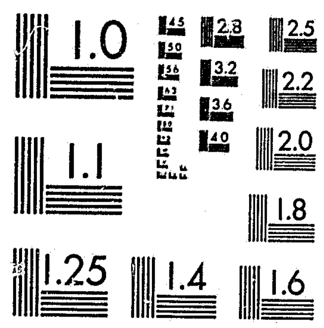

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OPERATIONAL MODEL FOR PREDICTING TIME BETWEEN LOSSES OF A VEHICLE IN A COMPUTER-TRACKED VEHICLE LOCATION SYSTEM

by

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TABLE OF SYMBOLS

t_Q	Time between updating estimated position of vehicle (sampling interval or unit of quantized time)
d	True vehicular travel distance measured on the odometer (from last update)
D(d)	Estimated position of vehicle on center-line maps, given that the vehicle has measured d miles of distance from the last update (zero-check)
x(d)	Random displacement of estimated vehicle position from its true position, as computed on center-line maps, after the vehicle has measured d miles of travel from the last update.
σ^2	Variance of the random displacement per unit of distance travelled
γ	Mean systematic displacement per unit of distance travelled
P_f	Probability that a lost vehicle can be successfully found
b	Length of the shortest possible city block
q	Probability of incurring an intersection after travelling a distance b
$\bar{\ell}$	Mean distance between intersections
r	Probability that the vehicle turns at any given intersection
P_ℓ	Probability of loss of a vehicle on a randomly selected turn
$\bar{D}_L$	Mean distance travelled between losses
$\bar{T}_L$	Mean time between losses
N	Number of bits transmitted containing angular (heading) information
α_Q	Quantized heading angle
α	Actual heading angle
a_i	The i^{th} divergence angle from an intersection
P_{α_Q}	Probability of loss of a vehicle at an intersection due to angular quantization

Table of Symbols

(continued)

d_Q	Unit of quantized distance
W	Window of positional uncertainty due to time quantization

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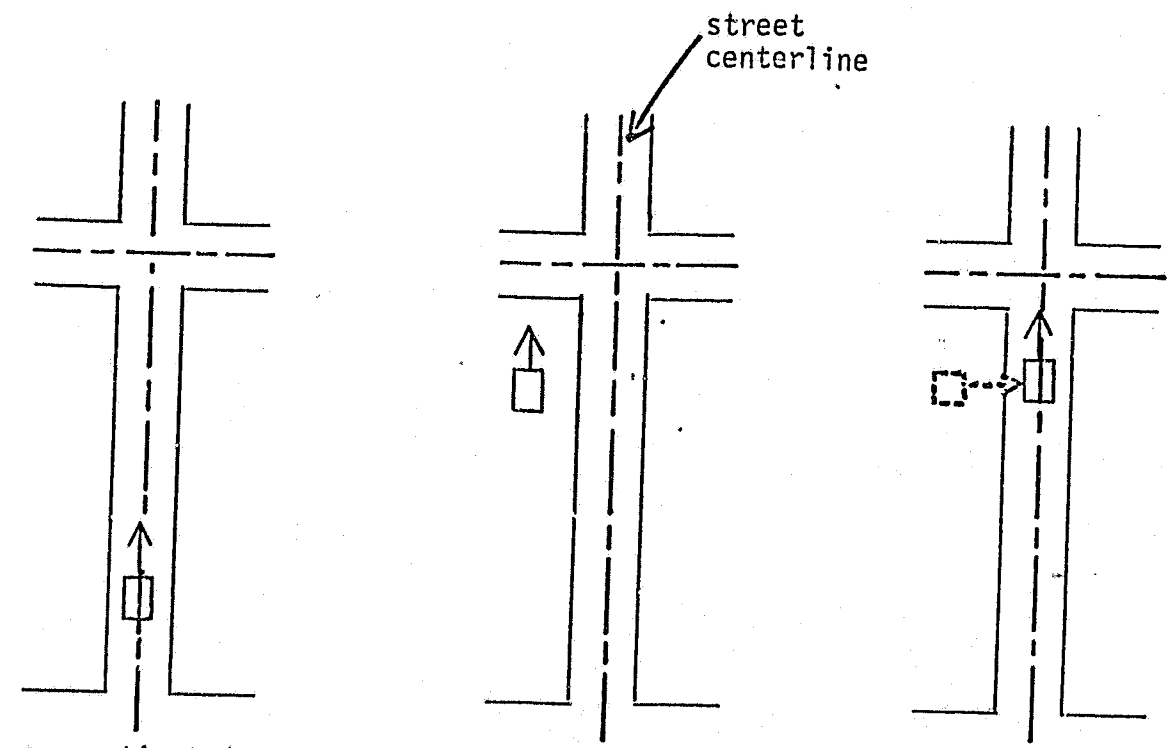
I. Introduction

In AVM systems such as FLAIR, an important characteristic of operational performance is some measure of the system error. Until recently this has usually been measured in feet or meters and stated in such forms as the "mean error is 100 feet," or "at least 95 percent of all position estimations are within 50 meters of the true position."

Computer-tracked vehicle location systems such as FLAIR pose new problems, however, in analyzing, modelling, and interpreting system errors. These systems use an in-car odometer and compass to provide a crude form of inertial guidance; the somewhat noisy information from the odometer and compass are transmitted periodically (every second in FLAIR) to a central receiver where it is processed. by a computer algorithm whose purpose is to update the estimated position of the vehicle. The update is performed with the aid of a detailed street map which is a collection of connected straight-line segments (representing street center lines) and "available" to the algorithm. In regular tracking, whenever the estimated position is infeasible, say off the street (perhaps in the center of an apartment complex), the computer "corrects" the estimated location back to the most likely center-line street position. This correction feature is depicted in Figure 1.

Figure A-1

Self-Correction Feature of FLAIR



a. estimated vehicle position at time $t = t_1$.

b. estimated vehicle position at time $t = t_1 + t_Q$, uncorrected (t_Q = time between updates)

c. estimated vehicle position at time $t = t_1 + t_Q$, corrected

II. Odometer Error: A One-Dimensional Error

To examine the error characteristics of this system, suppose for a moment that the vehicle always travels on a single road, never turning at intersections. Then position estimation error accumulates only in one dimension, that is along the direction of travel on the roadway. The accumulated error would be due to a collection of random phenomena that cause the odometer to yield inaccurate readings--bumps in the road; deviations from strict straight-line travel (e.g., lane switching); pebbles, rocks, sand and other conditions that cause the tires to skip, and--if viewed as uncorrectable--travel speed (which alters tire circumference). As argued in Chapter 5, random error can also arise along curved roads due to inaccuracies in the straight-line segment street map. In addition, there may be other phenomena that result in inaccurate odometer readings--but these may be systematic in some sense and, if detectable, correctable to some degree; examples include outside temperature (which alters in a predictable way the tire circumference), tire pressure, tire wear, and--if viewed as correctable--travel speed.

To summarize, the one-dimensional odometer error may be broken down into a strictly random component and a "systematic" (but perhaps still unknown) component.

II.1 Modelling the Random Error

In physical situations not unlike the current one researchers have found the Weiner process* to be an excellent model for the random component of the error. Historically, this stochastic process was first used to model the motion of a particle immersed in a liquid or gas, exhibiting

*See, for example, Emmanuel Parzen, Stochastic Processes, Holden-day, San Francisco, 1962, pp. 8, 26-29, 40, 67-68.

countless irregular motions. The central idea is that the particle is immersed in a field that offers continual bombardments of infinitesimal magnitude that cause the particle to become displaced from center. These bombardments show no preference for any particular direction (forwards or backwards in the case of one-dimensional displacement), so the net effect of the bombardments may be to move the particle in any of the possible directions (forwards or backwards in a two-dimensional case). This idea still applies in situations in which the particle is persistently moving in one direction, say due to wind currents or electrical currents (in the case of electrons in semiconductors). Then the random error is measured as random deviation from that position which would be obtained if the particle were governed only by the persistent movement.

In the vehicle location setting we must establish a frame of reference for the persistent movement and a measure of error from the anticipated position. We will measure the persistent movement by the true mileage d that the vehicle itself has measured since the last zero check (i.e., the last time the estimated and true position were known to coincide.) This measured mileage is accumulated over straight and winding roads, with and without lane switching, with and without slippage, etc. Associated with the traversed path of the vehicle is a sequence of connected straight-line segments representing street centerlines in the computer map. Suppose we measure a distance d along these connected segments, starting with the position of the last zero check. That process will yield a point on one of the segments representing the estimated position of the vehicle. The true position of the vehicle is presumably at some other (not-too-distant) point, most probably on the same segment. The location estimation error is the (center-line) distance between these two points. This method for determining location estimation error naturally incorporates errors due to both driving behavior and mapping procedures.

Invoking the Central Limit Theorem from probability theory, one assumes that the position of the particle (vehicle) about its anticipated position has a <u>Gaussian</u> or <u>Normal</u> distribution. This distribution is found in many applications of probability where the net effect of some process or activity is the sum of many small processes or activities. Moreover, we assume with the Weiner process model that the random perturbations in vehicle positioning occurring during one time interval or distance interval are independent of the perturbations occurring during another non-overlapping time or distance interval. For instance, we assume that the random error incurred while traversing one block is independent of the random error incurred while traversing the previous, the next, or any other block(s).

Finally, we would expect that as a vehicle (particle) travels further (i.e., exposed to more random perturbations), the accuracy of the position estimate deteriorates. This is exactly what happens with the Weiner process model--the <u>variance</u> of the distribution about the mean grows linearly in time (or distance).

To formalize our discussion to this point, we model the random component of the odometer error as follows:

Let $X(d)$ = the random displacement of the estimated vehicle position, as computed on center-line maps, after the vehicle has measured d miles of travel from the starting position (or last update)

$D(d)$ = estimated position of vehicle on center-line maps, given the vehicle has measured d miles of travel from its starting position

= $d = X(d)$.

By definition $X(0) = 0$. Now the Weiner process model requires that $X(d)$ have a Gaussian distribution with zero mean and variance that grows linearly with d. If

$f_X(x|d)$ = probability density function of $X(d)$,

then

$$f_X(x|d) = \frac{1}{\sqrt{2\pi\sigma^2 d}} e^{-x^2/2\sigma^2 d} \qquad -\infty < x < +\infty \tag{1}$$

where

σ^2 = a parameter indicating the intensity of the infinitesimal perturbations.

Here σ^2 can be considered to be the variance of the random displacement per unit of distance travelled. As one verifies from Eq. (1), the mean or expected value of the random displacement is zero, i.e.

$$E[X(d)] = 0 \tag{2}$$

and the variance ($\sigma^2_{X(d)}$) grows linearly with distance, i.e.,

$$\sigma^2_{X(d)} \equiv E[X(d) - E[X(d)]^2] = \sigma^2 d. \tag{3}$$

* Ignoring truncation errors, for the moment. (See Section IV.)

(3)

Thus the probability law of the Weiner process is specified by Eq. (1), which reveals the importance of the parameter σ^2. This parameter must be empirically measured in most applications, although occasionally a theory can be constructed that predicts σ^2 in terms of more fundamental quantities. For instance, in the case of the Weiner process model for Brownian motion, where σ^2 is the mean squared displacement of the particle per unit time, Einstein in 1905 showed that

$$\sigma^2 = \frac{4RT}{Nf},$$

where R is the universal gas constant, N the Avogadro number, T the absolute temperature, and f the friction coefficient of the surrounding medium. Unfortunately, we know of no similar relationship for odometer displacements, thereby revealing the need for empirical measurement.

Numerical Example

To illustrate an example of the use of the Weiner process model, suppose that we repeatedly drive a vehicle over a 10,000-foot test course and measure the map displacement error at the end of each 10,000-foot test drive. The Weiner process model predicts that the histogram of such errors would resemble a bell-shaped (normal or Gaussian) curve, symmetrically positioned about its mean of zero. Suppose as a result of the test runs we calculate the standard deviation of the error to be 50 feet. Then, the histogram would resemble the Gaussian curve depicted in Figure 2. From these data we can obtain an estimate

of σ^2, which is the mean square error displacement per unit distance (foot). We set the standard deviation of the Weiner process model equal to the measured value, thereby obtaining

$$\begin{aligned} \sqrt{\sigma^2 d} &= 50 \\ \sigma^2(10{,}000) &= 2{,}500 \\ \sigma^2 &= 0.25 \\ \sigma &= 0.50. \end{aligned}$$

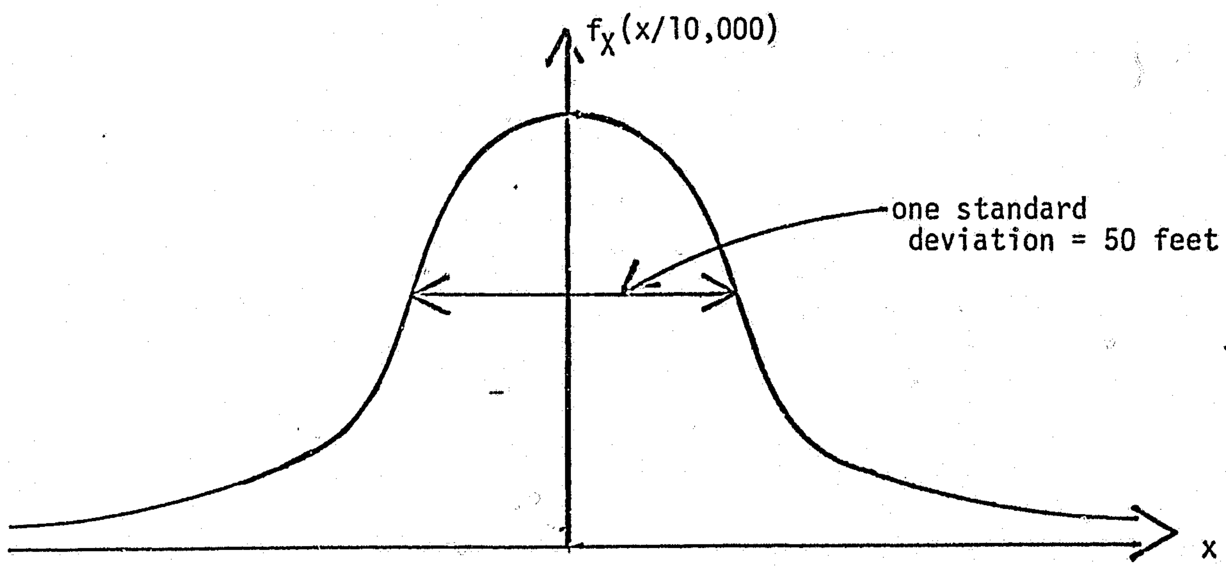
Table 1 presents a summary of the probabilities computed from the Gaussian probability law. Each entry in Table 1 gives a probability that a Gaussian random variable is within y standard deviations of its mean. For instance, using our example, the probability that the estimated position is correct to within ± 25 feet (corresponding to one half of a standard deviation on either side of the mean) is 0.383, assuming $\sigma^2 = 0.25$. The probability that the estimated position is within ± 50 feet (corresponding to one standard deviation on either side of the mean) is 0.6826. Note from Table 1 that it is quite likely (probability = 0.9974) that the estimated position is correct to within ± 150 feet (three standard deviations).

