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OPT2: AN INTERACTIVE COMPUTER PROGRAM FOR CONDUCTING BAYESIAN HYPOTHESIS TESTS

by

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OR 108-81 July 1981

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OPT2 is an interactive PL/I program written under National Institute of Justice Grant Number 80-IJ-CX-0048. The program is designed to assist evaluators in selecting decision rules for simple hypothesis tests involving normal distributions. PRIORS is likewise an interactive PL/I program written under this grant. The products of PRIORS may be useful in formulating decision rules with OPT2.

Hypothesis testing is no longer simply a laboratory tool. Today it effects the courses of thousands of lives and millions of dollars. FDA regulations are an especially tangible example of the present power of hypothesis testing. Admissions policies to public assistance programs, special education programs, limited medical facilities and psychiatric institutions are, intentionally or not, decision rules for hypothesis tests.

The problems involved in formulating such decision rules not to mention their consequences set hypothesis testing in social institutions apart from testing in laboratories. It is neither politically acceptable nor economically feasible to determine which citizens will receive public assistance according to the same formulas used to determine the effectiveness of malathion against Drosophila. Consider the problem of formulating requirements for admission to the following public assistance program.

The law requires we admit people solely on the basis of their present assets. We are however left with the problem of deciding how valuable a clients assets may be. We initiated an extensive retrospective study to determine the distribution of assets among applicants. Case workers judged whether each client was actually deserving of assistance. Extensive investigation of the histories of rejected applicants was undertaken as well. We found that half of all applicants could, denied admission, go on to support themselves. Unfortunately however there was no level of assets which could unambiguously distinguish these people. In fact the study showed the following distribution:

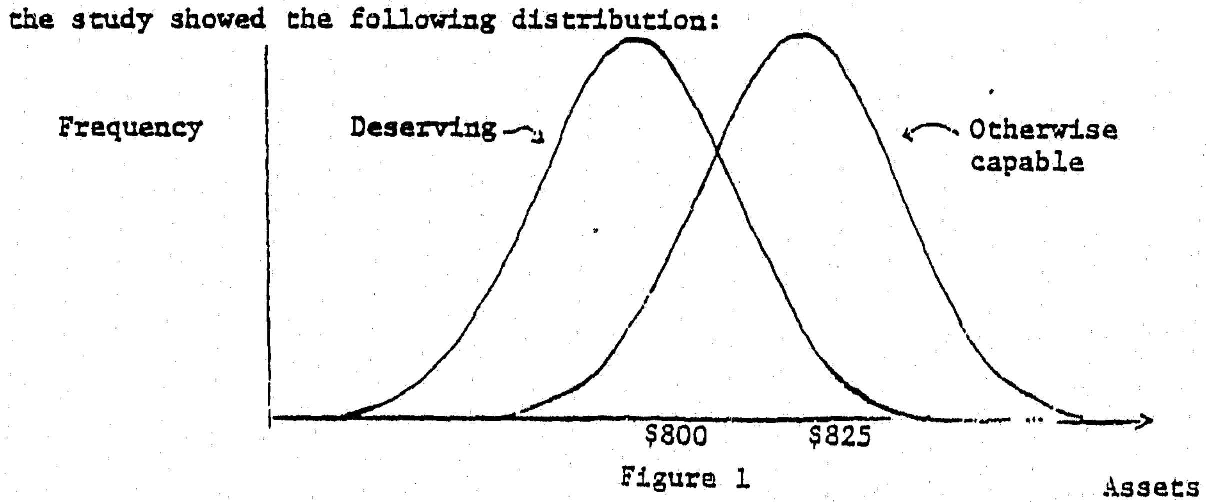


Figure 1

It is clear from Figure 1 that regardless of what level we choose we will reject deserving applicants, accept otherwise capable applicants or both. In this situation Classical Statistics would prescribe either the .05 alpha-level decision rule or the .05 beta-level decision rule. The .05 alpha-level decision rule is, roughly speaking, designed to ensure that the chances of turning away a deserving applicant remain below one in twenty. The .05 beta-level decision rule on the other hand ensures that the chances of accepting an applicant capable of supporting himself remain below the same figure.

