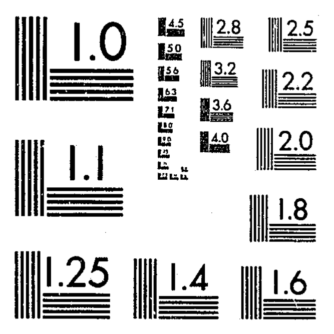

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A MODEL FOR WIDE-AREA PATROL POLICY EVALUATION

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Errata For

"A Model for Wide Area Police Patrol"

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Page 13, line 2: insert "for each replication" after "dollars."

Page 23, line 7: replace "4.16" by "4.10a, 4.15".

Pages 22, 25, 26, and 34: replace by new pages.

Page 50, line 14: replace "24" by "24.3".

line 15: replace "15" by "14.5".

replace "15.2" by "5.2".

Page 63, line 22: insert "or" between "little" and "no".

Page 74, lines 3 and 8: replace "variance" by "standard deviation".

Page 77, heading of column 2: replace "car" by "calls".

heading of column 4: replace "variance" by "standard deviation".

Page 91, line 1: replace "5" by "0.5".

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ACKNOWLEDGEMENT

The research reported on here was supported by a contract from the National Institute of Justice. Cooperation of personnel from the Washtenaw County (Michigan) Sheriff department allowed us to develop a realistic rural patrol model and helped us to confirm crucial assumptions about the reasonableness of using "patrol-switch probabilities" as a means of characterizing patrol policy. The department also provided us with data that allow a full-scale test of the resulting model. In particular Sheriff Minick graciously gave us ready access to his patrol planners, data, and communications staff. Commander Ron Schebil and Lt. Robert Marcoum were always available for valuable discussions.

Part of the computation reported on here was performed at the University of Michigan Computer Center.

1. SUMMARY

Police patrol allocation in *urban* areas has been extensively studied in the past, using a variety of quantitative and management techniques. Virtually all of these studies have been based on the assumptions that the region being served has a high population density and that immediate response is necessary for most calls for service. Police forces in *rural* or *suburban* areas have been unable to adapt these models due to substantial differences between the high density (low area) and low density (wide area) environments.

In this study we have formulated and developed a new model that is consistent with conditions in large, low population regions. Termed SWAP (Strategies for Wide Area Patrol), it has been tested using data and advice from the Washtenaw County (Michigan) Sheriff Department. The SWAP model should be useful for evaluating rural patrol allocation policies in thousands of county sheriff and small police departments across the country. It should prove particularly useful for maintaining effective patrols when resources made available to the policing agencies are reduced, or in evaluating the cost-effectiveness of providing contract police patrol services to unincorporated areas or small towns and villages.

The model currently has been implemented on a large main frame computer, in a form suitable for further development or use by patrol planners with access to operations analysts or software professionals. Implementation on a microcomputer, for wide-scale distribution, although an original secondary goal of the project has not been effected. We expect, however, that if the model is found to be useful to rural patrol planners, microcomputer implementation will be feasible.

2. STATEMENT OF PROBLEM

Police patrol forces operate in rural environments in thousands of counties and small towns across the country. Like fire departments and emergency ambulance services, they must provide the community with prompt and effective emergency service whenever it is requested. Deputies in patrol cars respond to calls for service throughout a region of primary responsibility, called the "response area" or "beat". The design of beats and the allocation of patrol cars to the beats are important decisions that face every police and emergency service department. The department's efficiency is strongly dependent on these decisions. Extensive studies have been made of the police patrol process in dense population *urban* areas, but the specific problems of wide-area patrol allocation have been rarely explored. Some aspects that make wide-area patrol different from urban patrol include: low population density, poor access to certain parts of the region being covered, and many distinct types of calls for service. This report addresses these considerations by developing a model -- termed SWAP (Strategies for Wide Area Patrol) -- suitable for use by rural patrol forces.

2.1. Comparison of Rural and Urban Patrol Models

Larson's [1] extensive studies first formalized the urban police patrol problem in terms of queueing models and optimization criteria. He developed a travel time model, patrol allocation algorithm and simulation model based on the assumptions of high population density areas. Kolesar and Blum [2] later developed a "square root law" to represent travel times for use in fire engine response areas. Based on these studies, Chaiken and others at the Rand Corporation developed three models. The first, PCAM (_P_atrol _C_ar _A_llocation _M_odel) [3] is primarily used to determine the number of units to allocate to individual pre-defined sectors. A second model, the Hypercube

Queueing Model [4,13], is used to determine the design of these sectors or "beats" within a larger region. The third, a simulation model [5], was also developed and used for the specific data and geography of New York City. Other analyses of urban patrol systems include the LEMRAS model developed at IBM [6], a UCLA model created for the Los Angeles Police Department [7], and a beat optimization model by Bammi [8].

All of the above models assume that large computing resources are available to the user. While large city police departments may have these resources, smaller departments rarely have sophisticated computers or staff available to them. Heller, et al. [9], have developed algorithms for planning with low-cost computer processors, but, again, their methods depend on the attributes of an urban environment.

There are numerous basic differences between urban and rural patrol. Urban patrol involves travel on a road grid with a high density of road mileage within each sector and small response times with respect to total service time. However, in rural areas, travel time is often the major component of total service time. Rural areas generally have poor road access, and travel time is often highly dependent on the location of the vehicle relative to a limited number of major thoroughfares. This property immediately excludes the use of models (such as the Hypercube Queueing Model) which rely on the assumption that travel time is small compared to total service time. Any model that is to be useful for rural analysis must include this heterogeneity of travel times across the region.

Although the PCAM and Hypercube Queueing Models have been used in many police departments [10], to our knowledge all of these uses have been in an urban environment. The great majority of the users have been city police departments. Each of the relatively few county sheriff department users has

apparently used the models only for patrol analysis in the urbanized areas within those counties. One analysis that relates specifically to the rural patrol problem is an English report [11] that gives only general guidelines for rural patrol manpower requirements. Within our own experience, an implementation of PCAM was attempted a few years ago in Washtenaw County, but for the reasons discussed above it produced results that were not useful to the county sheriff department's wide area patrol planning. In particular, PCAM provides information only on the number of cars needed in a region, but not on how (when and where) they should patrol in that region.

The rural setting also complicates the redeployment and repositioning of vehicles when one unit must "fill in" for a busy patrol car in an adjacent sector. In urban areas, several patrol cars are generally assigned to each beat, and beats are close together. This makes the backing up of a busy car relatively simple. The size of the rural regions significantly alters such behavior and requires a new allocation strategy that an urban force rarely needs.

Another consideration required in the rural context involves distinguishing among different types of calls for service. This is necessary to allow the dispatchers to pre-empt patrol cars from low priority calls to free them more quickly for service of emergency calls. The Hypercube Queueing Model and PCAM do not allow for pre-emptive queue disciplines. Longer travel times and fewer cars assigned to beats in rural areas exacerbate the effect of pre-emptive dispatching and limit the utility of models that do not consider it.

2.2. Generic Description of Wide Area Patrol Environment

The basic model of wide area patrol is developed in Sections 4 and 5. This model requires descriptive elements of the geography and patrol procedures of the area to be modelled. Useful descriptive terms are defined below.

a) The *jurisdiction* is the entire area to be covered by the patrol force. In this study the jurisdiction is Washtenaw County, Michigan, but it could be a portion of a county, a park, a subset of a State Highway network, or a suburban area.

b) A *region* is the smallest useful subdivision of a jurisdiction. It should be possible to associate the following attributes with each region:

- a geographical location (either as a point or a portion of area);
- a rate of calls for service;
- a measure of the service time for calls (if different from that of the entire jurisdiction);
- one or more patrol units with responsibility for responding to calls for service.

In this study, the regions are the twelve rural *townships* in western Washtenaw County.

c) A *patrol unit*, generally a squad car, patrols a specified beat and is available for dispatch to a call for service. For the purposes of modelling the patrol units are considered to be distinguishable from one another.

d) *Calls for service* ("CFS") are requests for some kind of in-person, on-site activity on the part of the patrol force, (see Larson [1]). For use in the models developed here, the assumption is made that hourly rate statistics are available for *each* region for the following three types of CFS:

- *routine* - calls which require an ordinary response by a patrol unit;
- *emergency* - calls which require a rapid, "lights-and-siren" response;
- *unfounded* - a routine CFS that turns out to be a false alarm, or one that otherwise requires essentially no time for servicing.

Appropriate use of the model is based on the presumption that calls for service that can be handled by *not* dispatching a patrol unit (i.e., by taking a report on the telephone) are not included in the call rates. Activities besides servicing CFS's that take up a patrol unit's time such as lunch breaks, delivering prisoners, and performing property checks are also excluded from the CFS rates.

A fundamental assumption made for the purposes of this study is that CFS rates are independent of the status or number of patrol units in a jurisdiction or region. This means that CFS rates are completely exogenous to the patrol policy. This assumption can be easily relaxed, however, to take into account such dependent calls as patrol initiated activities (Larson and McKnew [12]), or directed patrol-generated calls. In general it is assumed that CFS rates are readily available, or computable, from existing data sources.

e) *Travel time* distributions describe the likelihood of the possible time to travel between all pairs of regions. Obtaining these distributions is somewhat tedious, particularly if there are a large number of regions, yet they are essential for a realistic representation of the geographical features of a jurisdiction. Because of their importance, we describe in Appendix B a procedure that can be used to efficiently generate appropriate travel time distributions. The method involves subjective assessment of travel speeds and geometric computation of travel distances. Whether or not the procedures of Appendix B are used, the model requires an average travel time between all pairs of regions for responding to both routine and emergency calls for service. Average time to travel to routine and emergency calls within all regions is also required.

f) The *service time* for handling calls is assumed to be exponentially distributed, with an average time depending upon the region and type of call.

Unfounded calls are assigned appropriately small average service times.

g) *Dispatch procedures* built into the model are representative of most agencies responsible for wide area patrol. A single dispatcher receives calls from citizens or other law enforcement agencies. A unit responsible for responding to the region from which the call came is dispatched, if one is available. If no unit is available, and if the call is an emergency, any free unit in the jurisdiction may be dispatched. If no unit is available and the call is routine, then it is "stacked" in a "first-come first-served" queue to be serviced as soon as a responsible unit becomes available.

h) Each patrol unit is assigned a *coverage factor* (a number between 0 and 1) for every region in the jurisdiction, indicating that unit's responsibility for routine coverage in that region. A factor of 1 indicates that it is the only unit responsible for responding to routine calls in that region, a factor of 0 means it has no responsibility to respond, and a factor between 0 and 1 indicates shared responsibility with another unit.

2.3. Criteria Used for Policy - Making

A rural police department often faces a different set of objectives than urban police departments. For example, since average response time is longer in large areas with low population density, it may be more important to respond to serious crimes and accidents within a *desired* time than to minimize the *average* response time for all calls. This often leads to a formal or informal priority system for responding to calls.

With a priority queueing system, police effectiveness may be measured by average response time to "high priority" calls and by a different measure of effectiveness for lower priority calls. For example, the percentage of low priority calls answered within a suitable pre-arranged time interval may be an

appropriate measure of the police department's effectiveness.

Because of the unique concerns of rural police departments, we have concentrated on developing a model that will produce the following measures that could be used to evaluate patrol policies:

Travel time, which is defined as the time between departure of a patrol unit to respond to a CFS and arrival at the call's location. Since travel time is a random variable, appropriate measures are its expectations and cumulative distribution. It is useful to have these measures for each patrol unit and for each type of call.

Response time, which is defined as travel time plus any time during which the call was queued awaiting availability of a patrol unit. This is also a random variable, and it is desired for all units and types of calls.

Fraction of time each patrol unit spends *on patrol* in each region. When the duties of the police officer during patrol are specified by a supervisor, this time is referred to as *directed patrol*.

Fraction of time each patrol unit spends in each region (either on patrol or servicing calls).

Queue characteristics, including expected number of each type of call in queue, and fraction of time a queue exists, for each region and call type.

2.4. Policy Related Control Variables

In order to develop a model to help analyze various patrol policies, it is important to be able to represent a wide range of policy choices through a simple, understandable set of policy variables.

The general *dispatch procedure*, of course, is one aspect of policy that can be varied by structural changes in a model. In this study, we have chosen to allow two priority levels, "emergency" and "routine". Routine calls are assumed to be pre-emptable by emergency calls. (In Washtenaw County, "emergency" and "routine" calls are referred to as "immediate" and "expedite" -- see Appendix A).

The *distribution* of an individual unit in time and space is given in terms of a patrol-switch matrix X, and switch interval T, where,

x_{ij} = prob. { unit patrolling in region i will switch to region j at the end of the next interval of length T }/ T,
$i,j = 1, 2, \cdots N$.

Note that *each unit will have its own patrol switch matrix* X. The N^2 numbers in each X matrix are intended to represent the general instructions given by the patrol planner to each unit on how to patrol *in the absence of a CFS.* Thus, without any responses to calls, X itself could be used to calculate, for example, the average fraction of time spent by that unit in each region and the expected time spent on patrol in any region before going to the next.

The *coverage* matrix C represents the responsibility the units have for responding to calls in the various regions. Thus

c_{ij} = fractional responsibility unit i has for responding in region j,

where,

$$\sum_{i=1}^{N} c_{ij} = 1 \text{ for all } j \ .$$

This matrix is used to compute response probabilities. In particular the probability that an *available* (i.e. not servicing a call) unit i will respond to a

call in region j is $c_{ij} / \sum_i c_{ij}$, where now the sum is over *available* units.

3. PHILOSOPHY OF MODEL DEVELOPMENT

Whenever an analyst is asked to develop a mathematical model to help real decision making, there are always compromises to be made between: realism and solvability; data requirements and cost of collection; detailed results and gaining of general insight. In this section we briefly discuss these issues in the context of the approach we took to develop the models in Sections 4 and 5.

3.1. Analytical vs. Simulation.

Simulation is an extremely powerful modelling tool, most useful when trying to represent a real system that is characterized by complexity in the logical relationships among its various components. Police patrol in wide areas can be effectively simulated by any of a number of contemporary commercial simulation languages. To test the usefulness of simulation as a patrol policy planning tool, we developed a simulation model of wide area patrol specifically geared to Washtenaw County. Although the specific code used and the details of this SIMSCRIPT-based simulation, along with sample inputs and outputs are available from the authors, they are not included in this report. (Appendix C contains a brief discussion of the simulation model.)

On the basis of our experience, we recommend such a simulation *not* be used as the primary SWAP planning tool. There are a number of reasons for this conclusion:

a) Monte-Carlo simulations in general are useful only when interpreting "steady-state" results. These results in turn require large numbers of replications, or "runs" (starting with the same initial conditions) to provide reliable estimates of output measures of interest such as average response times or percent of time on patrol. Although we were able to obtain reasonable results after only a few hundred replications, to provide a high degree of statistical

confidence for these results, computer runs of thousands of replications, costing tens of dollars, would have been necessary for evaluating *each* combination of a patrol policy option and set of input parameters.

b) The computer software needed for developing and running these simulations is not necessarily available to many of the agencies who could potentially benefit by such analysis. Furthermore, there are few microcomputers available that have computational speed, storage capacity or even compilers to run these simulations in their present form.

c) A modest change in structure of the jurisdiction or policy options being simulated (e.g., addition of a region, allowing split responsibility for some cars, changing the priority scheme) would require a complete change in the simulation. Although this problem also holds for an analytical model, such changes are more easily implemented in the latter case.

c) The transient behavior of the system (i.e., the changing of performance measures over the length of a shift to another) is difficult and costly to simulate correctly.

Because of the above problems associated with simulation models, our efforts were concentrated on developing an analytical model that computes, as a function of patrol policy, the various performance measures of interest.

3.2. Steady-State vs. Transient Analysis

Since the major objective of our model is to provide decision makers with performance measures as a function of patrol policies, it is important to examine these measures closely. For example, consider the measure d = "fraction of time car 1 is available for directed patrol." Assume it is agreed and understood that directed patrol takes place whenever a car is not travelling to (or servicing) a call, engaged in some self-initiated activity, or otherwise occupied (e.g., in transporting warrants or prisoners, at the gas station, etc.). Suppose in addition that a car *always* spends the first half of a shift "available" and after four hours always becomes unavailable for the remaining four hours. Then $d = 1$ for the first half-shift, $d = 0$ for the second half-shift and $d = .5$ for the entire shift (the latter being valid if we interpret d to be the average of "fraction of time available for directed patrol" over the whole shift). This behavior becomes an issue of concern when a measure such as d is really a function of time, i.e., $d(t)$. If the measure is not relatively constant over time, then we need to know the specific times at which the decision maker is interested.

Steady-state analysis, on the other hand, both in analytical and simulation models, is based on the assumption that as t becomes "large enough", all measures such as $d(t)$ become constant. Use of a steady-state result then depends upon the assumption that t is indeed "large enough", an assumption that can be tested only by analysis of the transient (time dependent) form of the model.

The model we have chosen to develop for analysis of wide area patrol is essentially a steady state one. Our numerical computations showed that most measures of performance reach a steady-state value within a fraction of a typical patrol shift, quickly enough to allow us to argue that transient effects are not crucial. On the other hand, these particular results are data-dependent, and indeed are driven by the fact that in the examples we have examined the total call-for-service rates and service rates are reasonably large. For situations where this does not hold, a straightforward modification of the model allows the computation of transient performance measures.

3.3. Reality vs. Usefulness.

The analytical model presented here requires individual patrol units to be in one of a number of possible "states," corresponding to: patrolling, travelling to a call, servicing one of two types of calls, etc. In addition, allowance is made for the possibility that a call is queued (i.e., waiting to be serviced). The model could, of course, be extended to allow for distinguishing among more than two types of call, different travelling speeds depending upon time of day, distinction between traffic patrol and road patrol, etc. However, such extensions necessarily entail more computation, more data, and of course more possibilities for programming or conceptual errors in modelling.

The level of detail we chose for this study was dictated by two factors: data limitations and sensitivity of output measures. Our experience with Washtenaw County was that call arrival rates and service times--the fundamental numbers needed to "drive" the model--were only known (or more important, forecastable) to an accuracy of around 10%. This is certainly good enough for input into a general policy-making model, but not accurate enough to warrant the development of a more "realistic" multi-state model. In addition, we have chosen to represent the major feature of a patrol policy by the probabilistic switching process X. In the face of this useful but somewhat abstract depiction of what actually goes on, it did not seem sensible to insist, for example, upon a more precise travel time model, or to account for the extremely low probability events corresponding to multiple emergency calls in queue.

4. SINGLE PATROL UNIT MARKOV MODELS

The SWAP model we now describe is a Markov Process representation of calls arriving and being serviced by patrol units, or being queued to await attention by a patrol unit. The states of the process represent the different possible "snapshot" conditions that could represent the status of the system at any time. Although we will eventually *compute* output measures of interest by assuming the process is discrete (i.e., transitions between states occur at distinct points of time, separated by a length of time called a "transition period"), our initial model is posed as a continuous process for notational convenience.

Since the time to travel from one region to another to answer a call, or to perform directed patrol, represents a significant proportion of a patrol unit's activity, one of the states is an explicit "travel" state. This represents a significant difference from the high population density assumptions in such models as the hypercube model[4] where travel times are treated as negligible. In the models presented here, the assumption of exponentially distributed travel times is used to calculate travel *rates*. A semi-Markov model, which has the potential to represent a wide variety of travel time distributions, is discussed briefly in Appendix F.

In the development in this section, we describe situations for a *single patrol unit*. The combining of these models into multiple-unit patrols is presented in Section 5.

4.1. General Structure-All Calls Identical.

We first describe the model in the case where all calls for service are of the same type, possibly requiring different service times in different regions. In order to develop the model we need to define states and transition rates between these states. If N = the number of regions, in this simple case there

are $3N$ states, three for each region:

State	Description
$p(i)$	unit on patrol in region i;
$s(i)$	unit is in service in region i;
$t(i)$	unit is travelling *from* region i.

Note that the travel states $t(i)$ are indexed by the region from which the unit is travelling, and that travelling may be due to either responding to a call or switching to another region for directed patrol.

The transition rates among these states depend upon the system parameters:

λ_i = rate of calls in region i ;

μ_i = mean service time for a call in region i ;

t_{ij} = mean travel time from region i to region j ;

and upon the policy variables:

x_{ij} = rate at which the patrol unit switches from patrolling region i to patrolling region j .

The resulting transition rates for $i = 1, 2, \cdots N$ and $j = 1, 2, \cdots N$ are:

From	To	Rate
$p(i)$	$t(i)$	$\lambda \equiv \sum_{k=1}^{N} \lambda_k$
$p(i)$	$p(j)$	$\begin{cases} x_{ij} & , \quad i \neq j \\ 0 & , \quad i = j \end{cases}$
$s(i)$	$p(i)$	$\frac{1}{\mu_i}$

$$t(i) \quad s(j) \quad \lambda_j \Big/ \sum_{k=1}^{N} \lambda_k t_{ik} \ ,$$

with all non-listed rates being 0 .

The first three of these rates reflect straightforward transition events: an arriving call (from any region) causing the unit to travel; switching of patrol from one region to another; and completion of service in a region releasing the unit to patrol (in that region - by assumption).

The transition rate from $t(i)$ to $s(j)$ was set to satisfy two conditions:

a) The probability of going to region j from any travel state $t(i)$ should be λ_j / λ .

b) The expected time spent in the travel state $t(i)$ should be t_{ij} , weighted by the probabilities of going to region j (that is, λ_j / λ).

By defining

$$r_{ij} \equiv \text{rate}\left[t(i) \rightarrow s(j) \right] \equiv \lambda_j \Big/ \sum_{k=1}^{N} \lambda_k t_{ik} \ ,$$

we see that condition **a)** is satisfied, since

$$\text{prob.}\left\{ \text{going to region } j \text{ from } t(i) \right\} = r_{ij} \Big/ \sum_j r_{ij}$$
$$= \lambda_j / \lambda$$

and **b)** is satisfied since

$$\text{expected time spent in state } t(i) = \left[\sum_{j=1}^{N} r_{ij} \right]^{-1}$$
$$= \left[\lambda \Big/ \sum_{k=1}^{N} \lambda_k t_{ik} \right]^{-1}$$
$$= \sum_{k=1}^{N} t_{ik} \left[\frac{\lambda_k}{\lambda} \right]$$

Figure 4.1 shows a transition diagram for $N = 2$ regions.

4.2. Multiple Priority Calls for Service

Additional states can now be added for representing calls with different service rates and different priorities. Some calls, for instance, are "unfounded" and are discovered to require essentially no service time after arriving in a region. There is also generally a difference between the service time required for emergency calls (which demand immediate service) and routine calls that do not. This expanded model consists of $8N$ states. For each $i = 1, 2, \cdots N$, the states are:[1]

State	Description	Abbreviation[1]
$p(i)$	patrol in i;	PATR
$t(i)$	travel from i to a routine or unfounded call;	ETRV
$t^*(i)$	travel from i to an emergency call;	ITRV
$rs_0(i)$	service of a routine call in i with no calls waiting;	ESRV
$rs_1(i)$	service of a routine call in i with one call waiting;	ESVQ
$es_0(i)$	service of an emergency call in i with no calls waiting;	ISRV
$es_1(i)$	service of an emergency call in i with one call waiting;	ISVQ
$u(i)$	service of an unfounded call in i.	UNFS

Note that we have now included two queueing states for "stacked" calls. Since the probability of receiving a call while in such states is assumed to be low, only one call is allowed in the stack. The $t^*(i)$ travel state is used for travelling to an emergency call that pre-empts service of a routine call.

[1]These abbreviations are used in the specific version of the SWAP computer program for Washtenaw county. The notation is unfortunately potentially confusing since emergency calls are called "immediate" and routine calls are called "expedite", hence ITRV is "travel to immediate", etc.