If the vehicle travels 100,000 feet (about 19 miles) the standard deviation now becomes $\sqrt{.25(100{,}000)} = 158.1$ feet. Then, for instance, the likelihood that the estimated position is correct to ± 158.1 feet (one standard deviation) is 0.6826.

At the other extreme, if the vehicle travels 100 feet, the standard deviation is $\sqrt{.25(100)} = 5$ feet. It will be for longer distances (on

Figure A-2

Distribution of Odometer Error,
Given the Vehicle Has Travelled 10,000 Feet
(without Systematic Error)



one standard deviation = 50 feet = $\sqrt{\sigma^2 d}$

$$\sigma^2(10{,}000) = 2500$$

$$\sigma^2 = 0.25$$

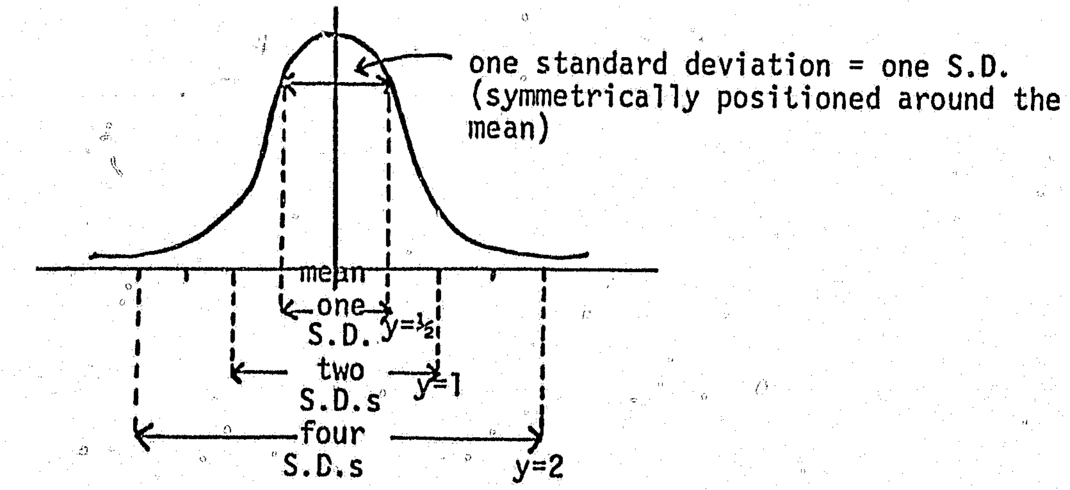
$$\sigma \approx 0.50$$

Table 1

Probability that a Gaussian Random Variable
Is within (+)y Standard Deviations of Its Mean*

y	Probability	y	Probability
0.00	0.00	1.60	0.8904
0.10	0.0796	1.70	0.9108
0.20	0.1586	1.80	0.9282
0.30	0.2358	1.90	0.9426
0.40	0.3108	2.00	0.9544
0.50	0.383	2.10	0.9642
0.60	0.4514	2.20	0.9722
0.70	0.516	2.30	0.9786
0.80	0.5762	2.40	0.9836
0.90	0.6318	2.50	0.9876
1.00	0.6826	2.60	0.9906
1.10	0.7286	2.70	0.9930
1.20	0.7698	2.80	0.9948
1.30	0.8064	2.90	0.9962
1.40	0.8384	3.00	0.9974
1.50	0.8664		

*"Within y standard deviations" means $\pm$ y standard deviations, as shown in this figure:



the order of one block length or more) that we will find most use for the Weiner process model.

2. Modelling the Systematic Error

The Weiner process model accounts for the zero-mean truly random error in the odometer. However, in applications one is likely to find large systematic errors that, if undetected and uncorrected, could dominate the random errors. The systematic errors could be due to outside temperature, tire pressure and wear, travel speed, etc.

We can model the systematic error of a vehicle operating under fixed conditions (i.e., constant temperature, speed, tire wear and pressure, etc.) by adding a bias term to the Weiner process probability law. With the bias, the expected value of the odometer displacement is no longer zero, but is given by

$$E[X(d)] = \gamma d, \qquad (4)$$

where γ is the mean systematic displacement per unit of distance travelled. Allowing for the bias, we still assume the same variance, i.e.,

$$\sigma^2_{X(d)} = \sigma^2 d, \qquad (5)$$

so that the probability law of the odometer displacement becomes

$$f_X(x|d) = \frac{1}{\sqrt{2\pi\sigma^2 d}} e^{-(x-\gamma d)^2/2\sigma^2 d} \quad -\infty < x < +\infty. \qquad (6)$$

The important point with this realistic modification to the model is that γ is usually a random variable, that is, its value is unknown

prior to testing and monitoring the odometer performance of each vehicle. Determining the value of γ for a particular vehicle corresponds to "calibrating" the odometer.* If a numerically large value of γ is left undetected and uncorrected (at least within the vehicle-tracking computer software), then the systematic error could "swamp" the random errors.

Numerical Example

Continuing with the numbers of our first example, suppose again that we repeatedly drive a vehicle over a 10,000-foot test course and measure the odometer error (displacement) at the end of each 10,000-foot test drive. Again, we assume that $\sigma^2 = 0.25$. But now we also assume a systematic error corresponding to $\gamma = 0.004$. Thus

$$E[X(d)] = 0.004d.$$

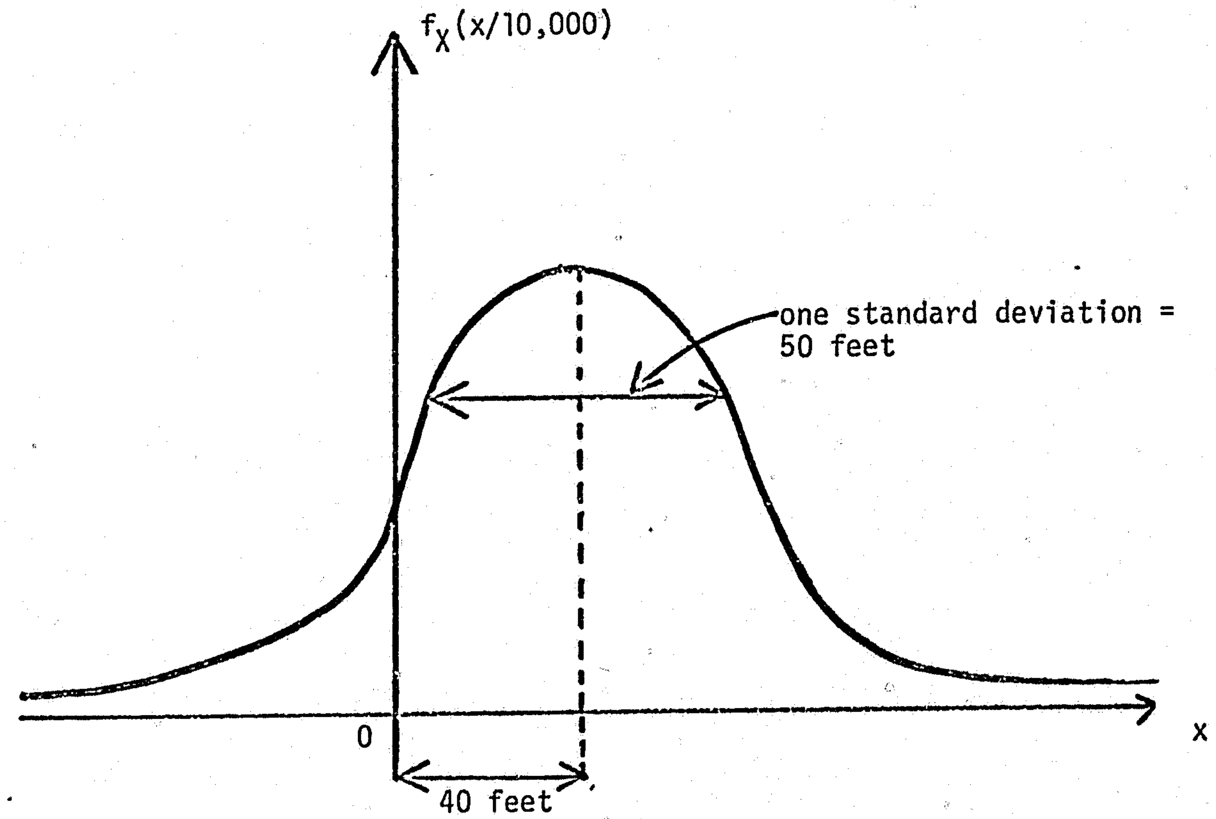
This means, for instance, that if the vehicle is driven 10,000 feet, the expected value (average value) of the odometer displacement is $E[X(10{,}000)] = 0.004(10{,}000) = 40$ feet. The Gaussian curve now indicating the distribution of odometer displacement is shifted to the right of zero by 40 feet, as indicated in Figure 3. Now the probability that the odometer reading is correct to ±50 feet is considerably reduced over that found earlier. The "±50 feet" converts to the region extending from 90 feet to the left of the mean to 10 feet to the right of the mean. This corresponds to 1.8 standard deviations to the left and 0.2 standard deviations to the right. We can obtain the appropriate probability estimate from Table 1.

---

*If the biasing effects of vehicular speed are viewed as correctable, then it may also be a function of time, varying in a systematic way with the speed of the monitored vehicle.

Figure A-3

Distribution of Position Estimation Error, Given the Vehicle Has Travelled 10,000 Feet (with Systematic Error)



one standard deviation = 50 feet = $\sqrt{\sigma^2 d}$

$\sigma^2 = 0.25$

$\gamma = 0.004$

which indicates that the probability of being within ($\pm$) 1.8 standard deviation is 0.9282, and dividing by 2 (yielding 0.4641) since we are only concerned with the side of the distribution to the left of the mean. A similar computation for the area to the right of the mean yields a probability equal to 0.1586/2 = .0793. Adding the two probabilities we discover that the probability that the odometer reading is correct to $\pm$50 feet is 0.4641 + 0.0793 = 0.5434 reduced from 0.6826 in the case of no systematic error (a reduction of 20.8 percent in this measure of accuracy).

Now consider the case in which the vehicle travels 100,000 feet. Here again the standard deviation is $\sqrt{.25(100,000)}$ = 158.1 feet. However, the bias in E[X(100,000)] = 100,000(0.004) = 400 feet. In this case, the likelihood that the odometer reading is correct to ±158.1 feet ($\pm$ one standard deviation) is approximately equal to the probability that the displacement falls in an interval to the left of the mean, starting at 3.5 standard deviations from the mean and ending at 1.5 standard deviations from the mean. This probability is approximately $0.5 - \frac{0.8664}{2} \approx 0.0668$, a reduction from 0.6826 in the case of no systematic error (a 90 percent reduction in this measure of accuracy).

Thus we see the importance of the systematic error term. A vehicle with even a small amount of systematic error can incur large position estimation errors as the driving distance from the last zero-check increases.

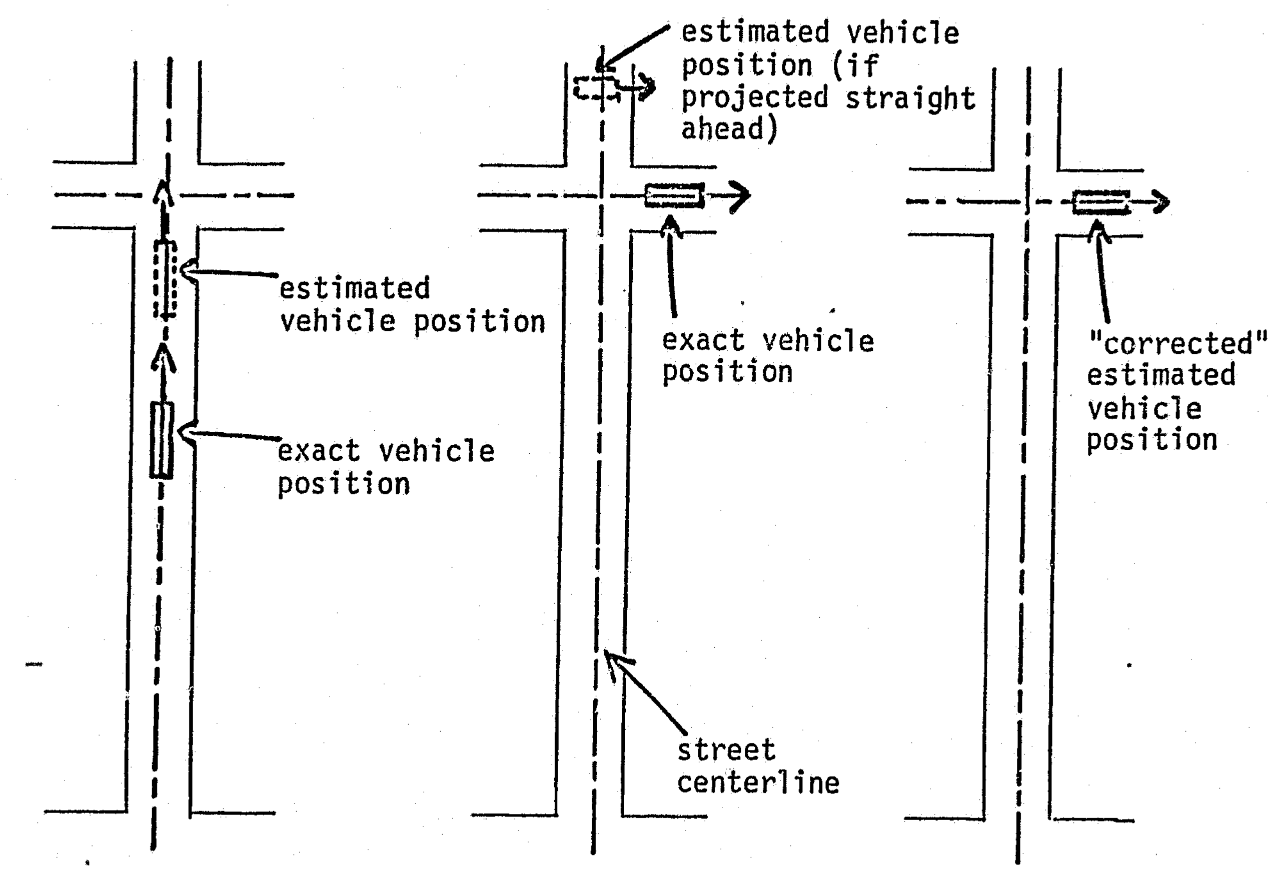
III. TIME BETWEEN LOSSES OF A VEHICLE

The Weiner process model applied to odometer readings in a computer-tracked vehicle monitoring system is a one-dimensional model; that is, it does not incorporate vehicles turning at intersections. However, it is this vehicular action which on the one hand allows very accurate position estimates to be sustained over long periods of time (even with σ^2 moderately large) and on the other hand gives rise to a unique type of position estimation error--the vehicle being "lost."

