about the precise nature of knowledge, theories, models, meaning, and many other topics which are central to this paper, there do appear to be some broad areas in which the general outlines are at least tentatively agreed upon. There are other areas in which the compelling conflicts for philosophers of science may be less relevant to the purpose we as social scientists might have for applying their insights, and may be cautiously, if not dangerously, avoided. An effort is made here to develop a view of the framework of scientific knowledge which, as much as possible, avoids taking idiosyncratic views which have been clearly rejected by the most competent philosophers of science. The goal is to find that core of agreement around which philosophers fight their major battles, but within which there are only minor skirmishes which, from a social scientist's point of view, are insignificant. A framework which appears to have some of these qualities is illustrated in Figure A1.1.

Perhaps the most fundamental characteristic of scientific

FIGURE A1.1
THE STRUCTURE OF SCIENTIFIC KNOWLEDGE



knowledge is the partitioning of it into two different regions or systems. Scientific knowledge simultaneously recognizes the validity of two different types of knowledge, the rational and the empirical. Rational knowledge is that which derives from the logical relationships of some formal logical system (e.g., the basis for knowing something may be due to its logical connection with other things which are known). Empirical knowledge is knowledge which is obtained by direct empirical observations (to the extent that there is such a thing). For example, something is known empirically because it was empirically seen to hold. Of course, there are many rigid procedures for insuring that empirical tests are valid, generalizable, falsifiable, and so on. In the framework proposed here these two types of knowledge are represented by the logical system and the empirical system respectively. These will be considered in turn.

The Logical System. Two key components of the logical system are concepts and relations among concepts. A concept may be viewed as an abstract construction from a family of meanings for particular uses of a word (Kaplan, 1964). A concept is then a generalized notion of the meaning of a word which holds for all uses of the word. Associated with each concept is a term or a name which refers to that concept. The assignment of a particular name to a concept is called nominal definition. This, by itself, lends no meaning but it is a critical step in establishing the concept for further reference.

Concepts may relate to each other in systematic ways. We may distinguish between analytic and synthetic relationships (Putnam,

1962). An analytic relation is one which is necessarily true given the logical system or language within which one is working (i.e., given the axioms of the system of knowledge under discussion). A synthetic relation is a contingent truth—that is a truth which is empirically the case but which is not logically necessary. For example, if one of the axioms of the theory states that men will make choices so as to maximize their utility, and the utility of one object, A, is defined as twice that of B, then the fact that men do always choose A over B is logically necessary given those earlier statements. This is an analytic relation between concepts. If, on the other hand, it is empirically found that men also are faster in choosing between A and B than they are in choosing among other alternatives, then that is a synthetic relation between concepts. The way these terms are used here it is possible for a relation to be at the same time both synthetic and analytic (e.g., it is logically necessary given the assumptions of the body of knowledge and it is found empirically to be the case). Or, a relation may be neither logically necessary nor empirically true. This is a key aspect of this view of knowledge: the two systems of knowledge are distinguished, but they are not viewed as incompatible and may occur jointly.

These concepts and relations between concepts exist within the context of a logical deductive structure. This may be loosely thought of as a vertical continuum in the diagram in Figure A1.1 which represents the level of abstraction. Concepts and relations near the top are most abstract and those near the bottom of the logical system are the least abstract. Concepts at the very top are

primitive concepts. Primitive concepts are those which are not explicitly defined in terms of other concepts but instead make up the base from which other concepts are defined (Reynolds, 1971). The remainder are derived concepts, concepts which are explicitly defined in terms of the primitive concepts and/or other derived concepts. (Reynolds, 1971). All concepts must be either primitive or derived. As we move down the diagram the concepts are progressively further down the chain of derivations and they are less abstract. All fit within the same logical structure of definitions which relates the concepts to each other. For example, if the concepts of "number" and "utility" are our primitive concepts (where "number" is just the amount of some commodity, A, possessed by person 1, and "utility" is the satisfaction that person derives from the possession of A) then we may explicitly define a third concept solely in terms of these two concepts. The "marginal utility" of A for person 1 is the amount person 1's utility would change with the additional possession of one more unit of commodity A.