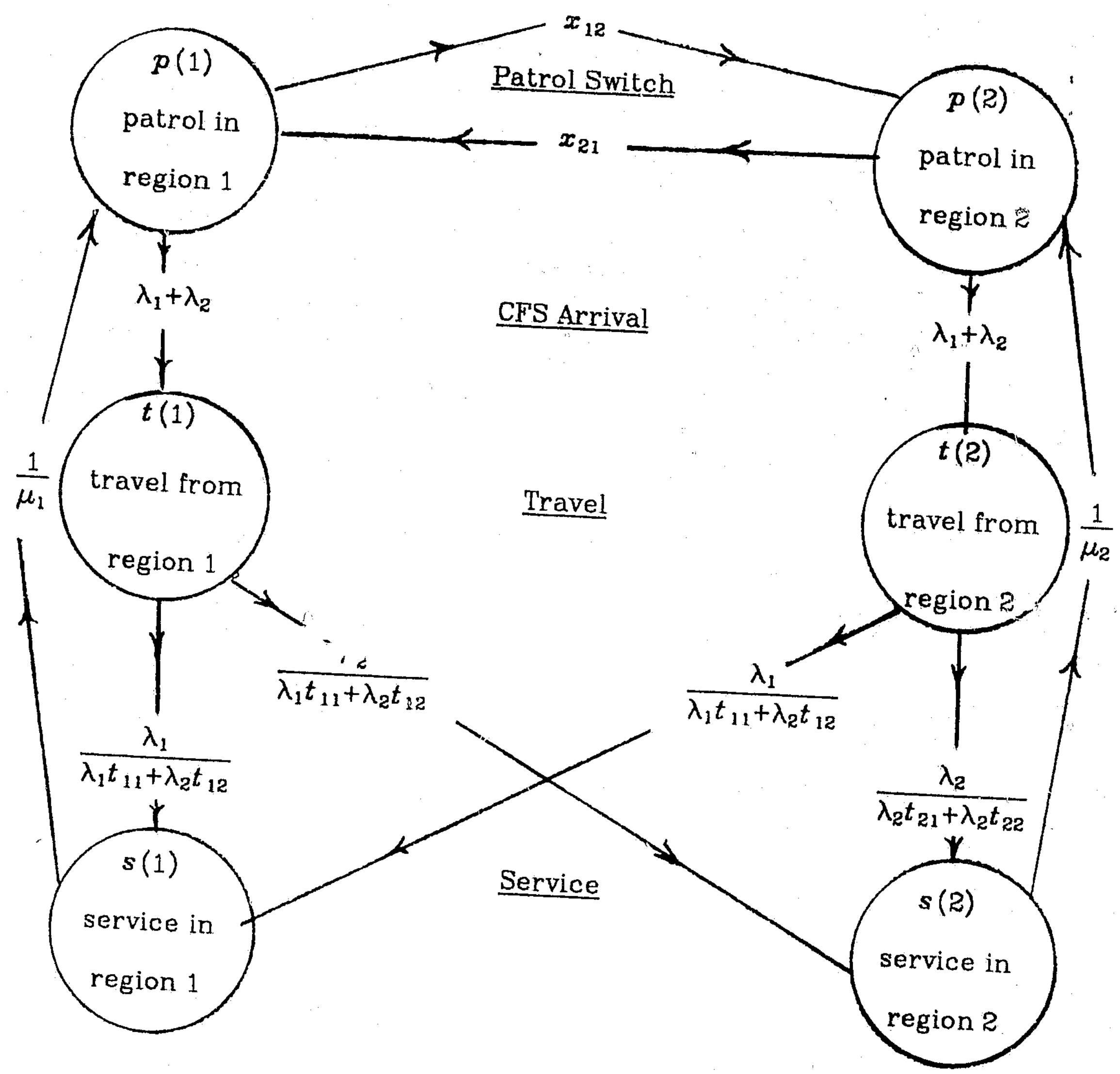

Figure 4.1
Markov model with $N=2$ regions and 1 patrol unit

In order to write transition rates for this extended model, we need to define some additional terms:

λ_i^1 = rate of emergency calls in region i;

λ_i^2 = rate of routine calls in region i;

λ_i^3 = rate of unfounded calls in region i;

μ_i^1 = mean service time for emergency calls in region i;

μ_i^2 = mean service time for routine calls in region i;

μ_i^3 = mean service time for unfounded calls in region i;

and, for $i = 1, 2, \cdots N$ and $p = 1, 2, 3$:

$$\lambda_i = \lambda_i^1 + \lambda_i^2 + \lambda_i^3 ;$$

$$\lambda^p = \sum_{i=1}^{N} \lambda_i^p ;$$

$$t_i^p = \sum_{k=1}^{N} \lambda_k^p \, t_{ik} ;$$

$$t_i = \sum_{p=1}^{3} t_i^p .$$

The transition rates then become, for all $i = 1, 2, \cdots N$ and $j = 1, 2, \cdots N$:

From	To	Rate	
$p(i)$	$t(i)$	$\lambda=\lambda^2+\lambda^3$	(4.1)
$p(i)$	$t^*(i)$	$\lambda=\lambda^1$	(4.2)
$p(i)$	$p(j)$	$\begin{cases} x_{ij} & i \neq j \\ 0 & i=j \end{cases}$	(4.3)
$t(i)$	$rs_0(j)$	$\lambda_j^2/(t_2+t_3)$	(4.4)
$t(i)$	$u(j)$	$\lambda_j^3/(t_2+t_3)$	(4.5)
$es_0(i)$	$es_1(i)$	λ	(4.6)
$es_0(i)$	$p(i)$	$1/\mu_i^1$	(4.7)
$rs_0(i)$	$p(i)$	$1/\mu_i^2$	(4.8)
$u(i)$	$p(i)$	$1/\mu_i^3$	(4.9)
$es_1(i)$	$t(i)$	$\left(1/\mu_i^1\right)\left(\frac{\lambda^2+\lambda^3}{\lambda}\right)$	(4.10)
$es_1(i)$	$t^*(i)$	λ^1/λ	(4.10a)
$rs_0(i)$	$rs_1(i)$	$\lambda^2+\lambda^3$	(4.11)
$rs_0(i)$	$t^*(i)$	λ^1	(4.12)
$rs_1(i)$	$t^*(i)$	λ^1	(4.13)
$t^*(i)$	$es_0(j)$	λ_j^1/t_i^1	(4.14)
$rs_1(i)$	$t(i)$	$1/\mu_i^2$	(4.15)

Again, these rates reflect straightforward transition behaviors from state to state. For example, the unit proceeds from patrol to regular travel, $t(i)$, whenever a routine or unfounded call occurs (4.1). If an emergency call occurs, the unit proceeds to emergency travel, $t^*(i)$ (4.2). Patrol switches from one region to another according to the policy variables, x_{ij} (4.3). The transition rates from travel to service states are weighted as in the simple model (4.4-4.5). The transition from the special travel state, $t^*(j)$, to servicing the pre-empting emergency call is found in the same manner(4.14).

From service states without queues, the transition can occur to the service state with a queue, if a call arrives (4.6, 4.11) or to the patrol state if service is completed (4.7, 4.8). If an emergency call occurs while the patrol unit is servicing a routine call (with or without a call waiting), then the unit enters the special travel state, $t^*(j)$ (4.12, 4.13). Service completion from the queue states results in transition to the travel state to go to the queued call (4.10, 4.16). From the unfounded call state it is assumed that the unit returns to patrol with high enough rate (4.9) that no other event can occur while the call is discovered to be unfounded.

4.3. Exercising the Model

The model of section 4.2 has been used to obtain steady state probabilities for each state. It is important to note that for steady-state calculations we do not need to assume that all probability distributions are exponential, simply that the rates are the reciprocal of appropriate times. (In fact, call arrival times have been found to be very close to exponential in a variety of settings [13]). Our analysis, however, has found travel times in wide areas to be more accurately approximated by a gamma or Erlang distribution (see Appendix B) [14].

Exponentiality of service time distributions was also investigated in Washtenaw County (see appendix A). It was found that many calls required essentially no service time and, so, were "unfounded". Instead of an exponential distribution, a "spiked" exponential distribution with an atom at $t=0$ provides a better fit to the data. We included this in the model by allowing the separate possibility of unfounded calls. Then the exponential assumption for the remaining calls becomes more reasonable. Again, when using the model for evaluating expected performance of the system, the exponential distribution assumption is not necessary.

Parameter values used in testing the model were taken from actual data from Washtenaw County. The procedure in Appendix B for determining travel times was applied to the county's characteristics and used for the travel time parameters.

Washtenaw County response procedures were also included in the exercising of the model. These procedures appear to be applicable in other wide-area regions. Priority dispatching had been implemented in Washtenaw County to respond more efficiently to emergency calls. In areas where travel times may be great, this seemed especially important. The primary feature of the dispatch policy is to differentiate between emergency calls that require immediate service ("immediate") and routine ("expedite") calls that do not. If a unit is servicing a routine call, then it may be pre-empted to serve an emergency call.

The county is also concerned with ***directed patrol*** in certain areas. The patrol switch probabilities x_{ij} represent a response to this need. As policy variables, they enable a patrol unit to "randomize" travel by switching from region to region according to x_{ij} . They also determine the long run state probabilities which can be used to determine the amount of directed patrol in each region absent calls for service.

Washtenaw County has low rates of calls for service compared to average service capability, and so calls were queued very rarely. This justified including only one call stacked in the queue. Additional queueing could be added, with a concomitant growth in the total number of states, for areas with higher call rates. We feel confident, however, that low call rates are common among wide-area low population regions and that single-call queueing is sufficient for such policy-oriented models.

In implementing the model, we had to determine a transition step size that would allow computation by approximating the continuous model with a discrete one. The value of the step size involves a trade-off between speed of convergence and computational accuracy. We chose five minute intervals because the expected number of events that occur within this interval is less than 0.1. Convergence to within 0.5 percent of steady-state solutions occurred in 15 to 35 transitions, a reasonably small number. With larger time intervals the model lost accuracy and with smaller intervals it converged more slowly. Solution by iteration was chosen instead of direct inversion of the transition matrix, in order to determine the time to convergence to steady-state, and to allow for more general transient analysis, such as finding the probability of being in a state after time t of a shift. Analyses could then be made of both the steady-state results and the intra-shift probabilities.

The steady state probabilities (or transient probabilities at given times) are used to evaluate average response time, delays in servicing calls, directed patrol frequency in each region, and other performance measures.

For example, the average emergency travel time $\bar{T}$ is found from the probabilities of being in the patrol states $p(i)$, or being in the "pre-emptable" service states $es_0(i)$ and $rs_1(i)$, or being in the state $es_1(i)$ *and* getting an emergency call (with rate λ^1). These are used to weight the average time spent in the resulting emergency travel state.

$$\bar{T} = \frac{\sum_{i=1}^{N} \left\{ \left[p(i) + es_1(i) + rs_0(i) + rs_1(i) \right] \left[\sum_{j=1}^{N} \left(\frac{\lambda_j^1}{\lambda^1} \right) t_{ij} \right] \right\}}{\sum_{k=1}^{N} \left[p(k) + es_1(k) + rs_0(k) + rs_1(k) \right]} \quad (4.17)$$

The travel time *distribution* can be found in an analogous way by weighting each intersector travel time distribution. Thus, if T is the random variable for overall travel time, and T_{ij} is the random variable for travel time from i

to j ,

$$P\left\{T \le t\right\} = \frac{\sum_{i=1}^{N} \left\{ \left[p(i) + es_1(i) + rs_0(i) + rs_1(i)\right] \left[\sum_{j=1}^{N} \left[\frac{\lambda^1 j}{\lambda^1}\right] P\left\{T_{ij} \le t\right\}\right]\right\}}{\sum_{k=1}^{N} \left[p(k) + es_1(k) + rs_0(k) + rs_1(k)\right]}$$

The average *response time* to an emergency call is found by adding the queue times:

$$\frac{es_1(i)}{p(i) + es_1(i) + rs_0(i) + rs_1(i)} \cdot \left[\text{average time in } es_1(i)\right]$$

to the term $\left[\frac{\lambda_j^1}{\lambda^1} t_{ij}\right]$ in equation (4.17).

Similiar expressions hold for routine calls.

Finally, the workload is calculated by

$$1 - \sum_{i=1}^{N} p(i) ,$$

where the total fraction of time on patrol is the proportion of directed patrol in each area.

Although numerical values for these computations are available, thus far we have only considered the case where there is a single patrol unit. The next section shows how a many-unit model may be developed by extending these results. Presentation of the numerical results of using the model are delayed until the multiple unit model is discussed.

5. MULTIPLE UNIT MODEL

The model developed in Section 4 was based on the assumption that there is only a single patrol unit capable of responding to calls for service within the jurisdiction. Although this may in fact be the case in some instances (i.e., for the midnight to 8:00 a.m. platoon on weekdays), it is more often true that there are many patrol units allocated to the jurisdiction, usually with implicit or explicit policies for the "sharing" of responsibility for responding to calls from among the different regions.

In theory the methods of Section 4 could be used to model the many-unit systems by appropriately defining "states" to represent the various possible configurations in which all units could be at any particular time. However, with K units this would require a total number of $(8N)^K$ states, the number of all possible combinations of patrol units, each in their own state. When $N \approx 15$, even with $K = 2$ units this would require the eventual manipulation of $14{,}400 \times 14{,}400$ matrices -- a formidable task for a main frame computer much less the microcomputer we envision being eventually used.

We chose instead to represent more than one patrol unit by means of an approximation method, newly developed for this study. This approximation effectively represents the behavioral aspects of the system without introducing more than a nominal amount of inaccuracy in the computation of important output values. This method can be most clearly explained by considering the extremely simple case of two patrol units in two regions.

5.1. Two Unit, Two Region Example.

The case of two patrol units in a two-region jurisdiction, although obviously not realistic, will serve here to illustrate the approximation method used to represent the general K-unit N-region system. To further clarify the discussion, it is also assumed that there is only a single priority class of calls for service. Finally, we assume that there are no queued calls -- all calls arriving when *both* units are busy are essentially "lost". (Again, we remind the reader that these gross simplifications are made in order to present the fundamental approach -- sufficient realism is re-introduced in the next section).

The fundamental approximation made is that *each unit* will behave according to its *own transition diagram* as in figure 4.1. The rates, however, will depend upon the other unit's parameters and state occupancy probabilities. This interaction between the two units is represented by the input variables (in addition to the arrival rates, service and travel times and patrol-switch probabilities introduced previously) called *coverage factors*. These were defined in Section 2 so that

$$c_{ki} = \text{prob.} \{ \text{unit } k \text{ responds to a call in region } i \text{ given } \underline{\text{both}} \text{ units are available} \}$$

where $c_{1i} + c_{2i} = 1$, so that *some* available unit must respond. Here, *available* means that the unit is on patrol in one of the two regions, and is neither travelling to nor servicing a call.

The procedure for incorporating the interaction between units -- in essence between transition structures of the type shown in Figure 4.1 -- is called PIMS (Parallel Iteration for Multiple Servers). Appendix D presents a more complete analysis of this method, which in outline is as follows.

First, define *availability* a_k for unit k to be the probability on patrol, i.e.,

$$a_k = \text{prob} \{p_k(1)\} + \text{prob} \{p_k(2)\} ,$$

where $p_k(i) =$ prob { unit k is on patrol in region i }. Then,

1) Set $a_1 = 1, a_2 = 1$.

2a) For unit 1, replace λ_1 by $\lambda_1(1-a_2c_{21})$ and replace λ_2 by $\lambda_2(1-a_2c_{22})$.

2b) For unit 2, replace λ_1 by $\lambda_1(1-a_1c_{11})$ and replace λ_2 by $\lambda_2(1-a_1c_{12})$.

3) Using any appropriate method, separately compute state probabilities for both units (using each unit's adjusted transition rates).

4) Check to see if either a_1 or a_2 (computed from $p_1(i)$ and $p_2(i)$) has changed. If one or both has, go to step 2. If not, stop.

The link between the two units is in step 2 where the general effect is to allow the calls "seen" by a unit to be reduced in proportion to the other's availability and coverage responsibility.

A number of issues related to this method are treated in Appendix D:

a) Whether, and how quickly, this procedure converges.

b) If it does converge, what does the solution at convergence mean?

c) The method makes overt use of the assumption that both units being available are independent events. Since this is clearly not true in general, how misleading are the results?

d) How important is the selection of a starting value for a_1 and a_2 (set equal to 1 in the above example)?

We have good computational experience with this procedure and have found that it provides output values readily useable for policy purposes. The important fact is that step 3 involves working with (and essentially inverting) two 6×6 matrices. In contrast, a straightforward extension of the model of Section 4 -- fully accounting for the dependence between the two units -- would involve a single 36×36 matrix. Although in this case the latter is still a reasonable size even for a microcomputer to handle, recall that this example did not allow for either different priorities or queued calls. Adding new states necessary for incorporating these, step 5 would involve two 16×16 matrices, while the full dependence model would require a 256×256 matrix.

It is also of interest to point out that although the procedure above was used, in this study, to compute only steady-state performance measures, step 3 could equally well be used to compute transient probabilities (for each time point of interest) and associated *non*-steady state measures.

5.2. K-units, N-regions and Two Priorities of Calls.

The PIMS procedure described above can be readily extended to the case of K units, N regions and two priorities of calls. First we define the following terms.

Overall availability for the k^{th} unit:

$$a_k = \sum_{n=1}^{N} \text{prob. } \{p(n)\} \,, \tag{5.1}$$

Emergency availability

$$a'_k = a_k + \sum_{i=1}^{N} [\text{prob.}\{rs_0(i)\} + \text{prob.}\{rs_1(i)\}] \,, \tag{5.2}$$

overall busy probability for the k^{th} unit

$$b_k = 1 - a_k \quad , \tag{5.3}$$

Emergency busy probability for

$$b'_k = 1 - a'_k \quad , \tag{5.4}$$

busy vector

$$\underline{\beta} = (\beta_1 \beta_2 \cdots \beta_k) \quad , \tag{5.5}$$

where

$$\beta_i = \begin{cases} 1 & \text{when unit } i \text{ is busy} \\ 0 & \text{when unit } i \text{ is available ,} \end{cases}$$

emergency busy vector $\underline{\beta}^*$

$$\underline{\beta}^* = (\beta^*_1 \beta^*_2 \cdots \beta^*_k) \quad .$$

unit i realization probability for routine calls

$$r_i\,(\underline{\beta}) = \prod_{\substack{k=1 \\ k \neq i}}^{K} [\beta_k b_k + (1-\beta_k)\,(1-b_k)] \tag{5.6}$$

unit i realization probability for emergency calls

$$r_i^*\,(\underline{\beta}^*) = \prod_{\substack{k=1 \\ k \neq i}}^{K} [\beta_k^* b'_k + (1-\beta_k^*)\,(1-b'_k)] \tag{5.7}$$

set of possible busy vectors

$$B = \{0,1\}^{K-1} \quad .$$

The rate of routine calls to region j for an available unit k, given it "sees" a busy vector $\underline{\beta}$, is

$$\varphi_{kj}\,(\underline{\beta}) = \frac{c_{kj}(\lambda_j^2)}{\sum_{l \neq k} (1-\beta_l)c_{lj} + c_{kj}} \quad , \tag{5.8}$$

while for emergency calls, the rate to region j for an available unit k given an emergency busy vector $\underline{\beta}^*$, is

$$\varphi^{\bullet}_{kj}(\underline{\beta}^{\bullet}) = \begin{cases} \dfrac{c_{kj}\,\lambda_j^1}{\sum_{l\neq k}(1-\beta_l^{\bullet})c_{lj} + c_{kj}} & \text{if } \overline{A} \\[2ex] \dfrac{\lambda_j^1}{K - \sum_{l\neq k}\beta_l^{\bullet}} & \text{if } A \end{cases} \quad (5.9)$$

where $A = \{c_{kj}=0 \text{ and } \sum_{l\neq k}(1-\beta_l^{\bullet})c_{lj}=0\}$.

The procedure then, in outline, is:

1. Set $b_k=0$ for $k=1,2,\cdots K$.
2. For unit k , replace λ_i^2 by

$$\sum_{\underline{\beta}\in B} \varphi_{kj}(\underline{\beta})\, r_k(\underline{\beta}) ,$$

and replace λ_i^1 by

$$\sum_{\underline{\beta}^{\bullet}\in B} \varphi^{\bullet}_{kj}(\underline{\beta}^{\bullet})\, r^{\bullet}_k(\underline{\beta}^{\bullet}) .$$

3. Using any appropriate method, separately compute state probabilities for each of the K units.
4. Compute a_k and a'_k for all k from equation (5.1) and see if they have changed. If so, go to step 2. If not, stop.

The logic behind these definitions and the procedure follows directly from the simpler case. Expressions (5.1) - (5.4) define a unit's availability as being the probability it is in a patrol state (or for emergency calls, also servicing a routine or unfounded call). For each possible busy-available combination of all units other than unit i (as given by all $\underline{\beta}\in B$), expressions (5.6) and (5.7) give the probability of that combination. Expressions (5.8) and (5.9) give an *effective* rate of calls for unit k . The denominator is the average total coverage of available units (given $\underline{\beta}$ or $\underline{\beta}^{\bullet}$), and the ratio of c_{kj} to this gives the

fraction of calls to which unit k will respond. If $c_{kj} = 0$, then unit k will *never* respond to a routine or unfounded call in region j , according to (5.8). However, even if $c_{kj} = 0$, the lower term in the bracket of (5.9) reflects an equally likely response of all units not busy on emergencies (their number being $K-\sum_{l=k}\beta_l^{\bullet}$) regardless of their not having assigned coverage in region j .

Note that step 2 weighs the rate conditional on a particular busy vector $\underline{\beta}$ by the probability that vector will be "seen" by unit k . The same issues about convergence and initial conditions for b_k and b'_k apply, in that their theoretical justification have yet to be established. Nonetheless, our computational experience indicates that, with care taken to assure actual convergence (see Appendix D), useful and practical results are obtained. In particular, step 3 involves K separate solutions of problems involving $(8N)\times(8N)$ matrices, rather than the solution of a problem with a single $(8N)^K \times (8N)^K$ matrix -- essentially impossible for $K\geq 2$ and $N\geq 3$.

6. EXAMPLE OF USE: WASHTENAW COUNTY, WESTERN PORTION

The model of Section 5 was applied to a jurisdiction comprising the western portion of Washtenaw County, an area consisting of twelve sparsely populated regions called townships. The geographic layout of these regions appears in Figure 6.1. The area's geography, service times, travel, and call rates, and current policies are described in Appendix A. The primary measures of interest were the fraction of time a unit is on patrol in each township, and the mean response times to emergency and routine calls. Patrol units are referred to as "cars".

The average number of calls for services in an hour for each region appear in Table 6.1. Table 6.2 contains the mean service times[2] in each region for emergency and routine calls. In the table, the Washtenaw County designations of "expedite" for routine calls and "immediate" for emergency calls are used. Service rates for unfounded calls were assumed to be the same as those for routine calls. Note that all regions have the same service time, and that service times for unfounded calls are zero. The average travel time from one region to another is given in Table 6.3. The diagonal term represents the average travel time between two sites *within* that region. The method for determining these times is described in Appendix B.

Five different basic patrol policies were examined. Each policy consisted of the number of cars k assigned to the entire area, and for each car the regions for which each car is responsible (The coverage matrix C), and the patrol behavior (patrol-switch matrix X). The car assigned to each region appears in Table 6.4a. If more than one car is responsible for a region then the average fraction of calls answered by each car is given in parentheses. In these

[2]A mean service time of 48.12 was used for this example, even though Figure A-3 suggests that, due to two outlines in this small data set, a mean of 40.5 (corresponding to the dashed line) might have been more appropriate.

basic policies, each car has an equal probability each hour of switching patrol from the current region to any other region in its area of responsibility. If a car is in a region outside its responsibility, it will switch to the nearest region for which it has responsibility with probability 1. For example the patrol-switch matrices given in Table 6.4b are for Policy 4.

Policies 1,2,3 and 4 represent allocating 1,2,3 or 4 cars respectively to the jurisdiction. In Policy 5, the fourth car is used exclusively in Dexter Township (area 2) to examine the value (in terms of reduced response time) of a car contracted for by that township.

Tables 6.5 through 6.9 show the fraction of time each car is expected to spend in each region for each activity. Tables 6.10 through 6.14 display the average response times for the Washtenaw County call types for each policy. Average response times for unfounded calls are the same as those for expedite calls in these tables.

The results indicate that two patrol cars substantially reduce average response times to all types of calls, when compared to using a single car. A third car in Township 6 substantially reduces average response times in that region, and a fourth car leads to further reduction in all mean response times.

The distribution of travel time is shown, for policy 4, in table 6.15. As is to be expected, the presence of the fourth patrol unit with 80% patrol responsibility in township 6 (Scio) provides this region with a .54 probability of having a travel time to a CFS of 6 minutes or less. By contrast, Sharon township (region 7) -- which shares patrol unit 2 with four other townships -- has only a .38 probability of having travel time to a CFS be 9 minutes or less.

1 Lyndon	2 Dexter	3 Webster
4 Sylvan	5 Lima	6 Scio
7 Sharon	8 Freedom	9 Lodi
10 Manchester	11 Bridgewater	12 Saline

Figure 6.1.

Western 12 Townships of Washtenaw County.

REGION	EXPEDITE	IMMEDIATE	UNFOUNDED
1.	0.038	0.017	0.003
2.	0.070	0.032	0.005
3.	0.041	0.019	0.003
4.	0.046	0.021	0.004
5.	0.036	0.017	0.003
6.	0.197	0.091	0.015
7.	0.012	0.006	0.001
8.	0.015	0.007	0.001
9.	0.038	0.017	0.003
10.	0.020	0.009	0.002
11.	0.009	0.004	0.001
12.	0.021	0.010	0.002

Table 6.1.

Hourly Call Rates.

REGION	EXPEDITE	IMMEDIATE
1.	27.36	48.12
2.	27.36	48.12
3.	27.36	48.12
4.	27.36	48.12
5.	27.36	48.12
6.	27.36	48.12
7.	27.36	48.12
8.	27.36	48.12
9.	27.36	48.12
10.	27.36	48.12
11.	27.36	48.12
12.	27.36	48.12

Table 6.2.

Mean Service Times in Minutes.