We are now ready to model the more realistic situation in which the vehicle occasionally makes turns at intersections. The situation of a turn is illustrated in Figure 4. Here the vehicle approaches the intersection from the south. The heading sensor (from the in-car compass) correctly gives a reading of "north." However, the estimated position of the vehicle on the street is two or three car-lengths north of the actual vehicle location. A time t_Q later (corresponding to the sampling interval) a new odometer reading is received and the direction of travel is now east. If the compass direction had not changed from north to east, the computer tracking algorithm would have placed the vehicle back on the north-south street center-line at a latitude projected from the new odometer reading. However, since the compass direction has changed, the algorithm "assumes" that the vehicle has turned at the nearest possible intersection and correctly places the vehicle on the appropriate east-west street (headed east) at a point very close to its actual

Figure A-4

Self-Correction Feature of FLAIR Vehicle Turning



a. estimated vehicle position at time $t = t_1$

b. estimated vehicle position at time $t = t_1 + t_Q$, uncorrected (t_Q = time between updates)

c. estimated vehicle position at time $t = t_1 + t_Q$, corrected position.* The important point to notice here is that virtually all** of the accumulated odometer error since the last zero-check is eliminated if the tracking algorithm correctly detects and interprets the vehicle's turn. Thus, each successfully monitored turn corresponds to an odometer zero-check. If all turns are monitored correctly, the system distance error does not build up indefinitely, but rather reaches some small average value as suggested by the Weiner process model (with or without systematic errors).

The major system accuracy problems occurs, however, when a turn is not detected or, if detected, not interpreted properly. This can occur in several ways, one of which is depicted in Figure 5. Here, the vehicle is headed north on a north-south street, but the estimated vehicle position is about two-thirds of a block length ahead (north) of the vehicle. When the vehicle turns east on street 2, the estimated position is now closer to the street immediately north of the vehicle, approximately only one-third of a block length from street 3, but two-thirds of a block length from street 2. Since the compass direction has suddenly changed from north to east, the tracking algorithm correctly detects that a turn has occurred. However, the estimated position of the unit is "corrected" to street 3, rather than street 2, resulting in the vehicle being "lost." This is the key error event in the system and one which we will attempt to model.

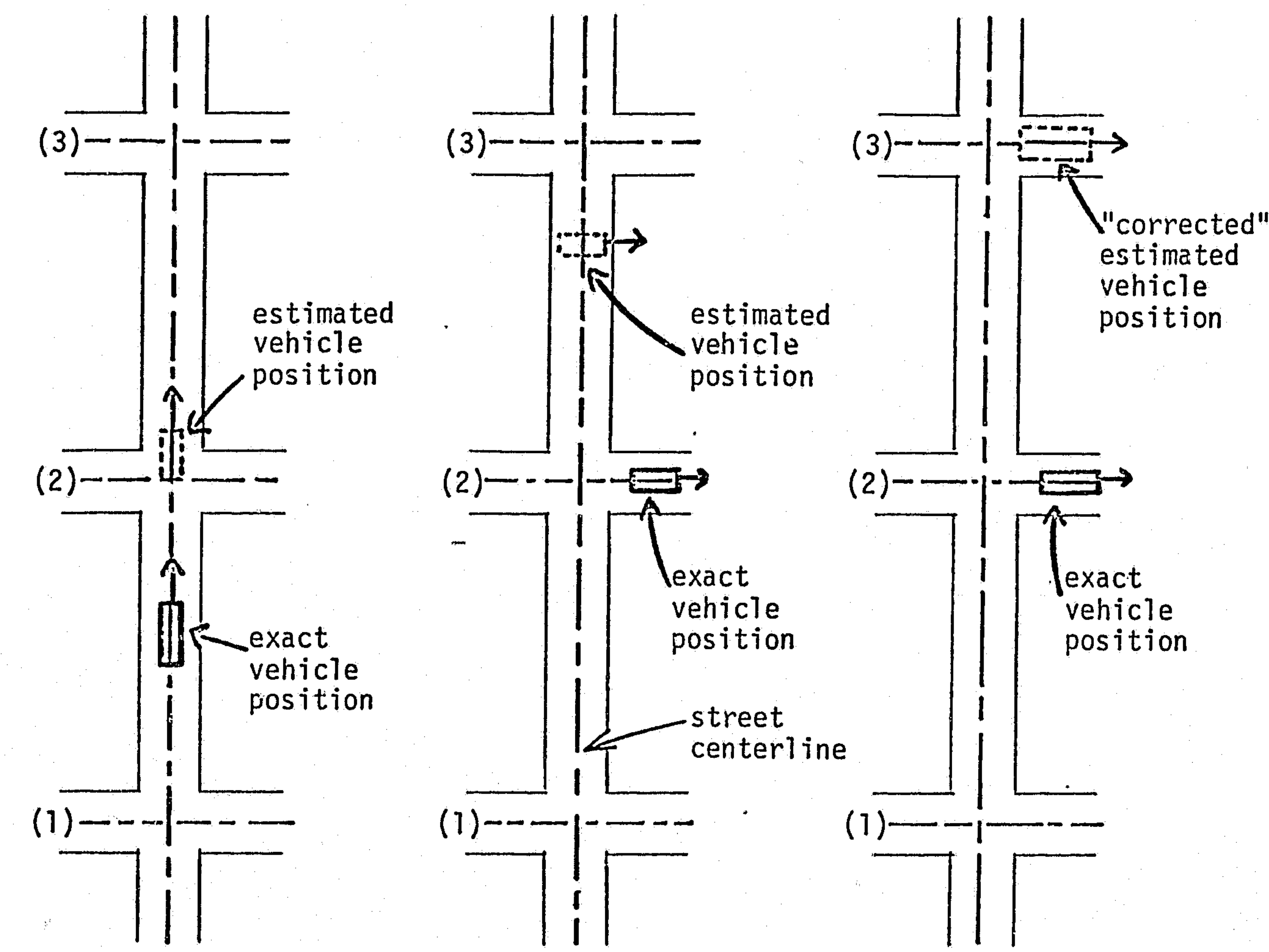
---

* See detailed discussion in Section IV.3.

** Again, see Section IV.3 which discusses a (usually) small amount of error that remains after the turn.

Figure A-5

Loss of Vehicle: A Vehicle Turn Incorrectly Interpreted



a. estimated vehicle position at time $t = t_1$

b. estimated vehicle position at time $t = t_1 + t_Q$, uncorrected (t_Q = time between updates)

c. estimated vehicle position at time $t = t_1 + t_Q$, "corrected;" vehicle now "lost."

While the event causing loss of the vehicle is shown in Figure A-5, the computer tracking algorithm may not detect the loss until sometime later (due to apparently infeasible turns executed by the vehicle). Assuming that the time from incorrectly interpreting a turn until detection of loss is very small (say, minutes) compared to the mean time between incorrectly interpreted turns (say, hours), we ignore the small intervening time span in the model; thus we say that a vehicle is lost as soon as the incorrectly interpreted turn occurs.

It is worth noting that sophisticated tracking algorithms can sometimes correct for a vehicle that is determined to be lost, that is, they can "find" a lost vehicle. We will not be concerned with the details of such finding procedures, but we will characterize the success of such an algorithm by a probability

p_f = probability that a lost vehicle can be successfully found.

Current computer software can usually find about 50 percent of lost vehicles, resulting in $p_f \approx 0.50$.

We now procede to the model formulation. We want to predict the mean and the variance (or more generally the probability law) of the time or distance between losses (for simplicity we will initially use distance rather than time). For the moment we will assume $p_f = 0$, thereby ignoring corrections after losses (we can easily incorporate a nonzero p_f after we have developed the model). We assume that each time a vehicle makes a turn and is correctly tracked, the accumulated odometer error goes to zero and this event is a <u>renewal event</u>. If the vehicle turns and is not tracked correctly, then the vehicle is <u>lost</u>; this is the event of interest.

We wish to incorporate in the model the following features:

1. Both systematic and random errors as discussed above.
2. The spacings between streets.
3. Some measure of the regularity or irregularity of the street pattern.
4. The frequency with which the tracked car makes turns at intersections.

To model both features 2 and 3, we assume that adjacent intersections are located kb units apart where

b = length of the shortest possible city block,

k = an integer random variable whose probability mass function is geometric.

Thus the probability law for k can be written

$$p\{k = v\} = (1 - q)^{v-1}q \qquad v = 1, 2, 3, \ldots \qquad (7)$$

There are several ways of interpreting this obviously simplified model of street positionings. In one interpretation, each time the tracked vehicle travels a distance b from the last intersection there is a probability q that it will incur another intersection; regardless of "success" or "failure" at finding an intersection at that point, the probability of incurring an intersection at a distance 2b from the original intersection is also q. In general, each time the vehicle travels b units of distance there is a probability q that an intersection will exist there.

Examining some limiting cases of the model, suppose q = 1. This corresponds to a situation in which the streets are designed in a regular square grid pattern, each (actual) block being exactly b units in length. This might be an accurate depiction of the streets in

Wichita, Phoenix, Tuscon and several other midwestern and far-western cities; where b typically is about 500 feet. At the other extreme, suppose $q = \varepsilon$, where ε is very small but positive. This would correspond to an almost totally random positioning of streets, with adjacent intersections positioned as in a Poisson process with mean "inter-arrival time" (mean distance between intersections) equal to b/ε. Here the parameter b (by itself) has little meaning, since in applications we would probably specify the ratio b/ε (which would correspond to the empirically measured mean distance between intersections), we would note the Poisson process nature of the street spacings, and we would set b and ε (keeping b/ε constant) sufficiently small so as to achieve the required accuracy in the model.

Having examined extreme values of q, we see that intermediate values correspond to intermediate degrees of regularity or irregularity in the street pattern, with higher values indicating greater regularity.

In actual applications, how do we determine numerical values for b and q? From the model we can compute that the mean distance between adjacent intersections is b/q and the variance is $b^2(\frac{1-q}{q^2})$. We can also compute empirical values for these quantities from a map of the city being modeled. Suppose the empirically calculated mean distance between intersections is $\bar{\ell}$ and the variance is σ_ℓ^2. Then set

$$\bar{\ell} = b/q \qquad \text{(a)}$$

and

$$\sigma_\ell^2 = b^2\left(\frac{1-q}{q^2}\right). \qquad \text{(b)} \qquad (8)$$

Manipulating these equations, we get

$$q = 1 - \frac{\sigma_\ell^2}{\bar{\ell}^2} \qquad \text{(a)}$$

$$b = \bar{\ell}\left(1 - \frac{\sigma_\ell^2}{\bar{\ell}^2}\right). \qquad \text{(b)} \qquad (9)$$

Note that in order for q to remain nonnegative, we must have $\sigma_\ell \leq \bar{\ell}$. This is just as we expect since the most random distribution of streets that we can model is the Poisson process distribution, and this corresponds to $\sigma_\ell = \bar{\ell}$. It is important to note that the parameter b now becomes the <u>unit of distance</u> in our model.

Feature 4 of the model, the frequency with which the vehicle makes turns, can be modelled simply by defining

$r \equiv$ probability that the vehicle turns at any given intersection.

We assume that the turning decision is made independently at each intersection and thus that turns occur as a Bernoulli process with parameter r.

We are now ready to compute the unconditional probability of "loss" of a vehicle on a randomly selected turn. Call this quantity p. Clearly,

$$p = \sum_{i=1}^{\infty} \text{Prob}\{\text{vehicle makes next turn } i \text{ units of distance from last turn}\}\text{Prob}\{\text{loss}|i\}.$$

If a vehicle is almost at a distance $d = i$ from the last turn, the probability of turning at i is simply equal to $q\,r$, the probability that a street intersection exists at $d = i$ multiplied by the probability of turning, given that an intersection exists. Thus the probability that the vehicle makes its next turn exactly i units of distance from the last turn is a geometrically distributed random variable with parameter qr, and we can write

$$p = \sum_{i=1}^{\infty} qr(1 - qr)^{i-1} \text{Prob}\{\text{loss}|i\}.$$

Our next task is to express Prob{loss|i} in terms of previously defined parameters. We assume that a vehicle is lost if it is estimated to be closer to an intersection other than the one at which it is actually turning. Figure 6 depicts "forward loss" of a vehicle, that is a situation in which the vehicle is estimated to be closer to an intersection "in front of the vehicle" than the one at which it is turning. In Figure 6 the vehicle turns at $d = i$, the next intersection ahead of the vehicle is located at $d = i + j$. If the estimated position of the vehicle is to the right of the halfway point between the two intersections $(\frac{2i + j}{2})$, then the vehicle has incurred <u>forward loss</u>. <u>Backward loss</u> occurs in a directly analogous fashion with the nearest intersection "behind" the vehicle at $d = i$. Utilizing the Weiner process model, the probability of incurring forward loss in this case is

