ALLOCATING POLICE OFFICERS ON A STATEWIDE BASIS:

A MATHEMATICAL MODEL FOR THE ILLINOIS STATE POLICE

DEPARTMENT OF LAW ENFORCEMENT
N OF ADMINISTRATION

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Illinois Department of Law Enforcement
Division of Administration

Richard A. Raub

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INTRODUCTION

Rational allocation of police officers always has presented a difficult problem. The number available to serve the population is small. The size is further reduced for two reasons. First, a portion of the officers serve the administrative needs of the organization. In all but emergencies, they do not serve the population directly. Secondly, the agency generally provides 24-hour, seven-day-per-week coverage. It takes between four and five officers to fill the three, 8-hour patrols. Even what may be a favorable ratio of police to public rapidly diminishes.

In an urban setting, the relatively small geographic area served allows overlapping coverage by the police officers on duty. Thus, more than one officer is available almost immediately to serve the public. With an organization such as the Illinois State Police, the wide geographic area served and lack of immediate assistance makes the placement of each officer critical. Such officer must be visible in patrol but also able to respond rapidly to calls for service.

The model described in this paper attempts to provide a method of assigning police personnel to maximize their service while minimizing delays and lack of visible presence. It is an outgrowth of work performed both on a state-wide basis and in the urbanized areas.

BACKGROUND

Until recently, the Illinois State Police (ISP) have allocated sworn personnel to the districts (the basic unit of command) based on compromise. Each district commander has requested officers according to their perceived needs. These needs have not been well defined nor adequately supported. At one point, there was an attempt to allocate manpower based on the work of Wilson.[1] This resulted in assignment of a large percentage of police officers to the populous six-county region surrounding and including Cook County (Chicago) and to a two-county region across from St. Louis, Missouri. Although serving 65 percent of the population of Illinois, such assignments covered only ten percent of the area. In other rural areas, there were not sufficient numbers of state police officers to supplement the relatively small local police forces.

Attempts to allocate police officers to districts based on proportional needs related to crime, population, miles of highway, vehicle miles, size of district, and the number of state police actions in terms of calls for service additionally have suffered from several drawbacks:

- The variables previously used have given population a heavy weight and have been inter-correlated. (Crimes, vehicle registrations, and vehicle miles all show strong positive correlations with population and with each other.) Such inter-correlation may lead to unpredictable results.
- Decisions in terms of the arrival rates of calls, how promptly they would be answered, and appropriate accounting for minimum response time could not be made.
- Administrative support (overhead) has not been adequately defined and the amount needed not adequately computed.

The literature describing allocation of police manpower in rural areas is limited. A study completed by the Arizona Department of Public Safety based allocations on response to accidents.[2] Systems Science Development Corporation of St. Louis prepared methodology for assigning linear patrols to Interstate 80 in Illinois.[3] Finally, the models of Lipseff and Arnold,[4] although designed for a "semi-rural police force", were more applicable to an urbanized area than to the rural patrol faced by the ISP.

That most of the work has been devoted to allocating manpower in urban areas is not unexpected. The areas are compact; they allow overlapping response. Traditionally, the urban police forces have been considered short of manpower. An adequate allocation of the limited resources throughout the city, therefore, has been important. Several theoretical approaches and models designed for specific cities are noted, including Chaiken,[5] Larson,[6] and LeGrande.[7] One of the more popular, the hypercube model designed by Richard Larson of M.I.T., has been used in part to help allocate officers in New Haven, Connecticut and St. Louis, Missouri. Even though not applicable directly to the needs for allocating state police officers in Illinois, these models have helped serve as a basis for the model presented in this paper.

