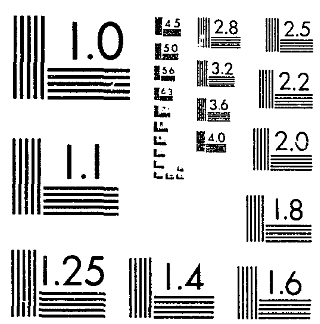

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

A METHOD OF PROJECTING ATTRITION OF SWORN PERSONNEL

Illinois Department of Law Enforcement
Division of Administration

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ABSTRACT

Projecting attrition of personnel, particularly retirement, by use of a survey of officers eligible for retirement does not provide adequate information. Expectations and subsequent actions vary too greatly. On the other hand projections derived from least squares analyses of attrition do not account adequately for variations in deaths, resignations, or retirements. In order to project attrition for the Illinois Department of Law Enforcement, a probabilistic model has been developed. This model incorporates death, resignation or termination, and retirement. It operates on the premise of most likely occurrence derived from historic data. By using the model, projections can be made for several years. Further, these projections can be altered by incorporating into the probabilities the effects of changes in such controlling influences as the economy. This model has shown a good fit with historic trends and been used to help project needs for replacement of officers in the Department.

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A METHOD OF PROJECTING ATTIRITION OF SWORN PERSONNEL

INTRODUCTION

Planning for cadet classes to provide officers who will replace departing sworn personnel requires a method for projecting attrition. In the past, the Illinois Department of Law Enforcement (DLE) has used surveys to help determine which officers plan to retire. The results have been highly variable. Although persons state in response to the survey that they plan to retire within one, two, or three years, their actions have differed substantially. Projections based on surveys have resulted in errors and inadequate planning. Moreover, such surveys have not accounted for deaths and resignations or terminations which, historically, have represented fifty percent of the attrition. To offset these problems, the Bureau of Planning and Development has generated and implemented a mathematical model of attrition. This model is described in this paper.

A model such as described must account for historic attrition. Yet, it can not be based solely on a least squares analysis of changes over time. Such does not adequately account for the stochastic nature of retirements and resignations. Further, a model must have flexibility in describing outcomes to account for major perturbations in attrition. These perturbations may generate large temporary increases or decreases in resignations and retirements, e.g. the large decreases that occurred during the most recent recession. Finally, such a model must be probabilistic. It should generate the most likely result.

DATA USED FOR THE MODEL

The information used to compute probabilities derives from the personnel records of the Department. A decision was made to extract data for a period starting in 1976. This also represents the period after the Department had been reorganized. At that time, the investigative and patrolling functions were brought into a single agency.

Three forms of attrition are used: death, resignation, and retirement. Resignations include terminations because the line between a resignation and termination sometimes is finely drawn. Further, the number of terminations is too small for computing probabilities. Retirements also include "early retirements". The statutes require 25 years of service and 50 years of age before drawing retirement benefits. However, 25 years can be earned as early as age 46. In the past, retirements prior to age 50 had been considered as resignations. The model described below considers a retiring officer as one who has at least 20 years of service and who is age 46 or older.

Not included in the base is the attrition of cadets. The computations apply only to those officers who have graduated from the training academy and begun patrol at the rank of Trooper. This decision is made because attrition of cadets results from a different set of reasons than attrition for officers on duty.

To perform the computations required, two sets of data need to be maintained. Because this model has been transferred to a microcomputer, these sets of data are considered as data files. The first contains a summary of attrition. The following information must be available:

1. Reason for departure (death, resignation, or retirement)
2. Month and year of birth
3. Month and year of start of service
4. Month and year of departure

The second file contains a current roster of personnel showing their date of birth and date of start of service. For use in this model, the DLE file is divided into uniformed and investigative personnel. This division allows separate accounting for differences in rates of resignation dependent upon the type of service (uniformed versus investigative) performed. (There are also differences in rates of retirement. However, so few persons have retired from the investigative branch that all personnel are combined for projecting retirements.)

If the number of departures was larger than the 600 that have occurred since 1976, further division of the historic data into rank, for example, would be useful. Generally, a higher ranking officer is older and has more years of service than a lower ranking person at retirement. They also are less likely to resign.

In the paper, the file containing data covering departures of sworn personnel is referred to as the "historic file". All information covering those who are currently employed is contained in the "current file".