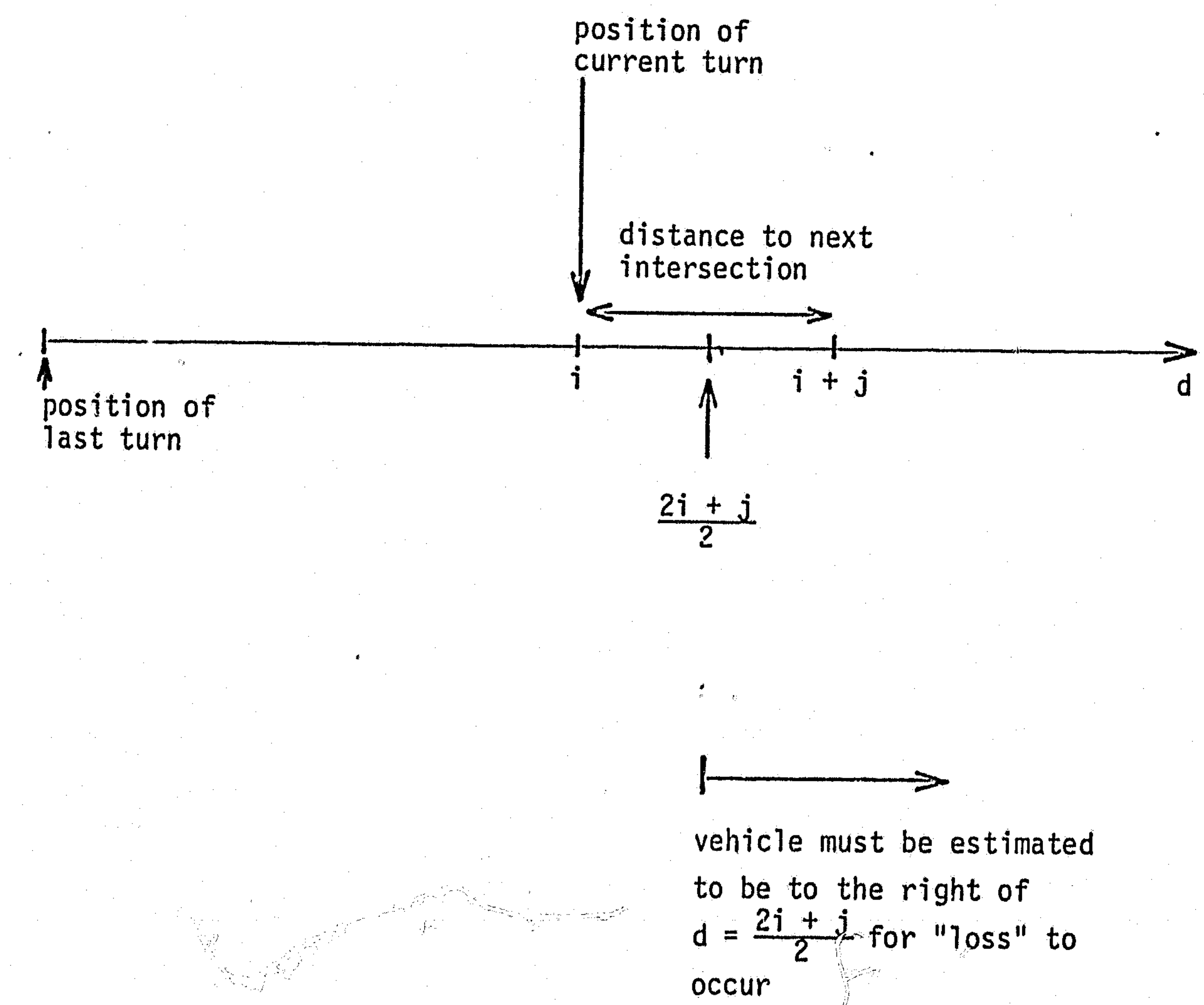
$$\int_{j/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i} \, dy.$$

The analogous probability of backward loss is

$$\int_{-i}^{-j/2} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i} \, dy.$$

Figure A-6

Forward Loss of a Vehicle



In most cases of practical interest, in which σ^2 and γ are sufficiently small to yield a small p, we can approximate p by changing -i (the lower limit on the last integral) to $-\infty$. Thus we approximate

Prob{loss x = i and next intersection j units in distance} $\approx$

$$\int_{j/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i}\, dy + \int_{-\infty}^{-j/2} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i}\, dy =$$

$$= 1 - \int_{-j/2}^{j/2} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i}\, dy.$$

Now, the probability that the next intersection is j units in distance is $q(1 - q)^{j-1}$, j = 1, 2, ... Thus

$$\text{Prob}\{\text{loss}|x = i\}\ \sum_{j=1}^{\infty} q(1 - q)^{j-1} \left[1 - \int_{-j/2}^{j/2} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i}\, dy\right].$$

Finally, the quantity of interest, p (the unconditional probability of loss on a randomly selected turn of the vehicle) is given by

$$p = \sum_{i=1}^{\infty} qr(1-qr)^{i-1} \sum_{j=1}^{\infty} q(1-q)^{j-1} \left[1 - \int_{-j/2}^{j/2} \frac{1}{\sqrt{2\pi\sigma^2 i}} e^{-(y-\gamma i)^2/2\sigma^2 i}\, dy\right] \quad (10)$$

Illustrative values of this probability have been tabulated with the assistance of a computer. (See Table 1.)

Since losses occur as in a Bernoulli process, the mean number of turns executed between losses is 1/p. The mean number of intersections

Table 2

Illustrative Values of Vehicle Loss Probability (p)

STREET PATTERN IRREGULARITY FIXED AT Q= 0.2

GAMMA = SYSTEMATIC BIAS PER UNIT DISTANCE
SIGMA = STANDARD DEVIATIØN ØF RANDØM BIAS PER UNIT DISTANCE
r = PRØBABILITY THAT VEHICLE TURNS AT RANDØM INTERSECTIØN

GAMMA	SIGMA	r	p
0.00	0.01	0.500	0.00000
0.00	0.01	0.250	0.00000
0.00	0.01	0.125	0.00000
0.00	0.02	0.500	0.00000
0.00	0.02	0.250	0.00000
0.00	0.02	0.125	0.00000
0.00	0.03	0.500	0.00000
0.00	0.03	0.250	0.00000
0.00	0.03	0.125	0.00000
0.00	0.05	0.500	0.00216
0.00	0.05	0.250	0.00862
0.00	0.05	0.125	0.02331
0.00	0.07	0.500	0.00819
0.00	0.07	0.250	0.02267
0.00	0.07	0.125	0.04835
0.00	0.10	0.500	0.02311
0.00	0.10	0.250	0.04933
0.00	0.10	0.125	0.08882
0.00	0.20	0.500	0.08973
0.00	0.20	0.250	0.14379
0.00	0.20	0.125	0.21206
0.01	0.01	0.500	0.00000
0.01	0.01	0.250	0.00000
0.01	0.01	0.125	0.00000
0.01	0.02	0.500	0.00000
0.01	0.02	0.250	0.00000
0.01	0.02	0.125	0.00000
0.01	0.03	0.500	0.00345
0.01	0.03	0.250	0.02271
0.01	0.03	0.125	0.07882
0.01	0.05	0.500	0.00765
0.01	0.05	0.250	0.03089
0.01	0.05	0.125	0.08679
0.01	0.07	0.500	0.01406
0.01	0.07	0.250	0.04180
0.01	0.07	0.125	0.09871
0.01	0.10	0.500	0.02807
0.01	0.10	0.250	0.06341
0.01	0.10	0.125	0.12387
0.01	0.20	0.500	0.09211
0.01	0.20	0.250	0.14962
0.01	0.20	0.125	0.22519
0.02	0.01	0.500	0.00000
0.02	0.01	0.250	0.00000
0.02	0.01	0.125	0.00000
0.02	0.02	0.500	0.00000
0.02	0.02	0.250	0.00000
0.02	0.02	0.125	0.00000
0.02	0.03	0.500	0.01924
0.02	0.03	0.250	0.07582
0.02	0.03	0.125	0.18761
0.02	0.05	0.500	0.02402
0.02	0.05	0.250	0.08020
0.02	0.05	0.125	0.18896
0.02	0.07	0.500	0.03010
0.02	0.07	0.250	0.08623
0.02	0.07	0.125	0.19151
0.02	0.10	0.500	0.04186
0.02	0.10	0.250	0.09908
0.02	0.10	0.125	0.19955
0.02	0.20	0.500	0.09907
0.02	0.20	0.250	0.16612
0.02	0.20	0.125	0.26018
0.03	0.01	0.500	0.00000
0.03	0.01	0.250	0.00000
0.03	0.01	0.125	0.00000
0.03	0.02	0.500	0.04328
0.03	0.02	0.250	0.13275
0.03	0.02	0.125	0.27923
0.03	0.03	0.500	0.04450
0.03	0.03	0.250	0.13332
0.03	0.03	0.125	0.27904
0.03	0.05	0.500	0.04811
0.03	0.05	0.250	0.13519
0.03	0.05	0.125	0.27858
0.03	0.07	0.500	0.05289
0.03	0.07	0.250	0.13801
0.03	0.07	0.125	0.27823
0.03	0.10	0.500	0.06198
0.03	0.10	0.250	0.14452
0.03	0.10	0.125	0.27899
0.03	0.20	0.500	0.11011
0.03	0.20	0.250	0.19084
0.03	0.20	0.125	0.30747

<u>Table 2</u>

(page 2 of 5)

<u>Illustrative Values of Vehicle Loss Probability (p)</u>

STREET PATTERN IRREGULARITY FIXED AT Q= 0.4

GAMMA = SYSTEMATIC BIAS PER UNIT DISTANCE
SIGMA = STANDARD DEVIATIØN ØF RANDØM BIAS PER UNIT DISTANCE
r = PRØBABILITY THAT VEHICLE TURNS AT RANDØM INTERSECTIØN

GAMMA	SIGMA	r	p
0.00	0.01	0.500	0.00000
0.00	0.01	0.250	0.00000
0.00	0.01	0.125	0.00000
0.00	0.02	0.500	0.00000
0.00	0.02	0.250	0.00000
0.00	0.02	0.125	0.00000
0.00	0.03	0.500	0.00000
0.00	0.03	0.250	0.00000
0.00	0.03	0.125	0.00000
0.00	0.05	0.500	0.00056
0.00	0.05	0.250	0.00431
0.00	0.05	0.125	0.01710
0.00	0.07	0.500	0.00381
0.00	0.07	0.250	0.01626
0.00	0.07	0.125	0.04436
0.00	0.10	0.500	0.01622
0.00	0.10	0.250	0.04523
0.00	0.10	0.125	0.09413
0.00	0.20	0.500	0.09510
0.00	0.20	0.250	0.16538
0.00	0.20	0.125	0.25224
0.01	0.01	0.500	0.00000
0.01	0.01	0.250	0.00000
0.01	0.01	0.125	0.00000
0.01	0.02	0.500	0.00000
0.01	0.02	0.250	0.00000
0.01	0.02	0.125	0.00000
0.01	0.03	0.500	0.00033
0.01	0.03	0.250	0.00686
0.01	0.03	0.125	0.04428
0.01	0.05	0.500	0.00202
0.01	0.05	0.250	0.01515
0.01	0.05	0.125	0.05946
0.01	0.07	0.500	0.00627
0.01	0.07	0.250	0.02761
0.01	0.07	0.125	0.07938
0.01	0.10	0.500	0.01899
0.01	0.10	0.250	0.05442
0.01	0.10	0.125	0.11815
0.01	0.20	0.500	0.09665
0.01	0.20	0.250	0.16909
0.01	0.20	0.125	0.26016

GAMMA	SIGMA	r	p
0.02	0.01	0.500	0.00000
0.02	0.01	0.250	0.00000
0.02	0.01	0.125	0.00000
0.02	0.02	0.500	0.00000
0.02	0.02	0.250	0.00000
0.02	0.02	0.125	0.00000
0.02	0.03	0.500	0.00379
0.02	0.03	0.250	0.03772
0.02	0.03	0.125	0.14071
0.02	0.05	0.500	0.00780
0.02	0.05	0.250	0.04676
0.02	0.05	0.125	0.14746
0.02	0.07	0.500	0.01384
0.02	0.07	0.250	0.05804
0.02	0.07	0.125	0.15672
0.02	0.10	0.500	0.02706
0.02	0.10	0.250	0.07961
0.02	0.10	0.125	0.17707
0.02	0.20	0.500	0.10122
0.02	0.20	0.250	0.17988
0.02	0.20	0.125	0.28239
0.03	0.01	0.500	0.00000
0.03	0.01	0.250	0.00000
0.03	0.01	0.125	0.00000
0.03	0.02	0.500	0.01252
0.03	0.02	0.250	0.08308
0.03	0.02	0.125	0.23479
0.03	0.03	0.500	0.01431
0.03	0.03	0.250	0.08523
0.03	0.03	0.125	0.23533
0.03	0.05	0.500	0.01960
0.03	0.05	0.250	0.09156
0.03	0.05	0.125	0.23717
0.03	0.07	0.500	0.02662
0.03	0.07	0.250	0.09983
0.03	0.07	0.125	0.24017
0.03	0.10	0.500	0.03983
0.03	0.10	0.250	0.11539
0.03	0.10	0.125	0.24805
0.03	0.20	0.500	0.10866
0.03	0.20	0.250	0.19682
0.03	0.20	0.125	0.31513

<u>Table 2</u>

(page 3 of 5)

<u>Illustrative Values of Vehicle Loss Probability (p)</u>

STREET PATTERN IRREGULARITY FIXED AT Q= 0.6

GAMMA = SYSTEMATIC BIAS PER UNIT DISTANCE
SIGMA = STANDARD DEVIATIØN ØF RANDØM BIAS PER UNIT DISTANCE
r = PRØBABILITY THAT VEHICLE TURNS AT RANDØM INTERSECTIØN

GAMMA	SIGMA	r	p
0.00	0.01	0.500	0.00000
0.00	0.01	0.250	0.00000
0.00	0.01	0.125	0.00000
0.00	0.02	0.500	0.00000
0.00	0.02	0.250	0.00000
0.00	0.02	0.125	0.00000
0.00	0.03	0.500	0.00000
0.00	0.03	0.250	0.00000
0.00	0.03	0.125	0.00000
0.00	0.05	0.500	0.00015
0.00	0.05	0.250	0.00218
0.00	0.05	0.125	0.01212
0.00	0.07	0.500	0.00173
0.00	0.07	0.250	0.01118
0.00	0.07	0.125	0.03808
0.00	0.10	0.500	0.01063
0.00	0.10	0.250	0.03855
0.00	0.10	0.125	0.09174
0.00	0.20	0.500	0.09174
0.00	0.20	0.250	0.17332
0.00	0.20	0.125	0.27325
0.01	0.01	0.500	0.00000
0.01	0.01	0.250	0.00000
0.01	0.01	0.125	0.00000
0.01	0.02	0.500	0.00000
0.01	0.02	0.250	0.00000
0.01	0.02	0.125	0.00000
0.01	0.03	0.500	3.41E-05
0.01	0.03	0.250	0.00211
0.01	0.03	0.125	0.02460
0.01	0.05	0.500	0.00056
0.01	0.05	0.250	0.00774
0.01	0.05	0.125	0.04177
0.01	0.07	0.500	0.00281
0.01	0.07	0.250	0.01850
0.01	0.07	0.125	0.06490
0.01	0.10	0.500	0.01229
0.01	0.10	0.250	0.04535
0.01	0.10	0.125	0.11069
0.01	0.20	0.500	0.09293
0.01	0.20	0.250	0.17617
0.01	0.20	0.125	0.27911

GAMMA	SIGMA	r	p
0.02	0.01	0.500	0.00000
0.02	0.01	0.250	0.00000
0.02	0.01	0.125	0.00000
0.02	0.02	0.500	0.00000
0.02	0.02	0.250	0.00000
0.02	0.02	0.125	0.00000
0.02	0.03	0.500	0.00064
0.02	0.03	0.250	0.01737
0.02	0.03	0.125	0.10244
0.02	0.05	0.500	0.00246
0.02	0.05	0.250	0.02702
0.02	0.05	0.125	0.11469
0.02	0.07	0.500	0.00637
0.02	0.07	0.250	0.03970
0.02	0.07	0.125	0.12996
0.02	0.10	0.500	0.01728
0.02	0.10	0.250	0.06462
0.02	0.10	0.125	0.15982
0.02	0.20	0.500	0.09645
0.02	0.20	0.250	0.18455
0.02	0.20	0.125	0.29589
0.03	0.01	0.500	0.00000
0.03	0.01	0.250	0.00000
0.03	0.01	0.125	0.00000
0.03	0.02	0.500	0.00266
0.03	0.02	0.250	0.04739
0.03	0.02	0.125	0.19251
0.03	0.03	0.500	0.00370
0.03	0.03	0.250	0.05069
0.03	0.03	0.125	0.19436
0.03	0.05	0.500	0.00731
0.03	0.05	0.250	0.06013
0.03	0.05	0.125	0.19998
0.03	0.07	0.500	0.01303
0.03	0.07	0.250	0.07207
0.03	0.07	0.125	0.20768
0.03	0.10	0.500	0.02551
0.03	0.10	0.250	0.09353
0.03	0.10	0.125	0.22358
0.03	0.20	0.500	0.10222
0.03	0.20	0.250	0.19794
0.03	0.20	0.125	0.32149