The size of Illinois, its relatively sparse population, long distances between major cities, and a need for enforcement of traffic laws as well as crime prevention throughout the state must be considered in allocating police officers. By law, the ISP currently are authorized 1600 sworn officers.[8] An additional number of state police officers are working under contract with the Toll Road Authority and are not included in the authorized strength. ISP officers have

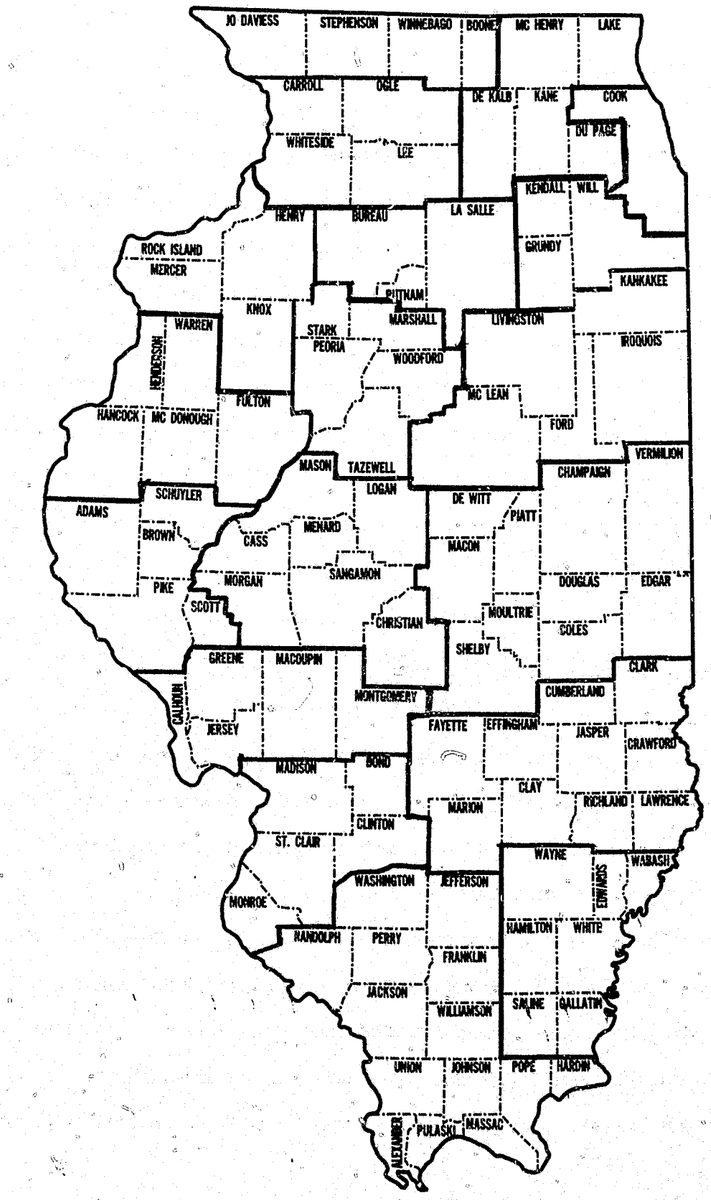
policing powers throughout Illinois and must provide services to the 11.6 million persons living in the 56,400 square miles (although the ISP generally does not provide services in the City of Chicago). Although the ISP jurisdiction covers the entire state, the availability of local law enforcement personnel in most communities with a population of 2,500 or more persons allow the state police to concentrate their activity outside these communities. They are supplemented by sheriffs' departments for patrol and policing rural areas. As a result, even though more than 65 percent of the population resides in eight Illinois counties, the major distribution of ISP officers must lie outside these counties. Patrol in the eight counties generally is limited to expressways and major highways. There are sufficient local police officers to provide other policing services.

Currently, officers from the ISP are assigned to all of the 102 counties. These offices report to the basic unit of command, a district, which comprises three or more counties except in Cook County which is divided into two districts. A map showing the state, counties, and districts is given as Figure 1. Distribution of personnel to the counties and the patrols within a district is the prerogative of district management. Generally, the majority of manpower available to the district is assigned near the centers of population.

In addition to the personnel assigned to patrol (including their immediate supervisors), a portion of the authorized strength handles administrative matters. Some are assigned to the central office in Springfield; a number also are assigned to security for the governor and top elected officials. Each district requires officers for district command, desk, and special assignments which serve the Department but which do not make the officer immediately available for

FIGURE 1

ILLINOIS STATE POLICE
DISTRICTS



patrol. The special assignments include: public information, equipment, ordnance, court, abandoned vehicle investigation, and fixed scale supervision.