R e g i o n

Region	1	2	3	4	5	6	7	8	9	10	11	12
1	5.90	9.70	15.10	10.90	11.50	17.40	18.90	20.10	21.80	22.60	26.20	35.90
2	9.70	4.70	7.90	14.60	10.10	11.00	21.20	16.50	16.30	24.80	23.50	26.20
3	15.10	7.90	5.40	18.40	11.80	9.90	21.50	17.30	16.90	29.80	23.90	23.60
R 4	10.90	14.60	18.40	7.60	9.10	13.70	13.00	15.20	18.20	19.40	24.90	32.40
e 5	11.50	10.10	11.80	9.10	6.10	8.90	14.10	10.80	11.80	16.80	19.30	20.70
g 6	17.40	11.00	9.90	13.70	8.90	5.60	19.30	12.80	9.90	23.60	19.80	17.60
i 7	18.90	21.20	21.50	13.00	14.10	19.30	6.90	11.60	18.70	11.90	16.60	25.50
o 8	20.10	16.50	17.30	15.20	10.80	12.80	11.60	5.90	9.00	13.60	10.80	15.20
n 9	21.80	16.30	16.90	18.20	11.80	9.90	18.70	9.00	5.50	19.20	13.60	11.20
10	22.60	24.80	29.80	19.40	16.80	23.60	11.90	13.60	19.20	5.80	10.00	15.90
11	26.20	23.50	23.90	24.90	19.30	19.80	16.60	10.80	13.60	10.00	6.50	10.80
12	35.90	26.20	23.60	32.40	20.70	17.60	25.50	15.20	11.20	15.90	10.80	6.60

Table 6.3. Mean Region to Region Travel Times (in Minutes)

Policy	1	2	3	4	5
Total No. of Cars	1	2	3	4	4
Region 1	1	1	1	1	1
2	1	1	1	1	4
3	1	2	2	3	3
4	1	1	1	1	1
5	1	1	1	1	1
6	1	2	2(.2),3(.8)	3(.2),4(.8)	3
7	1	1	1	2	2
8	1	1	1	2	2
9	1	2	2	3	3
10	1	1	1	2	2
11	1	1	1	2	2
12	1	2	2	2	2

Car Number Responsible For Region

Table 6.4a. Number of cars, and patrol cars responsible for each region, for five different policies. Parentheses show fraction of shared responsibilities.


```
HOURLY PATROL SWITCHING PROBABILITIES
CAR. 1
0.250 0.250 0.0   0.250 0.250 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.250 0.250 0.0   0.250 0.250 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.250 0.250 0.0   0.250 0.250 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.250 0.250 0.0   0.250 0.250 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0
CAR. 2
0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.200 0.200 0.0   0.200 0.200 0.200
0.0   0.0   0.0   0.0   0.0   0.0   0.200 0.200 0.0   0.200 0.200 0.200
0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.200 0.200 0.0   0.200 0.200 0.200
0.0   0.0   0.0   0.0   0.0   0.0   0.200 0.200 0.0   0.200 0.200 0.200
0.0   0.0   0.0   0.0   0.0   0.0   0.200 0.200 0.0   0.200 0.200 0.200
CAR. 3
0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.330 0.0   0.0   0.330 0.0   0.0   0.330 0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.330 0.0   0.0   0.330 0.0   0.0   0.330 0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0
0.0   0.0   0.330 0.0   0.0   0.330 0.0   0.0   0.330 0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0
CAR. 4
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   1.000 0.0   0.0   0.0   0.0   0.0   0.0
COVERAGE
CAR  REGIONS 1-12
 1 1.000 1.000 0.0   1.000 1.000 0.0   0.0   0.0   0.0   0.0   0.0   0.0
 2 0.0   0.0   0.0   0.0   0.0   0.0   1.000 1.000 0.0   1.000 1.000 1.000
 3 0.0   0.0   1.000 0.0   0.0   0.200 0.0   0.0   1.000 0.0   0.0   0.0
 4 0.0   0.0   0.0   0.0   0.0   0.800 0.0   0.0   0.0   0.0   0.0   0.0
```

Table 6.4b: Coverage Matrix and Patrol Switch Matrices for Policy 4.

CAR 1

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.042	0.011	0.004	0.009	0.002	0.006	0.003	0.000	0.077
2	0.053	0.012	0.004	0.016	0.004	0.011	0.006	0.000	0.106
3	0.043	0.009	0.003	0.010	0.002	0.007	0.003	0.000	0.077
4	0.045	0.011	0.004	0.011	0.003	0.007	0.004	0.000	0.084
5	0.041	0.007	0.002	0.008	0.002	0.006	0.003	0.000	0.070
6	0.100	0.021	0.007	0.046	0.011	0.032	0.016	0.001	0.234
7	0.032	0.009	0.003	0.003	0.001	0.002	0.001	0.000	0.051
8	0.033	0.007	0.002	0.004	0.001	0.002	0.001	0.000	0.051
9	0.042	0.009	0.003	0.009	0.002	0.006	0.003	0.000	0.074
10	0.035	0.012	0.004	0.005	0.001	0.003	0.002	0.000	0.062
11	0.031	0.010	0.003	0.002	0.001	0.001	0.001	0.000	0.049
12	0.036	0.012	0.004	0.005	0.001	0.003	0.002	0.000	0.063
ALL	0.535	0.131	0.044	0.127	0.031	0.087	0.044	0.003	1.000

Table 6.5. Fractions of time spent on different activities for Policy 1

CAR 1

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.086	0.007	0.003	0.012	0.001	0.009	0.001	0.000	0.119
2	0.117	0.010	0.004	0.022	0.002	0.016	0.002	0.000	0.174
3	0.008	0.001	0.000	0.0	0.0	0.001	0.000	0.0	0.010
4	0.090	0.007	0.003	0.015	0.002	0.011	0.002	0.000	0.129
5	0.099	0.007	0.003	0.011	0.001	0.009	0.001	0.000	0.132
6	0.010	0.001	0.000	0.0	0.0	0.005	0.000	0.0	0.016
7	0.071	0.007	0.003	0.004	0.000	0.003	0.000	0.000	0.089
8	0.086	0.008	0.003	0.005	0.001	0.004	0.001	0.000	0.106
9	0.008	0.001	0.000	0.0	0.0	0.001	0.000	0.0	0.010
10	0.076	0.009	0.003	0.006	0.001	0.005	0.001	0.000	0.100
11	0.083	0.011	0.004	0.003	0.000	0.002	0.000	0.000	0.103
12	0.007	0.003	0.001	0.0	0.0	0.000	0.000	0.0	0.011
ALL	0.742	0.071	0.026	0.078	0.008	0.065	0.009	0.001	1.000

CAR 2

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.002	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.003
2	0.002	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.004
3	0.151	0.013	0.005	0.013	0.002	0.010	0.002	0.000	0.196
4	0.002	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.003
5	0.002	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.003
6	0.258	0.018	0.007	0.063	0.009	0.049	0.012	0.001	0.417
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
8	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
9	0.148	0.012	0.004	0.012	0.002	0.009	0.002	0.000	0.189
10	0.002	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.003
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
12	0.137	0.017	0.006	0.007	0.001	0.005	0.001	0.000	0.174
ALL	0.707	0.064	0.023	0.095	0.014	0.078	0.018	0.002	1.000

Table 6.6. Fractions of time spent on different activities for Policy 2

CAR 1

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.088	0.007	0.002	0.012	0.001	0.009	0.001	0.000	0.121
2	0.119	0.010	0.003	0.023	0.002	0.017	0.002	0.000	0.177
3	0.007	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.008
4	0.093	0.008	0.003	0.015	0.002	0.011	0.002	0.000	0.132
5	0.100	0.007	0.002	0.012	0.001	0.009	0.001	0.000	0.132
6	0.006	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.008
7	0.073	0.007	0.002	0.004	0.000	0.003	0.000	0.000	0.090
8	0.087	0.008	0.003	0.005	0.001	0.004	0.001	0.000	0.107
9	0.007	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.008
10	0.077	0.009	0.003	0.006	0.001	0.005	0.001	0.000	0.102
11	0.084	0.011	0.004	0.003	0.000	0.002	0.000	0.000	0.103
12	0.006	0.002	0.001	0.0	0.0	0.000	0.000	0.0	0.010
ALL	0.746	0.072	0.024	0.079	0.009	0.060	0.009	0.001	1.000

CAR 2

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.004	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.005
2	0.004	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.005
3	0.202	0.010	0.003	0.015	0.001	0.012	0.001	0.000	0.244
4	0.004	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.005
5	0.004	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.005
6	0.217	0.008	0.003	0.023	0.000	0.015	0.000	0.000	0.266
7	0.004	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.004
8	0.004	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.004
9	0.198	0.009	0.003	0.014	0.001	0.011	0.001	0.000	0.237
10	0.004	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.005
11	0.004	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.004
12	0.184	0.011	0.004	0.008	0.001	0.006	0.001	0.000	0.215
ALL	0.831	0.041	0.014	0.060	0.003	0.047	0.003	0.001	1.000

CAR 3

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
2	0.001	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.002
3	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
4	0.001	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.002
5	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
6	0.840	0.018	0.007	0.062	0.001	0.053	0.001	0.001	0.984
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
8	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
10	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
12	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
ALL	0.851	0.019	0.008	0.062	0.001	0.057	0.001	0.002	1.000

Table 6.7. Fractions of time spent on different activities for Policy 3

CAR 1

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.181	0.008	0.003	0.014	0.001	0.011	0.002	0.000	0.220
2	0.214	0.010	0.003	0.026	0.002	0.020	0.003	0.000	0.280
3	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
4	0.197	0.011	0.004	0.017	0.002	0.013	0.002	0.000	0.246
5	0.201	0.009	0.003	0.013	0.013	0.001	0.011	0.002	0.241
6	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
8	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
10	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
12	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
ALL	0.805	0.041	0.013	0.070	0.007	0.055	0.008	0.001	1.000

CAR 2

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
2	0.002	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.002
3	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
4	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
5	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
6	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
7	0.178	0.005	0.002	0.005	0.000	0.004	0.000	0.000	0.195
8	0.194	0.005	0.002	0.006	0.000	0.005	0.000	0.000	0.212
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
10	0.179	0.004	0.002	0.008	0.000	0.006	0.000	0.000	0.200
11	0.166	0.003	0.001	0.004	0.000	0.003	0.000	0.000	0.177
12	0.180	0.005	0.002	0.009	0.000	0.007	0.000	0.000	0.204
ALL	0.906	0.023	0.008	0.032	0.001	0.028	0.002	0.001	1.000

Table 6.8.

Fractions of time spent on different activities for Policy 4

(continued on next page)

CAR 3

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
2	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.003
3	0.272	0.010	0.003	0.016	0.001	0.013	0.001	0.000	0.315
4	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
5	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
6	0.290	0.008	0.003	0.024	0.000	0.016	0.000	0.000	0.340
7	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
8	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
9	0.284	0.011	0.003	0.015	0.001	0.011	0.001	0.000	0.326
10	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
11	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
12	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
ALL	0.860	0.029	0.010	0.054	0.002	0.042	0.003	0.001	1.000

CAR 4

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
2	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
3	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
4	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
5	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
6	0.844	0.018	0.007	0.062	0.001	0.053	0.001	0.001	0.987
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
8	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
10	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
12	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
ALL	0.854	0.019	0.007	0.062	0.001	0.055	0.001	0.001	1.000

Table 6.8. (continued from previous page)

CAR 1

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.274	0.008	0.003	0.015	0.001	0.012	0.001	0.000	0.314
2	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
3	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
4	0.290	0.008	0.003	0.018	0.001	0.014	0.001	0.000	0.337
5	0.295	0.008	0.003	0.014	0.001	0.012	0.001	0.000	0.334
6	0.003	0.000	0.000	0.0	0.0	0.002	0.000	0.0	0.005
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
8	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
10	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
12	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
ALL	0.870	0.025	0.009	0.048	0.003	0.041	0.004	0.001	1.000

CAR 2

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
2	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
3	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
4	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
5	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
6	0.003	0.000	0.000	0.0	0.0	0.002	0.000	0.0	0.005
7	0.177	0.005	0.002	0.005	0.000	0.004	0.000	0.000	0.194
8	0.196	0.005	0.002	0.006	0.000	0.005	0.000	0.000	0.214
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
10	0.177	0.004	0.002	0.008	0.000	0.006	0.000	0.000	0.198
11	0.164	0.003	0.001	0.004	0.000	0.003	0.000	0.000	0.176
12	0.179	0.005	0.002	0.009	0.000	0.007	0.000	0.000	0.202
ALL	0.904	0.023	0.009	0.032	0.001	0.029	0.002	0.001	1.000

Table 6.9. Fractions of time spent on different activities for Policy 5

(continued on next page)

CAR 3

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
2	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
3	0.201	0.014	0.005	0.014	0.002	0.011	0.002	0.000	0.249
4	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
5	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
6	0.321	0.018	0.006	0.066	0.009	0.052	0.011	0.001	0.485
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
8	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
9	0.207	0.015	0.005	0.013	0.002	0.010	0.002	0.000	0.254
10	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
12	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
ALL	0.738	0.050	0.017	0.093	0.013	0.073	0.016	0.002	1.000

CAR 4

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
1	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
2	0.915	0.007	0.003	0.030	0.001	0.024	0.001	0.000	0.982
3	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
4	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
5	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
6	0.003	0.000	0.000	0.0	0.0	0.002	0.000	0.000	0.005
7	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
8	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
9	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
10	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
11	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
12	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
ALL	0.928	0.008	0.004	0.030	0.001	0.028	0.001	0.000	1.000

Table 6.9. (continued from previous page)

REGION	EXPEDITE	IMMEDIATE
1	37.858	23.463
2	34.016	20.395
3	35.546	21.538
4	36.041	22.070
5	31.000	18.166
6	32.056	18.840
7	37.805	23.663
8	32.917	19.828
9	33.560	20.173
10	40.465	25.779
11	39.031	24.614
12	42.027	26.766

Table 6.10. Policy 1 Average Response Time To Each Region (Minutes)

REGION	EXPEDITE	IMMEDIATE
1	25.488	16.030
2	24.609	14.963
3	22.936	13.782
4	24.111	15.009
5	21.146	12.443
6	18.626	10.515
7	24.955	15.842
8	22.589	13.686
9	19.827	11.569
10	27.279	17.646
11	28.465	18.233
12	25.978	16.601

Table 6.11. Policy 2 Average Response Time To Each Region (Minutes)

REGION	EXPEDITE	IMMEDIATE
1	25.062	15.545
2	24.281	14.548
3	21.266	13.818
4	23.712	14.500
5	20.820	12.030
6	9.593	6.900
7	24.557	15.444
8	22.286	13.363
9	17.629	11.175
10	26.888	17.321
11	28.168	17.985
12	23.184	15.627

Table 6.12. Policy 3 Average Response Time To Each Region (Minutes)

REGION	EXPEDITE	IMMEDIATE
1	16.288	10.385
2	16.327	10.080
3	16.836	11.130
4	17.571	11.171
5	15.605	9.428
6	8.758	6.386
7	20.638	14.663
8	16.553	11.466
9	16.603	10.891
10	16.869	11.950
11	16.258	11.431
12	21.190	15.104

Table 6.13. Policy 4 Average Response Time To Each Region (Minutes)

REGION	EXPEDITE	IMMEDIATE
1	14.874	9.979
2	7.482	5.152
3	18.719	11.101
4	14.400	9.623
5	13.891	9.049
6	15.278	8.497
7	20.652	14.671
8	16.548	11.492
9	18.575	11.044
10	16.938	12.006
11	16.305	11.512
12	21.221	15.228

Table 6.14. Policy 5 Average Response Time To Each Region (Minutes)

PROBABILITY TRAVEL TIME TO IMMEDIATE CALLS IS LESS THAN OR EQUAL TO
MINUTES

REGION	3	6	9	12	15	18	21	24	27
LYND	0.036	0.217	0.506	0.757	0.901	0.964	0.988	0.996	0.999
DEXT	0.081	0.268	0.509	0.720	0.855	0.929	0.966	0.985	0.993
WEBS	0.076	0.282	0.560	0.797	0.926	0.977	0.993	0.998	1.000
SYLV	0.033	0.188	0.432	0.657	0.812	0.904	0.954	0.980	0.991
LIMA	0.039	0.236	0.523	0.766	0.907	0.969	0.991	0.997	0.999
SCIO	0.141	0.542	0.813	0.933	0.977	0.993	0.998	0.999	1.000
SHAR	0.020	0.134	0.380	0.661	0.852	0.946	0.982	0.995	0.998
FREE	0.033	0.173	0.404	0.645	0.818	0.916	0.964	0.986	0.995
LODI	0.060	0.283	0.571	0.804	0.929	0.978	0.994	0.998	1.000
MANC	0.034	0.208	0.505	0.765	0.908	0.968	0.989	0.997	0.999
BRID	0.025	0.178	0.445	0.704	0.871	0.953	0.985	0.995	0.999
SALI	0.024	0.157	0.418	0.676	0.840	0.926	0.968	0.987	0.995

Table 6.15. Cumulative travel time distributions for Policy 4

7. GENERAL POLICY USES AND IMPLICATIONS

The model's primary use is the rapid computation of response times and coverage capabilities under different patrol policies. The decision maker can change the number of cars, regions of responsibility, and patrol policies within the regions. The average response times and coverage fractions then indicate the advantages of the different plans. This may be of particular interest to communities involved in evaluating either the benefits of increased service or the costs of decreased service.

In Washtenaw County, several townships contract for a car to be present in their township a certain percentage of the time. The fraction of time spent on patrol can be used to show that the township is adequately covered. Response times demonstrate the expected effect of that coverage. For example, a comparison of the results from Policy 3(Table 6.12) and Policy 5(Table 6.14) shows that an additional car with full responsibility for Township 2 (Dexter) results in a reduction from 24 to 7.5 minutes in the average time to respond to a routine call, and a reduction of from 15 to 15.2 minutes in the average time to respond to emergency calls. In addition, under policy 3 region 2 has a car patrolling only 12% of the time, whereas using policy 5 this figure rises to 96%. This information is of obvious use to that township in helping to decide whether to contract for a patrol car. This kind of information about the effects of adding or deleting a car is especially important considering the current financial plight of many communities. The merit of an additional car can be more accurately assessed and weighed against the costs of providing that car.

Another possibility for Dexter township would be to join with Townships 1, 4, and 5 and to contract for a car, as in Policy 4. In this case, a car will patrol Dexter 22% of the time, and the mean response times are 16 minutes for routine calls and 10 minutes for emergency calls. Given each of these alternatives

and the model's results, the township has a better idea of whether additional patrol is worthy of the cost of this service.

The Washtenaw County Sheriff Department considered these kinds of comparisons to be one of the major potential benefits of the model. They also thought that expressing patrol policy in terms of the patrol-switch probabilities from region to region was natural. Of particular interest to them was the fraction of time spent on patrol in each region. This time, called "directed patrol", is considered to be available for various crime prevention strategies. The ability of the model to compute directed patrol time, given the policy in terms of probabilities of switching from region to region, represented a major step to them in terms of quantifying their directed patrol capability.

The full model was implemented on the University of Michigan Amdahl V/8 computer. This large computer was especially useful to us, in that it has a virtually unlimited storage capacity. A microcomputer was initially used for the project but memory limitations made the large computer more convenient. With sufficient coding efficiency and use of disk storage for intermediate results the model will be implementable on a small machine. If this is done the model would be accessible to many departments. Two remaining difficulties would be in terms of the computational speed of the microcomputer and in obtaining useful and readable output formats. The latter may be improved with more expensive peripherals. The computation time should be in the order of tens of minutes for a 12-region jurisdiction for each policy, a practicable length of time for policy planning purposes. In general, however such implementation considerations could be addressed readily, and should offer no real technological challenge.

In implementing the model, users must of course be aware that the results are only as good as the input data. Thus actual policy decisions should not be based on model outputs without sufficient verification of both input data and the model's structure being a representation of the actual procedures of the patrol units in the jurisdiction.

The model, as presently coded, computes the distribution of travel times. Distributions of response times can also be included as mentioned in Section 4. These may be especially important where the tails of the distributions represent the problems perceived by the community and where their elimination would lead to an improvement in the community's perception of service.

8. CONCLUSIONS AND OPEN QUESTIONS

The SWAP model has been developed to be a policy tool for use in wide area police patrol. In those areas where travel time is a significant component in servicing a call, urban based models such as PCAM [3] and the hypercube queueing model [4] do not sufficiently represent reality. Our model explicitly incorporates travel time, making it a useful alternative.

Travel is incorporated by representing the patrol system in terms of a Markov process model, using a travel state for each region. Other states represent patrol, service for different priority calls, and service with calls waiting.

The parallel iteration (PIMS) solution approach allows this model to be used for multiple patrol units. In this method, calls that arrive in a region where the unit with coverage responsibility is busy are taken by a car from a neighboring region. The model successively computes the probability that each car is busy and modifies call rates to account for calls that cannot be handled by cars from their region of origin. The model iterates until these probabilities converge. This approach avoids solving a combinationally large problem for multiple cars and, in practice, has performed efficiently.

The Markovian nature of the model requires exponentiality of both arrival and travel time distributions. This assumption fits call data fairly accurately and can be modified for service time distributions by distinguishing between unfounded calls and others. Travel times, however, may have distributions closer to gamma or Erlang than exponential. The model can incorporate these distributions to produce steady-state results by using the methods of Section 6. A transient analysis in this case requires a semi-Markov model, and associated computational difficulties. Fortunately, our results have indicated that steady state is achieved fairly quickly and that transient analysis is not necessarily needed for patrol policy evaluation.

The exponential assumption helps to make calculations tractable. More reality may be added by extending the model, but policy experience and details idiosyncratic to a particular jurisdiction would be necessary before these details could be included. In addition to adding non-exponential travel times, for example, additional queueing states could be used. In Washtenaw County, these states were judged unnecessary because of the low probability of such states. The problems inherent in evaluating a more complex model, requiring more computational effort to solve, outweigh the need for this degree of model accuracy.

A potentially useful extension of our model would include the addition of "patrol initiated activities" (PIA's). These activities (see [12]) may occupy a large portion of the patrol unit's time. In fact, in the less populated areas of Washtenaw County, the occurrence rate of these activities may be substantially larger than the rate of calls for service. This behavior may be in fact typical of areas where calls for service are relatively infrequent.

PIA's may be easily integrated into the SWAP model by introducing transitions from the patrol states $p(j)$ directly to service state $es_0(j)$ or $rs_0(j)$. The rates for these transitions would be based upon data on the frequency of PIA's while a car is on patrol. Unfortunately, this type of data was not routinely gathered in Washtenaw County, and apparently is not in most other rural jurisdictions.

PIA's were specifically not included in our model because the car's patrol time in a region is a measure of "directed patrol". Thus, the fraction of time a car is patrolling represents the time that the car is available to initiate other activity. The number of PIA's in the region and the type of activity initiated may then be used to evaluate the effect of that directed patrol. The patrol policy may be modified to shift directed patrol, and the results for PIA's with that

policy may be compared with the previous policy.

Another potential extension and use of the model would be aimed at the generation of a set of "good" policies by a "semi-automatic" selection process. The model is currently purely descriptive and offers no prescriptive solutions. However a multiple objective or goal programming optimization, using criteria that the decision maker feels are relevant, could be developed. Since the changes in the steady-state distributions are not, in general, linear in the input parameters or policy variables, such an optimization scheme could become extremely complex. A method for linearizing the solutions is presented in Appendix G. This method can be used to formulate successive linear programs that will lead to a "good" but not necessarily optimal policy. Additional work in this area may prove extremely beneficial in enhancing the model's applicability.

The model as presented here is general and representative, but the specific computer code is a prototype, and practical use will most certainly require alterations especially, in its presentation. Problems of microcomputer implementation need to be addressed before the model is accessible to the great majority of the nation's low population, wide area law enforcement agencies. This effort should entail coding in a universal microcomputer language, such as BASIC, and should address efficient procedures for data storage and retrieval from disks or cassettes.

A properly coded use of the model would also require a user's manual to accompany the code that would explain, in terms accessible to a deputy sheriff or patrol officers, how the model works. This should also include extensive internal documentation of the code and examples of usage on different systems. In too many instances, software for policy evaluation is neglected because of the user's difficulty in understanding its function.

Detailed descriptions should also be given of the procedures needed to collect input data for the model and to construct the travel time distributions. These data may not be currently routinely gathered by the department, and so some description of how to gather it may be necessary.

This project has constructed a model of wide area-police patrol that should expand the possibilities for policy evaluation. It has been tested with data from one area but warrants verification by using da' a and procedures from other areas. Nonetheless we have concluded that wide area patrol may be modeled efficiently and additional effort spent in implementing it on a microcomputer may make it beneficial to many departments around the country.

REFERENCES

(1) Larson, R.C., *Urban Police Patrol Analysis*, The M.I.T. Press, Cambridge, Massachusetts, 1972.

(2) Kolesar, P. and E. Blum, "Square Root Law for Fire Engine Response Distances," *Management Science*, Vol. 19, 1973, pp. 1368-1378.

(3) Chaiken, J. and P. Permant, "Patrol Car Allocation Model: Executive Summary," The Rand Corporation, R-17861/1-HUD, September, 1975.

(4) Chaiken, J., "Hypercube Queueing Model: Executive Summary," The Rand Corporation, R-1688/1-HUD, July, 1975.

(5) Kolesar, P. and W. Walker, "A Simulation Model of Police Patrol Operations: Executive Summary," R-1625/1-HUD/NYC, The New York City Rand Institute, February, 1975.

(6) IBM Corporation, "LEMRAS Application Description Manual," Document H20-0629, Law Enforcement Manpower Resource Allocation System (LEMRAS) Program Description Manual, Program 5736-621, Document SH20-0695-0.

(7) "An Analysis of the Patrol Car Deployment Methods of the Los Angeles Police Department," Engineering School Report by Public Systems Analysis Class, University of California at Los Angeles, 1975.

(8) Bammi, D., "Allocation of Police Beats to Patrol Units to Minimize Response Time to Calls for Service," *International Journal of Computers and Operations Research*, Vol.2, pp. 1-12, 1975.

(9) Heller, N., Stenzel, W., Kolde, R. and Gill, A. "Police Planning with Low Costs ($300-$1000) Micro-Computers and Programmable Calculators," The Institute for Public Program Analysis, May, 1978.

(10) Chaiken, J., "Two Patrol Car Deployment Models' History of Use 1975-1979," The Rand Corporation, RAND/P-6458, March, 1980.

(11) Payne, D.F., "Study of Rural Beats," *Police Research Bulletin*, October, 1969, pp. 23-29.