Illustrative Values of Vehicle Loss Probability (p)

STREET PATTERN IRREGULARITY FIXED AT Q= 0.8

GAMMA = SYSTEMATIC BIAS PER UNIT DISTANCE
SIGMA = STANDARD DEVIATIØN ØF RANDØM BIAS PER UNIT DISTANCE
r = PRØBABILITY THAT VEHICLE TURNS AT RANDØM INTERSECTIØN

GAMMA	SIGMA	r	p
0.00	0.01	0.500	0.00000
0.00	0.01	0.250	0.00000
0.00	0.01	0.125	0.00000
0.00	0.02	0.500	0.00000
0.00	0.02	0.250	0.00000
0.00	0.02	0.125	0.00000
0.00	0.03	0.500	0.00000
0.00	0.03	0.250	0.00000
0.00	0.03	0.125	0.00000
0.00	0.05	0.500	4.26E-05
0.00	0.05	0.250	0.00113
0.00	0.05	0.125	0.00858
0.00	0.07	0.500	0.00076
0.00	0.07	0.250	0.00760
0.00	0.07	0.125	0.03202
0.00	0.10	0.500	0.00667
0.00	0.10	0.250	0.03198
0.00	0.10	0.125	0.08674
0.00	0.20	0.500	0.08486
0.00	0.20	0.250	0.17532
0.00	0.20	0.125	0.28593
0.01	0.01	0.500	0.00000
0.01	0.01	0.250	0.00000
0.01	0.01	0.125	0.00000
0.01	0.02	0.500	0.00000
0.01	0.02	0.250	0.00000
0.01	0.02	0.125	0.00000
0.01	0.03	0.500	3.51E-06
0.01	0.03	0.250	0.00066
0.01	0.03	0.125	0.01362
0.01	0.05	0.500	0.00016
0.01	0.05	0.250	0.00404
0.01	0.05	0.125	0.02970
0.01	0.07	0.500	0.00124
0.01	0.07	0.250	0.01244
0.01	0.07	0.125	0.05332
0.01	0.10	0.500	0.00768
0.01	0.10	0.250	0.03722
0.01	0.10	0.125	0.10259
0.01	0.20	0.500	0.08582
0.01	0.20	0.250	0.17769
0.01	0.20	0.125	0.29067

GAMMA	SIGMA	r	p
0.02	0.01	0.500	0.00000
0.02	0.01	0.250	0.00000
0.02	0.01	0.125	0.00000
0.02	0.02	0.500	0.00000
0.02	0.02	0.250	0.00000
0.02	0.02	0.125	0.00000
0.02	0.03	0.500	9.32E-05
0.02	0.03	0.250	0.00756
0.02	0.03	0.125	0.07259
0.02	0.05	0.500	0.00074
0.02	0.05	0.250	0.01545
0.02	0.05	0.125	0.08877
0.02	0.07	0.500	0.00285
0.02	0.07	0.250	0.02721
0.02	0.07	0.125	0.10836
0.02	0.10	0.500	0.01074
0.02	0.10	0.250	0.05239
0.02	0.10	0.125	0.14502
0.02	0.20	0.500	0.08868
0.02	0.20	0.250	0.18471
0.02	0.20	0.125	0.30440
0.03	0.01	0.500	0.00000
0.03	0.01	0.250	0.00000
0.03	0.01	0.125	0.00000
0.03	0.02	0.500	0.00042
0.03	0.02	0.250	0.02475
0.03	0.02	0.125	0.15378
0.03	0.03	0.500	0.00079
0.03	0.03	0.250	0.02822
0.03	0.03	0.125	0.15716
0.03	0.05	0.500	0.00250
0.03	0.05	0.250	0.03839
0.03	0.05	0.125	0.16696
0.03	0.07	0.500	0.00611
0.03	0.07	0.250	0.05164
0.03	0.07	0.125	0.17947
0.03	0.10	0.500	0.01593
0.03	0.10	0.250	0.07597
0.03	0.10	0.125	0.20281
0.03	0.20	0.500	0.09340
0.03	0.20	0.250	0.19603
0.03	0.20	0.125	0.32576

Illustrative Values of Vehicle Loss Probability (p)

STREET PATTERN IRREGULARITY FIXED AT Q= 1

GAMMA = SYSTEMATIC BIAS PER UNIT DISTANCE
SIGMA = STANDARD DEVIATIØN ØF RANDØM BIAS PER UNIT DISTANCE
r = PRØBABILITY THAT VEHICLE TURNS AT RANDØM INTERSECTIØN

GAMMA	SIGMA	r	p
0.00	0.01	0.500	0.00000
0.00	0.01	0.250	0.00000
0.00	0.01	0.125	0.00000
0.00	0.02	0.500	0.00000
0.00	0.02	0.250	0.00000
0.00	0.02	0.125	0.00000
0.00	0.03	0.500	0.00000
0.00	0.03	0.250	0.00000
0.00	0.03	0.125	0.00000
0.00	0.05	0.500	1.11E-05
0.00	0.05	0.250	0.00059
0.00	0.05	0.125	0.00609
0.00	0.07	0.500	0.00032
0.00	0.07	0.250	0.00514
0.00	0.07	0.125	0.02668
0.00	0.10	0.500	0.00400
0.00	0.10	0.250	0.02611
0.00	0.10	0.125	0.08075
0.00	0.20	0.500	0.07618
0.00	0.20	0.250	0.17397
0.00	0.20	0.125	0.29393
0.01	0.01	0.500	0.00000
0.01	0.01	0.250	0.00000
0.01	0.01	0.125	0.00000
0.01	0.02	0.500	0.00000
0.01	0.02	0.250	0.00000
0.01	0.02	0.125	0.00000
0.01	0.03	0.500	3.30E-07
0.01	0.03	0.250	0.00021
0.01	0.03	0.125	0.00755
0.01	0.05	0.500	4.10E-05
0.01	0.05	0.250	0.00213
0.01	0.05	0.125	0.02127
0.01	0.07	0.500	0.00052
0.01	0.07	0.250	0.00837
0.01	0.07	0.125	0.04388
0.01	0.10	0.500	0.00459
0.01	0.10	0.250	0.03022
0.01	0.10	0.125	0.09441
0.01	0.20	0.500	0.07697
0.01	0.20	0.250	0.17604
0.01	0.20	0.125	0.29797

GAMMA	SIGMA	r	p
0.02	0.01	0.500	0.00000
0.02	0.01	0.250	0.00000
0.02	0.01	0.125	0.00000
0.02	0.02	0.500	0.00000
0.02	0.02	0.250	0.00000
0.02	0.02	0.125	0.00000
0.02	0.03	0.500	1.16E-05
0.02	0.03	0.250	0.00316
0.02	0.03	0.125	0.05026
0.02	0.05	0.500	0.00021
0.02	0.05	0.250	0.00875
0.02	0.05	0.125	0.06836
0.02	0.07	0.500	0.00121
0.02	0.07	0.250	0.01860
0.02	0.07	0.125	0.09052
0.02	0.10	0.500	0.00640
0.02	0.10	0.250	0.04227
0.02	0.10	0.125	0.13179
0.02	0.20	0.500	0.07935
0.02	0.20	0.250	0.18217
0.02	0.20	0.125	0.30975
0.03	0.01	0.500	0.00000
0.03	0.01	0.250	0.00000
0.03	0.01	0.125	0.00000
0.03	0.02	0.500	4.92E-05
0.03	0.02	0.250	0.01191
0.03	0.02	0.125	0.11964
0.03	0.03	0.500	0.00014
0.03	0.03	0.250	0.01481
0.03	0.03	0.125	0.12444
0.03	0.05	0.500	0.00077
0.03	0.05	0.250	0.02388
0.03	0.05	0.125	0.13802
0.03	0.07	0.500	0.00269
0.03	0.07	0.250	0.03663
0.03	0.07	0.125	0.15478
0.03	0.10	0.500	0.00955
0.03	0.10	0.250	0.06152
0.03	0.10	0.125	0.18449
0.03	0.20	0.500	0.08328
0.03	0.20	0.250	0.19212
0.03	0.20	0.125	0.32831

traversed between <u>turns</u> is 1/r. Reinserting b as the minimal spacing between intersections, the mean distance between adjacent intersections is $\frac{b}{q}$. Combining these results, the mean distance travelled <u>between losses</u> is

$$\boxed{\overline{D}_L = \frac{b}{q}\frac{1}{r}\frac{1}{p}} \qquad (11)$$

If the vehicle travels at an average speed of s mph (accounting for stops at calls for service, meal breaks, intersections, etc.), then the <u>mean time</u> between <u>losses</u> is

$$\boxed{\overline{T}_L = \frac{b}{sqrp}} \qquad (12)$$

Let us now apply this equation to some hypothetical data to test its implications. Suppose our problem can be modelled with the following parameter values:

$\overline{d}_\ell$ = 528 feet = 0.1 mile

σ_ℓ = (1/2) $\overline{d}$

s = 10 mph

r = (1/3) (probability of a turn at a random intersection)

That is, the mean spacing between adjacent intersections is 528 feet, or one-tenth of a mile. The standard deviation of this spacing is

one-half of the mean. These two facts imply, using Equations 9 (a) and 9 (b), that

$$q = 3/4$$

$$b = 396 \text{ feet} = 0.075 \text{ mi.}$$

Substituting into Equation (12) for $\overline{T}_L$, we have

$$\overline{T}_L = \frac{0.075 \text{ mi.}}{(10 \text{ mi/hr})\frac{3}{4}\frac{1}{3}p} = \frac{0.03}{p}.$$

A popular system design objective calls for a mean time between losses not to be less than 12 hours. Thus,

$$\overline{T}_L \geq 12 \text{ hours.}$$

This implies a rather stringent requirement for the loss probability per turn, p, in the sense that

$$\frac{0.03}{p} \geq 12$$

or

$$p \leq \frac{0.03}{12} = 0.0025.$$

To put this in stronger terms, this means that the computer tracking software must correctly detect and interpret 99.75 percent of all turns made by the vehicle. This requirement would become even more stringent if we (1) increased the average travel speed above 10 mph; (2) decreased the mean spacings between intersections; (3) increased the probability of turning* above 1/3; (4) increased the variability (standard deviation) of the spacing between intersections above one-half of the mean.

IV. QUANTIZATION ERROR

The discussion to this point has assumed continuous tracking of the vehicle in time and space. In practice the time and space tracking are quantized, where the time quantization interval corresponds to the inverse of the polling rate per vehicle and the spatial quantization occurs both in the odometer (distance) and the heading sensor (angle). This section will discuss the ways in which these three types of quantizations increase the error probability predicted in Equation (10).

1. Angular Quantization

The heading sensor information is transmitted to the tracking computer as an N-digit binary number. This allows only 2^N different angular readings to be transmitted. It is customary to position uniformly the different quantized readings between 0 and 2π (radians), starting at 0. If we call α_Q the quantized angle, then α_Q can take on the values $0, 2\pi/2^N, 2\cdot2\pi/2^N,\ldots,(2^N - 1)\cdot2\pi/2^N$. Then, if the actual reading of the heading sensor is α, the value α_Q is transmitted, where α_Q is the quantized angle nearest α. In this way the set of possible

* Increasing the probability of turning naturally decreases p, since less distance is traversed between zero checks.

heading angles ranging from 0 to 2π is partitioned into quantization intervals $(-\pi/2^N, \pi/2^N)$, $(\pi/2^N, 3\pi/2^N)$, $(3\pi/2^N, 5\pi/2^N),\ldots,$ $((2^{N+1} - 3)\pi/2^N, (2^{N+1} - 1)\pi/2^N)$.

As an example, if N = 3, (bits), the angular information might be arranged as follows:

α_Q	Quantization Interval	Possible Binary Code
0	$(-\pi/8, \pi/8)$	000
$\pi/4$	$(\pi/8, 3\pi/8)$	001
$\pi/2$	$(3\pi/8, 5\pi/8)$	010
$3\pi/4$	$(5\pi/8, 7\pi/8)$	011
π	$(7\pi/8, 9\pi/8)$	100
$5\pi/4$	$(9\pi/8, 11\pi/8)$	101
$3\pi/2$	$(11\pi/8, 13\pi/8)$	110
$7\pi/4$	$(13\pi/8, 15\pi/8)$	111

This situation is depicted in Figure 7. Naturally, the greater the number of bits N, the greater is the accuracy of the transmitted information.