Just as some concepts may be derived from others, some relations among concepts may also be derived from other relations among concepts. We thus distinguish between axioms, relations which are not explicitly derived from other relations, and theorems, which are explicitly derived from other relations. For example, if one of the axioms of this body of knowledge is, as stated earlier, that men will make choices so as to maximize their utility, and a known contingent truth is that the utility of A for person 1 is greater than the utility for B, then a deduction from these two

relationships would be that person 1 would tend to choose A over B when he has the opportunity. This latter relation would then be a theorem.

Primitive concepts are represented in the diagram by circles and derived concepts by squares. Relations among concepts are represented by straight lines, with solid lines representing axioms and dashed lines representing theorems. An arrow indicates the direction of increasing abstraction.

In this view the logical system consists of concepts and logical relations among concepts. These form an interconnected system of relationships which are constrained by the rules of the logic used, or the logical calculus. That calculus may be any one of a number of available ones (Willer, 1967:13). It could be symbolic logic, some mathematical theory, causal theory, or even a computer program.

This logical system may be viewed from two perspectives: as a logical structure and as a logical process. The fully elaborated system with the possible relationships and the possible concepts identified and related to each other may be viewed as a structure of knowledge which represents what is known about some phenomenon. However, it is rare that a logical system is completely specified. There probably will always be new combinations of relationships and concepts to be created and developed and the system may be extended using these concepts and relations among concepts operating within the constraints of the logical calculus to produce new insights, new predictions, and new avenues for further exploration. The system then is used as a logical process to deduce such new

predictions and to extend knowledge.

The Empirical System (The Semantic Interpreted Component).

The empirical system is an alternative source of knowledge. It exists as a distinct, but not incompatible, supplement to the logical system for scientific knowledge. The major components of the empirical system are its analogues to the concepts and logical relations of the logical system: empirical indicators and empirical relationships among empirical indicators. Empirical indicators are directly observable phenomena, or the closest thing to them we can achieve in empirical research. A number of types of empirical indicators have been identified by Hage (1972) including variables which may vary either continuously or categorically and a variety of nonvariables including elements and qualities. Others have long distinguished between different levels of measurement for variables: the nominal, the ordinal, the interval, and the ratio scale of measurement (Stevens, 1970).

Just as concepts are logically related, empirical indicators may have empirical relationships or empirical linkages (Hage, 1972) among them. Such empirical relationships may take on many forms (e.g., linear, curvilinear, and so on), they may connect only two empirical indicators or many, and they may involve any of a variety of relations including interaction, additive effects, and so on (see Hage, 1972). They may vary in the certainty with which they are known to be true (e.g., they may be mere hypotheses, empirical generalizations, or empirical laws). There are, in short, a great many ways such linkages may be specified and explicated.

Operational Definitions. The empirical system is connected to the logical system by means of operational definitions--variously called "correspondence rules" (Carnap, 1956, and Margeneau, 1950), "epistemic correlations" (Northrop, 1947), or "coordinate dimensions" (Reichenbach, 1938). These operational definitions link the concepts of the logical system to the empirical indicators of the empirical system. For example, a concept, authoritarianism, may be measured by a number of empirical indicators such as reported tendencies to follow directions without question, to place a great deal of faith in the statement of superiors, and so on.