(12) Larson, R.C. and McKnew, M.A., "Police Patrol-Initiated Activities within a Systems Queueing Model," *Management Sci.*, Vol.28, pp.759-774, 1982.

(13) Larson, R.C., "Structural System Models for Locational Decisions: An Example Using the Hypercube Queueing Model," in K.B. Haley, ed., *Operations Research 1978*, Nath-Holland Publishing Co., Amsterdam, pp.1054-1091, 1979.

(14) Çinlar, E., *Introduction to Stochastic Processes*, Prentice-Hall, Englewood Cliffs, 1975.

(15) Kansas City, Missouri Police Department, "Response Time Analysis," Executive Summary" Volume I" Methodology; Volume II: Analysis, 1978.

(16) Schebil, Lieutenant R.J., Washtenaw County Sheriff Department, private communication, October 22, 1980.

(17) Hwang, C.L. and Masud, A.S., *Multiple Objective Decision Making Methods and Applications: A State of the Art Survey*, Springer-Verlag, New York, 1979.

(18) Pritsker, A.A.B., *The GASP IV Simulation Language*, John Wiley & Sons, New York, New York, 1974.

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REFERENCES

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(2) Kolesar, P. and E. Blum, "Square Root Law for Fire Engine Response Distances," *Management Science*, Vol. 19, 1973, pp. 1368-1378.

(3) Chaiken, J. and P. Permant, "Patrol Car Allocation Model: Executive Summary," The Rand Corporation, R-17861/1-HUD, September, 1975.

(4) Chaiken, J., "Hypercube Queueing Model: Executive Summary," The Rand Corporation, R-1688/1-HUD, July, 1975.

(5) Kolesar, P. and W. Walker, "A Simulation Model of Police Patrol Operations: Executive Summary," R-1625/1-HUD/NYC, The New York City Rand Institute, February, 1975.

(6) IBM Corporation, "LEMRAS Application Description Manual," Document H20-0629, Law Enforcement Manpower Resource Allocation System (LEMRAS) Program Description Manual, Program 5736-621, Document SH20-0695-0.

(7) "An Analysis of the Patrol Car Deployment Methods of the Los Angeles Police Department," Engineering School Report by Public Systems Analysis Class, University of California at Los Angeles, 1975.

(8) Bammi, D., "Allocation of Police Beats to Patrol Units to Minimize Response Time to Calls for Service," *International Journal of Computers and Operations Research*, Vol.2, pp. 1-12, 1975.

(9) Heller, N., Stenzel, W., Kolde, R. and Gill, A. "Police Planning with Low Costs ($300-$1000) Micro-Computers and Programmable Calculators," The Institute for Public Program Analysis, May, 1978.

(10) Chaiken, J., "Two Patrol Car Deployment Models' History of Use 1975-1979," The Rand Corporation, RAND/P-6458, March, 1980.

(11) Payne, D.F., "Study of Rural Beats," *Police Research Bulletin*, October, 1969, pp. 23-29.

(12) Larson, R.C. and McKnew, M.A., "Police Patrol-Initiated Activities within a Systems Queueing Model," *Management Sci.*, Vol.28, pp.759-774, 1982.

(13) Larson, R.C., "Structural System Models for Locational Decisions: An Example Using the Hypercube Queueing Model," in K.B. Haley, ed., *Operations Research 1978*, Nath-Holland Publishing Co., Amsterdam, pp.1054-1091, 1979.

(14) Çinlar, E., *Introduction to Stochastic Processes*, Prentice-Hall, Englewood Cliffs, 1975.

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(16) Schebil, Lieutenant R.J., Washtenaw County Sheriff Department, private communication, October 22, 1980.

(17) Hwang, C.L. and Masud, A.S., *Multiple Objective Decision Making Methods and Applications: A State of the Art Survey*, Springer-Verlag, New York, 1979.

(18) Pritsker, A.A.B., *The GASP IV Simulation Language*, John Wiley & Sons, New York, New York, 1974.

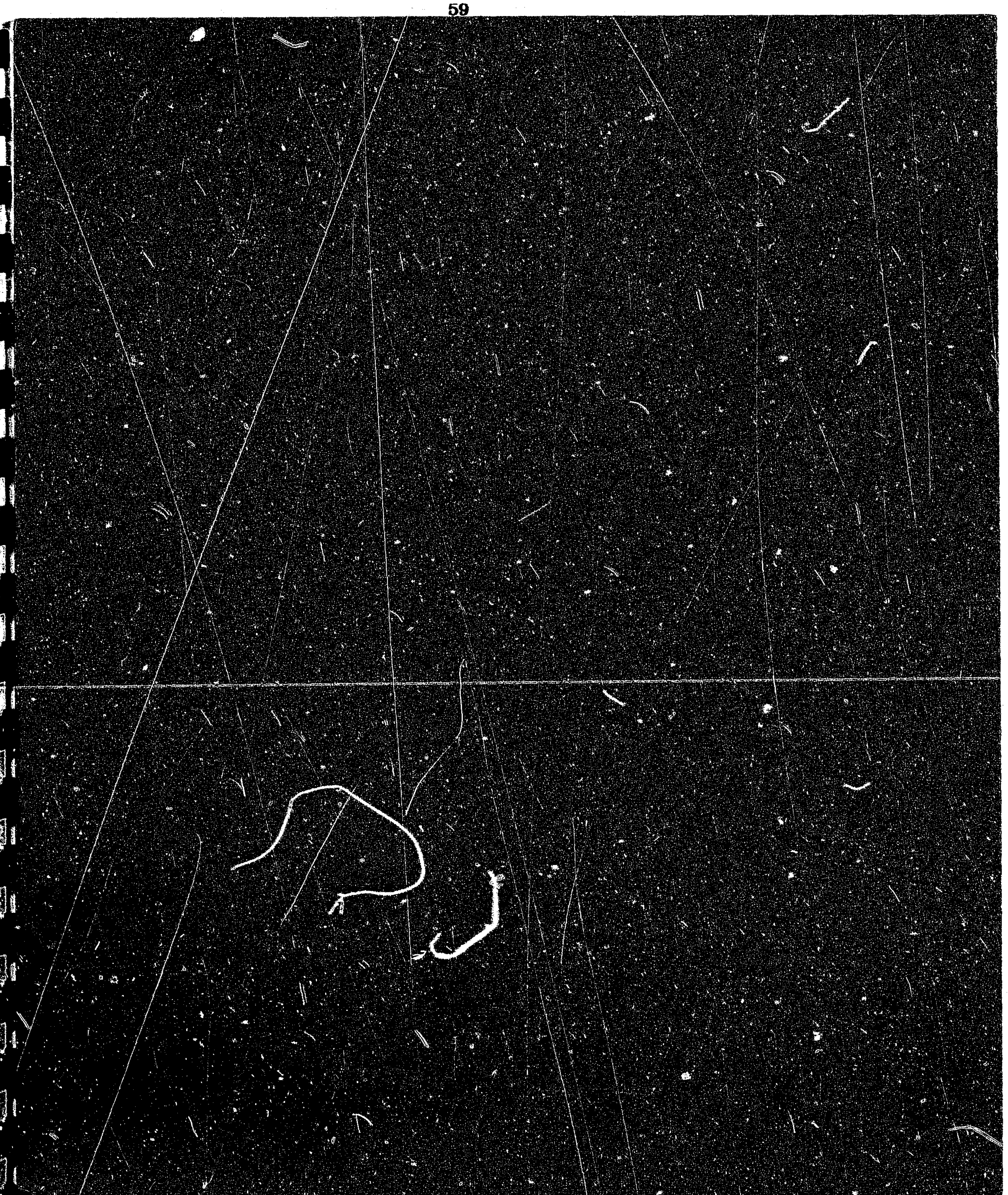
APPENDIX A: WASHTENAW COUNTY

A.1 Geography

Washtenaw County is rectangular, 30 miles wide in the east-west dimension and 24 miles wide in the north-south dimension. It is divided into 20 six mile by six mile townships. Figure A.1 shows the basic layout of the county. As shown in this figure, the largest cities in the county are Ann Arbor and Ypsilanti. Because of these cities, the townships of Ann Arbor and Ypsilanti are fairly urban in nature and are patrolled primarily by their city police departments. The other townships are relatively rural. The western 12 townships provide a convenient block of rural townships and were therefore chosen as a test area for this analysis.

There are five Sheriff's Department stations in the county: the main station, located between Ann Arbor and Ypsilanti, and four substations, in Ypsilanti, Dexter, Northfield, and Chelsea. Cars patrolling the western 12 townships generally work out of the Dexter and Chelsea substations. The dispatchers are located in the main station, as are the jail and administrative services. Some of the townships, such as Scio, have contracted with the Sheriff's Department for a patrol car during certain hours of the day. These contract cars are over and above the department's responsibility to patrol the portions of Washtenaw County not serviced by another police department. In addition to Ann Arbor and Ypsilanti, the cities and villages of Machester, Saline, Pittsfield, Chelsea, and Milan have their own police departments. These departments and the Sheriff Department cooperate when possible and back each other up in emergencies.

The roads in the western 12 townships include I-94, an east-west interstate highway, M-52, a north-south state highway, and US-12, a state highway that cuts diagonally through Saline township. The other roads consist of paved and unpaved county roads. The underlying pattern of these roads is an orthogonal



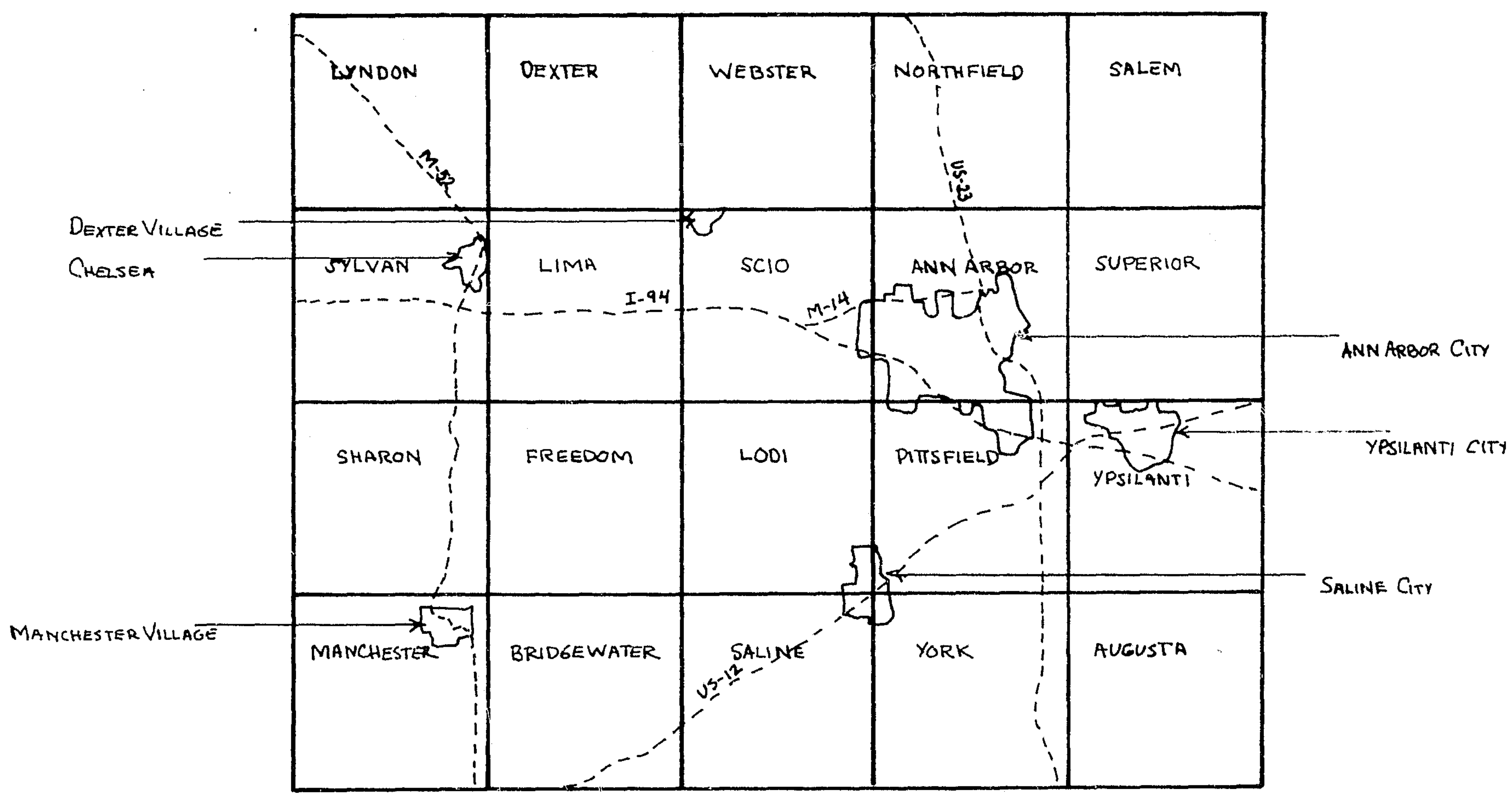


FIGURE A-1 WASHTENAW COUNTY, MICHIGAN

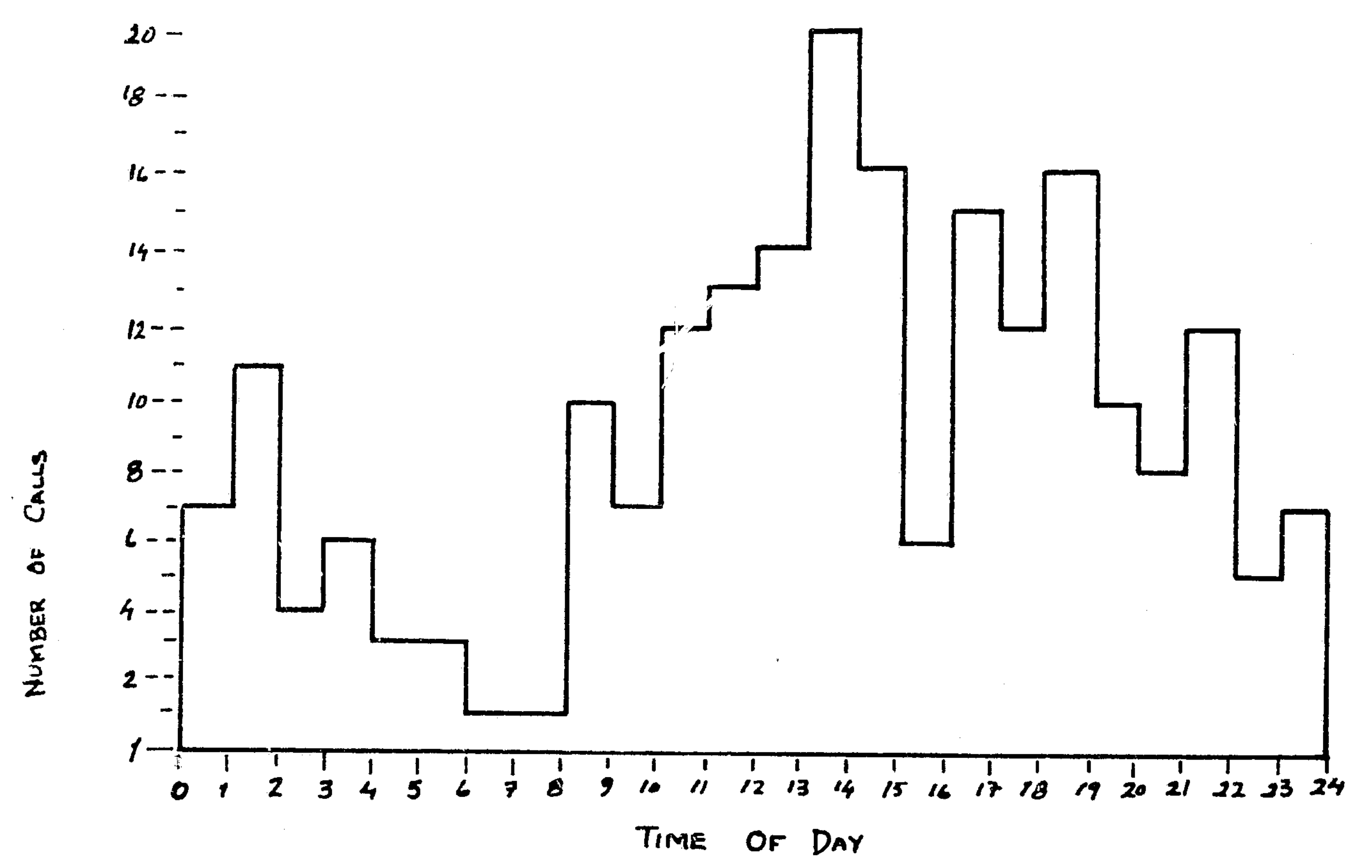


FIGURE A-2 TOTAL CALLS IN WESTERN 12 TOWNSHIPS OF WASHTENAW COUNTY BY TIME OF DAY REPORTED FOR 9 DAYS IN FEBRUARY 1982 grid with roads every mile on the mile. However, irregularities due to farms, lakes, and diagonal roads prevent finding travel distances between points by using the "Manhattan metric". The methods used to estimate travel times on these roads are discussed in Appendix B.

A.2 Call Rates and Service Times

The Washtenaw County Sheriff Department fills out a card on each call for service it receives. These cards give a description of the call type, car and officer assigned, location of incident, and other relevant data. In addition, these cards are punched by the dispatcher into a time clock four times: first, at the time the call is received; second, when a car is dispatched to the incident; third, when the officer radios that he/she has arrived at the scene of the incident; and fourth, when the officer radios that he/she has completed service. These cards were the main source of data on call rates by township and service times used to test the models in this study.

To estimate the call rates for each township the data from 20 months of 1981-82 cards were collected. Total calls during these 20 months, broken down by township, are presented in Table A.1. This table also shows the average calls per hour for each township. These averages were used as the call rates for the applications of the model to Washtenaw County. However, these overall averages are not completely representative of the call rates faced by the Washtenaw County Sheriff Department. This is because call rates vary throughout the day. Figure A.2, which presents 9 days of data from February 1982, showed significant fluctuation in call rates during the day. As would be expected, there were very few calls between 4 and 8 am, while the period from 1 to 7 pm registered a substantially greater number of calls. In realistic applications of the model developed in this study, separate runs should be made for different periods of the day, in order to reflect the variations in call rates.

Table A.1:
Call Rates by Township in
Washtenaw County
Total Calls

Township	Jan.-Dec., 1980	April-Nov., 1981	20 Month Total	Average Calls/hour[1]
Ypsilanti	17,328.	13,570.	30,898.	2.111
Scio	2,615.	1,820.	4,435.	0.303
Superior	2,609.	1,766.	4,375.	0.299
Ann Arbor	1,422.	1,113.	2,535.	0.173
Northfield	1,833.	1,275.	3,108.	0.212
York	864.	579.	1,443.	0.099
Pittsfield	676.	556.	1,232.	0.084
Augusta	825.	600.	1,425.	0.097
Dexter	977.	608.	1,585.	0.108
Sylvan	617.	422.	1,039.	0.071
Lima	491.	326.	817.	0.056
Lodi	491.	357.	898.	0.058
Lyndon	490.	362.	852.	0.058
Webster	511.	352.	863.	0.059
Salem	642.	329.	971.	0.066
Manchester	262.	188.	450.	0.031
Saline	290.	175.	465.	0.032
Freedom	207.	125.	332.	0.023
Sharon	111.	163.	274.	0.019
Bridgewater	112.	99.	211.	0.014
	33,373.	24,785.	58,158.	3.973

(1) Calculated by dividing column 3 by 14,640 = number of hours in 20 months (610 days)

In addition to call rates, the SWAP model requires input data on service times and travel times. However, the data cards in use by the Washtenaw County Sheriff Department at the time of this work did not clearly indicate priority of the call. Because it was felt that a correlation might exist between service times and call type, an experiment was conducted during the week of May 17, 1982. During this week, dispatchers marked all data cards as immediate

priority or expedite/normal priority. 89 calls were received during this week, 32 immediate priority and 57 expedite/normal priority. These data were used to estimate service times and travel times by priority type.

Table A.2 presents a summary of the data collected during the experiment. These travel times provide some standard of comparison for estimating travel times. However, because travel time is dependent on location of both car and call, as well as travel speed and availability and quality of roads, these simple estimates are not adequate for use in the model. The model requires a mean and variance of the travel time when a car goes from a specified township to another specified township. Analytical estimates of these parameters are discussed in Appendix B.

The service times in Table A.2 have means fairly close to standard deviations, and therefore might reasonably be approximated by exponential distributions. However, Figures A.3 and A.4 which show the complementary cumulative distribution $(1-F(x)$ vs. x, where x = service time in minutes) indicate a potential problem with using an exponential distribution. Both of these graphs show reasonably linear curves on the semi-log scale, indicating appropriateness of the exponential distribution. In Figure A.3, the fit is made closer by excluding the two outliers which took over 150 minutes, as the dashed line does. Note that the intercepts of the curves do not occur at the 100 percent point. This is because both immediate and expedite priority calls have a positive probability of requiring little no service time. In the period studied, 14 immediate priority calls and 20 expedite/normal priority calls had essentially zero service times. The officers radioed in arrival and completion at the same time (at least the dispatchers punched these two times simultaneously). These calls are called "unfounded" because, although an officer responds to them, they do not require service. If these unfounded calls are not considered, an exponential distribution provides a good fit for the service times of the remaining calls. A representative

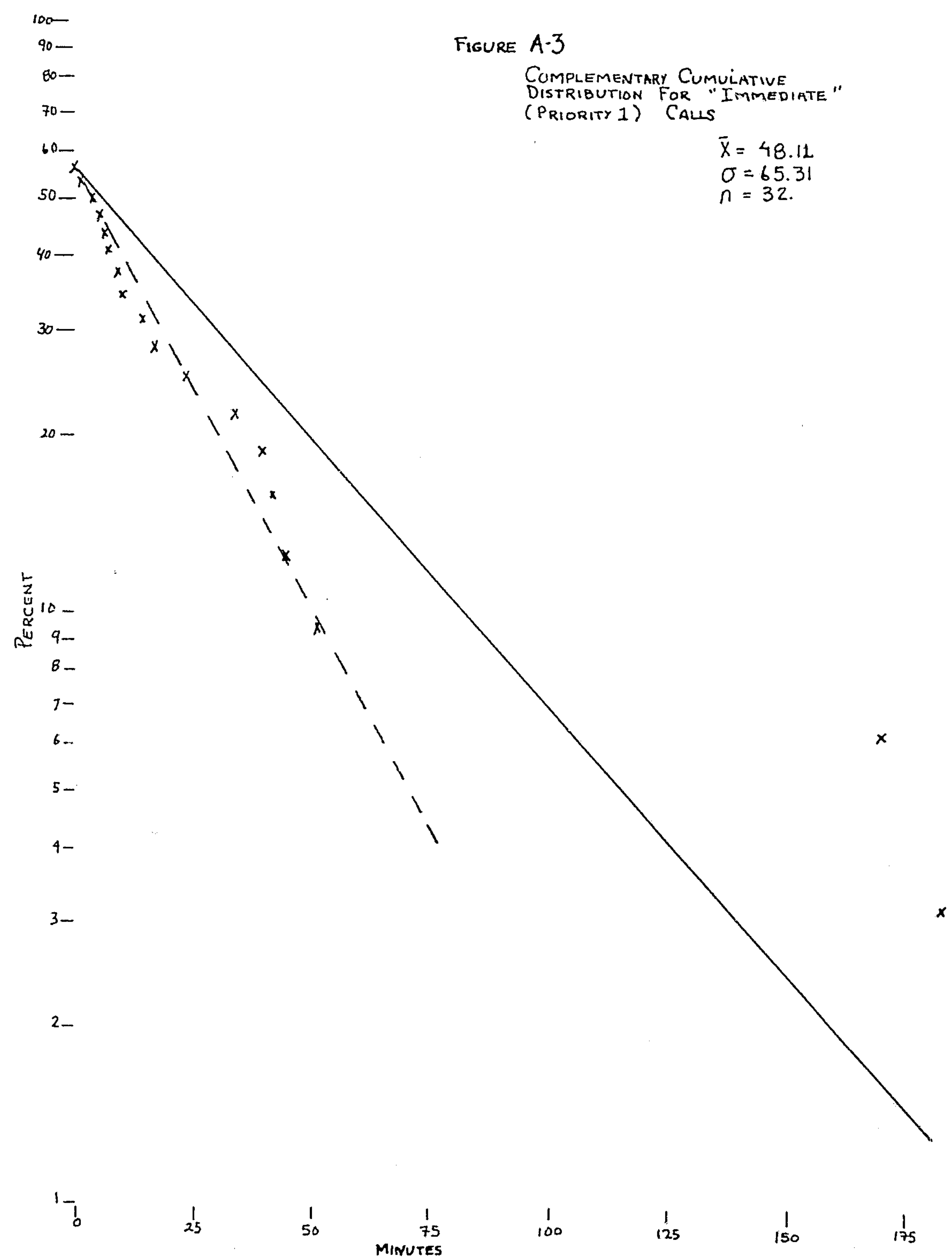


Figure A-3
Complementary Cumulative Distribution For "Immediate" (Priority 1) Calls
$\bar{X}$ = 48.11
σ = 65.31
n = 32.
Percent
Minutes
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
25
50
75
100
125
150
175