There appear to be two types of key errors that can occur due to angular quantization. The first is a consistent error that occurs while tracking a vehicle along a street whose actual angle is α but which is quantized as α_Q. This is illustrated in Figure 8(a), where a vehicle is travelling in a straight line at angle α (say $\alpha = 7\pi/16$)

Figure A-7

An Example of Angular Quantization, N = 3 bits

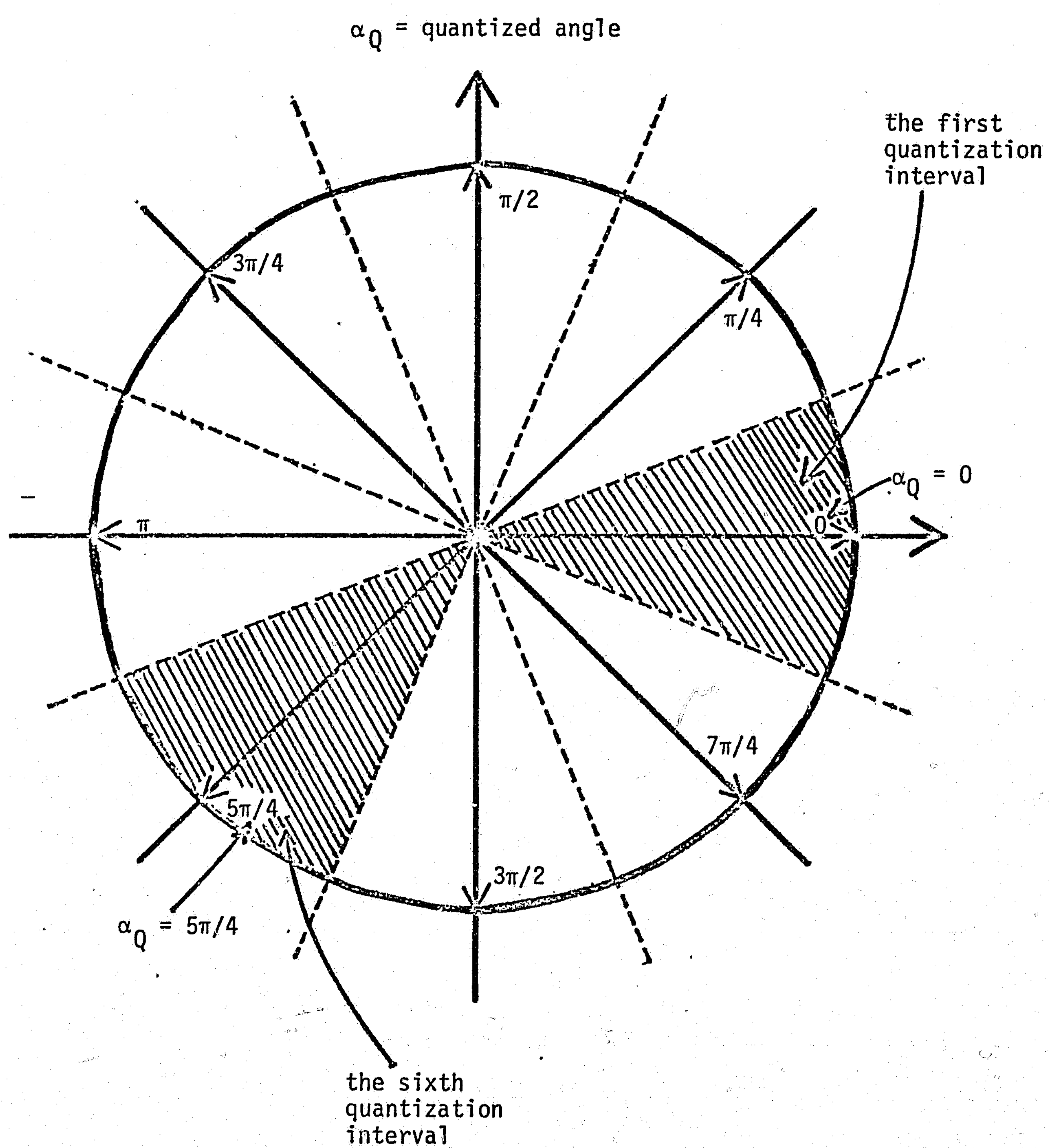
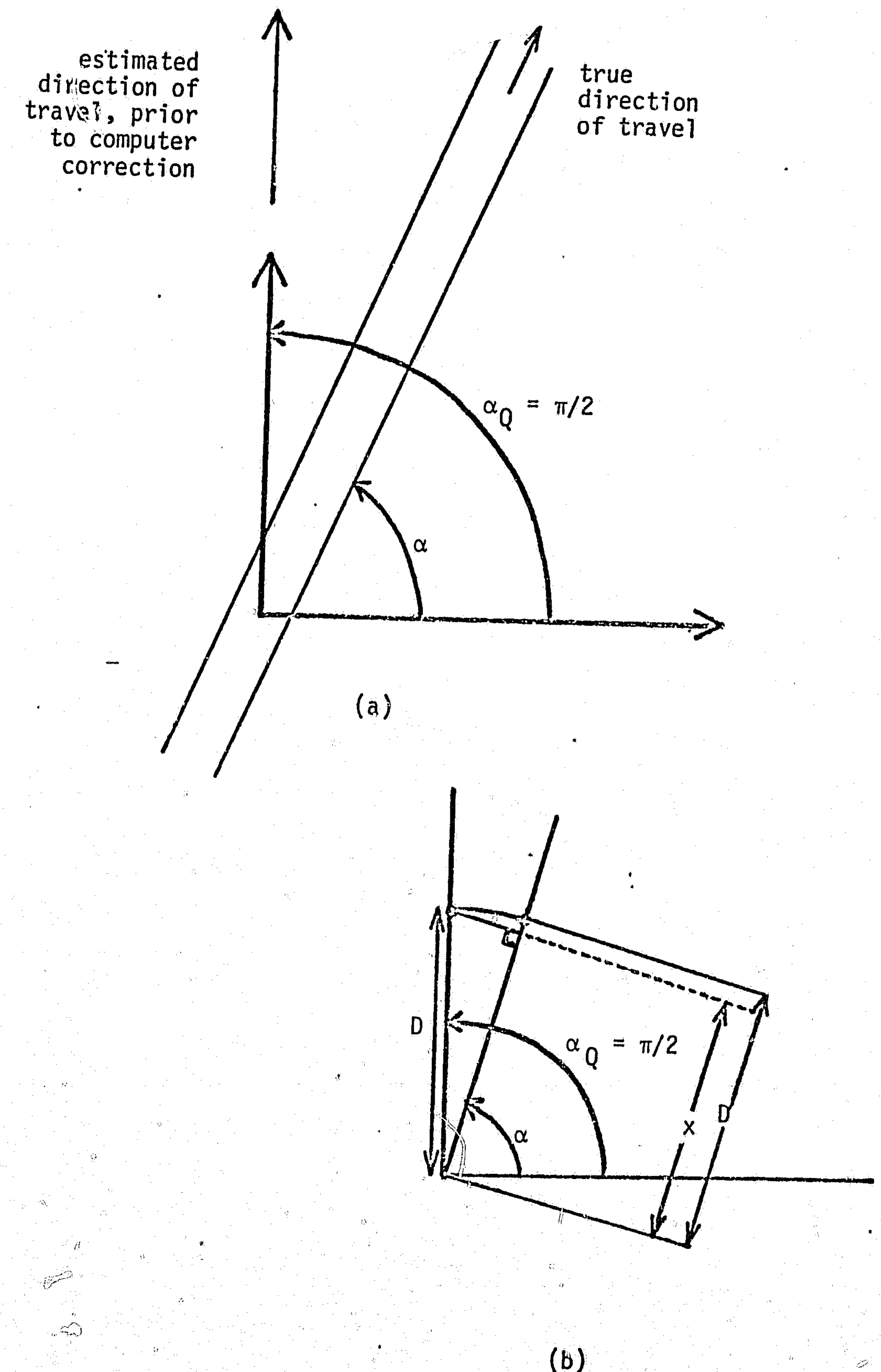


Figure A-8

Example of Consistent Tracking Error Due to Angular Quantization



but the quantized angle is α_Q(= $\pi/2$), yielding an angular error due to quantization of $|\alpha - \alpha_Q|$.

Developing this example, suppose the vehicle travels a distance D, then a "naive" tracking algorithm which did not take account of angular quantization might "correct" the position of the tracked vehicle back on the street at the point on the street closest to the current estimated (uncorrected) location of the vehicle. But this would yield a travelled distance on the actual street of only $x = D \cos|\alpha - \alpha_Q| \leq D$. (See Figure 8(b).) A distance estimation error would then be caused by the angular quantization; its magnitude would be $D - D\cos|\alpha - \alpha_Q| = D(1 - \cos|\alpha - \alpha_Q|)$. Obviously, the tracking algorithm need not be naive since the true angle of the street α is known and is maintained in the computer map. Thus, the correct procedure here is for the tracking algorithm to move the vehicle forward of its position as determined by a perpendicular to the street by an amount $D(1 - \cos|\alpha - \alpha_Q|)$; or, more simply, to move the vehicle along the street by a total amount D, not $D \cos|\alpha - \alpha_Q|$.

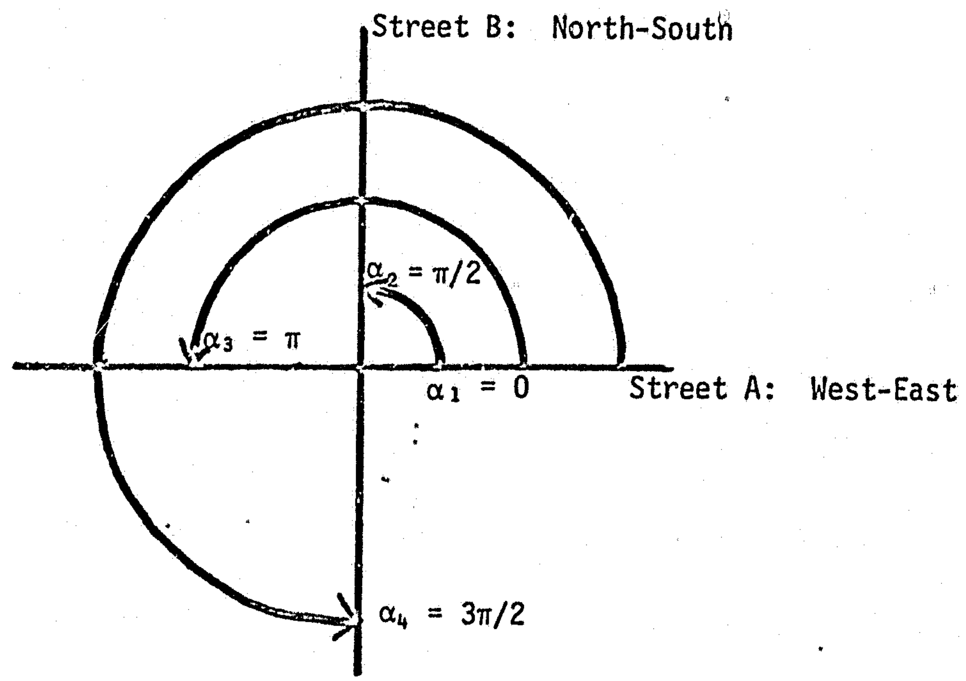
Since this type of consistent error can be easily corrected, we shall no longer concern ourselves with it. However, seeing the limited usefulness of the quantized angular information in tracking a vehicle along straight lines, it becomes apparent that the major purpose of angular information is to detect vehicular turns, when the vehicle changes streets on which it is travelling. So a question of concern is "How does angular quantization affect the ability of the tracking algorithm to detect vehicular turns?"

To answer this question, we need to introduce the notion of the divergence angle of street intersections. In Figure 9, we show two examples, a simple four-way perpendicular intersection and a complicated five-way intersection. Consider the simple example first. Imagine a vehicle entering the intersection from any one of the possible four directions. Upon exiting from the intersection, the tracking algorithm must determine if the vehicle has turned, that is if its angular direction has changed by $\pi/2$ or $-\pi/2$ radians (or by π, if u-turns are permitted). This it can readily do as long as there are at least four quantization intervals, corresponding to at least N = 2 bits. Now assume that the vehicle is entering the five-way intersection from any one of the incoming streets. Allowing u-turns, the computer tracking algorithm must determine which of the possible angles (α_1, α_2, α_3, α_4, or α_5) describes the motion of the exiting vehicle. Clearly if each of the actual angles α_i falls in a different quantization interval, then the direction of travel can be determined without error. This will be guaranteed to occur if the angles between all adjacent exiting streets, called divergence angles, are greater than the size of the quantization interval $2\pi/2^N$. Mathematically, the divergence angles in the example of Figure 9(b) are:

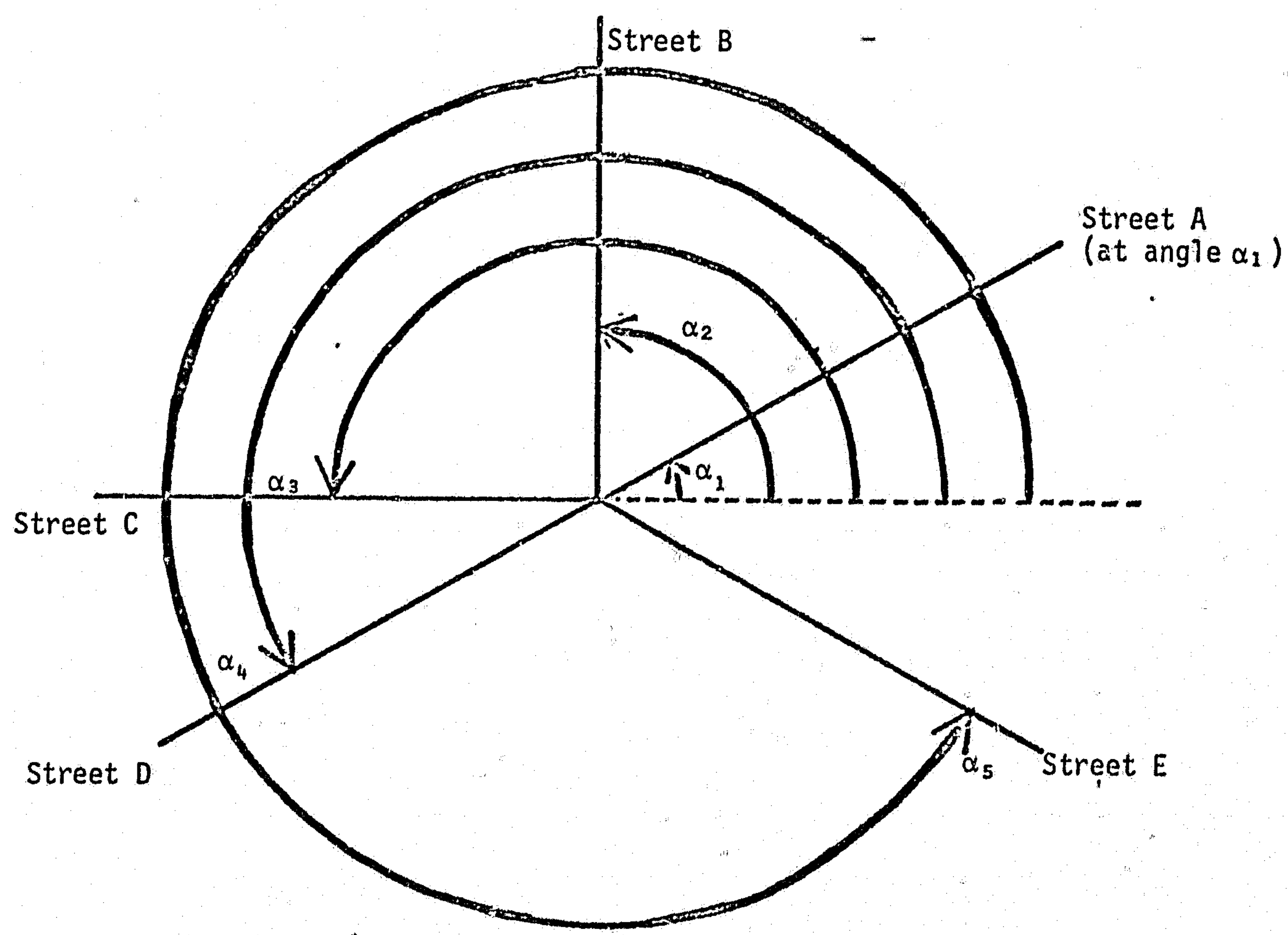
$$|\alpha_2 - \alpha_1| = a_1$$
$$|\alpha_3 - \alpha_2| = a_2$$
$$|\alpha_4 - \alpha_3| = a_3$$
$$|\alpha_5 - \alpha_4| = a_4$$
$$|\alpha_1 + 2\pi - \alpha_5| = a_5$$

Figure A-9

Street Angles at Intersections



(a) Simple Four-Way Perpendicular Intersection



(b) Complicated Five-Way Intersection

Thus, if $MIN\{a_i\} > 2\pi/2^N$, then the tracking algorithm can determine the street of exit from the intersection without error.