Those officers assigned to general patrol handle any police matter. Most of their time is spent enforcing traffic laws; however, they also respond to motor vehicle accidents, criminal complaints, and assist municipal and sheriffs' police as needed. Thus, any model prepared must account for all of these functions.

Other related police services including laboratory and forensic services, crime scene technicians, criminal intelligence, criminal identification, detective services, internal investigation, and training are provided by separate divisions within the Department of Law Enforcement. The Department is examining these functions separately for the purpose of determining better methods for allocating the affected personnel.

MODEL FOR ALLOCATING OFFICERS

Philosophy

The geographic size of patrols for the ISP are relatively large; many cover more than 1000 square miles. Response time can be measured in tens of minutes. Differences in one or two minutes become relatively unimportant. The major factor is the volume of calls. Where they are overlapping, one officer must be assigned to handle each call, or service to that call must be delayed. Further, that an officer is available in one county to answer calls generally does not make this officer available to answer calls in another county. Distances are too great.

This is the biggest difference between rural and urban allocation. In urban areas small increments in response time may be critical, particularly in criminal

matters.[9] As noted earlier, work by such researchers as Larson have also taken into account availability of nonobligated officers to handle calls outside their assigned boundaries.[10]

As a result, one part of the model for Illinois State Police must deal specifically with response to requests for service. At the same time, however, the model must account for preventative patrol (a visible presence). An officer not engaged in answering a call provides this patrol. The distance covered by the officer is inversely related to the volume of calls.

The model drawn for the ISP accounts for three categories of activity: administration, response, and general patrol. The model also incorporates the following premises.

1. Wherever possible, the variables used to allocate manpower are exogenous to the control of the agency.
2. These variables also are weakly correlated; multicollinearity is reduced.
3. The number of officers available for allocation is less than the total available (authorized strength); the remaining officers provide administrative support and are assigned by executive decision.
4. The amount of manpower to be assigned to a specific philosophy, e.g., answer calls for service, is not limited except by the total number of officers available for allocation.
5. The same techniques used for allocation can also be used for projecting needs.

Basic Model

The basic model for allocation contains three components:

- Overhead, called "administrative support"
- Response, called "calls for service"
- Patrol, called "policing and patrolling"

The sum of the officers assigned to each of these functions equals the number of police officers available. It can be expressed simply as:

$$N = O + C + D \qquad (1)$$

where:

N - Number of officers available

O - Officers assigned to administrative support as specified by management

C - Officers answering calls for service which is a function of accidents, criminal complaints, and response time
$C = f(A, C, Q)$

D - Officers policing and patrolling which is a function of highway mileage and traffic volume
$D = f(M, V)$

In the operation of the model, for equation (1) O is established for each district as well as central administrative activity. C and D are computed for each shift within a county (the smallest practical unit for data collection) and summed to the district level. Thus, more formally:

$$N = \sum (O_i + C_i + D_i) + O_c \qquad (2)$$

where:

i - District level

c - Central office level

Administrative Support

The administrative support needed and allocated is an executive decision. The support required could equal the number of officers available, but that would defeat the use of a model. In practice, the administrative support at the district level has included all personnel in the district at the rank of sergeant and above and a few selected special assignments, plus central office staff and executive security. The total in this category varies between 25 and 30 percent of the available force.

Administrative support is subtracted before other computations are made.