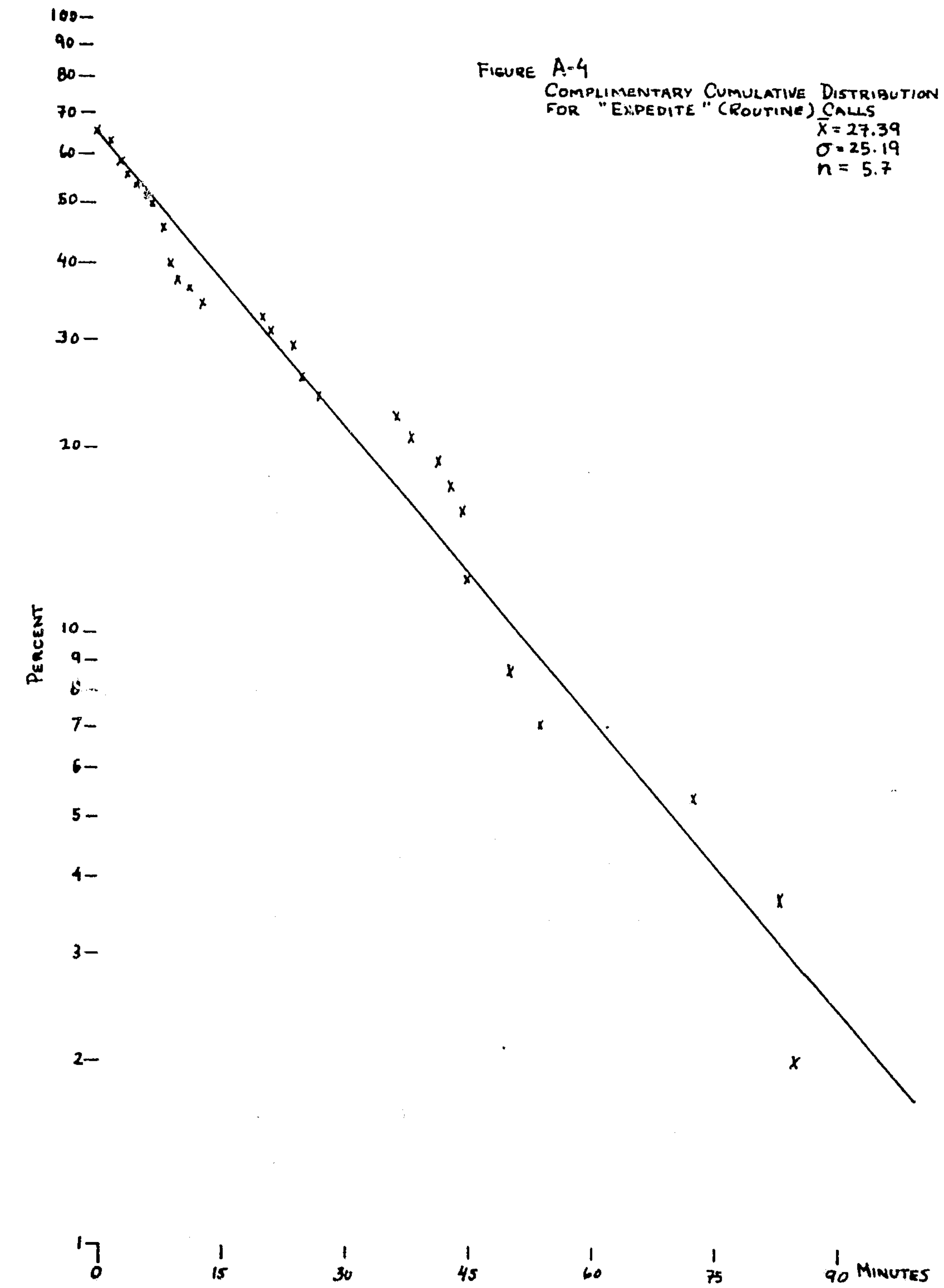


Figure A-4
Complimentary Cumulative Distribution For "Expedite" (Routine) Calls
$\bar{X}$ = 27.39
σ = 25.19
n = 5.7
Percent
Minutes
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
15
30
45
60
75
90

Table A.2:
Travel Time and Service Time Statistics
for 12 Western Townships in
Washtenaw County,
May 17-23, 1982.

	Immediate Priority	Expedite/Normal Priority	Total
Number of Calls	32	57	89
Travel Times			
Mean (Minutes)	15.12	19.00	17.92
Standard Deviation	21.81	15.90	17.88
Service Times			
Mean (Minutes)	48.12	27.39	34.05
Standard Deviation	65.31	25.19	43.43

model thus must have the capability to treat unfounded calls as a separate category, and thereby justify the use of exponential service times for the other calls.

A.3 Response Policy

In general, when the Washtenaw County Sheriff Department receives a call for service, the dispatchers assign a car to handle the call immediately. However, because such a simple policy could result in a misallocation of resources, in 1981 the department implemented a priority-based response policy. This policy assigns calls to four groups: immediate dispatch, expedite dispatch, routine dispatch, and deferred response. Immediate response calls take precedence over all others and can preempt a deputy from servicing a non-immediate call. (Thus they are the equivalent of "emergency" calls in the SWAP model) Cars can be called out of their assigned regions if necessary to cover immediate priority calls. Expedite calls are handled by a car assigned to that township as soon as it is available. Routine calls are handled essentially as expedite calls, with the

exception that their level of urgency is lower and therefore deputies may drive slower in responding to them. Deferred response calls don't require a patrol car and are therefore delayed and serviced by phone.

APPENDIX B: TRAVEL TIME DETERMINATION

B.1 Introduction

In rural police patrol systems, times to service calls are apt to be non-exponential due to non-exponential travel times, which comprise a significant portion of total service time. Any model of rural police patrol activities must therefore be provided with a characterization of travel times. This Appendix presents a practical method for approximating travel time distributions, and illustrates it by application to Washtenaw County.

There have been previous efforts made to characterize travel time distributions. Primarily, work by Larson [1] analyzed travel times in urban areas with roads configured in a Manhattan metric. Larson's method is not well-suited to the rural setting because of the low density of roads and the lack of a consistent Manhattan metric. Work has also appeared in the geography literature on travel time distributions in Euclidean space. This work is also not directly applicable to the rural police patrol problem because sparse roads make Euclidean travel a poor approximation. It would be possible to analytically combine the work of Larson and the geography literature. However, a simpler, more realistic numerical method for generating travel time distributions has been developed. This method transfers some of the burden from the analyst to the computer thereby reducing front-end effort. The procedure is described below.

B.2 Development of Model

The area under consideration (in this case, the county) is divided into subregions (in this case, townships). We wish to find travel time distributions both within (intra) townships and between (inter) townships.

Each township can be represented by a set of nodes. Nodes could be intersections, parking lots, police stations, etc. (The practical considerations of how

to choose a reasonable set of nodes is discussed in the next section). Once the set of nodes for a township has been developed, the travel times between all directly connected nodes must be estimated. Again, developing these travel times is conceptually simple but quite involved from a practical standpoint. Once this network of interconnected nodes has been developed, a number of available shortest path algorithms can be used to construct a travel time matrix, which gives the minimum travel time from each node to all others.

To generate an *intra*township travel time distribution, probabilities of the patrol car being at each node and the incident (call for service) being at each node must be assigned. The product of the probability of the car being at node x and the probability of the incident being at node y defines the probability of arc $x-y$ being traveled. Weighting all arcs in the travel time matrix by these probabilities and summing yields the mean travel time. Similarly, weighting squared times for each arc by the same probabilities and summing allows computation of the variance. The mean and variance could then be used to fit a function (e.g., a gamma distribution). The arcs could also be used to derive a cumulative probability distribution.

To find an *inter*township travel time distribution between any two townships the problem is to construct a travel time matrix of minimum travel times from each node in the township that contains the patrol car (exit township) to all nodes in the township where the incident occurs (entrance township). This can be done by identifying "exit ports" in the exit township and "entrance ports" in the entrance township. Exit and entrance ports are simply nodes where a patrol car leaves and enters the townships. These ports can be identified by considering all possible routes between the two townships. Common sense is required to prevent the number of such routes from becoming unmanageable. Travel times from each exit port to each entrance port must be estimated. Then, a matrix of travel times from nodes in the exit township to the nodes in the entrance township can be developed by exhaustively examining the sum of the travel times from a node in the exit township to each exit port to each entrance port to a node in the entrance township and choosing the minimum time. Minimum times from nodes to exit ports and minimum times from entrance ports to nodes have already been developed in the intratownship travel time matrices. Thus the intertownship travel time matrix can be generated easily using exhaustive search methods. By assigning probabilities of the car and incident being at individual nodes in the exit and entrance townships, respectively, mean and variance of the travel time distribution can be computed as in the intratownship case.

B.3 Practical Considerations of the Approach

The first practical problem in implementing the approach described above is representing a township by a set of nodes. At a maximum, all intersections and points of importance could be designated as nodes. However, the more nodes used the larger the travel time matrices will be. Large matrices will be cumbersome in the computer operations, particularly if a small system (e.g., a TRS-80) is used. Someone well-acquainted with the township should be involved in selecting a set of nodes to ensure that the township is realistically represented. The tradeoffs in choosing a set of nodes are:

- The more nodes used, the fewer arcs will be necessary to represent directly connected nodes. This must be balanced with the need to keep the travel time matrices small.
- If a large number of nodes is used, it is likely that many of the nodes represent obscure places that are unlikely to generate calls. The user must take care to counterbalance this by assigning low probabilities to these nodes to avoid artificially skewing the travel time distribution.

Once a set of nodes has been developed to represent each township, the next problem is to assign travel times between pairs of arcs directly connected by roads. Clearly in any realistic application, where there is likely to be on the order of 50 nodes per township, it is unreasonable to ask a member of the police department to estimate the time between all pairs of directly connected nodes. Instead, it is possible determine the speeds attainable on all roads in the townships under consideration by interviewing someone with patrol experience and to use this information, together with distances estimated from a map, to calculate travel times between nodes.

A sample protocol follows:

Analyst - To characterize travel times, I need to get some idea from you on how fast the police travel on the roads in the county. In particular, I'm interested in determining your effective average speed, including stops, on each road when you're in a hurry and traveling under siren.

Deputy - Well, there's a lot of variability of course. Weather, traffic, experience of the deputy, even farmers driving equipment on the county roads will greatly affect our effective speed.

Analyst - Okay. Let's just consider normal conditions and forget about snowstorms, etc. Can you give me a range of speeds for each road that considers varying traffic and deputy experience?

Deputy - I think so. Lets try some roads.

After considering a few roads a pattern began to emerge. The deputy had developed six classes of roads. These are shown below.

Range of Speeds	Average Speed	Road Speed
55 - 100	90	Highways
45 - 85	70	Very good paved
40 - 70	60	Good paved
30 - 60	40	Good unpaved
30 - 45	30	Poor roads, curves
< 30	20	Congested urban areas

Since many roads traverse multiple townships, reviewing all the roads was not overly time consuming. One issue that did arise was that attainable speeds do not remain constant over the entire length of a road. Changes in the quality of the road or bad curves must be identified in order to make the estimated speeds reflect reality. The method used in this study was to mark each road type on a map with a different color magic marker. This allowed the analyst to record the estimates made by the deputy rapidly enough to keep the discussion moving.

Once speeds on the roads inside and between townships are established it is useful to identify routes between townships. It is not necessary to identify routes between adjacent townships if nodes are defined so that both townships share a set of nodes at points where roads cross the border between the two townships. (The only exception to this would be a case where a patrol car might choose to take a route in another township. This case is probably not too common and if neglected would probably not change the results significantly.) Identifying travel routes between pairs of non-adjacent townships is very useful. While it might be possible to feed geographical orientation of the townships into a computer and have it examine all possible routes, this would almost certainly take a long time on a small computer. It would also require more complex software. The simpler approach we recommend is to simply ask a deputy what routes he or she might use to travel between pairs of townships.

A sample protocol follows:

Analyst - Given that you are patrolling somewhere in Dexter Township and get a call to go to Bridgewater Township. What roads would you use?

Deputy - It depends on where I am in Dexter and where the call is in Bridgewater.

Analyst - That's okay. I want to know all possible routes you might use.

Deputy - Well, those two townships are pretty far apart so I wouldn't use the minor roads, but there seem to be two basic routes. First, if either I or the call was on the east side of our township I'd probably take Werkner down to M-52 and M-52 to Austin. If I or the call was on the west part of the townships I'd take Island Lake or Dexter-Pinckney to Parker to Pleasant Lake to Schneider. If I was on the east and the call was on the west or vice versa it would depend on the specific location of me and the call.

Analyst - That's as it should be. When I give this information to the computer it will look at both routes to go from each point in Dexter to each point in Bridgewater.

In the process of identifying routes between all non-adjacent pairs of townships it became apparent that the routes from township X to township Y were just the reverse of the routes from Y to X. This would be the case unless there was a highway exit that didn't have an entrance or some other unusual road configuration. Another observation that saved interview time was the fact that the routes from a township X to a township Y were very similar to the routes from X to Z if Z was located "past" Y from X. All total, to ascertain road speeds and routes for a portion of county consisting of 12 townships took about 3 hours of time from an analyst and an experienced patroller.

B.4 Results of Computer Runs

A set of BASIC programs was developed to calculate shortest path travel time matrices for each of 12 townships in Washtenaw County and to use these matrices to calculate mean and variance of the travel time distribution from each township to the others. The results are presented in Table B.1. It was assumed that the locations of both call and incident were uniform in space, so that the resulting matrix is symmetric.[3] The matrix would also be symmetric for non-uniform car and call location as long as the distributions of the car and a call are the same in each township. Mean to variance ratios range from about 0.5 to 1.9.

[3]The smallest geographical location routinely kept for origin of a call was "township", so that finer resolution of "where" in the township a call originates from was impossible with the data we had available. The location of the car at time of a call being received was never routinely recorded.

TABLE B.1

INTRA- AND INTER-TOWNSHIP TRAVEL TIMES*

	1	2	3	4	5	6	7	8	9	10	11	12
1	5.9 (8.1)	9.7 (10.9)	15.1 (12.2)	10.9 (23.3)	11.5 (13.6)	17.4 (12.4)	18.9 (20.1)	20.1 (16.9)	21.8 (15.1)	22.6 (15.2)	26.2 (15.8)	35.9 (21.0)
2		4.7 (6.5)	7.9 (10.3)	14.6 (30.3)	10.1 (16.6)	11.0 (13.8)	21.2 (22.3)	16.5 (12.4)	16.3 (13.3)	24.8 (17.5)	23.5 (15.3)	26.2 (19.8)
3			5.4 (8.6)	19.4 (29.1)	11.8 (16.9)	9.9 (16.1)	21.5 (20.0)	17.3 (16.4)	16.9 (17.7)	29.8 (17.4)	23.9 (18.8)	23.8 (20.5)
4				7.6 (16.7)	9.1 (14.4)	13.7 (12.3)	13.0 (30.1)	15.2 (17.6)	18.2 (16.3)	19.4 (23.3)	24.9 (24.5)	32.4 (29.9)
5					6.1 (9.7)	8.9 (11.9)	14.1 (18.6)	10.8 (16.6)	11.8 (15.5)	18.8 (13.9)	19.3 (13.9)	20.7 (20.6)
6						5.6 (8.2)	19.3 (19.6)	12.8 (16.9)	9.9 (17.5)	23.6 (13.5)	19.8 (20.5)	17.6 (21.2)
7							6.9 (13.1)	11.6 (17.6)	18.7 (14.5)	11.9 (20.6)	16.8 (20.8)	25.5 (27.2)
8								5.9 (9.8)	9.0 (12.6)	13.6 (14.7)	10.8 (18.1)	15.2 (26.1)
9									5.5 (8.2)	19.2 (14.3)	13.8 (17.5)	11.2 (18.6)
10										5.8 (9.3)	10.0 (15.9)	15.9 (15.1)
11											6.5 (11.1)	10.8 (16.4)
12												6.6 (11.8)

* matrix is symmetric because distributions of call and incident are assumed to be uniform.

1-LYNDON	5-LIMA	9-LODI
2-DEXTER	6-SCIO	10-MANCHEST
3-WEBSTER	7-SHARON	11-BRIDGE
4-SYLVAN	8-FREEDOM	12-SALINE

B.5 Sensitivity Analysis

Depending on how the nodes are assigned and how police cars actually patrol, the assumptions of uniform spatial distributions for car and incident may or may not be valid. To test the sensitivity of the results to the choice of these distributions a number of cases were examined.

Intra-township travel times under a variety of probability distributions were calculated for Lyndon and Manchester townships. The results of these trials are presented in Table B.2. In general, variance seemed more sensitive than the mean to variations in the probability distributions. One intuitively realistic case had the probability of a call being 50% evenly distributed over 3 "hot spots" and the other 50% distributed over the remaining nodes. In both townships this case resulted in about a 10% increase in variance and, in Lyndon, variance increased by 15%. Clearly, the specific effect depends on the location of the "hot spots". These results do indicate, however, that uniform distributions might be reasonable approximations. Other observations include: a car can reduce mean travel time but increase the variance by patrolling only "hot spots", and a car can reduce both mean and variance by patrolling only fast roads.

Inter-townships travel times under different probability distributions were calculated for trips from Lyndon to Manchester and from Manchester to Lyndon. The results of these trials are presented in Table B.3. As in the intratownship case, the variance was considerably more sensitive than the mean to changes in the probability distributions. For the case where 50% of the probability is concentrated on 3 "hot spots" and the car patrols in accordance with the call distribution, a 6% reduction in mean and 20% reduction in variance were observed.

Considering the fact that it might be difficult to obtain estimates for the probabilities that should be assigned to nodes, since these estimates would have to come from subjective impressions (unless data collection procedures are

TABLE B.2

INTRA-TOWNSHIP TRAVEL TIMES-SENSITIVITY ANALYSIS

MANCHESTER TOWNSHIP

Distribution of Car	Distribution of Car	Mean (minutes)	Variance (Minutes)
Uniform	Uniform	5.8	9.3
Uniform	All calls at one node in center of Township on slow road	5.39	5.48
Uniform	All calls at one node in corner of Township on fast road	5.54	10.16
Uniform	50% of calls concentrated on three nodes & 50% distributed over remaining 36 nodes	5.56	7.92
50% of time on three nodes, 50% on remaining 36	Same as car distribution	5.11	8.44
Car always at central node on slow road (node is one of the three "hot spots")	50% of calls on three nodes 50% on remainder	4.28	6.46
Car patrols only M52 & Austin (fast roads)	50% of calls on three nodes 50% on remainder	5.04	8.36

changed) the assumption of uniform spatial distribution within a region is not unreasonable.

As in the intratownship case, a patrol car can reduce both mean and variance by exclusively patrolling fast roads. Of course, such a procedure would conflict with the need for performing thorough preventive patrol.

B.6 Switching Times Between Townships

One final issue concerns the time to travel from one township to another during routine patrol. (i.e. when called for by the patrol-switch probabilities) Clearly travel times are longer than in the cases where the patrol cars are traveling at high speeds with "lights and sirens". However, routine travel times would not simply be the high speed travel times multiplied by a constant. There are two reasons for this. First, while a deputy might slow down from 90 to 50 mph on good roads he might only slow from 30 to 25 on bad roads, a much smaller percentage decrease. Second, if a deputy decides to change townships and is not in any extreme hurry, he is apt to choose the easiest route and avoid driving rapidly on poor roads.

To characterize travel times during routine patrol, new speed classes for roads were devised. Those are as follows:

Road Type	Speed (mph)
Highways	55
Good Paved Roads	50
Good Gravel Roads	40
Urban Areas	20

Using these roads speeds and the list of main routes between townships obtained during the interview of the deputy, single number estimates of inter-township

travel times during routine patrol were made. These are presented in Table B.4. To account for variability in conditions, etc., a variance equal to 15% of the mean was assigned for each average travel time.

TABLE B.3

INTER-TOWNSHIP TRAVEL TIMES- SENSITIVITY ANALYSIS

LYNDON to MANCHESTER

Distribution of Car in Exit Township	Distribution of Call in Entrance Township	Mean	Variance
Uniform	Uniform	22.6	15.2
Uniform	All calls at one "hot spot"	25.6	6.0
Uniform	50% of calls on one "hot spot" 50% on remaining 38 nodes	24.1	13.1
Uniform	50% of calls on three "hot spots" 50% on remaining 36 nodes	22.9	12.7
50% of time on three "hot spots" 50% on remaining 40 nodes	50% of calls on three "hot spots" 50% on remaining 36 nodes	24.4	8.8
All time on one node, central "hot spot"	50% of calls on three "hot spots" 50% on remaining 36 nodes	23.5	6.7
Car patrols only M52 (fast roads)	Uniform	19.8	12.0
Car patrols on M52 (fast roads)	50% of calls on three "hot spots" 50% on remaining 36 nodes	20.2	9.5
Uniform	Uniform	22.6	15.2

—TABLE B.3— CONTINUED

Uniform	50% of calls on three "hot spots" 50% on remaining 40 nodes	23.6	14.5
50% on three "hot spots" 50% on remaining 36 nodes	50% of calls on three "hot spots" 50% on remaining 40 nodes	24.0	12.0
Car patrols only M52 and Austin (fast roads)	50% of calls on three "hot spots" 50% on remainder	22.4	8.7
Car patrols on M52 and Austin (fast roads)	Uniform	21.4	9.5

TABLE B.4

INTER-TOWNSHIP SWITCHING TIMES

	1	2	3	4	5	6	7	8	9	10	11	12
1	0 0	0 0	8.4 (1.3)	0 0	1.8 (0.3)	10.1 (1.5)	7.6 (1.1)	14.8 (2.2)	22.6 (3.4)	15.4 (2.3)	20.4 (3.0)	28.8 (4.3)
2		0 0	0 0	7.6 (1.1)	0 0	1.0 (0.15)	15.2 (2.3)	21.8 (3.3)	27.6 (4.1)	23.0 (3.4)	28.0 (4.2)	17.8 (2.7)
3			0 0	12.8 (2.0)	2.0 (0.3)	0 0	16.4 (2.5)	8.6 (1.3)	8.6 (1.3)	24.2 (3.6)	17.8 (2.7)	17.8 (2.7)
4				0 0	0 0	8.2 (1.2)	0 0	6.6 (1.0)	15.0 (2.2)	7.8 (1.2)	12.8 (1.9)	21.2 (3.2)
5					0 0	0 0	6.0 (0.9)	0 0	0 0	13.8 (2.1)	9.6 (1.4)	12.0 (1.8)
6						0 0	11.8 (1.8)	0 0	0 0	19.6 (2.9)	9.6 (1.4)	12.0 (1.8)
7							0 0	0 0	8.4 (1.3)	0 0	5.0 (0.8)	13.4 (2.0)
8								0 0	0 0	4.2 (0.6)	0 0	3.1 (0.5)
9									0 0	12.0 (1.8)	5.4 (0.8)	0 0
10										0 0	0 0	8.4 (1.26)
11											0 0	0 0
12												0 0

1-LYNDON 4-SYLVAN 7-SHARON 10-MANCHESTER
2-DEXTER 5-LIMA 8-FREEDOM 11-BRIDGEWATER
3-WEBSTER 6-SCIO 9-LODI 12-SALINE

APPENDIX C: SIMULATION MODEL

A simulation model developed in the course of this study used the SIMSCRIPT simulation language. SIMSCRIPT is a high-level simulation language that has built-in features to facilitate the modeling of systems with inter-event times that follow specified distributions. This police patrol simulation model is an event simulation that assumes exponential inter-call times, exponential service times, and Erlang travel times. The model runs for a specified amount of (simulated) time, generates "calls" and "service times" according to the above distributions, and tallies statistics on the performance of the police patrol system for that particular run.

A number of structural assumptions, in addition to the inter-event time distribution, are built into the model. The model allows three types of calls (immediate, expedite, and unfounded), each with its own mean service time. The model assumes that cars patrol for an amount of time (specified by the user) in a township and then switch to another township according to user-specified probabilities.

Each township has one car assigned to it. When a call for service comes, the car assigned to that township services it if the car is not busy. If the car assigned to the township requiring service is busy, the behavior of the model depends on the type of call:

a) Expedite priority calls are queued when the car serving their township is busy. They are then serviced in the order they were received when the car becomes available.

b) Immediate priority calls preempt the car assigned to the township from servicing any non-immediate priority calls. If the car is already

CONTINUED

1 OF 2

servicing an immediate priority call, then the closest[4] available car from another township is assigned to the immediate call.

The following input data are required for the simulation model.

- Number of Cars
- Number of Regions (Townships)
- Call Arrival Rates
- Service Time Means
- Car Assignments to Townships
- Fraction of Calls of Each Type
- Patrol Stay Time Mean
- Simulation Batch Length.

The input data was formatted using the Michigan Terminal System EDITOR, but an interactive front-end program was developed to complement the simulation model.

A SIMSCRIPT listing of the simulation model is available from the authors.

[4]"Closest" is defined in terms of the average travel time from the township containing the car to the township containing the call. Because travel times are not deterministic, it is possible that another car might turn out to have a shorter actual travel time. However, since the model only "knows" the township containing each car and not the location of the cars within the townships, average travel time is used.

APPENDIX D: PARALLEL ITERATION FOR MULTIPLE SERVERS

This appendix deals with the Parallel Iteration for Multiple Servers (PIMS) approach to the analysis of Markovian service systems. The method is an approximate one, and depends upon an assumption of independence among servers (in the case of this report, police patrol units) that in fact does not exist. Nevertheless useful results are attainable, ones that hold intuitive appeal and moreover have been borne out by numerical experimentation using the SWAP model.

D.1 General Approach.