If, on the other hand, there are two adjacent streets whose divergence angle is less than $2\pi/2^N$, then they may or may not be in the same quantization interval. For the N = 3 example, where the quantization interval has size $\pi/4$ radians, consider two adjacent streets with angles $3\pi/16$ and $5\pi/16$. Here the divergence angle $a = 5\pi/16 - 3\pi/16 = \pi/8 < \pi/4$. Yet, the first street falls in the quantization interval with $\alpha_Q = 0$ and the second falls in the one with $\alpha_Q = \pi/4$. So, in this case, no error will occur when distinguishing between the first and second streets as exiting streets. However, suppose two other adjacent streets were directed at angles $\pi - \pi/16$ and $\pi + \pi/16$; then we still have a divergence angle $a = \pi/8 < \pi/4$, but the two streets both fall in the same quantization interval with angle $\alpha_Q = \pi$. In such a case in which two streets are contained within the same quantization interval, then the tracking algorithm must "guess" the correct street, and it would be reasonable to assume that the conditional probability of error would be 1/2.* (We will ignore the unlikely cases in which three or more streets are in the same quantization interval.)

To complete our discussion of angular quantization, we seek to find a way to compute

* By utilizing statistics on turning probabilities and frequency of street usage, this conditional error probability presumably could be reduced below 1/2. However, we will ignore such sophistication.

P_{Q_A} ≡ probability of loss of a vehicle at an intersection due to angular quantization.

One way to compute P_{Q_A} would be to examine each intersection in the city and determine by inspection which intersections have diverging streets falling within the same quantization interval. If there are found to be 10,000 pairs of diverging streets in the city and 13 of them had angles falling within the same quantization interval, then we would estimate

$$P_{Q_A} \approx \frac{1}{2}\,\frac{13}{10{,}000} = 0.00065.$$

Of course this estimation procedure could be refined by incorporating data on street usage and (if available) turning probabilities. However, in the absence of such information, this simple calculation is not an unreasonable way to proceed.

A second method, particularly appropriate for very large cities, would be to estimate the probability distribution of the divergence angles by sampling a representative subset of them. Suppose

$F_a(x)$ = Fraction of divergence angles less than or equal to x.

Then, for a randomly selected divergence angle,

$$Fa(x) = \text{Prob}\{a \le x\}.$$

In this context it is natural to call

$$f_a(x) = \frac{d}{dx} F_a(x)$$

the probability density function of divergence angles. The final concept we need here is that of the probability of loss of the vehicle due to angular quantization, given the value of a,

$P_{Q_A}(x)$ ≡ Prob{vehicular loss due to angular quantization$|a = x$}.

In the earlier exhaustive way of computing P_{Q_A}, this probability was always either 0 or 1/2. Now, given that we are only sampling the divergence angles, we will assume that the absolute angle of rotation of the streets at a random intersection is uniformly distributed between 0 and 2π. That is, at intersection j, the i^{th} street leaving the intersection is situated at an angle $\alpha_{ij} + \theta_j$ for all i, and θ_j is uniformly distributed between 0 and 2π. (Note that such a random rotation leaves unchanged the divergence angles such as $a_{1j} = |\alpha_{2j} + \theta_j - (\alpha_{1j} + \theta_j)| = |\alpha_{2j} - \alpha_{2j}|$.) In such a case, P_{Q_A} can take on values between 0 and 1/2. In fact, it is easy to see that

$$P_{Q_A}(x) = \begin{cases} \frac{1}{2} - \frac{1/2}{(2\pi/2^N)}\,x & 0 \le x \le 2\pi/2^N \\ 0 & \text{otherwise.} \end{cases} \qquad (13)$$

That is, the conditional probability of error drops linearly from 1/2 to 0 as the divergence angle increases to the length of the quantization interval $2\pi/2^N$; once above that value, the conditional error probability remains at zero. For the N = 3 case, we gave two examples in which the divergence angle was a = π/8, one yielding an error probability of 0 and the other 1/2. In this new setting in which the absolute street rotations are considered random, we would have

$$P_{Q_A}(\pi/8) = \frac{1}{2} - [\frac{1/2}{2\pi/2^3}]\frac{\pi}{8}$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Thus, streets whose divergence angles are one-half of the length of the quantization interval have a 50-50 chance of falling within the same quantization interval; given that they do, the conditional error probability is 1/2. The unconditional error probability is therefore $\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$, so our result checks with intuition.

Finally, utilizing each of the above concepts, the unconditional probability of vehicular loss at an intersection due to angular quantization is

$$P_{Q_A} = \int_{x=0}^{2\pi/2^N} \frac{1}{2}[1 - \frac{1}{(2\pi/2^N)} x] f_a(x)dx \qquad (14).$$

This formula provides a relatively easy way for a city whose angular characteristics are summarized in $f_a(x)$ to compute vehicular loss probability (due to angular quantization) as a function of the number of bits given for angular information N.

2. Distance Quantization

In a manner paralleling angular information, distance information is also transmitted digitally, therefore necessitating a distance quantization interval d_Q. Thus, in a moving vehicle, if the odometer

reading has just changed (by adding 1 bit to the previous reading) then the next odometer change will occur after the vehicle has travelled a distance equal to d_Q. Clearly if d_Q is of the same order of magnitude as block lengths then this type of quantization could severely increase the loss probability. However, typically d_Q is 25 feet or less (at least one order of magnitude less than a typical block length).

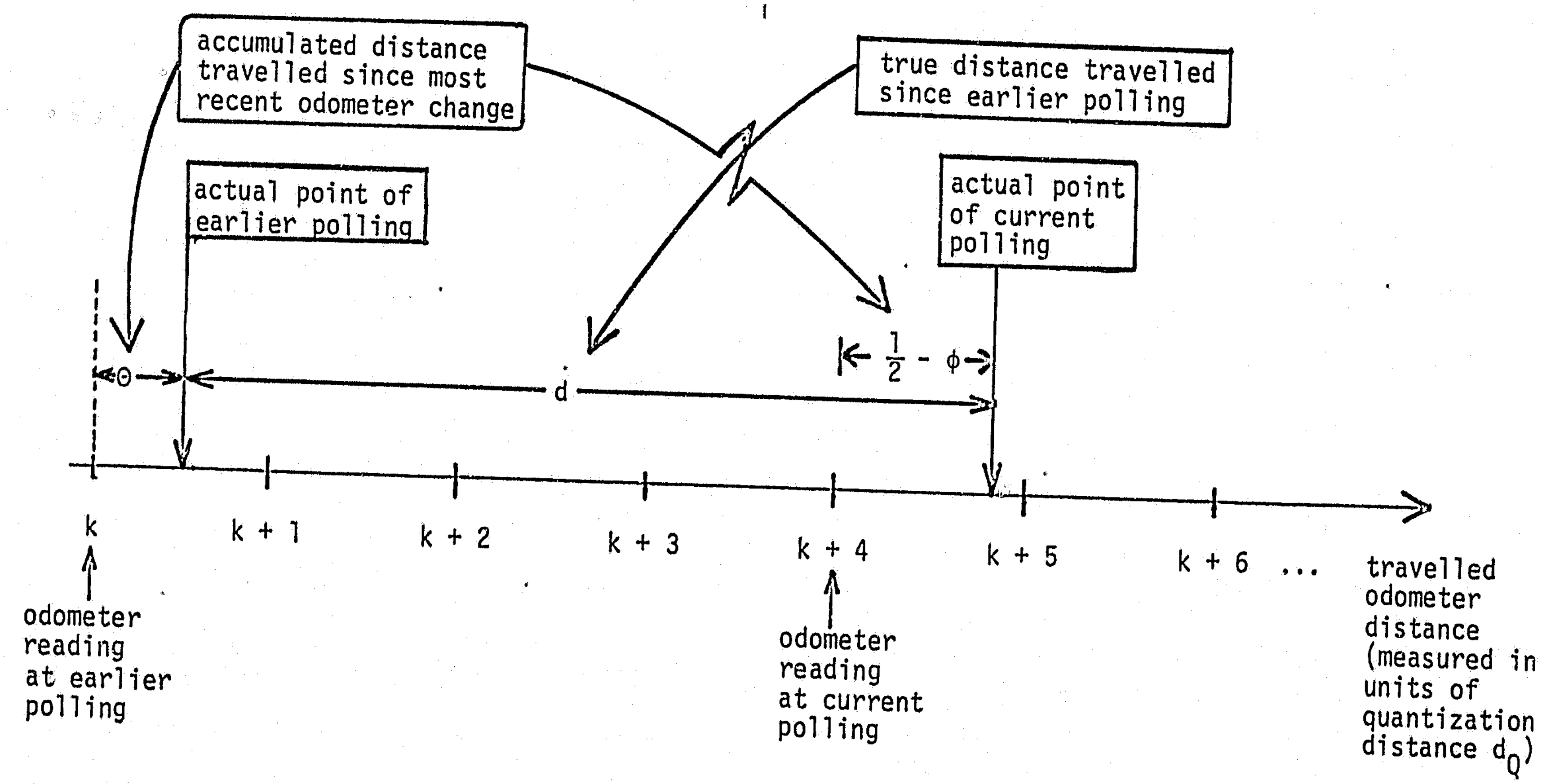
The overall effect of distance quantization can be understood by examining Figure 10. We focus on the current and an earlier polling of the vehicle. At the earlier polling the odometer reading is some (arbitrary) integer k. But at the actual point of polling the vehicle had travelled θ units (where distance is measured in units of quantization distance d_Q) since the odometer reading changed to k. (Obviously, $0 \leq \theta < 1$.) The uncertainty in the value of θ increases our uncertainty regarding the position of the vehicle. The vehicle then travels an exact odometer distance equal to d, at which point the current polling takes place. The odometer reading must be an integer, so we round down to the integer which represents the current odometer reading. The amount by which we round down is $\frac{1}{2} - \phi$, where $-\frac{1}{2} \leq \phi < \frac{1}{2}$. As we will see, this rounding off procedure also causes uncertainty in our estimate of the vehicle's location. Summarizing, the odometer reading of the vehicle between any two arbitrary* pollings is given by

$$\hat{d} = d + \theta + \phi - \frac{1}{2}.$$

*Assuming that the vehicle is moving, to avoid degenerate cases.

Figure A-10

Key Variables in Distance Quantization



The following assumptions regarding the two random variables θ and ϕ and the variable d seem reasonable:

1. θ is uniformly distributed between 0 and 1.
2. ϕ is uniformly distributed between $-\frac{1}{2}$ and $+\frac{1}{2}$.
3. θ is independent of the subsequent value of d.

Clearly, ϕ is dependent on $d + \theta - \frac{1}{2}$ since ϕ is determined by the non-integer part of the latter quantity.

The polling procedures are obviously unbiased since

$$E[\hat{d}] = d + E[\theta] + E[\phi] - 1/2$$
$$= d + 1/2 + 0 - 1/2 = d.$$

Thus d is an unbiased estimator of the measured odometer distance D.

Following the argument of Section II.1, the updated map center-line distance between any two pollings, given that the odometer has measured d units of travel, is

$$D(d) = \hat{d} + X(d)$$
$$= d + \theta + \phi - 1/2 + X(d),$$

where X(d) is the Gaussian error term of Section II.1.

Assuming $\sigma = 0$ (for convenience of presentation),

$$E[D(d)] = d,$$

as expected. However, we wish to compute the variance of D(d) to determine the manner in which the polling procedure adds to position estimation uncertainty at intersections at which the vehicle may turn. This variance is:

$$\sigma^2_{D(d)} = E[(d + \theta + \phi - 1/2 + X(d) - E[D(d)])^2$$

After some straightforward manipulation we obtain

$$\sigma^2_{D(d)} = \sigma^2_{X(d)} + \sigma^2_{\theta} + \sigma^2_{\phi} + 2E[\phi(X(d) + \theta - 1/2)].$$

As one can see, the uncertainty of vehicular position is increased over that due solely to random odometer error ($\sigma^2_{X(d)}$) by (1) the unknown odometer distance travelled since the most recent odometer change at the last polling (σ^2_{θ}), (2) the integer round-off procedure (σ^2_{ϕ}) and (3) the dependence of ϕ on the other variables.

Here, assuming d is at least a block length (which should be several units of distance--measured in terms of d_Q), we can assume that (approximately) ϕ is independent of $X(d) + \theta - 1/2$, thus reducing the above equation to

$$\sigma^2_{D(d)} \approx \sigma^2_{X(d)} + \sigma^2_{\theta} + \sigma^2_{\phi}.$$

Since

$$\sigma^2_{\theta} = \sigma^2_{\phi} \doteq 1/12,$$
$$\sigma^2_{D(d)} = \sigma^2_{X(d)} + 1/6.$$

Since this derivation has been carried out in units of d_Q, if we switch back to feet (or some other absolute standard of distance) we obtain

$$\sigma^2_{D(d)} = \sigma^2_{X(d)} + d_Q^2/6 \qquad (16)$$

In practice we can use this result in a very simple and straightforward way. We invoke the facts that X(d) is a Gaussian random variable and that D(d) is the sum of random variables. Since usually $\sigma^2_{X(d)} > d_Q^2/6$, the Central Limit Theorem should apply quickly here, indicating that D(d) can be treated as a Gaussian random variable, having mean 0 and variance $\sigma^2_{X(d)} + d_Q^2/6 = d\sigma^2 + d_Q^2/6$.

Thus, applying this result to the two pollings associated with two successive turns, the increase in the vehicle loss probability at a random turn due to distance quantization could be estimated by adding $d_Q^2/6$ to the Weiner process variance ($\sigma^2 i$) in Equation (10).

To obtain an intuition for the numbers involved, suppose a turn occurs after 10,000 feet and suppose the Weiner process variance is $\sigma^2(10,000) = 2,500$ (as in the example in Section II.1.). Suppose further that the quantization interval is $d_Q = 25$ feet. Then $d_Q^2/6 = 625/6 \approx 104$. Thus the total variance of the estimated distance travelled is

$$\sigma^2_{\hat{D}}\,(1,000) = 2,500 + 104.$$

As can be seen even with this simple example, reasonably small values for the distance quantization interval d_Q should result in little degradation in system performance (as measured by vehicle loss probability). Note, however, that a larger quantization interval of $d_Q = 100$ feet would result in a significant increase of the total variance (from 2500 to 4166).