METHODOLOGY

Matrix of Currently Employed Officers

All projections in the model are based on a matrix of currently employed officers, given both age in years (age) and years of service (tenure). As attrition is projected, the number of persons in this matrix decreases. Age is shown in rows and is described by the subscript i. The minimum age for sworn personnel which is 21 years and is noted as i = 21. A maximum age, identified as i_{max}, will be assumed to be 60 years; although, any year can be substituted.

Length of service is represented by columns and is identified by the subscript j. The first year of service is j = 1. The maximum, j_{max}, is assumed to be 40 years of service. The maximum age generally is less. After 30 years of service, there is no increase in pension; most officers retire once they reach 30 years.

The age and tenure of every employed officer can be computed at any given month and year (rounded up to the nearest year) and frequencies accumulated in the appropriate cells of the matrix. The number of officers in any cell will be called O_{ij} where i varies from 21 to 60 and j from 1 to 40. In addition, a total for each row and column is computed for use in the model. These totals are:

$$O_{.j} = \sum_{i} O_{ij} \tag{1a}$$

$$O_{i.} = \sum_{j} O_{ij} \tag{1b}$$

$$O = \sum_{i} \sum_{j} O_{ij} \tag{1c}$$

where:

O_{ij} - Number of officers at age i and with j years of service

When projections are made for multiple years, each of the O_{ij} in the matrix is reduced by attrition computed for the previous year. New officers may be added. Such additions always occur under the column j = 1 because there is no lateral transfer into DLE. For each succeeding year of projection, both age and tenure is increased simultaneously by 1 year. An officer who is 40 years old and has 12 years of service in 1984 will be 41 years old and have 13 years of service in 1985. Each successive year included in the computations then follows the same pattern as shown in Equation (2).

$$O_{i+1,\, j+1} = O_{ij} \tag{2}$$

for all i and j

Probability of Death

The probability of attrition resulting from death is computed for age only. Because few officers die while employed, there is not enough information to compute a probability given both the age and length of service of individuals. Standard tables could be used. However, using historic data recognizes the possibility that deaths of sworn personnel could differ from those in the population as a whole.

Probabilities computed for this model are cumulative by age given that a death occurs. The probability of death at any age, i, is equal to the sum of deaths from age 21 through age i, divided by the total deaths from age 21 through i_{max}. This is demonstrated mathematically in Equation (3) and an example is shown in Figure 1.

$$P(D)_i = \sum_{k=21}^{i} D_k / \sum_{k=21}^{i_{max}} D_k \qquad (3)$$

where:

$P(D)_i$ - Cumulative probability of death at any age i, given that a death occurs

D_k - Actual number of deaths at each k age from 21 through i or i_{max}

i_{max} - Maximum number of years of age for any officer, for this model 60 years

FIGURE 1

SAMPLE OF PROBABILITY OF DEATH

Age	Number of Deaths	Cumulative Total	$P(D)_i$ Probability
21	0	0	0.00000
22	1	1	0.02632
23	0	1	0.02632
24	2	3	0.07895
...	...	...	...
60	0	38	1.00000

The second part of the computation is the expected number of deaths in any year. Because the number of officers employed from year to year varies, the model uses an average rate of death. For each year of data in the historic file, the number of deaths is divided by the average number of officers employed (in thousands). This yields an annual rate per 1,000. The model used for DLE bases the count on those officers employed at the midpoint of the year. If the base contains a partial year, the number of deaths are annualized. All rates are

summed; an average rate and deviation then is computed. This average rate is used in subsequent calculations.

$$D'_y = 1000 \, a_y \, D_y \, / \, O_y \qquad (4)$$

where:

D'_y - Rate of deaths in year y per 1,000 officers

a_y - Adjustment for partial year from 12/m, where m is the number of months of data represented by that partial year. For full years, $a_y = 1$

D_y - Number of deaths in year y

O_y - Average number of officers in year y

The average rate and deviation.

$$\overline{D'} = \sum_y D'_y \, / \, Y \qquad (5a)$$

$$s_{D'} = \sqrt{\sum_y {D'_y}^2 \, / \, Y - \overline{D'}^2} \qquad (5b)$$

where:

$\overline{D'}$ - Average rate of death for all years.

D'_y - Rate of deaths in year y, from equation (4).

Y - Number of years of data.

$s_{D'}$ - Standard deviation in the rate of deaths.