We assume that there are K service units, each of which can be in one of M states. Transition between states are governed for the k^{th} unit by the Markov transition matrix $P^{(k)}$, $k=1,2, \cdots K$, producing state occupancy probability vectors $\pi^{(k)}$. In order to account for interaction among the service units, some of the elements of $P^{(k)}$ explicitly depend upon the state occupancy probabilities of the other units, that is $P^{(k)} = P^{(k)}[\pi^{(1)}, \pi^{(2)}, \cdots \pi^{(k)}]$. When steady-state probabilities are of interest, the problem reduces to finding the simultaneous solutions to

$$\begin{aligned} \pi^{(k)}P^{(k)} &= \pi^{(k)} \quad k = 1,2, \cdots K \\ \pi^{(k)}e &= 1 \quad\quad k = 1,2, \cdots K \end{aligned} \tag{D.1}$$

where $e = (1,1, \cdots ,1)^t$.

The iterative method of solution is to define $\pi_n^{(k)}$ to be the solution to

$$\begin{aligned} \pi_n^{(k)}A_n^{(k)} &= \pi_n^{(k)} \quad k = 1,2, \cdots K \,;\, n = 1,2, \cdots \\ \pi_n^{(k)}e &= 1 \end{aligned} \tag{D.2}$$

where

$$A_n^{(k)} = P^{(k)}\,[\pi_{n-1}^{(1)}, \pi_{n-1}^{(2)}, \cdots \pi_{n-1}^{(k)}] \qquad k = 1,2, \cdots K \,;\, n = 1,2, \cdots \tag{D.3}$$

The iteration starts with an initial set of vectors $\pi_0^{(1)}, \pi_0^{(2)}, \cdots \pi_0^{(k)}$. Equation (D.3) is used to compute $A_1^{(k)}$ for $k = 1,2, \cdots K$. Then equations (D.2) are solved to find $\pi_1^{(1)}, \pi_1^{(2)}, \cdots \pi_1^{(k)}$, which are used in (D.3) to find $A_2^{(k)}$, etc.

The fundamental assertions behind this method are that for all $k = 1, 2, \cdots K$

a) $\pi_n^{(k)} \to \pi^{(k)}$ as $n \to \infty$, independent of $\pi_0^{(k)}$.

b) The convergence of a) is rapid enough, and regular enough, to allow numerical computation of $\pi^{(k)}$ by a sufficiently small number of iterations

c) $\pi^{(k)}$ can be interpreted to be steady-state probabilities of interest to a policy maker.

In the following sections these assertions are examined from both an analytical and computational perspective, in the context of two simple examples. For these examples it is possible to obtain analytical solutions to both exact representations and PIMS-like approximations. Comparisons between the two solution methods show the advantages and potential problems with the approach.

D.2 Example 1: Two servers, two regions, no queues.

Consider a system with two servers and two regions. The rate of calls for region i is λ_i, the total rate is thus $\lambda = \lambda_1 + \lambda_2$. The (exponential) service rate for server i is μ_i, and there is *no* travel to service. The policy is such that if both servers are free (i.e. not servicing a call) then server i responds to calls from region i. If one server is busy, then the other services *any* arriving call. If both servers are busy, then an arriving call is "lost".

This system is most conveniently represented as a continuous parameter Markov process (rather than as a discrete process as discussed in section D.1).

However, the approach is the same.

D.2.1. Exact Solution

First, for comparison to the PIMS approximation an *exact* analytic solution is easily obtained. The states of the process are:

0 = no servers busy
1 = server 1 busy
2 = server 2 busy
B = both servers busy

The transition *rate* matrix[5] is:

$$R = \begin{array}{c|cccc|} & 0 & 1 & 2 & B \\ \hline 0 & * & \lambda_1 & \lambda_2 & 0 \\ 1 & \mu_1 & * & 0 & \lambda \\ 2 & \mu_2 & 0 & * & \lambda \\ B & 0 & \mu_2 & \mu_1 & * \end{array}$$

(Note that a discrete time transition matrix P may be obtained from the rate matrix by multiplying all off-diagonal terms by the transition period Δ, and then appropriately making the diagonal terms such that row sums are one.)

The resulting *exact* steady state probabilities (obtained from $R\pi = 0, \pi e = 1$) are

[5]In all rate matrices, the diagonal terms -- indicated by * -- are the negative sum of the other row terms.

$$\pi(0) = \left\{1 + \frac{(\mu+\lambda)}{\mu(2\lambda+\mu)}\left[\frac{\lambda^2+\mu\lambda_1}{\mu_1} + \frac{\lambda^2+\mu\lambda_2}{\mu_2}\right]\right\}^{-1}$$

$$\pi(1) = \frac{1}{2\lambda+\mu}\left[\frac{\lambda^2+\mu\lambda_1}{\mu_1}\right]\pi(0)$$

$$\pi(2) = \frac{1}{2\lambda+\mu}\left[\frac{\lambda^2+\mu\lambda_2}{\mu_2}\right]\pi(0) \qquad \text{(D.4)}$$

$$\pi(B) = [\pi(1)+\pi(2)]\,\frac{\lambda}{\mu}$$

where $\mu \equiv \mu_1+\mu_2$. These are the desired variables needed to evaluate the system's performance.

D.2.2. PIMS Solution

The PIMS approximation treats the two servers separately. In particular, server 1 is represented by a two-state Markov process with states

F_1 = server 1 is free
B_1 = server 1 is busy ,

and $b_1 \equiv \text{prob}\{B_1\} = 1 - \text{prob}\{F_1\}$

Similarly, server 2 is assumed to be in one of two states

F_2 = server 2 is free
B_2 = server 2 is busy

and $b_2 \equiv \text{prob}\{B_2\} = 1 - \text{prob}\{F_2\}$. We see that b_1 and b_2 are variables of interest to a decision maker.

Since an empty server 1 will receive calls from region 2 only when server 2 is busy, the transition rate matrix for server 1 is, according to the PIMS approximation,

$$R^{(1)} = \begin{array}{c} \\ F_1 \\ B_1 \end{array} \begin{array}{cc} F_1 & B_1 \\ \left|\begin{array}{cc} * & \lambda_1+b_2\lambda_2 \\ \mu_1 & * \end{array}\right| \end{array}$$

Similarly for server 2 the rate matrix is

$$R^{(2)} = \begin{array}{c} \\ F_2 \\ B_2 \end{array} \begin{array}{cc} F_2 & B_2 \\ \left|\begin{array}{cc} * & \lambda_2+b_1\lambda_1 \\ \mu_2 & * \end{array}\right| \end{array}$$

The variables of interest, b_1 and b_2, can now be determined by simultaneously solving the steady-state equations

$$(b_1, 1-b_1)R^{(1)} = (0, 0)$$

$$(b_2, 1-b_2)R^{(2)} = (0, 0)$$

which yield:

$$b_1 = \frac{\lambda_1+b_2\lambda_2}{\mu_1+\lambda_1+b_2\lambda_2} \qquad \text{(D.5a)}$$

$$b_2 = \frac{\lambda_2+b_1\lambda_1}{\mu_2+\lambda_2+b_1\lambda_1} \qquad \text{(D.5b)}$$

These non-linear equations in this simple example can be, in fact, solved analytically since equations (D.5) produce quadratic equations in b_1 or b_2.

The more general iterative approach to the solution of equations (D.5) makes use of the fact that we can define the sequence $\bar{b}_1(n)$ and $\bar{b}_2(n)$, $n=0,1,2\cdots$ with initial values $\bar{b}_1(0)$, $\bar{b}_2(0)$, so that

$$\bar{b}_1(n) = \frac{\lambda_1+\bar{b}_2(n-1)\lambda_2}{\mu_1+\lambda_1+\bar{b}_2(n-1)\lambda_2} \qquad \text{(D.6a)}$$

$$\bar{b}_2(n) = \frac{\lambda_2+\bar{b}_1(n-1)\lambda_1}{\mu_2+\lambda_2+\bar{b}_1(n-1)\lambda_1} \qquad \text{(D.6b)}$$

If equations (D.5) represent a contraction mapping taking the unit interval into itself, then the contraction mapping theorem (see, for example, Edwards, C.H., *Advanced Calculus of Several Variables*, Academic Press p.181) gives:

$$\lim_{n\to\infty} \bar{b}_1(n) \to b_1 \quad ; \lim_{n\to\infty} \bar{b}_2(n) \to b_2$$

The conditions under which (D.5) is a contraction mapping are readily found by noting that $\bar{b}_1(n)$ can be gotten from (D.6), by solving

$$\bar{b}_1(n+2) = \frac{\gamma+\beta\bar{b}_1(n)}{\alpha+\delta\bar{b}_1(n)}$$

where:

$$\alpha = \lambda_1\lambda_2 + \lambda_1\mu_2 + \lambda_2^2$$
$$\beta = \lambda^2 + \lambda_1\lambda_2$$
$$\gamma = \lambda_1\lambda_2 + \lambda_1\mu_2 + \lambda_2^2 + \lambda_2\mu_1 + \mu_1\mu_2$$
$$\delta = \lambda_1^2 + \lambda_1\lambda_2 + \lambda_1\mu_1$$

The equation

$$f(x) = \frac{\alpha+\beta x}{\gamma+\delta x}$$

is a contraction mapping when $|\beta\gamma-\alpha\delta| < \gamma^2$, a condition which can be shown to hold for all $\lambda_i, \mu_i \geq 0$.

D.2.3. Comparison of Exact and PIMS Solution

The accuracy of the PIMS approximate solution can be demonstrated by comparing output variables of interest obtained from solutions of (D.4) with (D.5). A first comparison involves the variable prob. { server 1 is busy }. This is b_1 in the PIMS approximation, and is $\pi(1) + \pi(B)$ in the exact solution. Numerical analysis shows that the percent difference between these is less than 5% for a wide range of values of λ_i and μ_i, including all those typical of real service systems that would be load-sharing between regions (i.e. $0.2 \leq \lambda_1/\lambda_2 \leq 5$, $0.5 \leq \mu_1/\mu_2 \leq 2$). In particular, when $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$ the *maximum* error of 4.5% is attained when $\lambda/\mu = 1$, and this falls to less than

3% for $\lambda/\mu < 5$.

In order to compare computations of prob. { both servers are busy }, which equals $\pi(B)$ in the exact model, it is necessary to examine the dependence between the servers implicit in the PIMS approach. In particular

prob { both servers busy } = prob { server 1 busy }·
prob { server 2 busy | server 1 busy }

$$= b_1 \left[\frac{\lambda_2+\lambda_1}{\lambda_2+\lambda_1+\mu_2}\right] \qquad \text{(D.7)}$$

The first term in equation (D.7) comes directly from the simultaneous solutions of equations (D.5). The second term comes from the second equation of (D.5) with $b_1 = 1$ (i.e. the "given" that server 1 is busy).

Similarly, it is possible to compare $\pi(0)$ of the exact solution to

prob { both servers are free } = prob { server 1 free }·
prob { server 2 free | server 1 free }

$$= (1 - b_1)\left[\frac{\lambda_2}{\mu_2+\lambda_2}\right] \qquad \text{(D.8)}$$

Again, numerical computation shows that both (D.7) and (D.8) differ from the exact values ($\pi(B)$ and $\pi(0)$) by less than 5% for all reasonable values of λ_i and μ_i.

Thus, even in this case where arrivals are lost to the system when both servers are busy -- a case that will exacerbate errors introduced by the PIMS independence assumption -- the results are definitely usable for policy purposes.

D.3. Example 2: Two servers, two regions with queues.

We now compare the exact and PIMS approach in the case of two servers and two regions, with queued calls allowed -- the queue being "shared" between the two servers. Again (for convenience of discussion) there is no travel to service. If both servers are free then server i responds to calls from region i. If one server is busy then the other services *any* arriving call. If both servers are busy, then arriving calls enter a queue from which they are serviced by the next server to become free.

D.2.3.1. Exact Solution

For the *exact solution*, consider the states:

0 = no servers busy
$1a$ = only server 1 is busy
$1b$ = only server 2 is busy
2 = both servers are busy, none in queue
n = both servers are busy, $(n-2)$ calls are in the queue, $n = 3,4,\cdots$

and again let $\lambda = \lambda_1 + \lambda_2$, $\mu = \mu_1 + \mu_2$

The transition rate matrix is

$$R = \begin{array}{c|cccccccc|} & 0 & 1a & 1b & 2 & 3 & \cdot & \cdot & \cdot \\ 0 & * & \lambda_1 & \lambda_2 & 0 & 0 & \cdot & \cdot & \cdot \\ 1a & \mu_1 & * & 0 & \lambda & 0 & \cdot & \cdot & \cdot \\ 1b & \mu_2 & 0 & * & \lambda & 0 & \cdot & \cdot & \cdot \\ 2 & 0 & \mu_2 & \mu_1 & * & \lambda & 0 & \cdot & \cdot \\ 3 & 0 & 0 & 0 & \mu & * & \lambda & \cdot & \cdot \\ \cdot & & & & 0 & & & & \\ \cdot & & & & & & & & \\ \cdot & & & & & & & & \end{array}$$

and the resulting steady-state probabilities are

$$\pi(0) = \left\{1 + \frac{\mu}{(\mu-\lambda)(2\lambda+\mu)}\left[\frac{\lambda^2+\mu\lambda_1}{\mu_1} + \frac{\lambda^2+\mu\lambda_2}{\mu_2}\right]\right\}^{-1}$$

$$\pi(1a) = \frac{1}{2\lambda+\mu}\left[\frac{\lambda^2+\mu\lambda_1}{\mu_1}\right]\pi(0)$$

$$\pi(1b) = \frac{1}{2\lambda+\mu}\left[\frac{\lambda^2+\mu\lambda_2}{\mu_2}\right]\pi(0) \qquad \text{(D.9)}$$

$$\pi(n) = \left[\pi(1a) + \pi(1b)\right]\left[\frac{\lambda}{\mu}\right]^{n-1}$$

We note that two variables of interest are

prob. { server 1 is free } = $\pi(0) + \pi(1b)$
prob. { server 2 is free } = $\pi(0) + \pi(1a)$

D.2.3.2. PIMS Solution

Again we force interdependence between the servers to appear only as a adjustments to the arrival rate "seen" by each server. The state space for server 1 is:

0 = server 1 free
n = server 1 busy, $(n-1)$ calls are in the queue,

with an associated rate matrix:

$$R^{(1)} = \begin{array}{c|ccccccc|} & 0 & 1 & 2 & 3 & \cdot & \cdot & \cdot \\ 0 & * & \lambda_1+b_2\lambda_2 & 0 & 0 & \cdot & \cdot & \cdot \\ 1 & \mu_1 & * & \lambda_1 b_2 & 0 & \cdot & \cdot & \cdot \\ 2 & 0 & \mu_1 & * & \lambda_1 b_2 & 0 & \cdot & \cdot \\ 3 & 0 & 0 & \mu_1 & * & \lambda_1 b_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \end{array}$$

where b_2 = prob { server 2 is busy }.

A similar argument holds for the rate matrix $R^{(2)}$ of server 2, with of course the subscripts properly adjusted. Steady-state solutions then give

$$b_1 = \text{prob } \{ \text{ server 1 is busy } \} = \frac{\lambda_1+b_2\lambda_2}{\mu_1+\lambda_1+b_2(\lambda_2-\lambda_1)}$$

$$b_2 = \text{prob } \{ \text{ server 2 is busy } \} = \frac{\lambda_2+b_2\lambda_1}{\mu_2+\lambda_2+b_2(\lambda_1-\lambda_2)} \qquad \text{(D.10)}$$

Again, although it is easy to reduce equations (D.10) to two separate quadratic equations, one in b_1 and one in b_2, it is possible to solve them iteratively by defining

$$\bar{b}_1(n) = \frac{\lambda_1+\bar{b}_2(n-1)\lambda_2}{\mu_1+\lambda_1+\bar{b}_2(n-1)(\lambda_2-\lambda_1)}$$

$$\bar{b}_2(n) = \frac{\lambda_2+\bar{b}_1(n-1)\lambda_1}{\mu_2+\lambda_2+\bar{b}_1(n-1)(\lambda_1-\lambda_2)} \qquad \text{(D.11)}$$

and using initial values $\bar{b}_1(0)$ and $\bar{b}_2(0)$. These also represent a contraction mapping, and so convergence to the solution of equations (C.10) is assured.

D.2.3.1. Comparison of Exact and PIMS Solution

Numerical computations were performed to compare results obtained from the exact solutions (D.9) and PIMS approximations (D.11). The variables of interest are

variable	exact	PIMS
prob {server 1 is busy}	$1-\pi(0)-\pi(1b)$	b_1
prob {server 2 is busy}	$1-\pi(0)-\pi(1a)$	b_2
prob {both servers are free}	$\pi(0)$	$\left\{ \begin{array}{l} (1-b_1)\left[\frac{\mu_2}{\mu_2+\lambda_2}\right] \text{ or} \\ (1-b_2)\left[\frac{\mu_1}{\mu_1+\lambda_1}\right] \end{array} \right.$

Table D.1

The PIMS computation of the probability that both servers are free is once more obtained from prob {server 1 is free} · prob { server 2 is free | server 1 is free}. Again, the PIMS approximation is quite accurate. Errors are less than 3% for $0.5 \le \mu_1/\mu_2 \le 2$ and $0.2 \le \lambda_1/\lambda_2 \le 5$, (as long as both $\lambda_1/\mu_1 < 1$ and $\lambda_2/\mu_2 < 1$, the usual conditions for stability).

For the special case $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$, equations (D.9) reduce to the well known $M/M/2$ queue results

$$\pi(0) = \frac{1-\rho}{1+\rho}$$

$$\pi(1a) = \pi(1b) = \rho\pi(0)$$

$$\pi(n) = 2\rho^n\pi(0)$$

where $\rho = \lambda/\mu$. The PIMS equations (D.10) give $b_1 = b_2 = \rho$. Using these in table D.1 shows that, for the variables of interest listed above, the PIMS approach gives *exact* solutions.

Thus in the case where queueing is possible, the PIMS approximation is extremely attractive, producing accurate or even exact results.

D.4. Convergence of PIMS and Selection of Initial Conditions.

For the full scale SWAP model used in section 5 of this report, the iterative solution method was necessary due to the size of the state space involved. Two practical issues remain to be completely resolved, although our numerical experience to date has been encouraging.

The first issue concerns the rate of convergence of the iterations. For a wide variety of input data, our experience has been that at most 5 or 6 iterations were needed to have all $\pi_n^{(k)}$ sufficiently close to $\pi_{n-1}^{(k)}$ so that convergence is assured. Although the theoretical basis for this rapid convergence, and results guaranteeing error bounds, still remain to be established, we are satisfied that the approach is sound and applicable to well-balanced patrol systems (that is systems which *a priori* attempt to *roughly* equalize total call rates per responding unit).

The second issue involves the choice of initial conditions $\pi_0^{(k)}$. An unfortuitous choice *may* effect the type of convergence to $\pi^{(k)}$ (whether monotone or oscillatory), and therefore could effect the accuracy with which values of $\pi_n(k)$, gotten at the end of an "absolute difference" termination criterion, reflect those of $\pi^{(k)}$. Numerical experimentation has shown that results are sometimes sensitive to the initial conditions. However, by judiciously selecting a number of *different* initial conditions and comparing results at convergence, it is possible to rapidly "bracket" the true solutions to equations (D.1) to any degree of accuracy. In particular, selecting for initial conditions ones that imply either of the extremes

a) prob {unit k is patrolling } = 1
or b) prob {unit k is serving } = 1

produces a good "bracket" of the values of $\pi^{(k)}$.

APPENDIX E: SWAP COMPUTER PROGRAM

The SWAP program uses a Markov chain that has 8 states for each region for each car. The 8 states are patrol (PATR), expedite[6] travel (ETRV), immediate travel (ITRV), expedite service (ESRV), expedite service with a queue (ESVQ), immediate service with a queue (ISVQ), unfounded service (UNFS). As described in section 5, the Markov models for each car are run in parallel until the entire system reaches equilibrium.

The input data for the SWAP program is entered interactively by the user. The required data items are:

- Number of Regions
- Number of Cars
- Travel Time Means (minutes) from each region to all others.
- Hourly call rates of each call type in each region
- Mean service times of each call type in each region
- Hourly patrol switching probabilities
- Assigned coverage of each car to each region

The SWAP program echoes the input data and copies both input and output into a file. An example of a file from a run for Washtenaw County follows.

A listing of the FORTRAN code for the SWAP program is also included in this appendix.

[6]This notation is due to Washtenaw County's use of the terms "immediate" for "emergency", and "expedite" for "routine".

E.1

Sample Input and
Output from the SWAP
Computer Program

```
TEST12.4
NUMBER OF REGIONS
12
NUMBER OF CARS
 4
TRAVEL ALPHA VALUES
 0.68  0.93  1.26  0.46  0.87  1.38  0.95  1.19  1.42  1.50  1.64  1.70
 0.93  0.64  0.76  0.48  0.59  0.82  0.94  1.33  1.23  1.41  1.53  1.34
 1.26  0.76  0.56  0.67  0.68  0.61  1.07  1.04  0.95  1.71  1.26  1.18
 0.46  0.48  0.67  0.39  0.66  1.09  0.46  0.86  1.10  0.82  1.00  1.08
 0.87  0.59  0.68  0.66  0.66  0.79  0.78  0.65  0.76  1.33  1.40  1.01
 1.38  0.82  0.61  1.09  0.79  0.71  0.98  0.78  0.61  1.74  0.96  0.85
 0.95  0.94  1.07  0.46  0.78  0.98  0.58  0.69  1.28  0.59  0.83  0.94
 1.19  1.33  1.04  0.86  0.65  0.78  0.69  0.68  0.67  0.96  0.56  0.59
 1.42  1.23  0.95  1.10  0.76  0.61  1.28  0.67  0.73  1.35  0.80  0.62
 1.50  1.41  1.71  0.82  1.73  1.74  0.59  0.96  1.35  0.69  0.60  1.07
 1.64  1.53  1.26  1.00  1.40  0.96  0.83  0.56  0.80  0.60  0.62  0.65
 1.70  1.34  1.18  1.08  1.01  0.85  0.94  0.59  0.62  1.07  0.65  0.61
TRAVEL TIME K VALUES
     4     9    19     5    10    24    18    24    31    34    43    61
     9     3     6     7     6     9    20    22    20    35    36    35
    19     6     3    13     8     6    23    18    16    51    30    28
     5     7    13     3     6    15     6    13    20    16    25    35
    10     6     8     6     4     7    11     7     9    25    27    21
    24     9     6    15     7     4    19    10     6    41    19    15
    18    20    23     6    11    19     4     8    24     7    14    24
    24    22    18    13     7    10     8     4     6    13     6     9
    31    20    16    20     9     6    24     6     4    26    11     7
    34    35    51    16    25    41     7    13    26     4     6    17
    43    36    30    25    27    19    14     6    11     6     4     7
    61    36    28    35    21    15    24     9     7    17     7     4
HOURLY CALL RATES
REGION  EXPEDITE IMMEDIATE UNFOUNDED
 LYND   0.038       0.017     0.003
 DEXT   0.070       0.032     0.005
 WEBS   0.041       0.019     0.003
 SYLV   0.046       0.021     0.004
 LIMA   0.036       0.017     0.003
 SCIO   0.197       0.091     0.015
 SHAR   0.012       0.006     0.001
 FREE   0.015       0.007     0.001
 LODI   0.038       0.017     0.003
 MANC   0.020       0.009     0.002
 BRID   0.009       0.004     0.001
 SALI   0.021       0.010     0.002
MEAN SERVICE TIMES
REGION  EXPEDITE IMMEDIATE UNFOUNDED
 LYND   27.36       48.12
 DEXT   27.36       48.12
 WEBS   27.36       48.12
 SYLV   27.36       48.12
 LIMA   27.36       48.12
 SCIO   27.36       48.12
 SHAR   27.36       48.12
 FREE   27.36       48.12
 LODI   27.36       48.12
 MANC   27.36       48.12
 BRID   27.36       48.12
 SALI   27.36       48.12
HOURLY PATROL SWITCHING PROBABILITIES
```

CAR. 1

0.250	0.250	0.0	0.250	0.250	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250	0.250	0.0	0.250	0.250	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250	0.250	0.0	0.250	0.250	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.250	0.250	0.0	0.250	0.250	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CAR. 2

0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.200	0.0	0.200	0.200	0.200
0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.200	0.0	0.200	0.200	0.200
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.200	0.0	0.200	0.200	0.200
0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.200	0.0	0.200	0.200	0.200
0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.200	0.0	0.200	0.200	0.200

CAR. 3

0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.330	0.0	0.0	0.330	0.0	0.0	0.330	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.330	0.0	0.0	0.330	0.0	0.0	0.330	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
0.0	0.0	0.330	0.0	0.0	0.330	0.0	0.0	0.330	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0

CAR. 4

0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0

COVERAGE

CAR REGIONS 1-12

1	1.000	1.000	0.0	1.000	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	1.000	1.000	0.0	1.000	1.000	1.000
3	0.0	0.0	1.000	0.0	0.0	0.200	0.0	0.0	1.000	0.0	0.0	0.0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.800	0.0	0.0	0.0	0.0	0.0	0.0