A great deal of thought has been given to the importance of these connections between empirical indicators and concepts, and as a result a number of distinctions and concepts may be pointed out. Concepts are said to be interpreted when they are connected by operational definitions to empirically observable indicators, or uninterpreted when they have no direct connections with empirical indicators (Greer, 1969). Two types of meaning are distinguished for concepts: systemic and referential. Systemic meaning is derived from the relationships of a concept with other concepts in the logical system (Kaplan, 1964:64)--e.g., suicide may be defined and given meaning by relating it to other concepts which define it and by explicating its analytic or synthetic relationships with other concepts. Referential meaning is derived from the connections between a concept and its empirical indicators. Such meaning "consists of points of contiguity between (a term) as a linguistic symbol and the observable attributes, properties, and relations that it represents" (Lachenmeyer, 1971:10)--e.g.,

alienation may be measured by a reported sense of helplessness, despair, depression, and so on. There may be some concepts in a particular theory which are not directly related to any empirical indicators. The usefulness of such terms is an issue (e.g., see Braithwaite, 1970 and many others). Such uninterpreted terms clearly cannot have referential meaning. But they may have systemic meaning derived from their logical connections with other concepts.

Operational definitions are not the only connections between the empirical and the logical systems. There is also a sense in which the relationships found in an empirical system can be compared with the logical relationships of the logical system. In addition to the operational definitions, there must be some operationalization of relationships. However, that aspect of the similarity between the two systems has not been dealt with so explicitly as the connection between concepts and empirical indicators, and little more can be said about that at this time.

Isomorphism. Both of these systems of knowledge are abstractions in many senses removed from the phenomena to which they pertain. The logical system, perhaps most clearly, imposes upon the knowledge the categories associated with the concepts and their possible logical relations. It, in essence, re-creates the phenomenon in terms of the concepts and possible relationships we imagine to be there. It is problematic to what extent this re-creation is more a reflection of our own predispositions than of the phenomenon itself. Similar considerations suggest that the empirical system is also abstracted and removed from the

phenomenon by our own categories of perception, and its relationships to that phenomenon must also be subject to scrutiny. The nature of the empirical indicators and the categories which are created for measurement, the specific questions or items which measure them, the types of relations which are examined; all of these again are channels through which our own predispositions can come to play in our perception of the phenomenon. It doesn't require a sociologist of knowledge to realize that there is some room for distortion here.

Each of these systems of knowledge then, must relate not only to each other (through operational definitions and the equivalent operationalization of relationships), but also to a third component, the phenomenon. The relationship of each of these to the phenomenon must be viewed as problematic. At a very minimum there must be some basis for viewing the systems as related to the phenomenon, and at a maximum, the systems would be seen as bordering on being identical to the phenomenon. We may posit some sort of isomorphism between each of these systems of knowledge and the phenomenon being studied. That is, there must be something which these systems and the phenomenon have in common. Each of these systems is true in some respects. It is with regard to those aspects which they have in common with the phenomenon that they may serve to increase our knowledge about the phenomenon.

The Model Building Perspective

Different specific frameworks of knowledge could be distinguished on the basis of these components and the relationships among

them. That is, they could be distinguished on the basis of the following:

- 1) characteristics of the logical system,
- 2) characteristics of the empirical system,
- 3) characteristics of the phenomenon,
- 4) the operational connections between the logical and empirical systems, and
- 5) the isomorphism between the logical and empirical systems and the phenomenon.

The specific relationships among these components of frameworks of knowledge are governed by a number of constraints. One of the most significant of these constraints is the incompatibility between the simplicity required for the development of both the logical and empirical systems and the complexity which appears to characterize most sociological phenomena. To create a body of knowledge there must be some simplification. Without it the logical and empirical systems simply would not be manageable. It is not possible to include everything which goes on even in a simple experimental setting. Furthermore, it is not useful to do so. All knowledge must abstract and simplify from the complex reality with which it deals in order to be manageable. Yet, if knowledge is too abstract and too far removed from the reality then we may have missed the most critical aspects and our work will be of little consequence. Clearly there is a need both to maximize the realism of our knowledge (i.e., the quality for the isomorphism it has with the phenomenon) and the simplicity of the knowledge so that it can be tested empirically and systematized

logically. Precisely where to balance these two conflicting goals is absolutely critical and has profound implications for our methodology.