Calls for Service

Two types of calls have been selected for use in the model: accidents and criminal complaints. The number of calls answered can be documented. They also can be projected. For either category is known:

1. The number that occurs for each eight-hour shift in a given county.
2. The average time taken to service the call.

Computation of the number of calls expected during the period, any one call is being serviced then is computed through a Poisson function:

$$P(x) = e^{-m} m^x/x! \qquad (3)$$

where:

m - Is the average number of calls occurring during the service interval, t, for any one call.

$$m_A = A_r\, t_A\, p_A/2920 \qquad (3a)$$

$$m_B = B_r\, t_B\, p_B/2920 \qquad (3b)$$

where:

A - Accidents (annual)

B - Criminal complaints (annual)

t - Time taken to service A or B

p - Proportional factor used to increase or decrease the number of accidents or criminal complaints handled

2920 - Number of annual hours in an eight-hour shift

The number of officers needed for any one shift is the "x" at the point where P(x) from equation (3) is equal to or greater than some service level. Service level is expressed as a portion of calls handled immediately. Any call received after the level is reached is placed in a queue that is serviced by delaying response or requesting assistance from another agency. (All computations assume that only one officer services a call.) In the model, the service level is a parameter. While a 100 percent service level is feasible, it is not practical. In many cases this would cause the model to add one officer to serve one percent or less of the expected number of calls received.

Because calls for service can be so few, P (0) for both accidents and criminal complaints can exceed the service level. No officer is assigned. However, the police philosophy is that an officer must be available to respond, in a reasonable time, even for one call per year. Therefore, equation 3 is modified.

$$C' = (X_A + X_B) \geq f(Q) \qquad (4)$$

where:

C' - Positions required to handle calls for service

X_A, X_B - Total number of officers assigned in one shift to meet a requisite service level for accidents and criminal complaints

f(Q) - Response time which is a function of the area served

Note: In equation (4), the larger of the two sides $X_A + X_B$ or f(Q) is used.

$$F(Q) = Q/2d'^2 \qquad (4a)$$

$$d' = d\, f(c) \qquad (4b)$$

where:

Q - Area in square miles

d' - Driving time adjusted for congestion

d - Desired response time

f(c) - The ratio of actual speed of the responding vehicle, given congestion, to the potential speed.

$0 \leq f(c) \leq 1.0$

Note: In all but urbanized areas, f(c) 1.0 (see Appendix A for the formula used).

One further adjustment of C' from equation (4) is required. What is computed is the number of officers required to satisfy the parameters. However, to provide this service 365 days a year requires more than one person to fill the position created. For the ISP, a work year of 220 eight-hour days was selected. Given 8760 hours in one year, it takes 1.66 persons to fill that one position per shift. Thus, C is adjusted by this amount.

$$C = C' s \qquad (5)$$

where:

C - Number of officers required to answer calls for service

s - Adjustment of positions to achieve annual coverage, 1.66 for this model.

From the total strength, then is subtracted both administrative support and calls for service. If the result is negative, either the parameters used for calls for service (desired level of service) must be reduced or the administrative support reduced. Resulting from this action should be a body of persons not assigned. These are distributed according to policing and patrolling needs.

$$D = N - O - C \qquad (6)$$

$$D \geq 0.0$$

Policing and Patrolling

Policing and patrolling refers to the preventative aspect of policing. In theory, a visible patrol vehicle will help prevent crimes or motorist violations of

traffic laws. Given a population to be served and highways to be patrolled, a basis for measuring patrol is available. Chosen for this model was both mileage and persons served per patrolling officer.

The two parameters are distinct. The mileage is patrolled to prevent violations of traffic law, serve the motorist, and, in the end, to reduce accidents. The objective is to maximize that mileage. In preparing the model, three highway patrolling philosophies were used: patrol of Interstate highways, of state highways with moderate to high volume of traffic, and of low volume, county, and township roads. Frequent patrol is required on Interstate highways. This means short patrolling lengths. More infrequent patrol occurs on higher volume, two-lane roads and very infrequent patrol on all other highways. The model separates these three highways but uses the same mathematical philosophy the compute length of patrol.