Projecting Deaths for Any Year

The projection of deaths for any year is dependent upon the number of persons currently employed. A two-step process is used to make the projection. First is computed the expected number of deaths. This will be equal to the average rate plus or minus a normally distributed, but randomly selected, deviation times the number of officers employed in thousands. The resulting expected number is rounded to the nearest integer.

$$\hat{D} = (\overline{D'} + z\, s_{D'})\, O / 1000 \qquad (6)$$

where:

$\hat{D}$ - Expected number of deaths (rounded to the nearest integer)

$\overline{D'}$ and $s_{D'}$ - Average and deviation in the rate of deaths, from equations (5a and 5b)

O - Number of officers currently employed, from equation (1c)

z - Normally distributed z score which can be computed from an algorithm

$$z = \mathrm{COS}\,(2\pi r_2) \sqrt{-2 \ln r_1} \qquad (7)$$

where:

ln - Natural log

r_1, r_2 - Random numbers lying between 0.0 and 1.0

π - 3.1415962 (pi)

Ages are then computed and matrix of currently employed officers decreased. The age (i) at death is based on the cumulative probability of death

and selected from a normally distributed random number lying between 0.0 and 1.0. The tenure (j) is selected randomly. If there is at least one frequency ($O_{ij} \geq 1$) in the selected cell, that frequency is decreased by one. This is repeated until $\hat{D}$ is reached. The process for this is described below:

1. Obtain a random number lying between 0.0 and 1.0. Compare this number to the cumulative probability $P(D)_i$ (from Equation 3) at each age i until the cumulative probability exceeds that random number. This is the age i selected. If there are no frequencies in the row $O_{i.}$ for that age, select another random number and compute a new age. Figure 2 summarizes the process.

2. Within that age, the year of service, j, is determined by computing a random j which lies between 1 and 40. If O_{ij} is greater than zero, reduce O_{ij} by one. Otherwise, repeat step 2 until an O_{ij} greater than zero is found. Reduce $O_{i.}$ by one. Reduce the expected number of deaths, $\hat{D}$, by one.

3. Repeat steps 1 and 2 until $\hat{D}$ is equal to zero.

Probability of Resignation

The projection of resignations is similar to that performed for deaths. Because there are more resignations to use as a base, both age and tenure are given probabilities of occurrence. Sufficient information is not available to compute a joint probability given both age and tenure. Further, the probabilities of resignation and termination are computed for all uniformed personnel as a group. However, the average rates of resignation differ between the uniformed and

FIGURE 2

COMPUTATION OF
ATTRITION FOR
DEATHS*

1. Select a random number between 0.0 and 1.0, e.g. 0.421

2. Compare to cumulative probabilities of death.

Age	Cumulative Probability
....	....
32	.3618
33	.3814
34	.4058
35	.4270
36	.4479

3. At i equals 35, the cumulative probability of death is 0.4270 which just exceeds the random number, 0.421. Therefore, one death could be expected at age 35.

4. If there are no frequencies at $O_{35,.}$ repeat steps 1 through 4.

5. Select a random number from 1 through 40, e.g., 26.

6. If $O_{35, 26}$ is greater than or equal to 1 decrease $O_{35, 26}$ by 1 and $O_{35,.}$ by 1. If there are no frequencies repeat step 5 until a frequency is found.

7. Decrease $\hat{D}$ by 1.

8. Repeat steps 1 through 7 until $\hat{D}$ is equal to zero.

*Similar procedure is followed for resignations.

investigative branches. It is the average rate for each branch which serves as the base for projections. For this paper O and O_{ij} will stand for either branch as will the probabilities and expected rates which are described by the letter T. All computations are the same regardless of the branch selected.

There are two probabilities: resignation given age and resignation given tenure. Each is computed separately using i for age and j for years of service.

$$P(T)_i = \sum_{k=21}^{i} T_k \Big/ \sum_{k=21}^{i_{max}} T_k \qquad (8a)$$

$$P(T)_j = \sum_{k=1}^{j} T_k \Big/ \sum_{k=1}^{j_{max}} T_k \qquad (8b)$$

where:

$P(T)_i$, $(P(T)_j$ - Probabilities of resignation given age i and given tenure j.

T_k - Actual resignations at either the kth age or kth year of service, starting from k = 21 for age and k = 1 for years of service.

i_{max}, j_{max} - Maximum age or tenure.

The computation of expected resignations then follows the same procedure as for deaths. Substitute the variable T for D in equations (4), (5a), and (5b). All other variables are the same. The average rates and deviations are expressed as: $\overline{T'}$ and $s_{T'}$ (in the same manner as equations 5a and 5b).