2	DEXT
3	WEBS
4	SYLV
5	LIMA
6	SCIO
7	SHAR
8	FREE
9	LODI
10	MANC
11	BRID
12	SALI

POLICE PATROL MODEL OUTPUT

CAR 1 — FRACTION OF TIME BY ACTIVITY

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
LYND	0.181	0.008	0.003	0.014	0.001	0.011	0.002	0.000	0.220
DEXT	0.214	0.010	0.003	0.026	0.002	0.020	0.003	0.000	0.280
WEBS	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SYLV	0.197	0.011	0.004	0.017	0.002	0.013	0.002	0.000	0.246
LIMA	0.201	0.009	0.003	0.013	0.001	0.011	0.002	0.000	0.241
SCIO	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SHAR	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
FREE	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
LODI	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
MANC	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
BRID	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SALI	0.001	0.001	0.000	0.0	0.0	0.000	0.000	0.0	0.002
ALL	0.805	0.041	0.013	0.070	0.007	0.055	0.008	0.001	1.000

CAR 2 — FRACTION OF TIME BY ACTIVITY

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
LYND	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
DEXT	0.002	0.000	0.000	0.0	0.0	0.001	0.000	0.0	0.002
WEBS	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SYLV	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
LIMA	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SCIO	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
SHAR	0.178	0.005	0.002	0.005	0.000	0.004	0.000	0.000	0.195
FREE	0.194	0.004	0.002	0.006	0.000	0.005	0.000	0.000	0.212
LODI	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
MANC	0.179	0.004	0.001	0.008	0.000	0.006	0.000	0.000	0.200
BRID	0.166	0.003	0.001	0.004	0.000	0.003	0.000	0.000	0.177
SALI	0.180	0.005	0.002	0.009	0.000	0.007	0.000	0.000	0.204
ALL	0.906	0.023	0.008	0.032	0.001	0.028	0.002	0.001	1.000

CAR 3 — FRACTION OF TIME BY ACTIVITY

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
LYND	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
DEXT	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.003
WEBS	0.272	0.010	0.003	0.016	0.001	0.013	0.001	0.000	0.315
SYLV	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
LIMA	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SCIO	0.290	0.008	0.003	0.024	0.000	0.016	0.000	0.000	0.340
SHAR	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
FREE	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
LODI	0.284	0.010	0.003	0.015	0.001	0.011	0.001	0.000	0.326
MANC	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
BRID	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
SALI	0.002	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
ALL	0.860	0.029	0.010	0.054	0.002	0.042	0.003	0.001	1.000

CAR 4 — FRACTION OF TIME BY ACTIVITY

REGION	PATR	ETRV	ITRV	ESRV	ESVQ	ISRV	ISVQ	UNFS	ALL
LYND	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
DEXT	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.002
WEBS	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
SYLV	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
LIMA	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
SCIO	0.844	0.018	0.007	0.062	0.001	0.053	0.001	0.001	0.987

SHAR	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
FREE	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
LODI	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
MANC	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
BRID	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
SALI	0.001	0.000	0.000	0.0	0.0	0.000	0.000	0.0	0.001
ALL	0.854	0.019	0.007	0.062	0.001	0.055	0.001	0.001	1.000

AVERAGE RESPONSE TIME TO EACH REGION (MINUTES)

REGION	EXPEDITE	IMMEDIATE	UNFOUNDED
LYND	16.264	10.368	16.264
DEXT	16.335	10.088	16.335
WEBS	16.762	11.076	16.762
SYLV	17.583	11.182	17.583
LIMA	15.602	9.408	15.602
SCIO	8.780	6.405	8.780
SHAR	20.651	14.675	20.651
FREE	16.525	11.446	16.525
LODI	16.539	10.846	16.539
MANC	16.840	11.929	16.840
BRID	16.231	11.411	16.231
SALI	21.191	15.106	21.191

AVERAGE TIME IN QUEUE (MINUTES)

REGION	EXPEDITE	IMMEDIATE	UNFOUNDED
LYND	3.612	0.000	3.612
DEXT	3.612	0.000	3.612
WEBS	2.615	0.000	2.615
SYLV	3.612	0.000	3.612
LIMA	3.612	0.000	3.612
SCIO	0.354	0.000	0.354
SHAR	1.768	0.000	1.768
FREE	1.768	0.000	1.768
LODI	2.615	0.000	2.615
MANC	1.768	0.000	1.768
BRID	1.768	0.000	1.768
SALI	1.768	0.000	1.768

PROBABILITY TRAVEL TIME TO IMMEDIATE CALLS IS LESS THAN OR EQUAL TO MINUTES

REGION	3	6	9	12	15	18	21	24	27
LYND	0.036	0.217	0.506	0.757	0.901	0.964	0.988	0.996	0.999
DEXT	0.081	0.268	0.509	0.720	0.855	0.929	0.966	0.985	0.993
WEBS	0.076	0.282	0.560	0.797	0.926	0.977	0.993	0.998	1.000
SYLV	0.033	0.188	0.432	0.657	0.812	0.904	0.954	0.980	0.991
LIMA	0.039	0.236	0.523	0.766	0.907	0.969	0.991	0.997	0.999
SCIO	0.141	0.542	0.813	0.933	0.977	0.993	0.998	0.999	1.000
SHAR	0.020	0.134	0.380	0.661	0.852	0.946	0.982	0.995	0.998
FREE	0.033	0.173	0.404	0.645	0.818	0.916	0.964	0.986	0.995
LODI	0.060	0.283	0.571	0.804	0.929	0.978	0.994	0.998	1.000
MANC	0.034	0.208	0.505	0.765	0.908	0.968	0.989	0.997	0.999
BRID	0.025	0.178	0.445	0.704	0.871	0.953	0.985	0.995	0.999
SALI	0.024	0.157	0.418	0.676	0.840	0.926	0.968	0.987	0.995