It is this issue which appears to be the basis for many of the differences between the use of mathematical models and other forms of knowledge. A model building approach is a special case of general approaches to the development of scientific knowledge. The framework of knowledge created through such an approach has all of the elements identified earlier for frameworks of scientific knowledge: a logical system, an empirical system, operational connections between these two, and an isomorphism of both of these systems with the phenomenon being studied. It is distinct from other frameworks of knowledge not in its components but in the balance maintained among those components. In a model building approach the balance between simplicity and realism is chosen to favor simplicity at the expense of realism. That is, in model building the simplicity of the empirical and logical systems is maximized at the expense of the quality of the isomorphism between those systems and the phenomenon being modeled.

The maximization of the simplicity of the system at the expense of the isomorphism with the phenomenon being modeled has a number of very important implications for the development of knowledge. It drastically affects the role the logical system plays in that framework of knowledge. It raises a number of key issues which must be addressed in empirically validating such a framework of knowledge (issues which, incidentally, are rarely successfully addressed in the literature reviewed here). And, in general, it

radically restructures the balance among the components of these frameworks of knowledge causing them to have strengths and potential pitfalls which are much different from other social science knowledge frameworks. These are so different, in fact, that the model building approach is best viewed as a major alternative methodological approach significantly different from other social science approaches to the development of scientific knowledge.

The major advantage of the model building approach is that it makes the logical system a much stronger component of the framework of knowledge than it is for most other approaches commonly used in the social sciences. In the more common social science approaches the logical systems are mere shadows of what they might be. They generally perform only a minor checking role for assessing the logical consistency of propositions in the knowledge base. They rarely provide a systematizing framework or typologies and they almost never provide the capacity to deduce new hypotheses and propositions beyond common sense for the effective exploration and extension of the knowledge base. However, when the model building approach is utilized it is much more likely that a rigorous logical system may be developed which not only provides clear typologies and a logical framework within which to classify various aspects of the phenomenon, but also provides a powerful deductive system for checking the logical consistency of propositions and allows for the deduction of precise, empirically testable hypotheses. The logical system, if sophisticated enough may quickly be extended beyond the common sense predictions and allow the exploration of interesting new elements of that system of knowledge.

But these advantages do not occur without their price. The major area where the problems of this approach occur is in the empirical testing of these systems of knowledge. The model building approach raises a number of issues there which have created serious problems that have yet to be effectively resolved in the literature. For the traditional social science approaches, the only viable source of truth is the empirical testing of the model. But when the logical system is rigorous and precise (as is possible when parsimony is maximized) the logical system becomes a viable alternative source of truth. Then it is necessary to address the issue of how the two systems of knowledge may be reconciled (i.e., what is to be done when logically true statements are found to be empirically false). Here a number of interesting issues are raised. These issues are the source of much of the difference between this approach and other approaches to knowledge building. For such a system of knowledge, for example, it is an issue whether empirical verification is even required; how such a model might be empirically tested while recognizing the validity of both empirical and logical knowledge; and how such models may be tested when there is only a weak isomorphism between the models and the phenomenon they model.

There are two antithetical views of the need for verification of models which are commonly discussed in the philosophy of science literature: rationalism and empiricism. Rationalism is the view that a model or theory is a system of logical deductions from a series of synthetic premises of unquestionable truth not subject to empirical verification. Such premises which do not require or

admit of empirical verification are what Kant called the synthetic a priori. This view holds that such statements are obvious and do not require empirical verification. However, as Naylor suggests (1971:155), attempts to spell out in detail such assumptions soon leads to a point where they are no longer at all obvious. Some (e.g., Reichenbach) even deny that there exist any statements which are synthetic a priori.

Empiricism, on the other hand, is in complete opposition to rationalism. Empiricists suggest that empirical science instead of mathematics is the ideal form of knowledge. Any statements which are not subject to empirical verification should not be considered and have no value.