Additionally, some police personnel have to be available to serve the rural population as a whole regardless of highway mileage. The presence of any police satisfy this need; ISP officers augment the local efforts. In effect, this yields a fourth parameter for patrol. As shown in equation (7), there are four elements to be solved.

$$D = D_f + D_t + D_o + D_r \qquad (7)$$

where:

O - Officers available for patrol

D_f, D_t, D_o, D_r - Officers needed to patrol four-lane (Interstate), two-lane (state marked roads with volumes exceeding 1500 vehicles per day, 1500 ADT), other roads, and augmented service to the rural population

Because the officers needed to answer calls for service in equation (5) are not always busy, their free time also can be devoted to the patrol specified in equation (7). Therefore, before computing and distributing personnel, D is adjusted for that free time.

$$D' = 1760D + (1760C - A\,p_A\,t_A - B\,p_B\,t_B) \qquad (8)$$

where:

D' - Hours available for patrol

C - From equation (5)

A, p_A t_A,

B, p_B, t_B - Described in equation (3)

1760 - Manhours per officer at 220 man-days per year

The first step is to arrive at a rational decision regarding the distribution of the available officers (or hours) among the four patrols. This is done by management. It reflects the emphasis placed on urban or rural orientation. Emphasis given to Interstate highway patrol is urban oriented because of the volume of traffic on those roads. Further, two-lane and other mileage is grouped as one unit, two-lane patrol. This is done to solve for unknowns. In equation (9) the p (f, t, r) represents the proportion of 1.0 assigned to each of the three.

$$D = (p_f D + p_t D + p_r D)\ /1760/1.66 \qquad (9)$$

where:

p_f, p_t, p_r - Proportion of available officers to be distributed among Interstate, two-lane (state and other), and rural patrol.

1760 - Man hours per officer per year

1.66 - Persons per position per shift (see equation (5))

The number of officers needed to serve a segment of highway (the basic unit is mileage in a county), is dependent upon the miles of highway to be patrolled and the miles of highway that can be patrolled per hour. The latter is dependent upon the speed of the patrol vehicle, congestion, and the number of traffic stops made by the officer. Only miles of highway and volume of traffic are independent variables in the equation. Multiple regression analysis has shown that these variables are highly related to other measures such as population, registered vehicles, accidents, citations written, etc., but are not related to each other.

While the model shows an amount of miles in a patrol, the mileage that will be patrolled in any county during any shift will be less when congestion is present. This adjustment is derived mathematically. It is given in Appendix A.

- Four-Lane Patrol

One of the two divisions used for determining patrol is four-lane (generally Interstate) highways. The State Police provides sole patrol for most of this mileage throughout Illinois. The number of positions available to work four-lane roads is shown in equation (10). It is derived from a decision by management to assign p_f proportion to that highway.

$$D_f = p_f\,D'\ /1760/1.66 \qquad (10)$$

where:

D_f - Positions available per shift to patrol the four-lane highway

p_f - Specified as a parameter, proportion of officers to be given to four-lane patrol

D' - From equation (8)

Because D_f and total miles of four-lane roads are known, what is solved is the miles of patrol for any one officer assigned to patrol of four-lane roads. If, at the conclusion of processing, the miles of patrol are too high, then either the available officers, D', must be increased, or the proportion of officers assigned to four-lane roads, p_f, increased.

$$H_f = f(s)\, M_f / D_f \qquad (11)$$

where:

H_f - Miles of patrol per position

M_f - Miles of four-lane roads

D_f - From equation (10)

f(s) - Correction factor for speed as shown in Appendix A. In most rural counties for all shifts, $f(s) \approx 1.0$. In Cook County for shifts covering rush hour f(s) may exceed 4.0

In practice, H_f is solved for all counties for all shifts. The assignment of positions in any one county is then performed by equation (12).

$$D_{fi} = M_{fi}\, f(s)_i / H_{fi} \qquad (12)$$

(subscript i stands for the individual county and shift within that county.)

- Two-Lane Patrol

The same technique is used to assign patrol to two-lane roads. Because both state marked highway and other types of two-lane highway are included and because only one equation is available, the miles of patrol for other highways is fixed. What has been used in the model is 6000 miles. This is approximately equivalent to one pass of the patrol vehicle once every month.

$$D_t = p_t D' \,/\, 1760 \,/\, 1.66 \qquad (13)$$

$$H_t = f(s)\, M_t \,/\, (\, D_t - M_o \,/\, 6000\,) \qquad (14)$$

where:

H_t - Miles of patrol of a two-lane (state marked highway)

f(s) - As shown in Appendix A

D' - From equation 8

M_t - Miles of two-lane (state marked) higher volume roads

D_t - Persons assigned to patrol of two-lane highway

p_t - Proportion of persons to be assigned other highway patrols

M_o - Miles of other roads and state marked roads with less volumes than 1500 ADT

6000 - Miles of patrol for other highways.