Projecting Resignations for Any Year

This procedure is similar to that used for projecting deaths except that both the tenure and age used for projecting resignation are derived randomly from the cumulative probabilities. Further, the computations are performed separately for uniformed and investigative branches. The first computation to be made yields the expected number of resignations, $\hat{T}$. This is found by substituting $\overline{T'}$ and s_T for $\overline{D'}$ and s_D in equation (6). Then, a resignation is computed for any i, j cell as described below:

1. Determine the age by generating a random number between 0.0 and 1.0 and comparing that number to the cumulative probability of resignation at age i, $P(T)_i$ from Equation (8a). If there are no officers in that row, $O_{i.}$, repeat the process. This was demonstrated for deaths in Figure 2 and is the same for resignations.

2. Compute the tenure at resignation by the same process as in step 1, using instead $P(T)_j$ from equation (8b) instead of $P(T)_i$. If O_{ij} is zero, repeat step 2 until an O_{ij} is found that is greater than zero.

3. Reduce both O_{ij} and $O_{i.}$ by one. Also reduce $\hat{T}$, the expected resignations, by one.

4. Repeat steps 1 through 3 until $\hat{T}$ is equal to zero.

Because individual submatrices were made for uniformed and investigative branches, a $\hat{T}$ has been computed for the uniformed officers and for the

investigative officers. The reduction of O_{ij} and $O_{i.}$ should occur both in the appropriate submatrix as well as in the main matrix.

Probability of Retirement

Computation of retirement probabilities is based on the premise that by i_{max} or j_{max} one hundred percent of all officers will be retired. Thus, the probability of retirement at any other point in the matrix given age i and tenure j will be less than or equal to 1.0.

Let Z_a stand for the sum of frequencies along any diagonal of the retirement matrix which starts at years of service equal to 1. Therefore, the subscript "a" represents a starting age, e.g., 21 at the first year of service (j = 1). Progression in the matrix is by i + 1 and j + 1 simultaneously, e.g. 21 and 1, 22 and 2, etc., until the maximum age, i_{max} or maximum tenure, j_{max} is reached. For the Department of Law Enforcement, i_{max} always is reached first because retirement is mandatory at age 60 which represents 40 years of service (j_{max}) or less. Figure 3 shows a portion of a sample retirement matrix and some computations of probabilities. Because officers do not retire before age 46 all cells in the rows from i = 21 through i = 45 are equal to zero. All Z_a are computed.

$$Z_a = \sum_{k=a,\ m=1}^{i_{max},\ j_{max}} R_{km} \qquad (9)$$

where:

Z_a - Sum of the occurrences along a diagonal vector the retirement matrix starting at age a and years of service equal to 1 a - Age of the officer at the first year of service

R_{km} - Number of retirements in any k, m cell where k = a and m = 1 to start. Both k and m increase by 1 simultaneously until i_{max} or j_{max} is reached, (for DLE i_{max} always is reached first): $R_{24,\,1} + R_{25,\,2} + \ldots R_{i_{max},j}$

FIGURE 3

RETIREMENT MATRIX

Age at Retirement (i)	Years of Service (j)											
	1	...	20	21	22	23	24	25	26	27	28	29
21	-	...	-	-	-	-	0	0	0	0	0	0
...	...	...	...	...	...	...	...	...	...	...	...	...
47	-	...	-	-	-	-	0	1	1	0	0	0
48	-	...	-	-	-	-	0	0	0	0	0	0
49	-	...	-	-	-	-	0	0	1	1	0	0
50	-	...	-	-	-	-	0	1	0	1	1	0
51	-	...	-	-	-	-	1	0	1	0	0	0
52	-	...	-	-	-	-	0	2	0	2	0	1
53	-	...	-	-	-	-	0	3	2	1	0	1
54	-	...	-	-	-	-	0	2	3	3	1	0
55	-	...	2	1	1	0	1	3	4	5	2	1

The probability of a retirement at a given age and tenure is equal to the sum of retirements along the vector starting at age a at the first year of service up to age i of retirement and tenure j divided by the sum of all frequencies in that vector Z_a. This is shown in equation (10). Both the age and tenure increase by 1 simultaneously. This also can be seen in Figure 3.

$$P(R)_{ij} = \sum_{k=a,\,m=1} R_{km} / Z_a \tag{10}$$

where:

$P(R)_{ij}$ - Probability of retirement at age i and tenure j

R_{km} - Number of retirements in any cell. The summation in this case starts at k = a and m = 1, each subscript increasing simultaneously by 1 until the i,j is reached, e.g., $R_{29,\,1} + R_{30,\,2} + \ldots + R_{i,\,j}$

Z_a - The sum of the diagonal starting at age a with tenure equal to 1

Examples:

$$Z_{27} = (R_{27,\,1} + \ldots + R_{53,\,27} + R_{54,\,28} + R_{55,\,29} + \ldots + R_{60,\,34}) = 3$$

$$P(R)_{54,\,28} = (R_{54,\,28} + R_{53,\,27} + \ldots + R_{27,\,1})/Z_{27} = 2/3 = 0.6667$$

$$Z_{29} = (R_{29,1} + \ldots R_{52,24} + R_{53,25} + R_{54,26} + R_{55,27} + \ldots + R_{60,32}) = 11$$

$$R(R)_{54,\,26} = (R_{54,\,26} + R_{53,\,25} + \ldots + R_{29,\,1}) / Z_{29} = (3 + 3) / 11 = 0.5455$$

Where the number of retirements are relatively few, the probability of retirement can be computed from a weighted rate. One method is to count the frequencies for the current tenure plus and minus one year (j, j + 1, j - 1). The computation is similar to equation (10).

$$P(R')_{ij} \;\; (\sum R_{k,\,m-1} + \sum R_{k,\,m} + \sum R_{k,\,m+1}) / (Z_{a-1} + Z_a + Z_{a+1}) \tag{11}$$

where:

$P(R')_{ij}$ - Weighted probability of retirement at i and j

R_{km} - Number of retirements in any cell at ages k and tenure m - 1, m, and m + 1

Z_a - Number of retirements for the vectors including $R_{k, m-1}$, $R_{k, m}$, and $R_{k, m+1}$ starting at a - 1, and a + 1

Note: Where a = 21 there are no frequencies along a diagonal starting at a - 1 (a person must be age 21 to join the sworn personnel). Therefore, only a, and a + 1 are used.

Projecting Retirements for Any Year

The projection of retirements is a single step unlike the two steps required for projecting deaths and resignations. The expected number of retirements at any i, j age and tenure is equal to the number of officers currently employed (after deleting deaths and resignations) times the probability of retiring at that age and tenure. The equation below is performed for all i and j.

$$R_{ij} = O_{ij} \, P(R)_{ij} \qquad (12)$$

Note: $P(R')_{ij}$ may be substituted

OPERATION OF THE MODEL

Weighting the Resignations and Retirements

Computations can be performed rapidly on a computer. What must be maintained is up-to-date historic and current files. With the historic file, the number of months in each year for which data are collected must be recorded. This allows adjustment of the rates of attrition where partial years are collected.

During initial testing and operation of the model on a micro computer, two elements were found to be important: a weighting of historic activity such that deaths, resignations, and retirements in which the current years are more heavily weighted than in earlier years, and the ability to increase or decrease the probability of resignations and retirements to reflect expected changes in such influencing exogenous factors as the economy. Weighting of historic data was accomplished by assigning all years a weight equal to the year in which the action such as a death took place minus the current year plus a maximum weighting factor. Where the result is less than 1, it is set equal to 1 (as shown in equation 13). This weight is then multiplied by the number of deaths (D), resignations (T), and retirements (R) in each year prior to computing the appropriate equations.

$$w_y = Y_{yr} - Y_{cur} + w_{max} \qquad (13)$$

such that: $1 \quad w_y \quad w_{max}$

where:

w_y - Weight for the year to be weighted, y, limited to w_y = 1

Y_{yr} - Year in which the attrition occurred

Y_{cur} - Year to be weighted

w_{max} - The maximum weight to be assigned. Using w_{max} = 4 in the examples below

w_{83} = 1982 - 1983 + 4 = 3

w_{79} = 1978 - 1983 + 4 = -1 (set equal to 1).