Straight forward as these rules may seem their consequences may be intolerable to many planners and decision makers. In our case the .05 alpha-level decision rule would admit people with assets not exceeding $840. Anyone else would be rejected. It is clear from Figure 2 that some applicants who could otherwise support themselves would be accepted into our program. In fact 72% of this group would be accepted. If each client in the program costs us $1,200.00 then these people alone will cost our program over four million dollars for every ten thousand applicants.

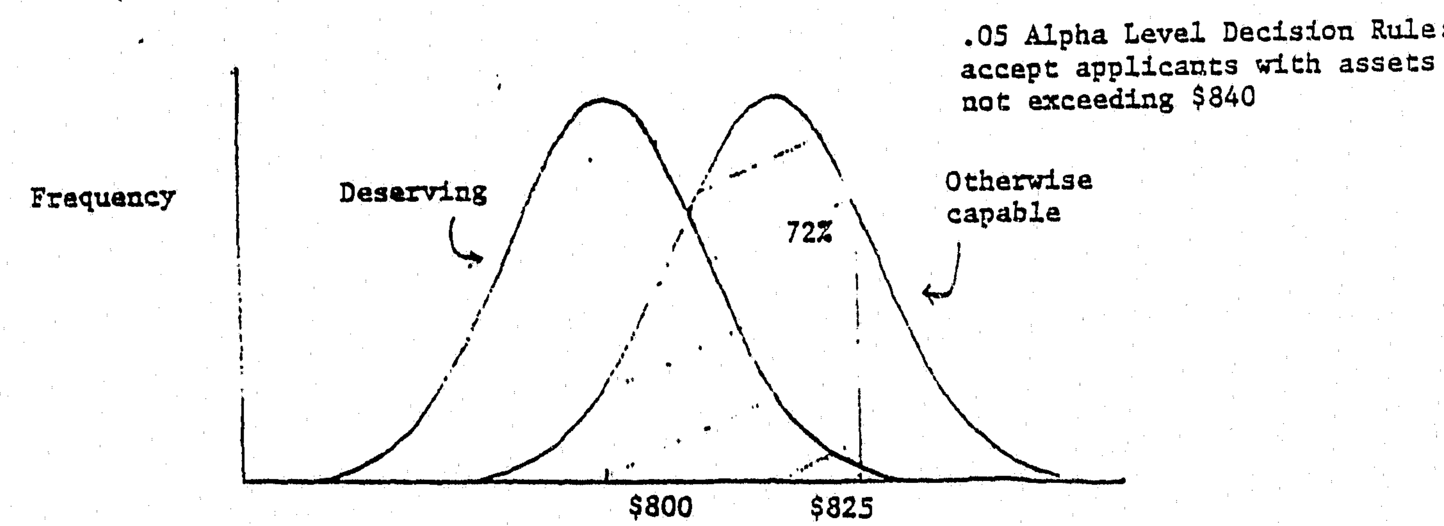


Figure 2

The .05 beta-level (Figure 3) rule will on the other hand prevent this situation. However the consequence of being so parsimonious is that nearly eighty deserving applicants will be turned out in the cold for every one hundred applying. The costs of this policy are certainly no less than those of the overly generous .05 alpha-level rule.