Similar to four-lane patrol, H_t, is solved given all counties and all three shifts. In an equation similar to (12), the positions are assigned to each county.

$$D_{ti} = M_{ti}\, f(s)_i / H_{fi} \qquad (15)$$

$$D_{oi} = M_{oi} / 6000 \qquad (16)$$

where:

P_{oi} - Patrol of other highways

i - Subscript denoting each county

- Rural Patrol

Some allowance is made for assisting local law enforcement in terms of additional personnel to supplement their strength. First determined is a ratio of population to police officers. State police officers are added to number of local officers to help meet this ratio. Two conditions are set. First, state police officers would not be removed from a location if there are more than a sufficient number of other police officers. Second, the number of additional state police officers added per shift also should be limited. One officer per shift has been chosen as that limit.

Setting limits, however, means that the actual distribution can not be computed directly by solving for the unknown, number of persons served. Rather a solution is estimated and the computations carried through to determine whether the number of officers assigned matches the number to be assigned. If the equality is not found, the number of persons served is increased or decreased and iterations are continued.

$$D_r = p_r D' / 1760 / 1.66 \qquad (17)$$

$$D_{ri} = R_i / H_r / 1.66 - L / 4.98 \qquad (18)$$

where:

D_r - Positions assigned to rural patrol

R - Rural population

D' - From equation 8

H_r - Number of persons per officers

L - Local police officers

i - Subscript standing for any county

1.66 - Number of officers required to support one position per shift

4.98 - Number of officers needed to support three shifts

Note: H_r is adjusted until $D_r = \sum D_{ri}$

Adjustment For Officers Assigned to Calls For Service

All of the computations have been performed with a variable, D', which has included free time of officers assigned by the model to calls for service. To obtain the actual count of officers assigned to policing and patrolling requires a reduction of D by the free time that had been included. The ratio of D to D then can be multiplied directly through the positions assigned, the positions added, and multiplied by officers per shift.

$$D_i = 1.66D (D_{ri} + D_{ti} + D_{ri}) / D \qquad (19)$$

where:

D_i - Number of officers allocated to policing and patrolling in a county

SUMMARY

The model has been transferred to an IBM 360 at the Department of Law Enforcement and has been in use for more than one year. During that time it has been upgraded from the original described in Raub and Sweat.[11] What this upgrading has yielded has been refinements in the allocations and a better balance between urban and rural patrol.

The Department has used this model to evaluate the effect of several philosophies in terms of changes in strength for all districts. Its use has helped support a decision by the Division of State Police to put greater emphasis on rural patrol and to spend more patrol on state highways as opposed to Interstate highways.

Additionally, the Department has examined the size of a force required to meet improvements in services. Most recently, it has determined to what extent services would be curtailed if there where reductions in the number of police officers. Finally, in conjunction with its present policing philosophies, the Department is assigning graduating cadets to districts depending upon relative need.

The next step will be to use premises developed for this model to identify allocations of detectives. Here, instead of highway and volume, case load would serve as the basis. What needs to be determined, yet, are the independent variables required for projection of case load. Also, unlike policing services, the type of case undertaken has a large bearing on the amount of time and, therefore, need for investigative efforts. This also must be resolved.

Given the limited information originally available, the model devised appears to be a practical tool. While it has been limited to police services, its three-fold concept of overhead, reaction, and prevention can be applied to other fields.

NOTES

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(2) J. Stoneberger and L. Deitch, Manpower Requirement and Allocation Study, Arizona Department of Public Safety, Phoenix, Az. (undated).

(3) A Compendium of Illinois I-80 Project, Systems Science Development Corporation, St. Louis, Mo., June 1970.