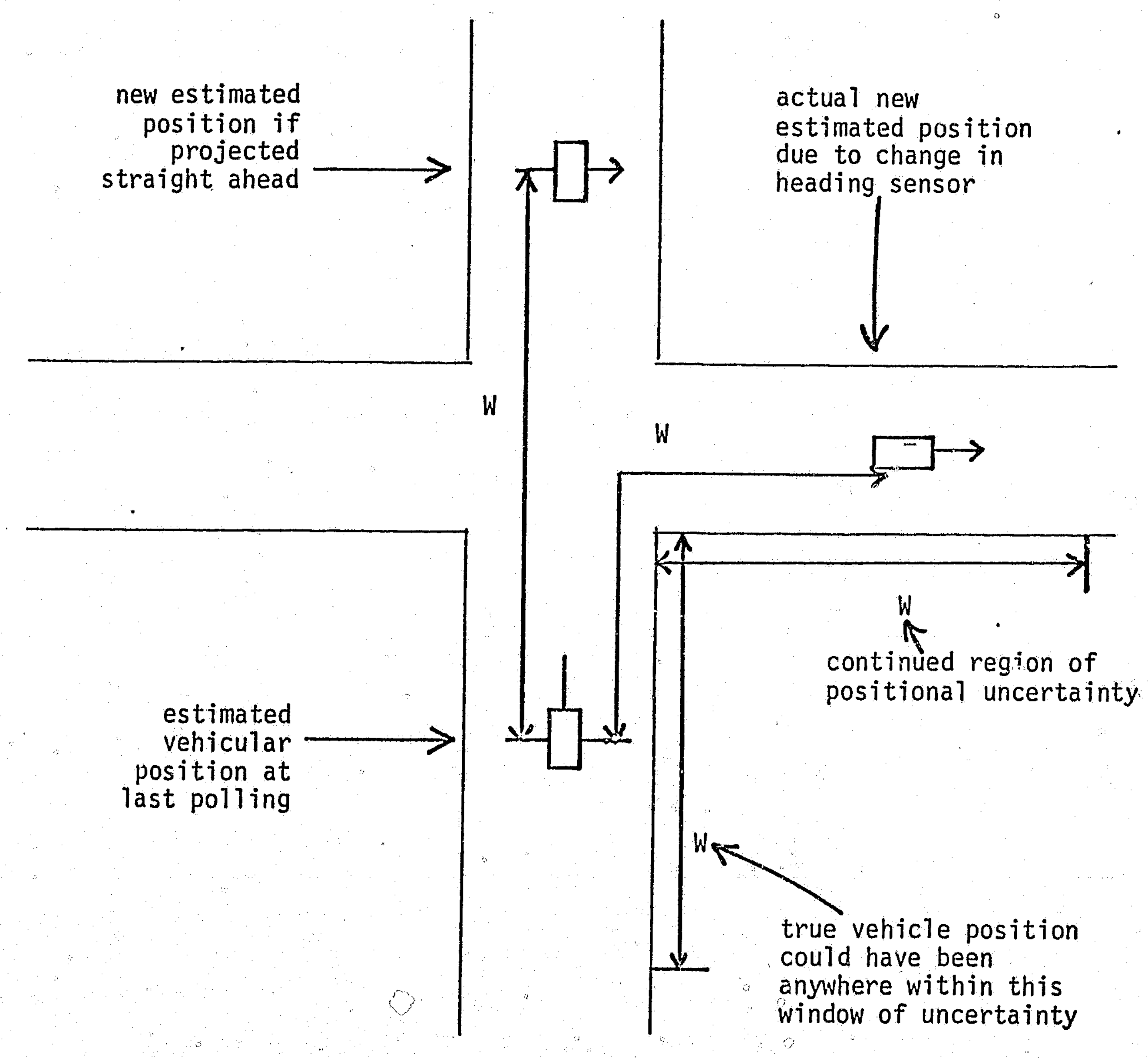
3. Time Quantization

Like angular and distance quantization, time quantization too causes additional uncertainty in the estimate of a vehicle's location and thus increases the loss probability p. The unit of time quantization is t_Q, which means the vehicle is polled every t_Q seconds to obtain new distance and heading readings. Typically t_Q is one or two seconds.

Time quantization's effect on positional uncertainty at a turn can

Figure A-11

Positional Uncertainty at a Turn
Due to Time Quantization



$W = t_Q \cdot$ (speed of vehicle)

be seen in Figure 11, at the last polling the vehicle was estimated to be south of the intersection, headed north. At the current polling, the unit has travelled a distance W, which is equal to the speed of the vehicle times t_Q, and its heading has changed from north to east. If the computer tracking algorithm simply projected the vehicle north a distance W, the vehicle would be on a north-south street headed east, an obvious inconsistency. Thus, the algorithm assumes that a turn has occurred and positions the vehicle a travel distance W from the last estimated position* but on the east-west street headed east away from the intersection. Assuming that this particular street is the correct street on which the vehicle turned, the fact that the heading sensor changed between pollings means that the turn could have occurred at any time during the time interval t_Q. Thus, since the vehicle travelled a distance W during t_Q, the actual position of the vehicle at the last polling could have been anywhere south of the intersection up to a distance W away. Thus, the new (current) position of the vehicle could be anywhere east of the intersection up to a distance W away. As a numerical example, if t_Q = 2 seconds and the vehicular speed = 30 mph = 44 feet/second, then W = 2·44 = 88 feet.

If we imagine the vehicle entering the region of the intersection with estimated location described by a Gaussian random variable with variance $\sigma_d^2 + d_Q^2/6$, then part of this uncertainty persists after leaving the intersection. In the worst imaginable case, yet assuming a correctly

*This is one reasonable procedure for positioning the vehicle on the east-west street. Another, which has been utilized in FLAIR, is to position the vehicle exactly at the exit point of the intersection, heading east.

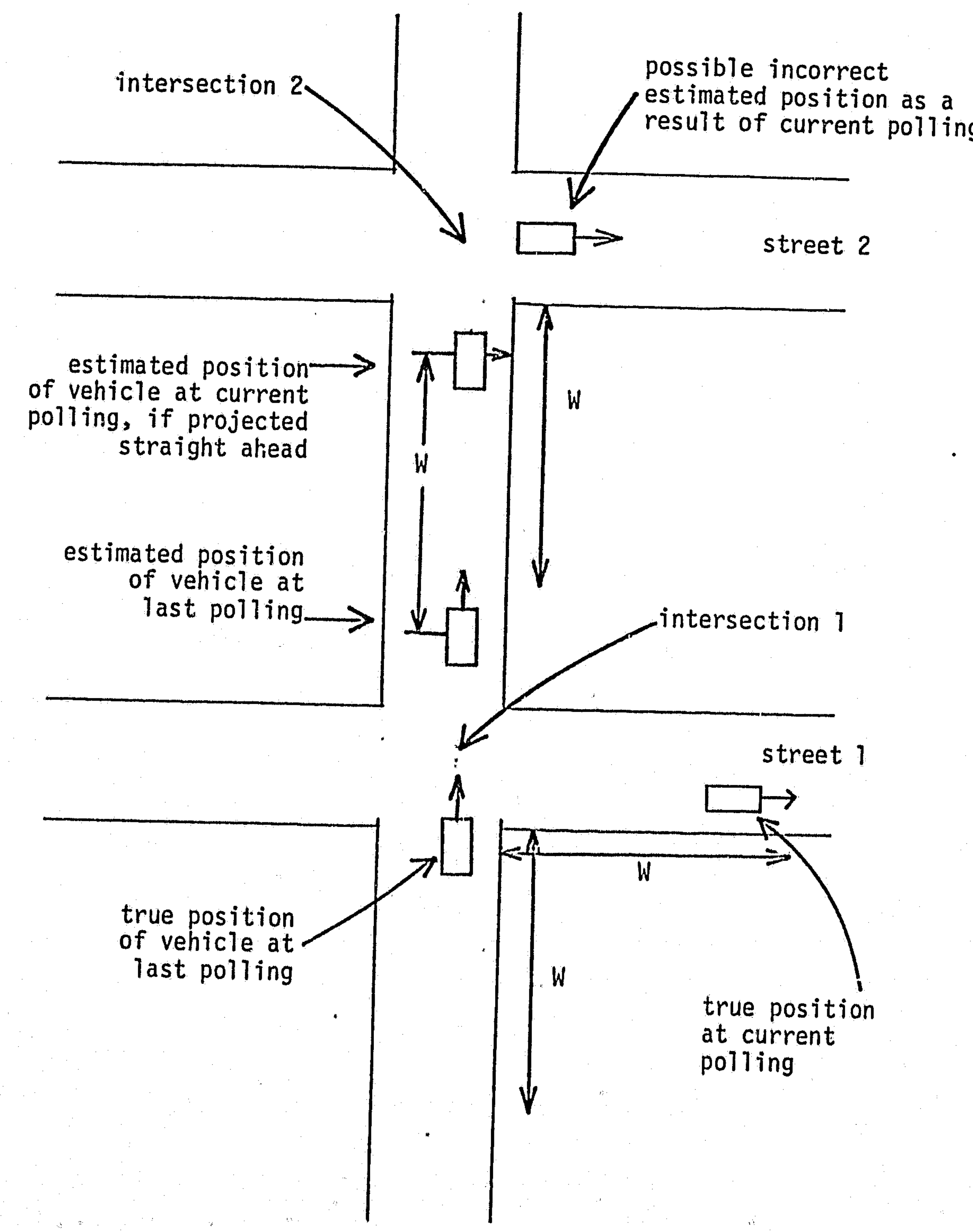
interpreted turn, the persisting positional uncertainty could be described as a uniformly distributed random variable over the west-east interval W (extending from the intersection). This gives the vehicle an initial variance in estimated position of $W^2/12$, rather than 0 as is assumed in the renewal theory model of Section III. Upon entering the next intersection where a turn is to take place, after travelling a distance d', the variance in the position estimate will be $W^2/12 + \sigma_{d'}^2 + d_Q^2/6$. For reasonably small values of t_Q, the addition to the variance due to W (which is proportional to t_Q) should not be very large.

Additional insight on the effect of time quantization on loss probabilities can be gained by examining Figure 12. Here at the last polling the vehicle was just about to enter the intersection and execute a right turn. However, the estimated position of the vehicle was somewhat north of the intersection (heading sensor still reading north), perhaps one or two standard deviations from the mean (or perhaps nearer the mean in a system with systematic error). At the current polling the vehicle has travelled a distance W, and the heading has changed from north to east. Given that the vehicle has turned right, the computer tracking algorithm is confronted with a decision: Did the vehicle turn on street 1 (the first east-west street) or street 2? There are two alternative hypotheses: at the time of the last polling the vehicle was in the window of length W just south of either intersection 1 or intersection 2. For a vehicle such as this one which is estimated ahead of its actual position, the greater the value of W, the more likely it is that the computer tracking algorithm will choose (incorrectly) intersection 2 (and thus street 2). This is due to the fact that as W increases the southern tip of the window of length W from intersection 2 gets closer to the last estimated position of the vehicle, while the window from intersection 1 (while getting larger) remains at a constant distance from this last estimated position. Thus, as W increases, it becomes more and more plausible that the vehicle was actually at the southern tip of the intersection 2 window rather than at the northern tip of intersection 1's.

Obviously, for fast moving vehicles moving on streets with relatively short block lengths (perhaps engaged in a criminal pursuit), these effects of time quantization could cause a measurable increase in vehicular loss probability.

Figure A-12

Possible Loss of Vehicle Due Directly to Time Quantization



V. Discussion

In this appendix we have developed several highly simplified models in order to analyze the factors that contribute to vehicle loss probability. Briefly summarizing, we have found the following:

(1) One component of vehicle drift from its true location is due to random error. This is due to many factors including tire slippage on streets, irregular (non-straight line) driving patterns, map errors, and, if uncorrected in the tracking algorithm, speed variations which change the tire circumference. This net effect of such random error is summarized in the parameter σ^2 which is the mean squared random displacement per unit of distance travelled.

(2) A second, often dominating component of vehicle drift is due to systematic error. This type of error creates a bias in the odometer readings and its magnitude is determined by temperature, tire wear and pressure, and speed (when the effect of speed on drift is viewed as correctable). The bias term is γ, which is the mean systematic displacement per unit of distance travelled.

(3) The vehicle loss probability will depend strongly on the particular street patterns of the city in question. In general the loss probability increases as the mean spacings between streets decreases, as the street pattern becomes more irregular (implying more very short blocks), and as the diverging angles at intersections become small (the definition of small depending on the number of bits used to transmit angular information).

(4) The number of binary digits (bits) used to transmit information on vehicular heading and distance can markedly affect vehicular loss probability. One can virtually guarantee no increase in loss probability due to angular quantization if the corresponding number of bits N is sufficiently large so that $2\pi/2^N$ is smaller than the smallest diverging street angle in the city. The effect of distance quantization is to add to the variance of the random error a term proportional to the square of the distance quantization interval.

(5) The magnitude of the sampling interval (in time) can also affect the loss probabiilty. For those turns which are tracked correctly, the magnitude of the sampling interval determines the size of a window of positional uncertainty which characterizes the vehicle's estimated position until it next turns; this can often be crudely characterized as an increase in the variance of the estimate of position. However, the window of positional uncertainty can also have a direct effect on contributing to an incorrect interpretation of a turn; the larger the window [which means the larger the sampling interval], the larger is the probability of incorrect decision.

(6) In most cases we have developed simple equations to estimate at least the first order effects on vehicle loss probability of each of the key factors.

There are at least two important topics that also bear on system performance that have not been discussed in this appendix. The first is open loop tracking which occurs whenever the tracked vehicle leaves a mapped street or alleyway and enters a parking lot, an industrial property, etc. With open loop tracking, the tracking algorithm cannot use well-mapped street patterns to correct certain drifts in the vehicle's location. Thus, the estimation error becomes a two-dimensional error rather than a one-dimensional one. Moreover, angular, spatial, and temporal quantization can markedly increase the chance of losing a vehicle that is being tracked in the open loop mode. Recognizing the extent of imperfect information received in the open loop mode, the tracking software in our currently implemented system automatically signals "Lost vehicle" as soon as the measured odometer distance in an open-loop situation exceeds some prespecified threshold value.

The second topic is <u>system subvertability</u>, which is defined as the susceptability of the system to deliberate acts aimed at increasing loss probability. These include reporting an incorrect address at time of "loss correction" (or "reinitialization"), momentarily switching off the power of the unit located in the vehicle. The system subvertability is increased by the presence of magnetic anomolies that create faulty (uncorrectable) heading sensor readings and the presence of intersections whose diverging street angles are sufficiently small so as to create a high chance of vehicular loss. This topic is discussed at greater length in Chapters V, XII of the main report.

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