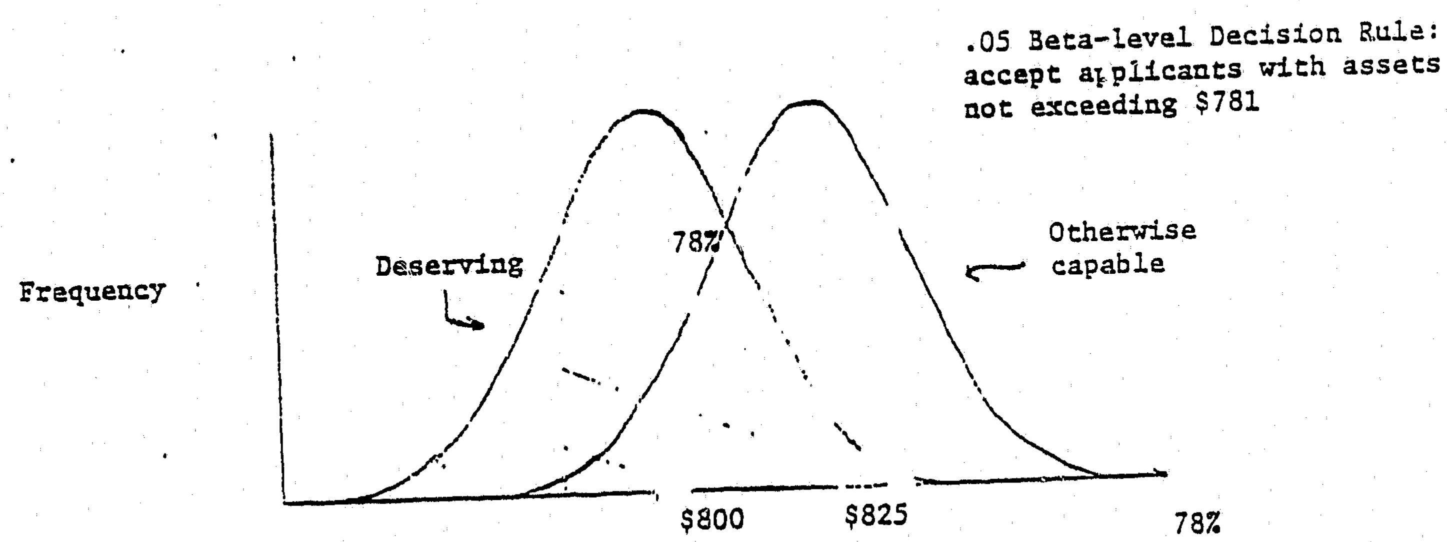


Figure 3

The obvious difficulty with classical statistical decision rules is that they ignore the consequences of the various possible outcomes. OPT2 incorporates policy-makers' estimates of the relative value associated with various outcomes into the formulation of decision rules. In this example we might estimate the value of accepting a deserving applicant at 1200. This number need not be interpreted monetarily — it might for example represent years or lives — anything. It is only a relative measure of the value of admitting a deserving applicant.

We might similarly estimate the value of turning away someone who will as a result of our decision become self-supporting at 2000. The penalty on the other hand for accepting such a person must reflect not only the expense to our program, but also the lost opportunity costs to the individual. We might estimate this at -3200. Finally we estimate the tragic consequences of turning away a

deserving applicant at -5000. Given these values, OPT2 will determine a decision rule which has the largest expected value. The best decision rule based on the values we assigned is pictured in Figure 4. This rule plays off the penalty for accepting an applicant who would otherwise support himself against that of rejecting a deserving applicant. As shown in Figure 4 the best rule accepts approximately forty per cent of the applicants who could otherwise support themselves in order to ensure that no more than twenty-two per cent of the deserving applicants are rejected.

The values we assigned to the various possible outcomes of a decision are by no means absolute. In fact if we halve or even double all of our estimates we will arrive at the same decision rule. These estimates are as we mentioned before only measures of relative value. In the case outlined above the expected value of a decision based on:

the .05 alpha-level decision rule		-422.81
the .05 beta-level decision rule	is	-933.22
the optimal decision rule		256.98

The difference between the expected value of the optimal decision rule in one-thousand applications 256,980 and that for say even the .05 alpha-level decision rule -422,810 should certainly make one reconsider classical statistical rules.

Using OPT2

Once you have logged on and loaded OPT2 you will receive a short description of the program. In order to use OPT2 correctly you must be making a simple hypothesis test between two known normal distributions. Should you not know the distributions involved we recommend you to the related program PRIORS to help with estimating them.

Now that you know the normal distributions you wish to select between you must choose one to be the null hypothesis. Whereas this distinction is crucial to classical statistical decision rules, it is only a matter of convenience in formulating a best rule. In the example above, the null hypothesis was that an applicant was deserving.

OPT2 will first ask for the mean of the two distributions. Simply type your responses after each question and hit the return key. The mean of the distribution under the null hypothesis in the previous example was 800.00. It is not important what units you use as long as you are consistent.

Next OPT2 will ask for the variances of the distributions. The variance of a distribution is some number greater than zero which measures the dispersion of the distribution about its mean. The variance of the distribution under the alternative hypothesis was 700.00 in our example.

Now that you have characterized the choices OPT2 will ask you to estimate the values of the possible outcomes of a decision. Your answers to these questions may be any number negative or positive. However OPT2 will interpret your answers to reflect the relative values of the outcomes — that with the most positive value being the most desirable. In our example the value of correctly selecting the null hypothesis was the value of accepting a deserving applicant — 1200. whereas the value of incorrectly rejecting the null hypothesis was the value of turning such an applicant away — namely -5000.