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(5) J. M. Chaiken, Patrol Allocation Methodology for Police Departments, Report R-1852-HUD, Corporation, Santa Monica, Ca., September 1975.

(6) R. C. Larson, A Hypercube Queuing Model for Facility Location and Redistricting Emergency Services, Report R-1238-HUD, RAND Corporation, Santa Monica, Ca., March 1973.

(7) J. L. LeGrande, "Distribution of Municipal Police Patrol Manpower," Planning and Research Bulletin, No. VI, School of Police Administration and Public Safety, Michigan State University, Lansing, July 1968.

(8) Illinois Revised Statutes, 121-307.1, Amended by PA 76-270, July 1, 1969.

(9) C. Clawson and S. K. Chang, "The Relationship of Response Delays and Arrest Rates," Journal of Police Science and Administration, 5:1, March 1977, pp. 53-68.

(10) Larson, supra.

(11) Richard A. Raub and George L. Sweat A Method for Allocating State Police Officers In Illinois, Illinois Department of Law Enforcement, Springfield, Il., June 1981.

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APPENDIX A

MATHEMATICAL FORMULAE FOR ALLOCATION MODEL

1. MODEL

$$N_i = O_i + C_i + D_i \qquad (1)$$

such that: $T = \sum N_i$

variables:

N_i - Number of officers allocated

O_i - Officers performing administrative support

C_i - Officers answering calls for service

D_i - Officers patrolling

T - Total number of persons available for allocation

subscript

i - any county and within any county, for each shift

Note: If the model is used for projection, this portion of the formula is not used.

2. CALLS FOR SERVICE

(Note: Each of the steps below is performed for each county, as the smallest unit of area, and for each of three shifts within that county.)

$$C_i = S C_1 \text{ or } S C_2 \text{ (whichever is greater)} \qquad (2)$$

where:

C_i -Number of officers for each shift in a county

S - Number of officers required to staff one position, one shift for a 365-day period

$S = 8760/Mh/3$

where:

Mh - Man hours of work per year for one officer

8760 - Number of hours in a year

3 - Number of shifts

Calls

$$C_1 = A + B \qquad (3)$$

$$C_A = f(A;m) \quad \text{(Accidents)} \qquad (3a)$$

where for each shift: C_A results from the Poisson distribution

$$P(x) = e^{-m} m^x / x!$$

$$m = A\ p_A\ t_A / 2920$$

such that: $\sum P(x) \leq (1.0 - q_A)$

$$C_B = f(B;m) \quad \text{(Criminal Investigation)} \qquad (3b)$$

where for each shift: C_B results from the Poisson distribution

$$P(x) = e^{-m} m^x / x!$$

$$m = B\ p_B\ t_B / 2920$$

such that: $\sum P(x) \leq (1.0 - q_B)$

where:

C_A - Officers needed to respond to accidents

C_B - Officers needed to respond to criminal complaints

A - Accidents occurring during that shift

B - Criminal complaints arising during that shift

p_A, p_B - Proportional factor used to adjust the number of accidents and criminal complaints

t_A, t_B - Time taken to service an accident and criminal complaint

q_A, q_B - Proportion of accidents and criminal complaints that will be delayed or given to another agency.

e - Natural log, 2.7183 . . .

Minimum Response Time

$$C_2 = Q / 2d'^2 \qquad (4)$$

where:

C_2 - Officers needed to achieve a minimum response time.

Q - Area of county in square miles

d' - Driving distance during a given time of response. At 60 mph d miles is the same as response timed. For congestion, d must be adjusted

$$d' = d\,(60 - g_t) / 60 \qquad (5)$$

where:

d - Desired response time

g_t - Adjustment for congestion on two-lane road, see equation (8a)

3. POLICING AND PATROLLING

(Note: This portion is solved for each county and, within each county, for each shift.)

$$D = T - \sum O_i - \sum C_i \qquad (6)$$

$$D_i = 1.66\ D_{fi} + 1.66\ D_{ti} + 1.66\ D_{ri} \qquad (6a)$$

Note: Allocation for any county is the sum of the allocation of manpower to four-lane (Interstate) patrol, other highway (two-lane and local) patrol, and patrol for the rural population. D and subsequently D_{fi}, D_{ti}, and D_{ri} are known. What is solved is their basis, miles of patrol of highways and persons served in rural populations.