The second adjustment that should be incorporated affects the probability used for retirement. For a given age, i, and service, j, the computed probabilities P(R) normally would be used. However, if conditions are such that officers are delaying retirements by extending their service and are older at retirement, P(R) may not represent the appropriate probability of retirement. Therefore, the model should allow for increases or decreases in this probability. Empirical analysis of retirement data has shown that a delay in retirement by one year could be simulated by using the probability of retirement along the diagonal one year earlier than the age for which retirement is being computed. For example, retirement at age 55 with 25 years of service can be represented by using the probability of retirement at age 54 with 25 years of service.

$$\hat{R} = P(R)_{55,\,25}\, O_{55,\,25} \text{ (expected under normal conditions)} \qquad (14a)$$

$$\hat{R}' = P(R)_{54,\,25}\, O_{55,\,25} \text{ (delayed retirement)} \qquad (14b)$$

where:

$\hat{R}$ - Expected number of retiring officers, normal conditions

$\hat{R}'$ - Expected number of retiring officers, retirement delayed one year

$P(R)_{55,\,25}$ - Probability of retirement at age 55 with 25 years of service

$P(R)_{54,\,25}$ - Probability of retirement at age 54 (one year earlier) with 25 years of service

$O_{55,\,25}$ - Number of officers currently employed at age 55 with 25 years of service

Computation of Attrition

The operation of a model starts at some base year and month. The age and tenure matrix contains all officers on duty at that starting period. The period used as a base for projection of attrition arbitrarily is set at a point six months later. This represents a point at mid-year for officers currently employed. For example, given a starting date of January 1984, the model works at the point July 1984. Once the historic data and current list of officers is on file, only three parameters are required.

1. The maximum weighting factor, w_{max}, which can range from 1.0 upward. It has been shown in equation (13). The w_y are applied to deaths (D), resignations (T) and retirements (R) before they are used in the equations (3), (8), and (9).

2. The adjustment for early or delayed retirement in terms of plus or minus t years where t is positive when the expected age of retirement increases and is negative when the expected age decreases. The probability of retirement is expressed as $P(R)_{i-t,j}$

3. Maximum of age and tenure at retirement. In this last situation, a limit on the age of retirement and in years of service is established. The probability of retirement $P(R)_{ij}$ equals 1.0 when i or j is equal to or greater than the maximum ages given:

$$P(R)_{ij} = 1.0 \quad \text{when } i \text{ or } j \geq i_{max} \text{ or } j_{max} \qquad (15)$$

The model can be run for one or more years. Output from the model should show the number of officers at the start of each year, a summary of departures by death, resignation, and retirement, and the number of officers remaining at the end of the year. The type of output available from the computer is shown in Figure 4. Optional is a list by age showing the numbers of deaths, resignations, and retirements for each age.

A final option should be the ability to increase the number of officers on duty. This accounts for the addition of new officers. All new officers have a tenure of 1 year. The distribution in ages of these new officers can be computed based upon a table showing the percentage distribution of new officers by age from historic data. Such additional officers should be included prior to projecting deaths, resignations, and retirements. They become a permanent part of the matrix in that run of the model.

Computer Operation

A program written in BASIC is available for the APPLE II Plus microcomputer used by DLE Bureau of Planning and Development. The model has been tested against prior departures and found to be in close agreement. It also has been used for the past two years to project attrition to help in preparing the departmental budget. At this point, the first fiscal year for which projections were made has not been completed. However, the actual number of departures for all three elements, death, resignation, and retirement, are running relatively close to the projections.

FIGURE 4

SAMPLE OUTPUT

Starting	Year - 1983 Month - July	
Number of Officers		1,840
	Added	0
	Total - 7/83	1,840
Attrition		
	Death	4
	Resignation	28
	Retirement	115
	Total	147
Officers at end of year - 6/84		1,693
Age at Retirement Increased by 1 year.		
Age at Resignation Unchanged.		
	Maximum Age	60
	Years of Service	32
	Weighting Factor	4

(Optional)
Summary For 7/83 to 6/84

		Attrition		
Age	Death	Resignation	Retirement	Total
21	0	2	0	2
...	...	...	...	...
38	1	2	0	3
...	...	...	...	...
54	0	0	4	4
...	...	...	...	...
Total	4	28	115	147

SUMMARY

The model for attrition for the Illinois Department of Law Enforcement is based on the probability of occurrence of three forms of attrition, death, resignation, and retirement occurring. It represents a simple mathematical treatment of the historic trends in each of these categories. In each case, projections are made given that the event occurs. For deaths and resignations, an expected frequency of occurrence is computed based upon average rate and deviation in that rate. Retirements are computed from the matrix of officers currently employed. Such a computation assumes that all those currently employed will retire by some specific date (expressed in terms of age or length of service).

The model is quite useful where for projecting retirement in a closed body with a definite limit in retirement. Thus, it is most appropriate for such organizations as the armed forces, police, and fire. If it were to be applied to other organizations, more investigation would be required in all attrition to determine if the mathematical treatment described in this paper is useable.

END