Finally OPT2 will ask you to estimate the a priori probability that the null hypothesis is true. In our example, since half of all the applicants were deserving, this was .5. This probability need not always be so objective. It is often both necessary and prudent to incorporate more subjective information such as advice of experts or past experience with related situations into your estimate of this probability. Again, should you have trouble, we recommend you to PRIORS for assistance.

The decision rules OPT2 formulates are (assuming the means of the distributions are different) threshold decision rules. The four possibilities are:

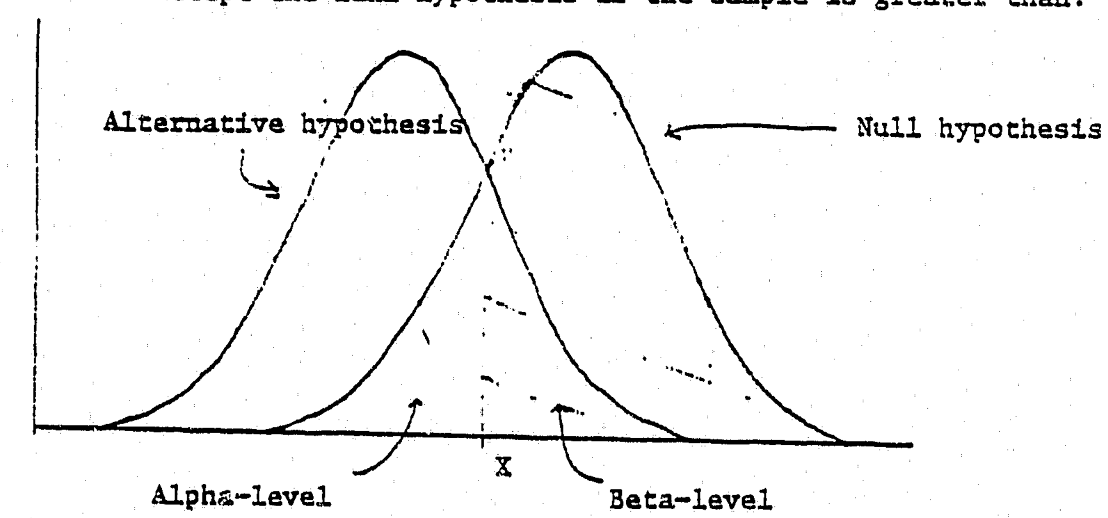
1. Accept the null hypothesis always.
2. Accept the null hypothesis never.
3. Accept the null hypothesis if the sample is less than: X
4. Accept the null hypothesis if the sample is greater than: X

Should the two distributions have the same mean, OPT2 will formulate rules:

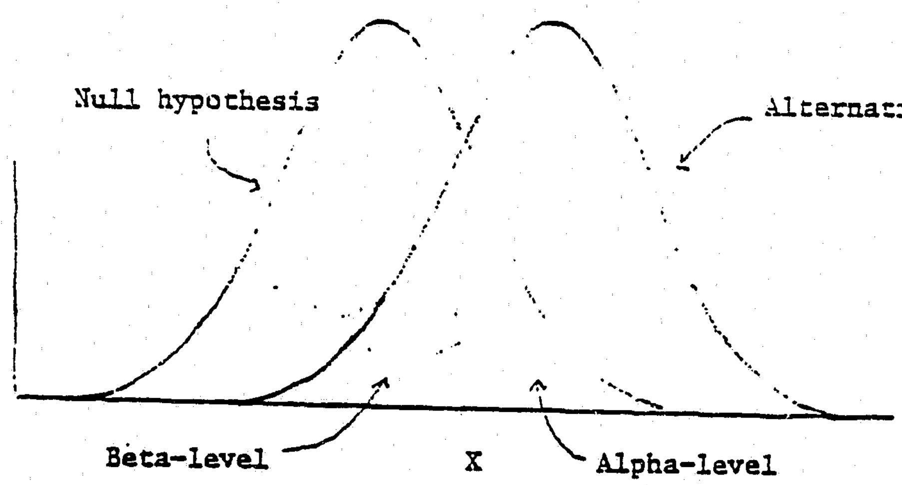
1. Accept the null hypothesis always.
2. Accept the null hypothesis never.
3. Accept the null hypothesis if the sample is less than: X
   or greater than: Y
4. Accept the null hypothesis if the sample is less than: X
   and greater than: Y