The view of models taken here suggests that both empirical and rational knowledge are valid. They are mutually compatible and complimentary, and models should have both simultaneously. From this perspective it is necessary that models, apart from their logical validity, be tested empirically to assess their empirical validity. Virtually everyone who addresses this issue also agrees that models must be tested empirically (e.g., Ashby, 1970: passim; Black, 1962:223; Naylor, 1971:153; Maslov, 1962:15; Bartos, 1967:321; Cohen & Cyert, 1961; Inbar & Stoll, 1972:281; Willer, 1967:629; and Grunberg, 1957:passim). As Coleman (1964_a:53) says, mathematics is "a tool of social science rather than... an end in itself." It is not enough that an elegant logical framework has been constructed which is appealing and logically sound. As Ashby says, "ultimately the raw facts are final" (1970:95), and the operational test is the last court of appeal" (1970:104).

There are many consequences which arise from this. First, it requires that models be made empirically testable. At least some of the terms of the models must have corresponding empirical indicators which may be examined empirically. It also makes relevant all of the constraints typically associated with empirical tests of knowledge. These include the general problems of confirmationism, hypothesis testing, the general requirements of research, the necessary procedures for statistical tests, and so on. The recognition of an empirical basis for judging the validity of models also implies that extensive knowledge of the phenomenon is required for the development of models.

Another very serious source of constraints upon model verification arises from the simultaneous recognition of the validity of both empirical and rational knowledge. This view is a common one, as evinced by the common recognition of the need for empirical verification and the frequent reference to desirable characteristics which are peculiar to rational knowledge, such as logical validity, logical rigor, and so on.

One consequence of recognizing the validity of both rational and empirical knowledge is that the empirical verification of any particular proposition has more of an implication for the growth of knowledge if many logical consequences of that proposition are of importance for the theory. For this reason testing assumptions which are the basis for many later propositions in the theory may provide more knowledge than testing particular deductions of the theory which serve as the logical basis for very few other propositions. The earlier a statement occurs in the deductive chain the

more impact it has, and the more knowledge is gained by testing it empirically. Of course, some contribution is made no matter where the proposition is located in the model.

Another issue arises when there are concepts in the model which have no empirical referents (see Rosenberg, 1968:183). These are called uninterpreted concepts. The nature of such concepts and propositions containing them can only be approached from the view of rational knowledge. That is, the propositions can be tested indirectly by testing logically related propositions. If, as is often the case, propositions are deduced from propositions containing uninterpreted concepts the empirical verification of those deduced propositions does not necessarily imply the validity of the propositions from which they are deduced. But if, as is less often the case, the propositions of interest are deduced from empirically testable propositions, then the empirical verification of those propositions would also imply the empirical validity of these nontestable propositions.

Yet another consequence of the simultaneous recognition of the validity of both rational and empirical knowledge is that it is not necessarily helpful to attempt to test entire models (both assumptions and deductions) empirically. Coleman (1960) and Diesing (1971) both make this argument. Such a problem arises when a theorist "proposed to test his model by setting up the conditions specified in the postulates, as near as practicable, and then seeing whether the predictions of the model, its derivations, agree with the experimental data" (Diesing, 1971:85). The problem arises in the supposition that there is a merely "hypothetical

or contingent" connection between the postulates of the model and its predictions. This is not true for logically rigorous models in which the predictions follow necessarily from the assumptions. If the conditions corresponding to the postulates are set up, then the results of the deductions must necessarily occur. If they are found, then nothing has been contributed to our knowledge. And if they are not, then we know only that the conditions were not properly established as we had thought. This problem arises when there is another source of truth in the model, the logical truth. Such a truth can compliment that of empirical truth and there is no need to reaffirm it empirically. This problem of course, does not arise in less rigorous models for which the logically necessary conclusions of assumptions cannot be established.

This problem is particularly acute in experimental research in which the conditions corresponding to the assumptions of the model are created artificially. As Coleman (1960:144-145) points out, the connection between reality and the model becomes confused.