Because there is time available from those officers assigned to answer calls for service, equation (6) is adjusted and the D resulting from equation (6b) used:

$$D' = D + 2920 (C - A p_A t_A - B p_B t_B) / s \qquad (6b)$$

where:

D - Number of officers remaining after subtracting officers assigned to administrative support and to calls for service

D' - Adjusted number of officers given free time from calls for service

Note: Other variables have been explained above.

Four-Lane Patrol

$$D_{fi} = p_f D \qquad (7)$$

Given for all three shifts summed:

$$H_f = 55 \Sigma \left[M_f / (55 - g_f) \right] i / \left[p_f D / 1.66 - 13.75 \Sigma (Y_f / (55 - g_f)) i \right]$$

$$D_{fi} = 55 M_f / \left[H_f (55 - g_f) \right] + 13.75 Y_f / (55 - g_f)$$

Note: Unless projections are being computed, H_f must be solved first.

For each shift:

$$g_f = V_f p_w / 8000 \qquad (7a)$$

for $V_f p_w \leq 10300$

$$g_f = 3.88 + (V_f p_w - 10300) / 1300$$

for all other $V_f p_w$

$$Y_f = 6.0 W_f p_w / 2920 \qquad (7b)$$

where:

p_f - Proportion of policing and patrolling to be assigned to four-lane highways

H_f - Miles of patrol for one officer on a four-lane highway

M_f - Miles of four-lane highways in a county

Y_f - Reduction in patrolling speed resulting from traffic stops

V_f - Daily volume on four-lane roads (ADT)

W_f - Volume expressed in thousands of vehicle miles, $V_f * M_f$

g_f - Reduction in speed occurring from congestion

p_w - Proportion of volume (ADT) occurring during a given eight-hour shift

Other Highway Patrol

$$D_{ti} = p_t D \qquad (8)$$

Given for each shift:

$$H_t = 55 M_t H_o / \left[p_f D_i H_o (55 - g_t) - 55 M_o - 13.75 H_o (Y_t + Y_o) \right]$$

$$D_{ti} = \left[55 (M_t H_o + M_o H_t) + 13.75 H_t H_o (y_t + Y_o) \right] / H_t H_o (55 - g_t)$$

For each shift:

$$g_t = V_t p_w / 2000 \qquad (8a)$$

for $V_t p_w \leq 3400$

$$g_t = 5.15 + (V_t p_w - 3400) / 70$$

for all other $V_t p_w$

$$Y_t = 8.75 W_t p_w / 2920 \qquad (8b)$$

$$Y_o = 1.83 W_o p_w / 2920 \qquad (8c)$$

where:

H_t - Miles of patrol on a two-lane roads

H_o - Miles of patrol on all other highways. This is a constant supplied to the model.

M_t, M_o - Miles of two-lane and other highways

Y_t, Y_o - Reduction in patrolling speed resulting from traffic stops on two-lane and other roads

W_h, W_o - Volume expressed in thousands of vehicle miles and two-lane and other roads

V_t, V_o - Volume (ADT) on two-lane and other roads

g_i - Reduction in speed occurring from congestion

p_w - Proportion of volume

Patrol For Rural Population

$$D_{ri} = R / H_r / s - L/3s \quad (9)$$

Given for each shift and each county

$$P_r = 1.0 - p_f - p_t$$

where:

R - Rural population

L - Local police including deputies and those police officers in municipalities of less than 1500 persons

H_r - Number of persons served per rural police officer

Note: D_{ri} for each shift within each county is constrained to a minimum and maximum. Therefore, H_r is substituted into equation and the sum of all state police officers compared to p_r D. This step is performed iteratively until:

$$\sum D_{ri} = p_r D$$

In practice: $\sum D_{ri} = p_r D \pm 0.01\ p_r D$

END