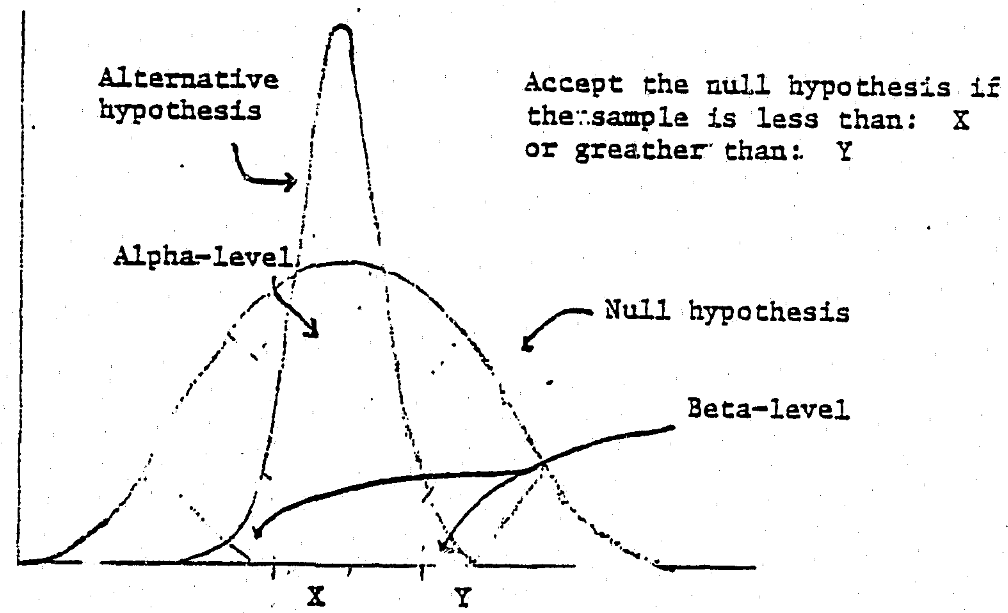
The following diagrams illustrate the different rules.

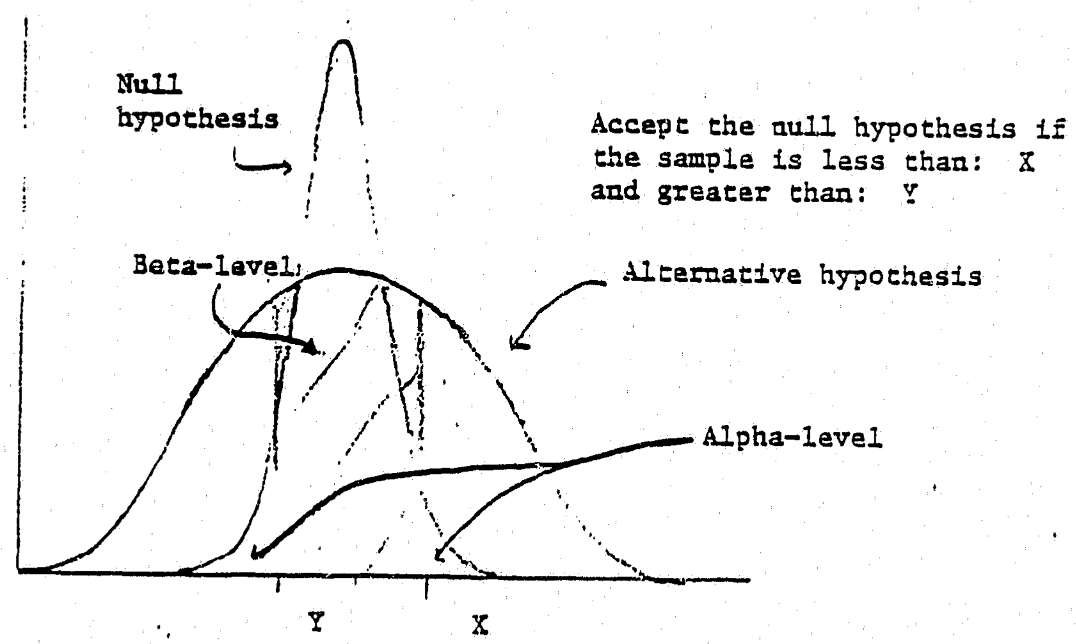
Accept the null hypothesis if the sample is greater than: X