...the "reality" which it attempts to mirror is itself a completely constructed and artificial situation. Thus a model is not being constructed to correspond to certain phenomena in the real world, but conversely, phenomena are constructed to correspond to the model. In such a situation it is difficult to know where the ingenuity lies--in construction of the model, or in manipulation of people so that they will behave in accordance with it. It is hard to see just what the goal is in such a model.

This problem is generally not present for approaches to knowledge building other than models because the additional constraints of trying to mirror reality exactly generally make it impossible to develop theories which have logically necessary implications.

A normative model may be a special case of the type of model in which the logical necessity derives not from the assumptions but from some arbitrarily imposed criterion for some outcome which is used to logically derive the necessary conditions for achieving the optimal value of some outcome. For a normative model, the same problems of verifying both the assumptions and the predictions would be found in attempts to empirically test them. For such models verification would be very difficult indeed. As it happens, however, for these models empirical verification is not the objective and is largely irrelevant to the models. The goal is not to describe empirically occurring phenomena, after all, but to prescribe how certain objectives may be met.

This general problem of verifying models which at the same time may have both a rational and an empirical knowledge base has generated a controversy among researchers as to which parts of the models should be tested. Should assumptions be tested? Should predictions be tested? Or should both be tested? Bartos (1967) argues that only the assumptions of a model need to be tested (321). Testing the implications does not contribute much to the knowledge by itself. By "showing that some implications of the model are empirically true (one) has not proved that the model itself is necessarily true" (316). However, testing the implications of a model may be useful as an indirect test of the assumptions. Particularly, in those frequent cases where it is not possible to conclusively prove the assumptions correct, the model may be tested by testing both its assumptions and its implications (322).

Another view is held by Kendall (1967:11). In this view

(which is also held by Milton Friedman) it is suggested that "the validity of a model depends not only on the validity of the assumptions on which the model rests (as Hutchinson would have one believe) but, also on the ability of the model to predict the behavior of the dependent variables that are treated by the model." This argument is that empirical tests need only be applied to the predictions of the model and not to its assumptions. He "seems to be saying that it makes no difference whatever to what extent the assumptions falsify reality" (Naylor, 1971:156).

Coleman (1960:141-2) suggests that a critical difference in assessing the fruitfulness of models is at what point they make a connection with empirical phenomena. Many models have uninterpreted terms and make connections with empirical indicators only in a few places. Two types of models which are very different are those for which the empirical connections occur only at the level of assumptions or postulates and those for which the empirical connections occur only at the level of theorems or propositions which have been derived from the assumptions. This distinction parallels the distinction between explanatory and synthetic models.

In general, though it is true that tests of logically necessary predictions offer little in the way of increased knowledge, it is rarely true that the logical necessity is irrefutably established, and unless it is there should properly be empirical tests of both the assumptions and the deductions of models. Reasons why logical necessity may not be established vary. There may be some critical assumptions which are not verifiable. Perhaps there are other variables or processes entering into the system which are not

recognized. Or it may be that a lack of perfect isomorphism between the model and the phenomenon may obviate the logical necessity. In addition, the phenomenon may be stochastic in nature requiring several empirical tests, and there may be measurement errors and other problems which occur. If the deductions are confirmed there is still a need to verify the postulates for similar reasons. If the deductions are confirmed there is still a great part of the theory which has not yet been confirmed (including other deductions not yet tested and the postulates which lead to them). Also, it is important that the postulates be established because there are many possible alternative explanations for any particular set of deductions.

Yet another issue is the isomorphism between the model and the phenomenon being modeled (Brodbeck, 1958:380; Meyer, 1951:118). Because the isomorphism is not complete between the model and the phenomenon, any discrepancy between the empirical phenomenon and the predictions of the model might well be due to the lack of fit between the model and the phenomenon. Care must be exercised in generalizing from the model to the phenomenon itself. Empirical results predicted by the model and confirmed empirically might arise from characteristics of the model which are not isomorphic to the phenomenon and should not be interpreted unequivocally as in support of that model of the phenomenon or as indications of properties possessed by that phenomenon.