Accept the null hypothesis if the sample is less than: X







CLASSICAL STATISTICS PRESCRIBES THE .05 ALPHA-LEVEL OR THE .05 BETA-LEVEL DECISION RULE FOR SOLVING SIMPLE HYPOTHESIS TESTS. THIS MEANS THE CLASSICAL STATISTICIAN DECIDES IN ADVANCE TO STRONGLY FAVOR ONE HYPOTHESIS. THIS PROGRAM ILLUSTRATES THE DIFFERENCE BETWEEN THE CLASSICAL DECISION RULES AND A BAYESIAN DECISION RULE BASED ON COSTS. THE SIMPLE HYPOTHESIS TEST CONSIDERED HERE IS: DOES AN OBSERVATION COME FROM A POPULATION NORMALLY DISTRIBUTED ABOUT U0 WITH VARIANCE V0 (THE NULL HYPOTHESIS) OR DOES IT COME FROM A POPULATION NORMALLY DISTRIBUTED ABOUT U1 WITH VARIANCE V1 (THE ALTERNATIVE HYPOTHESIS).

THE PROGRAM WILL ASK YOU FOR SPECIFIC INFORMATION. AFTER EACH QUESTION IT WILL TYPE A PERIOD INDICATING IT IS READY FOR YOUR ANSWER. TYPE IN YOUR ANSWER USING DECIMAL POINTS WHENEVER APPROPRIATE. YOU MAY CORRECT ANY MISTAKE BY TYPING '@' TO ERASE YOUR LAST DIGIT AND THEN TYPING THE CORRECT DIGIT. WHEN YOUR ANSWER IS COMPLETE HIT RETURN.

IF YOU DO NOT UNDERSTAND A QUESTION TYPE '?'.

WHAT IS THE MEAN OF THE DISTRIBUTION UNDER THE NULL HYPOTHESIS?
.800.00

WHAT IS THE MEAN OF THE DISTRIBUTION UNDER THE ALTERNATIVE HYPOTHESIS?
.825.00

WHAT IS THE VARIANCE OF THE DISTRIBUTION UNDER THE NULL HYPOTHESIS?
.600.00

WHAT IS THE VARIANCE OF THE DISTRIBUTION UNDER THE ALTERNATIVE HYPOTHESIS?
.700.00

WHAT IS THE REWARD FOR CORRECTLY SELECTING THE NULL HYPOTHESIS?
.1200.00

WHAT IS THE REWARD FOR CORRECTLY SELECTING THE ALTERNATIVE HYPOTHESIS?
.2000.00

```
WHAT IS THE REWARD FOR INCORRECTLY SELECTING THE NULL HYPOTHESIS?
:
.-32000@.0

WHAT IS THE REWARD FOR INCORRECTLY SELECTING THE ALTERNATIVE
HYPOTHESIS?
:
.-5000.00

WHAT IS THE APRIORI PROBABILITY THAT THE NULL HYPOTHESIS IS TRUE?
:
..5

THE .05 ALPHA-LEVEL DECISION RULE IS:
ACCEPT THE NULL HYPOTHESIS
IF THE SAMPLE IS LESS THAN:                    840.294
THE BETA-LEVEL IS:                             0.718387
THE EXPECTED REWARD FOR THIS RULE IS:         -422.805

THE .05 BETA-LEVEL DECISION RULE IS:
ACCEPT THE NULL HYPOTHESIS
IF THE SAMPLE IS LESS THAN:                    781.477
THE ALPHA-LEVEL IS:                            0.775231
THE EXPECTED REWARD FOR THIS RULE IS:         -933.217

THE OPTIMAL DECISION RULE IS:
ACCEPT THE NULL HYPOTHESIS
IF THE SAMPLE IS LESS THAN:                    818.457
THE ALPHA-LEVEL IS:                            0.225576
THE BETA-LEVEL IS:                             0.402334
THE EXPECTED REWARD FOR THIS RULE IS:          256.979

DO YOU WANT TO CONTINUE? TYPE 'YES' OR 'NO': .9
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END