Appendix 2

A TEST FOR MEAN RECURRENCE TIME

One prediction of Markov models which is commonly tested empirically as part of efforts to validate such models is a comparison of the predicted and observed recurrence times. The recurrence time for a particular state is simply the time it takes for the system of interacting individuals to return to that state after once having been in it. For example, in the sequence of interactions,

$$O_1 C_1 O_1 C_2 O_2 C_2 O_1 C_3 O_1 C_1$$

only one of the second-order states (states including actions by both participants--i.e., $O_1 C_1$ is one second-order state), $O_1 C_1$ occurs more than once. That state occurs first in the chain and last in the chain. The recurrence time for this state is then the time between its first appearance and its next appearance.

One of the reasons that this test has been so popular is that Feller (1968) has shown that for irreducible, aperiodic Markov chains with states having nonzero probabilities of occurrence at equilibrium, the mean recurrence time for each state is the reciprocal of the probability of that state occurring at equilibrium:

$$r_k = 1/u_k, \text{ where } u_k = \lim_{n \rightarrow \infty} p_{jk}^{(n)}$$

where r_k is the mean recurrence time of event E_k .

Unfortunately, in some cases, the empirical estimation of recurrence times is not as straightforward as the theoretical prediction. To empirically estimate the mean recurrence time of each state one could simply determine the time it takes for each event to recur wherever it does recur and then find that mean. Unfortunately, when there are many categories the mean recurrence time for a particular event may be quite long relative to the length of the encounters examined (e.g., one predicted mean recurrence time was 90.91 while many of the encounters examined had considerably fewer interactions than that). When this occurs there may be a large number of events which occur and never recur. When the encounter ends it is not possible to determine when such an event would have next recurred.

When the length of encounters examined is short relative to the length of the mean recurrence time for a particular state it is not possible to get an adequate estimate of recurrence times because the observed recurrences are part of a highly truncated distribution—only the earliest recurrences occur in time to get measured—resulting in a gross underestimation of the mean recurrence times. For example, event O_1C_1 has a predicted recurrence time of 52.63. Of the 40 times when this event occurred only 11 of those were recurrences of the event within the same encounter. The resulting estimate of recurrence times is only 7.45 (see Tables A2.1 and A2.2)

Because of this problem of estimation, the test of predicted mean recurrence times is not included in this study as a test of the Markov model.

TABLE A2.1
OBSERVED DISTRIBUTION OF EVENTS AND RECURRENCES OF EVENTS FOR
SECOND-ORDER MODEL

Event	Number of Times Event Occurred	Number of Times Event Occurred	Proportion of Times Event Recurred
01C1	40	11	.275
01C2	529	434	.820
01C3	43	18	.419
02C1	62	37	.597
02C2	983	895	.910
02C3	165	33	.200
03C1	34	8	.235
03C2	295	178	.603
03C3	128	64	.500
C101	38	14	.368
C102	78	48	.615
C103	27	4	.148
C201	452	364	.805
C202	973	885	.910
C203	318	198	.623
C301	69	30	.435
C202	75	42	.560
C303	89	40	.449

TABLE A2.2

PREDICTED AND OBSERVED MEAN RECURRENCE
TIMES FOR SECOND-ORDER EVENTS

<u>Event</u>	<u>Mean Recurrence Times</u>	
	<u>Predicted</u>	<u>Observed</u>
O1C1	52.63	7.45
O1C2	4.42	3.82
O1C3	52.63	6.39
O2C1	34.48	4.14
O2C2	2.10	1.81
O2C3	33.33	3.67
O3C1	66.67	9.88
O3C2	7.58	5.49
O3C3	18.18	5.92
C1O1	58.82	7.14
C1O2	28.57	7.04
C1O3	90.91	6.75
C2O1	4.67	3.77
C2O2	2.16	1.83
C2O3	6.62	5.03
C3O1	33.33	8.43
C3O2	30.30	3.67
C3O3	25.64	4.80

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