E.2

SWAP Program

Listing

```
C THIS PROGRAM ACCEPTS INPUT DATA FOR THE TRANSITION MATRIX FOR THE
C   C-CAR MARKOV MODEL WITH THREE TYPES OF CALLS AND QUEUEING.
      DIMENSION XLAM(20,3),XCHNG(20,20,5),TTMN(20,20),SRVMN(20,2)
      DIMENSION P(5,160,160),PI(160),TLAM(3,5),A(20,20)
      DIMENSION COVP(5,20),TTLAM(5),BUSI(5),BUSY(5),PROB(5,160)
      DIMENSION COVI(5,20),COVE(5,20),RPROB(5,20),APROB(5,20),TCPROB(5)
      DIMENSION TLAMP(3,5),TTLAMP(5),OBUSY(5),OBUSI(5)
      DIMENSION NX(32,4), ESTA(3,5,20), INAME(20)
      INTEGER DATA(4),KP(20,20)
      DATA IY/'Y   '/
      COMMON /X1/XLAM,XCHNG,TTMN,SRVMN,P,PI,TLAM,DATA,N,COVP,NC,
     1 BUSY,BUSI,COVE,COVI,ESTA,TLAMP,TTLAMP,NX,M,PROB
      COMMON /X2/INAME,A,KP
C
C    XLAM - CALL ARRIVAL RATES 1- EXPEDITE
C                              2- IMMEDIATE
C                              3- UNFOUNDED
C
C    XCHNG - SWITCH PROBABILITIES
C
C    TTMN - MEAN TRAVEL TIMES
C
C    SRVMN - MEAN SERVICE TIMES 1- EXPEDITE
C                               2- IMMEDIATE
C
C
C    P  - TRANSITION MATRIX
C
C    PI - LONG RUN PROBABILITIES
C
C    PINEW - WORK VECTOR FOR LONG RUNS
C
C   TLAMP - CAR C'S ORIGINAL RATES FOR EXP, IMM, AND UNF CALLS
C
C   TLAM - CAR C'S TOTAL EFFECTIVE RATES FOR EXP, IMM, AND UNF CALLS
C
C   TTLAM - CAR C'S TOTAL EFFECTIVE RATE FOR ALL CALLS
C
C
C   COVP(C,J) - COVERAGE FOR CAR C IN REGION J
C
C   DATA - NAME OF DATA SET TO WRITE TO
C
C----------------------------------------------------------------
C
C  START INPUTING DATA
C
C CHOOSE TO MODIFY OR CREATE A NEW FILE
      WRITE(6,1)
 1    FORMAT(' DO YOU WANT TO MAKE A NEW DATASET ?')
```

```
C THIS PROGRAM ACCEPTS INPUT DATA FOR THE TRANSITION MATRIX FOR THE
C   C-CAR MARKOV MODEL WITH THREE TYPES OF CALLS AND QUEUEING.
      DIMENSION XLAM(20,3),XCHNG(20,20,5),TTMN(20,20),SRVMN(20,2)
      DIMENSION P(5,160,160),PI(160),TLAM(3,5),A(20,20)
      DIMENSION COVP(5,20),TTLAM(5),BUSI(5),BUSY(5),PROB(5,160)
      DIMENSION COVI(5,20),COVE(5,20),RPROB(5,20),APROB(5,20),TCPROB(5)
      DIMENSION TLAMP(3,5),TTLAMP(5),OBUSY(5),OBUSI(5)
      DIMENSION NX(32,4), ESTA(3,5,20), INAME(20)
      INTEGER DATA(4),KP(20,20)
      DATA IY/'Y   '/
      COMMON /X1/XLAM,XCHNG,TTMN,SRVMN,P,PI,TLAM,DATA,N,COVP,NC,
     1 BUSY,BUSI,COVE,COVI,ESTA,TLAMP,TTLAMP,NX,M,PROB
      COMMON /X2/INAME,A,KP
C
C    XLAM - CALL ARRIVAL RATES 1- EXPEDITE
C                              2- IMMEDIATE
C                              3- UNFOUNDED
C
C    XCHNG - SWITCH PROBABILITIES
C
C    TTMN - MEAN TRAVEL TIMES
C
C    SRVMN - MEAN SERVICE TIMES 1- EXPEDITE
C                               2- IMMEDIATE
C
C
C    P  - TRANSITION MATRIX
C
C    PI - LONG RUN PROBABILITIES
C
C    PINEW - WORK VECTOR FOR LONG RUNS
C
C   TLAMP - CAR C'S ORIGINAL RATES FOR EXP, IMM, AND UNF CALLS
C
C   TLAM - CAR C'S TOTAL EFFECTIVE RATES FOR EXP, IMM, AND UNF CALLS
C
C   TTLAM - CAR C'S TOTAL EFFECTIVE RATE FOR ALL CALLS
C
C
C   COVP(C,J) - COVERAGE FOR CAR C IN REGION J
C
C   DATA - NAME OF DATA SET TO WRITE TO
C
C---------------------------------------------------------------
C
C  START INPUTING DATA
C
C CHOOSE TO MODIFY OR CREATE A NEW FILE
      WRITE(6,1)
1     FORMAT(' DO YOU WANT TO MAKE A NEW DATASET ?')
      READ(5,2) IM
2     FORMAT(A1)
      IF(IM.EQ.IY) GO TO 5
      CALL CHNGDT
      GO TO 21
5     CONTINUE
      WRITE(6,10)
10    FORMAT(' INPUT THE NUMBER OF  REGIONS (I2)')
      READ(5,11) N
C
```

```
C      N - NO OF REGIONS
C
11     FORMAT(I2)
C
C .    NC - NUMBER OF CARS
C
       WRITE(6,40)
40     FORMAT('ENTER THE NUMBER OF CARS (I2)')
       READ(5,11) NC
C
C  LOOP THROUGH ALL REGIONS
C
       DO 20 I=1, N
C
C START CALL RATE INPUT
C
       WRITE(6,12) I
12     FORMAT(' ENTER THE EXP, IMM, UNF CALL RATES PER HOUR IN REGION ',
      1 I2,' (3F5.3)')
       READ(5,13) (XLAM(I,J),J=1,3)
13     FORMAT(3F5.3)
C
C START TO INPUT SERVICE MEANS
C
       WRITE(6,14) I
14     FORMAT(' ENTER THE EXP AND IMM SERVICE MEANS IN ',I2,
      1 ' IN MINUTES (2F6.2)')
       READ(5,15) SRVMN(I,1),SRVMN(I,2)
15     FORMAT(2F6.2)
C
C INPUT PATROL SWITCH PROBABILITIES
C
       DO 201 K=1,NC
       DO 201 J=1,N
       WRITE(6,16) K, I,J
  16   FORMAT(' ENTER THE ONE-HR PATROL SWITCH PROBS FOR CAR ',I2,
     1  ' FROM ',I2,' TO ',I2, ' (F5.3)')
       READ(5,17) XCHNG(I,J,K)
17     FORMAT(F5.3)
201       CONTINUE
C
C   INPUT COVERAGE
C
       DO 203 K=1,NC
       WRITE(6,161) K,I
161    FORMAT(' ENTER FRACTION COVERAGE FOR CAR ',I2,' IN REGION ',I2,' (F5.3)')
       READ (5,17) COVP(K,I)
203    CONTINUE
C
C INPUT THE ALPHA VALUES FOR TRAVEL TIMES
C
       DO 20 J=1,N
       WRITE(6,18) I,J
18     FORMAT(' ENTER THE ALPHA VALUES FOR TRAVEL TIME FROM ',I2,' TO ',I2,
      1 ' IN MINUTES(F6.3)')
       READ(5,19) A(I,J)
19     FORMAT(F6.3)
20     CONTINUE
C
C  INPUT K VALUES FOR TRAVEL TIMES
```

```
C
      DO 2001 J=1,N
      WRITE(6,2002) I,J
2002  FORMAT(' ENTER K VALUE FOR TRAVEL FROM ',I2,' TO ',I2,
     1 ' IN MINUTES(I2)')
      READ(5,2003) KP(I,J)
2003  FORMAT(I2)
2001  CONTINUE
C
C  INPUT REGION NAMES
C
      DO 2020 I=1,N
      WRITE (6,2010) I
2010  FORMAT (' ENTER 4 LETTER NAME FOR REGION ',I2)
      READ (5,2011) INAME(I)
2011  FORMAT (A4)
2020  CONTINUE
C
21    CONTINUE
C
C
C -----------------------------------------------------------------------------
C
C  CHOOSE DATA SET
C
C -----------------------------------------------------------------------------
C
      WRITE(6,3000)
3000  FORMAT(' DO YOU WANT TO PUT THIS ON A NEW FILE ?')
      READ(5,3001) NK
3001  FORMAT(A1)
      IF(NK.NE.IY) GO TO 3005
      WRITE(6,301)
301   FORMAT(' WHAT FILE WILL THIS BE ON ?')
      READ(5,302) DATA
302   FORMAT(4A4)
C
C  ASSIGN 2 TO THE CHOSEN FILE
C
      CALL FTNCMD('$CRE ?',6,DATA)
3005  CONTINUE
      CALL FTNCMD('ASSIGN 2=?',10,DATA)
C
C WRITE OUT DATA
C
C MAME OF DATA SET
C
      WRITE(2,3020) DATA
3020  FORMAT (1X,4A4)
C
C NUMBER OF  REGIONS
C
      WRITE(2,303) N
303   FORMAT(' NUMBER OF REGIONS',/,I3)
C
C   NUMBER OF CARS
C
      WRITE(2,306) NC
  306 FORMAT (' NUMBER OF CARS',/,I3)
C
```

```
C
      DO 2001 J=1,N
      WRITE(6,2002) I,J
2002  FORMAT(' ENTER K VALUE FOR TRAVEL FROM ',I2,' TO ',I2,
     1 ' IN MINUTES(I2)')
      READ(5,2003) KP(I,J)
2003  FORMAT(I2)
2001  CONTINUE
C
C  INPUT REGION NAMES
C
      DO 2020 I=1,N
      WRITE (6,2010) I
2010  FORMAT (' ENTER 4 LETTER NAME FOR REGION ',I2)
      READ (5,2011) INAME(I)
2011  FORMAT (A4)
2020  CONTINUE
C
21    CONTINUE
C
C
C ---------------------------------------------------------------------------
C
C  CHOOSE DATA SET
C
C ---------------------------------------------------------------------------
C
      WRITE(6,3000)
3000  FORMAT(' DO YOU WANT TO PUT THIS ON A NEW FILE ?')
      READ(5,3001) NK
3001  FORMAT(A1)
      IF(NK.NE.IY) GO TO 3005
      WRITE(6,301)
301   FORMAT(' WHAT FILE WILL THIS BE ON ?')
      READ(5,302) DATA
302   FORMAT(4A4)
C
C  ASSIGN 2 TO THE CHOSEN FILE
C
      CALL FTNCMD('$CRE ?',6,DATA)
3005  CONTINUE
      CALL FTNCMD('ASSIGN 2=?',10,DATA)
C
C WRITE OUT DATA
C
C MAME OF DATA SET
C
      WRITE(2,3020) DATA
3020  FORMAT (1X,4A4)
C
C NUMBER OF  REGIONS
C
      WRITE(2,303) N
303   FORMAT(' NUMBER OF REGIONS',/,I3)
C
C   NUMBER OF CARS
C
      WRITE(2,306) NC
  306 FORMAT (' NUMBER OF CARS',/,I3)
C
```

```
C TRAVEL PARAMETERS
C
      WRITE (2, 3061)
3061  FORMAT (' TRAVEL ALPHA VALUES ')
C
      DO 305 I=1,N
      WRITE(2,304) (A(I,J),J=1,N)
304   FORMAT (12F6.2)
305   CONTINUE
C
      WRITE(2,3051)
3051  FORMAT('TRAVEL TIME K VALUES')
      DO 3052 I=1,N
      WRITE(2,3053) (KP(I,J),J=1,N)
3052  CONTINUE
3053  FORMAT (12I6)
C
C CALL RATES
      WRITE (2, 307)
307   FORMAT (' HOURLY CALL RATES' / ' REGION  EXPEDITE IMMEDIATE UNFOUNDED')
      DO 315 I=1,N
      WRITE(2,3071) INAME(I), (XLAM(I,J),J=1,3)
3071  FORMAT (2X,A4,3X,F5.3,5X,F5.3,5X,F5.3)
315   CONTINUE
C
C SERVICE MEANS
C
      WRITE (2, 3151)
3151  FORMAT.(' MEAN SERVICE TIMES' / ' REGION  EXPEDITE IMMEDIATE UNFOUNDED')
C
      DO 320 I=1,N
      WRITE(2,308) INAME(I), (SRVMN(I,J),J=1,2)
308   FORMAT (2X,A4,3X,F5.2,5X,F5.2,5X,F5.2)
320   CONTINUE
C
C SWITCH PROBABILITIES
      WRITE (2, 309)
309   FORMAT (' HOURLY PATROL SWITCHING PROBABILITIES')
      DO 330 K=1,NC
      WRITE (2, 310) K
310   FORMAT (' CAR.',I2)
      DO 330 I=1,N
      WRITE(2,311) (XCHNG(I,J,K),J=1,N)
311   FORMAT (12F6.3)
330   CONTINUE
C
C   COVERAGE MATRIX
C
      WRITE (2, 3301) N
3301  FORMAT (' COVERAGE' / ' CAR  REGIONS 1-',I2)
C
      DO 340 K=1,NC
      WRITE(2,312) K, (COVP(K,J),J=1,N)
312   FORMAT (I3,12F6.3)
340   CONTINUE
C
C  REGION NAMES
C
      WRITE (2,3410)
3410  FORMAT (' REGION NAMES')
```

```
C
      DO 350 I=1,N
      WRITE (2,3415) I, INAME(I)
3415  FORMAT (I3,5X,A4)
350   CONTINUE
C
C   END   OF   INPUT
C
C-----------------------------------------------------------------
C
C   INITIALIZE PARAMETERS FOR FIRST CALCULATION OF
C   THE TRANSITION MATRIX
C
C-----------------------------------------------------------------------
C
C . M - SIZE OF THE MATRIX
C
C   TTLAM - TOTAL CALL RATE FOR ALL TYPES
C
C
C  CALCULATE TRAVEL TIME MEANS
C
      DO 951 I=1,N
      DO 951 J=1,N
      TTMN(I,J)=KP(I,J)/A(I,J)
951   CONTINUE
C
C
C   CONVERT TO 5 MIN PERIODS
C
      DO 950 I=1, N
      XLAM(I,1)=XLAM(I,1)/12.0
      XLAM(I,2)=XLAM(I,2)/12.0
      XLAM(I,3)=XLAM(I,3)/12.0
      SRVMN(I,1)=SRVMN(I,1)/5.
      SRVMN(I,2)=SRVMN(I,2)/5.
      DO 950 J=1, N
      TTMN(I,J)=TTMN(I,J)/5.0
      DO 950 K=1, NC
      XCHNG(I,J,K)=XCHNG(I,J,K)/12.0
  950 CONTINUE
C
C  START COVI AND COVE MATRICES WITH COVP VALUES
C
      DO 970 I=1,N
      DO 960 J=1,NC
      COVI(J,I) = COVP(J,I)
      COVE(J,I) = COVP(J,I)
960   CONTINUE
970   CONTINUE
C
C   NX - MATRIX OF 0'S AND 1'S FOR PARALLEL ITERATIONS
C   CREATE NX MATRIX
C
      LINE = 0
      DO 700 I=1, 2
      DO 700 J=1, 2
      DO 700 K=1, 2
      DO 700 L=1, 2
      LINE = LINE + 1
```

```
      NX(LINE,1) = L - 1
      NX(LINE,2) = K - 1
      NX(LINE,3) = J - 1
      NX(LINE,4) = I - 1
700   CONTINUE
C
C  CALCULATE PERMANENT RATES
C
      DO 391 K=1,NC
      TLAMP(1,K)=0.0
      TLAMP(2,K)=0.0
      TLAMP(3,K)=0.0
        DO 391 I=1,N
      TLAMP(1,K)=TLAMP(1,K)+XLAM(I,1)*COVP(K,I)
      TLAMP(2,K)=TLAMP(2,K)+XLAM(I,2)*COVP(K,I)
      TLAMP(3,K)=TLAMP(3,K)+XLAM(I,3)*COVP(K,I)
      TTLAMP(K)=TLAMP(1,K)+TLAMP(2,K)+TLAMP(3,K)
391   CONTINUE
C
C  INITIALIZE TEMPORARY EFFECTIVE RATES
      DO 392 K=1,NC
      DO 392 J=1,3
      TLAM(J,K)=TLAMP(J,K)
392   CONTINUE
C
C  START WITH BUSY AND BUSI EQUAL TO ONE AS THE SEED
C
      DO 393  I=1,NC
      BUSY(I)=1.0
      BUSI(I)=1.0
393   CONTINUE
C
      M=8*N
C
C ------------------------------------------------------------------------
C
C  FIND STEADY STATES (TMATRIX) AND ITERATIVELY REVISE
C  CALL RATES (PARIT) UNTIL PROBABILITIES OF BEING BUSY
C  CONVERGE
C
C ------------------------------------------------------------------------
C
C  STORE OLD BUSY PROBABILITIES
C
      ICON = 1
      ICOUNT = 0
3930  DO 394 K=1,NC
      OBUSY(K)=BUSY(K)
      OBUSI(K)=BUSI(K)
394   CONTINUE
C
      ICOUNT = ICOUNT + 1
      WRITE (6,3941) ICOUNT
3941  FORMAT (' ITERATION NUMBER ',I2)
C
C  CHECK FOR CONVERGENCE
C
      IF (ICON .EQ. 1) CALL TMATRX
      IF (ICON .EQ. 1) CALL PARIT
      ICON = 0
```

```
      C
      C  FIND DIFFERENCE BETWEEN OLD AND NEW BUSY PROBABILITIES
      C
            DO 395 K=1,NC
            DIFF1=ABS(OBUSY(K) - BUSY(K))
            DIFF2=ABS(OBUSI(K) - BUSI(K))
            IF (DIFF1 .GE. 0.001) ICON=1
            IF (DIFF2 .GE. 0.001) ICON=1
      395   CONTINUE
      C
            IF (ICON .EQ. 1) GOTO 3930
      C
      C  CALCULATE PROBABILITIES SUMMED OVER REGIONS AND AVTIVITIES
      C
      C  RPROB(I) - TOTAL FRACTION OF TIME CAR K SPENDS IN REGION I
      C  APROB(I) - TOTAL FRACTION OF TIME CAR K SPENDS ON ACTIVITY J
      C
            DO 3990 K=1,NC
      C
            DO 3965 I=1,N
            RPROB(K,I)=0
      C
            DO 3960 IJ=1,8
            JJ=8*(I-1)+IJ
            RPROB(K,I)=RPROB(K,I)+PROB(K,JJ)
      3960  CONTINUE
      C
      3965  CONTINUE
      C
            DO 3975 J=1,8
            APROB(K,J)=0
      C
            DO 3970 LJ=1,N
            LL=8*(LJ-1)+J
            APROB(K,J)=APROB(K,J)+PROB(K,LL)
      3970  CONTINUE
      C
      3975  CONTINUE
      C
            TCPROB(K)=0
      C
            DO 3980 I=1,N
            TCPROB(K)=TCPROB(K)+RPROB(K,I)
      3980  CONTINUE
      C
      3990  CONTINUE
      C
      C  WRITE OUT FINAL PROBABILITIES
      C
            WRITE (6,3890)
            WRITE (2,3890)
      3890  FORMAT ('1')
            WRITE (6,3900)
            WRITE (2,3900)
      3900  FORMAT (72X)
            WRITE (6,3901)
            WRITE (2,3901)
      3901  FORMAT (' POLICE PATROL MODEL OUTPUT')
      C
            DO 3950 K=1,NC
```

```
      WRITE (6,3900)
      WRITE (2,3900)
      WRITE (6,3903) K
      WRITE (2,3903) K
3903  FORMAT (' CAR',I2,'              FRACTION OF TIME BY ACTIVITY')
      WRITE (6,3905)
      WRITE (2,3905)
3905  FORMAT (' REGION  PATR  ETRV  ITRV  ESRV  ESVQ  ISRV  ISVQ  UNFS   ALL')
C
      DO 3940 I=1,N
C
      IFIRST = 8*I-7
      ILAST = 8*I
C
      WRITE (6,3907) INAME(I), (PROB(K,IJ), IJ=IFIRST,ILAST), RPROB(K,I)
      WRITE (2,3907) INAME(I), (PROB(K,IJ), IJ=IFIRST,ILAST), RPROB(K,I)
3907  FORMAT (2X,A4,2X,9F6.3)
3940  CONTINUE
C
      WRITE (6,3908) (APROB(K,J), J=1,8), TCPROB(K)
      WRITE (2,3908) (APROB(K,J), J=1,8), TCPROB(K)
3908  FORMAT ('  ALL',3X,9F6.3)
C
3950  CONTINUE
C
C  CALCULATE RESPONSE TIMES
C
      CALL RESPNS
C
      END
C
C  END OF MAIN PROGRAM
C
C ***********************************************************************
C
C  SUBROUTINE TO CALCULATE TRANSITION MATRIX AND STEADY STATES
C  FOR INDIVIDUAL CARS
C
C ***********************************************************************
C
      SUBROUTINE TMATRX
C
      DIMENSION XLAM(20,3),XCHNG(20,20,5),TTMN(20,20),SRVMN(20,2)
      DIMENSION P(5,160,160),PI(160),PINEW(160),TLAM(3,5)
      DIMENSION COVP(5,20),TTLAM(5),BUSI(5),BUSY(5),PROB(5,160)
      DIMENSION COVI(5,20), COVE(5,20),A(20,20)
      DIMENSION TLAMP(3,5), TTLAMP(5)
      DIMENSION NX(32,4), FRACT(3,5,20), ESTA(3,5,20), INAME(20)
      INTEGER DATA(4),STATE,M1,M2,M3,M4,M5,M6,M7,M8,KP(20,20)
      DATA M1/'PATR'/,M2/'ETRV'/,M3/'ITRV'/,M4/'ESRV'/,M5/'ESVQ'/,
     1 M6/'ISRV'/,M7/'ISVQ'/,M8/'UNFS'/,IY/'Y   '/,M9/'OUTS'/
      COMMON /X1/XLAM,XCHNG,TTMN,SRVMN,P,PI,TLAM,DATA,N,COVP,NC,
     1 BUSY,BUSI,COVE,COVI,ESTA,TLAMP,TTLAMP,NX,M,PROB
      COMMON /X2/INAME,A,KP
C
C   START KTH CAR ITERATION
C
      DO 200 K=1, NC
      TTLAM(K)=TLAM(1,K)+TLAM(2,K)+TLAM(3,K)
C
```

```
C*DEBUG      WRITE (6,50) K
   50 FORMAT (' CAR ',I2)
C
C ITERATE THROUGH THE  REGIONS
C
       DO 100 I=1,M
        DO 100 J=1,M
       P(K,I,J)=0.0
100     CONTINUE
C
       DO 200 I=1,N
C
C
C
C FIND PROBS FROM PATROL
C
       L=8*(I-1)+1
C
C L - LEAVING STATE
C
       P(K,L,L)=1.0
C
C TO TRAVEL (EXPEDITE OR UNFOUNDED)
C
       P(K,L,L+1)=TLAM(1,K)+TLAM(3,K)
C
C TO TRAVEL (IMMEDIATE)
C
       P(K,L,L+2)=TLAM(2,K)
C
C TO SELF
C
       P(K,L,L)=1-P(K,L,L+1)-P(K,L,L+2)
C
C TO OTHER PATROL
C
       DO 110 J=1,N
       IF(I.EQ.J) GO TO 110
       LL=8*(J-1) + 1
       P(K,L,LL)=XCHNG(I,J,K)
       P(K,L,L)=P(K,L,L)-P(K,L,LL)
110    CONTINUE
C
C
C
C FIND PROBS FROM TRAVEL (EXPEDITE OR UNFOUNDED)
C
       L = 8*(I-1) + 2
C
C FIND DENOM FOR CALCULATION
C
       DEN=0.0
       P(K,L,L)=1.00
       DO 120 J=1,N
       DEN1=XLAM(J,1)*TTMN(I,J)*COVE(K,J)*1.3
       DEN3=XLAM(J,3)*TTMN(I,J)*COVE(K,J)*1.3
       DEN=DEN+DEN1+DEN3
120    CONTINUE
C
C*DEBUG      WRITE (6,55) DEN
```

```
   55 FORMAT (' DEN 1,3 = ',F5.3)
C
C TO SERVICE
C
      DO 130 J=1,N
      LL=(J-1)*8+4
C
C  EXPEDITE SERVICE
C
      P(K,L,LL)=XLAM(J,1)*COVE(K,J)/DEN
      P(K,L,L)=P(K,L,L)-P(K,L,LL)
C
C UNFOUNDED SERVICE
C
      LL=LL+4
      P(K,L,LL)=XLAM(J,3)*COVE(K,J)/DEN
      P(K,L,L)=P(K,L,L)-P(K,L,LL)
 130  CONTINUE
C
C
C
C FIND PROBS FROM TRAVEL (IMMEDIATE)
C
      L = 8*(I-1) + 3
      P(K,L,L)=1.0
C
C FIND DENOM FOR CALCULATION
C
      DEN=0.0
      DO 131 J=1,N
      DEN=DEN+XLAM(J,2)*TTMN(I,J)*COVI(K,J)
 131  CONTINUE
C
C*DEBUG      WRITE (6, 60) DEN
   60 FORMAT ( 'DEN 2 = ',F5.3)
C
C TO SERVICE (IMMEDIATE)
C
      DO 132 J=1,N
      LL=(J-1)*8 + 6
      P(K,L,LL)=XLAM(J,2)*COVI(K,J)/DEN
      P(K,L,L)=P(K,L,L)-P(K,L,LL)
 132  CONTINUE
C
C
C
C FIND PROBS FROM UNFOUNDED
C
      L=I*8
      LL=L - 7
      P(K,L,LL)=1.0
C
C
C
C FIND PROBS FROM EXPEDITE SERVICE (NO QUEUE)
C
      L=(I-1)*8 + 4
C
C TO SERVICE AND QUEUE
C
```

```
C  CALCULATE PROBABILITY THAT OTHER CARS WITH RESPONSIBILITY FOR
C  REGION I ARE ALL BUSY, THUS CAUSING A QUEUE TO FORM
C
      XPROD=1.0
      DO 135 IK=1,NC
      IF (IK .EQ. K) GOTO 135
      IF (COVP(IK,I) .EQ. 0) GOTO 135
      XPROD=XPROD*BUSY(IK)
135   CONTINUE
C
C  MULTIPLY RATES OF TYPE 1 AND 3 CALLS BY XPROD TO GET PROB OF QUEUE
C
      P(K,L,L+1)=(TLAMP(1,K)+TLAMP(3,K))*XPROD
C
C  TO TRAVEL (IMMEDIATE)
C
      P(K,L,L-1) = TLAM(2,K)
C
C  TO SELF
C
      P(K,L,L)=1.0-P(K,L,L+1)-P(K,L,L-1)
C
C
C TO PATROL   .
C
      P(K,L,L-3)=1/SRVMN(I,1)
C
C UPDATE SELF
C
      P(K,L,L)=P(K,L,L)-P(K,L,L-3)
C
C
C
C FIND PROBS FROM IMMEDIATE SERVICE (NO QUEUE)
C
      L = 8*(I-1) + 6
      P(K,L,L)=1.0
C
C TO SERVICE WITH QUEUE
C
C  CALCULATE PROBABILITY THAT ALL OTHER CARS ARE BUSY
C
      ZPROD=1.0
      DO 140 IK=1,NC
      IF (IK .EQ. K) GOTO 140
      ZPROD=ZPROD*BUSI(IK)
140   CONTINUE
C
C  MULTIPLY RATE OF TYPE 2 CALLS BY ZPROD AND MULTIPLY TYPE 1
C  AND TYPE 3 CALLS BY XPROD TO GET PROB OF QUEING CALLS
C
      P(K,L,L+1)=(TLAMP(1,K)+TLAMP(3,K))*XPROD+TLAMP(2,K)*ZPROD
      P(K,L,L)=P(K,L,L)-P(K,L,L+1)
C
C TO PATROL
C
      P(K,L,L-5)= 1/SRVMN(I,2)
      P(K,L,L)=P(K,L,L)-P(K,L,L-5)
C
C
```

```
C
C  FROM EXPEDITE SERVICE WITH QUEUE
C
      L = (I-1)*8 + 5
      P(K,L,L)=1.0
C
C TO TRAVEL (EXPEDITE) IF DONE
C
      P(K,L,L-3)=1/SRVMN(I,1)
      P(K,L,L)=P(K,L,L)-P(K,L,L-3)
C
C
C
C FROM IMMEDIATE SERVICE WITH QUEUE
C
      L = L + 2
      P(K,L,L)=1.0
C
C TO TRAVEL (EXPEDITE) IF DONE
C
      P(K,L,L-5)=(1/SRVMN(I,2))*(TLAM(1,K)+TLAM(3,K))/TTLAM(K)
C
C TO TRAVEL (IMMEDIATE) IF DONE
C
      P(K,L,L-4)=(1/SRVMN(I,2))*TLAM(2,K)/TTLAM(K)
C
      P(K,L,L)=P(K,L,L) - P(K,L,L-5) - P(K,L,L-4)
200   CONTINUE
C
C  WRITE OUT TRANSITION PROBABILITIES
C
      GOTO 2001
      DO 2000 K=1,NC
      DO 2000 L=1,M
      DO 2000 LL=1,M
      WRITE (6,201) K,L,LL,P(K,L,LL)
201   FORMAT (' P(',I2,',',I2,',',I2,')= ',F5.3)
2000  CONTINUE
      STOP
2001  CONTINUE
C
C  END DATA ENTRY
C
C-------------------------------------------------------------------
C
C  BEGIN TO COMPUTE STEADY STATE PROBABILITIES
C
C ------------------------------------------------------------------------
C
C  INITIALIZE COUNTERS FOR ACCUMULATING PROB OF CAR I BEING IN REGION J.
C
      DO 3900 I=1,NC
C
C  SET FRACT AND ESTA TO ZERO
C
      DO 3800 IJ=1,3
      DO 3800 J=1,N
      FRACT(IJ,I,J)=0
      ESTA(IJ,I,J)=0
3800  CONTINUE
```

```
C
3900  CONTINUE
C
C  START CAR K LOOP
C
      DO 435 K=1,NC
C
C  SET ACCUMULATORS TOTE AND TOTI TO ZERO
C
      TOTE=0
      TOTI=0
C
      DO 400 I=1,M
      PINEW(I)=0.0
400   CONTINUE
      DO 401 I=1,N
      L=8*I-7
      PINEW(L)=1/FLOAT(N)
C*DEBUG      WRITE (6,4001) L, PINEW(L)
4001  FORMAT ( 'PINEW(',I2,')= ',F6.4)
401   CONTINUE
C
C MULTIPLY AS LONG AS DIFF > .001
C
      NITR = 0
C
C NITR - NO OF ITERATIONS
C
410     CONTINUE
      NITR = NITR + 1
      DO 412 I=1,M
      PI(I)=PINEW(I)
412   CONTINUE
C
C MULTIPLY OUT THE VECTOR
C
      DO 420 I=1,M
      PINEW(I)=0.0
      DO 420  J=1,M
      PINEW(I)=PINEW(I) + PI(J)*P(K,J,I)
420   CONTINUE
C
C CHECK FOR ACCURACY
C
      DO 425 I=1,M
      DIFF = ABS(PI(I)-PINEW(I))
      IF(DIFF.GE.0.001) GO TO 410
425   CONTINUE
C
C HERE ALL PROBS ARE WITHIN .001 OF PREVIOUS ITERATIONS
C
C  DEBUGGING AID - PRINTS OUT PROBS ON EACH ITERATION OF TMATRIX
C
      GOTO 4352
C
C WRITE OUT HOW LONG IT TOOK AND THE PROBS
C
      WRITE (6,429)
      WRITE (2,429)
429   FORMAT (72X)
```

```
      WRITE(6,430) K, NITR
      WRITE(2,430) K, NITR
430   FORMAT('CAR',I2,'. ',I5,' ITERATIONS FOR .001 DIFFERENCE')
C
      DO 4351 I=1,M
C
C FIND THE PROPER STATE
C
      KS = MOD(I,8)
      IF(KS.EQ.1) STATE=M1
      IF(KS.EQ.2) STATE=M2
      IF(KS.EQ.3) STATE=M3
      IF(KS.EQ.4) STATE=M4
      IF(KS.EQ.5) STATE=M5
      IF(KS.EQ.6) STATE=M6
      IF(KS.EQ.7) STATE=M7
      IF(KS.EQ.0) STATE=M8
433   CONTINUE
      ZI=(I-1)/8.0
      IF(I.LE.8*N) IJ=INT(ZI)+1
      IF(I.GT.8*N) IJ=N+(I-8*N)
C
      WRITE(6,434) STATE,IJ,PI(I)
434   FORMAT(A4,I4,' PROB =',F5.3)
4351  CONTINUE
C
4352  CONTINUE
C
      DO 500 J=1,M
      PROB(K,J)=PI(J)
500   CONTINUE
C
C  COMPUTE PROBABILITY CAR K IS BUSY (ANY CALL)
C
      SUM = 0
      DO 436 IJ=1, N
      L=8*IJ-7
      SUM = SUM + PI(L)
436   CONTINUE
      BUSY(K)=1-SUM
C*DEBUG      WRITE (6,4361) K, BUSY(K)
4361  FORMAT (' BUSY(',I1,')= ',F5.3)
C
C  COMPUTE PROBABILITY THAT CAR K IS BUSY (IMMEDIATE CALL)
C
      BUSI(K)=0
      DO 437 IJ=1,N
      L1=8*IJ-2
      L2=8*IJ-1
      BUSI(K)=BUSI(K)+PI(L1)+PI(L2)
437   CONTINUE
C*DEBUG      WRITE (6,4371) K, BUSI(K)
4371  FORMAT (' BUSI(',I1,')= ',F5.3)
C
C  COMPUTE PROB CAR K IS IN REGION IJ AND AVAILABLE FOR CALLS OF TYPES 1,2,3
C
C  FRACT(L,K,IJ) - FRACTION OF TIME CAR K IS IN IJ AVAILABLE FOR TYPE L CALLS
C  TOTE (TOTI), - TOTAL TIME CARS ARE AVAILABLE FOR EXPEDITE (IMMEDIATE) CALLS
C  ESTA(L,K,IJ) - PROB CAR K IS IN IJ AVAILABLE FOR TYPE L CALLS
C
```

```
      DO 4353 I=1,M
      KS=MOD(I,8)
      ZI=(I-1)/8.0
      IF(I.LE.8*N) IJ=INT(ZI)+1
      IF(I.GT.8*N) IJ=N+(I-8*N)
C
      IF (KS .NE. 6 .AND. KS .NE. 7) FRACT(2,K,IJ)=FRACT(2,K,IJ)+PI(I)
      IF (KS .NE. 6 .AND. KS .NE. 7) TOTI=TOTI+PI(I)
      IF (KS .EQ. 1) FRACT(1,K,IJ)=FRACT(1,K,IJ)+PI(I)
      IF (KS .EQ. 1) TOTE=TOTE+PI(I)
C
C*DEBUG      WRITE(6,4340) FRACT(1,K,IJ), TOTE
4340  FORMAT (' FRACT1= ',F5.3,' TOTE= ',F5.3)
C*DEBUG      WRITE (6,4341) FRACT(2,K,IJ), TOTI
4341  FORMAT (' FRACT2= ',F5.3,' TOTI= ',F5.3)
4353  CONTINUE
C
      DO 4362 IJ=1,N
      ESTA(1,K,IJ) = FRACT(1,K,IJ)/TOTE
      ESTA(2,K,IJ) = FRACT(2,K,IJ)/TOTI
      ESTA(3,K,IJ) = ESTA(1,K,IJ)
C*DEBUG      WRITE (6,4360) ESTA(1,K,IJ)
4360  FORMAT (' ESTA1= ',F5.3)
C*DEBUG      WRITE (6,4370) ESTA(2,K,IJ)
4370  FORMAT (' ESTA2= ',F5.3)
4362  CONTINUE
C
435   CONTINUE
C
      RETURN
      END
C
C  END OF TMATRX
C
C ************************************************************************
C
C  SUBROUTINE TO PERFORM PARALLEL ITERATIONS.  CALL RATES ARE
C  UPDATED DURING EACH ITERATION THROUGH COVI, COVE,  BUSY, AND BUSI
C
C ************************************************************************
C
      SUBROUTINE PARIT
C
      DIMENSION XLAM(20,3),XCHNG(20,20,5),TTMN(20,20),SRVMN(20,2)
      DIMENSION P(5,160,160),PI(160),TLAM(3,5)
      DIMENSION COVP(5,20),TTLAM(5),BUSI(5),BUSY(5),PROB(5,160)
      DIMENSION COVI(5,20), COVE(5,20), EXIMM(5,20), EXEXP(5,20)
      DIMENSION TLAMP(3,5), TTLAMP(5),A(20,20)
      DIMENSION NX(32,4), ESTA(3,5,20), INAME(20)
      INTEGER DATA(4),KP(20,20)
      COMMON /X1/XLAM,XCHNG,TTMN,SRVMN,P,PI,TLAM,DATA,N,COVP,NC,
     1 BUSY,BUSI,COVE,COVI,ESTA,TLAMP,TTLAMP,NX,M,PROB
      COMMON /X2/INAME,A,KP
C
      IF (NC .EQ. 1) RETURN
C
C
C  CALCULATE NUMBER OF ROWS OF NX TO BE USED
C
      NROWS = 2 ** NC
```

```
C
C  INITIALIZE EXIMM AND EXEXP TO ZERO
C
      DO 806 I=1,N
      DO 805 KK=1,NC
      EXIMM(KK,I)=0
      EXEXP(KK,I)=0
805   CONTINUE
806   CONTINUE
C
      DO 810 I=1, N
      DO 800 KK=1, NC
C
C  LOOP THROUGH NC ROWS OF NX MATRIX
C
      DO 770 IJ=1,NROWS
C
C  NO EXCESS CALLS IN CASE WHERE CAR KK IS BUSY
C
       IF (NX(IJ,KK) .EQ. 1) GOTO 770
C
      SUMI = 0
      SUME = 0
      SUMX = 0
      PRODI = 1.0
      PRODE = 1.0
C
C  GO THROUGH ROWS OF NX TO REPRESENT POSSIBLE BUSY, NOT
C  BUSY COMBINATIONS
C
C  SUMI - TOTAL COVERAGE OF BUSY CARS FOR REGION I (IMM CALLS)
C  SUME - TOTAL COVERAGE OF BUSY CARS FOR REGION I (EXP CALLS)
C  SUMX - NUMBER OF BUSY CARS
C  PRODI - PROB CARS ARE BUSY SERVING IMMEDIATE CALLS
C  PRODE - PROB CARS ARE BUSY SERVING EXPEDITE CALLS
C
      DO 760 IK=1, NC
      IF (IK .EQ. KK) GOTO 750
      SUMI=SUMI+NX(IJ,IK)*COVP(IK,I)
      SUME=SUME+NX(IJ,IK)*COVP(IK,I)
      SUMX=SUMX+NX(IJ,IK)
      PRODI=PRODI*(NX(IJ,IK)*BUSI(IK)+(1-NX(IJ,IK))*(1-BUSI(IK)))
      PRODE=PRODE*(NX(IJ,IK)*BUSY(IK)+(1-NX(IJ,IK))*(1-BUSY(IK)))
C
C*DEBUG      WRITE (6,7510) SUMI, SUMX, PRODI
C*DEBUG      WRITE (6,7511) SUME, PRODE
7510  FORMAT (' SUMI= ',F5.3,' SUMX= ',F5.3,' PRODI= ',F5.3)
7511  FORMAT (' SUME= ',F5.3,' PRODE= ',F5.3)
750   CONTINUE
760   CONTINUE
C
C  ADD UP COVERAGE OF ALL FREE CARS
C
      DENOM = 0
      DO 765 KJ=1,NC
      DENOM=DENOM+(1-NX(IJ,KJ))*COVP(KJ,I)
765   CONTINUE
C
C*DEBUG      WRITE (6,7651) DENOM
7651  FORMAT (' DENOM= ',F5.3)
```

```
C
      IF (DENOM .EQ. O) COEFF = 1/(NC-SUMX)
      IF (DENOM .NE. O) COEFF = COVP(KK,I)/DENOM
      EXIMM(KK,I)=EXIMM(KK,I)+(1-BUSI(KK))*PRODI*SUMI*COEFF
      IF (DENOM .NE. O) EXEXP(KK,I)=EXEXP(KK,I)+(1-BUSY(KK))*PRODE*SUME*COEFF
C
770   CONTINUE
C
C*DEBUG      WRITE (6,766) KK, I, EXIMM(KK,I)
C*DEBUG      WRITE (6,767) KK, I, EXEXP(KK,I)
766   FORMAT (' EXIMM(',I1,',',I1,')= ',F5.3)
767   FORMAT (' EXEXP(',I1,',',I1,')= ',F5.3)
C
800   CONTINUE
810   CONTINUE
C
C  UPDATE COVI BY ADDING EXIMM TO COVP AND UPDATE
C  COVE BY ADDING EXEXP TO COVP
C
      DO 8100 I=1,N
      DO 8100 KK=1,NC
      COVI(KK,I)=COVP(KK,I)+EXIMM(KK,I)
      COVE(KK,I)=COVP(KK,I)+EXEXP(KK,I)
C
C*DEBUG      WRITE (6,7601) KK, I, COVI(KK,I)
7601  FORMAT (' COVI(',I2,',',I2,')= ',F5.3)
C*DEBUG      WRITE (6,7602) KK, I, COVE(KK,I)
7602  FORMAT (' COVE(',I2,',',I2,')= ',F5.3)
8100  CONTINUE
C
C  UPDATE EFFECTIVE CALL RATES
C
      DO 8111 K=1,NC
      TLAM(1,K)=0.0
      TLAM(2,K)=0.0
      TLAM(3,K)=0.0
C
      DO 8110 I=1,N
      TLAM(1,K)=TLAM(1,K)+XLAM(I,1)*COVE(K,I)
      TLAM(2,K)=TLAM(2,K)+XLAM(I,2)*COVI(K,I)
      TLAM(3,K)=TLAM(3,K)+XLAM(I,3)*COVE(K,I)
8110  CONTINUE
C
C*DEBUG      WRITE (6,8101) I, TLAM(1,K), TLAM(2,K), TLAM(3,K)
8101  FORMAT (' REGION ',I1,': TLAM1= ',F5.3,' TLAM2= ',F5.3,' TLAM3= ',F5.3)
C
8111  CONTINUE
C
      RETURN
      END
C
C  END OF PARIT
C
C ****************************************************************************
C
C  THIS SUBROUTINE CALCULATES EXPECTED RESPONSE TIME FOR CAR K TO TYPE L CALLS
C  AND PLACES THIS VALUE IN RESPON(K,L)
C
C ****************************************************************************
C
```

```
      SUBROUTINE RESPNS
C
      DIMENSION XLAM(20,3),XCHNG(20,20,5),TTMN(20,20),SRVMN(20,2)
      DIMENSION P(5,160,160),PI(160),TLAM(3,5),A(20,20),CDF(20,9)
      DIMENSION COVP(5,20),TTLAM(5),BUSI(5),BUSY(5),PROB(5,160)
      DIMENSION ERESP(20,3), TESTA(3,5), COVI(5,20), COVE(5,20)
      DIMENSION RBUS(20), TLAMP(3,5), TTLAMP(5), Q(20,3)
      DIMENSION NX(32,4), RESPON(20,5,3), ESTA(3,5,20), INAME(20)
      INTEGER DATA(4),KP(20,20)
      COMMON /X1/XLAM,XCHNG,TTMN,SRVMN,P,PI,TLAM,DATA,N,COVP,NC,
     1 BUSY,BUSI,COVE,COVI,ESTA,TLAMP,TTLAMP,NX,M,PROB
      COMMON /X2/INAME,A,KP
C
C  SET RESPON TO ZERO
C
      DO 8200 I=1,N
      DO 8200 K=1,NC
      DO 8200 L=1,3
      RESPON(I,K,L)=0
 8200 CONTINUE
C
C  NORMALIZE COVE AND COVI MATRICES
C
C
      DO 8302 I=1,N
C
      DEN1=0
      DEN2=0
      DO 8301 K=1,NC
      DEN1=DEN1+COVE(K,I)
      DEN2=DEN2+COVI(K,I)
 8301 CONTINUE
C
      DO 8300 K=1,NC
      COVE(K,I)=COVE(K,I)/DEN1
      COVI(K,I)=COVI(K,I)/DEN2
 8300 CONTINUE
C
 8302 CONTINUE
C
C  CALCULATE TOTAL PROB OF CAR K BEING AVAILABLE FOR TYPE L CALLS
C
      DO 8201 k=1,NC
      DO 8201 L=1,3
      TESTA(L,K)=0
 8201 CONTINUE
C
      DO 8202 L=1,3
      DO 8202 K=1,NC
      DO 8202 J=1,N
      TESTA(L,K)=TESTA(L,K)+ESTA(L,K,J)
 8202 CONTINUE
C
C  SET CUMULATIVE TRAVEL TIME DISTRIBUTION TO ZERO
C
      DO 8203 I=1,N
      DO 8203 ICT=1,9
      CDF(I,ICT)=0
 8203 CONTINUE
C
```

```
C  LOOP FOR EACH CALL TYPE
C
      DO 840 L=1, 3
C
C  LOOP FOR EACH CAR
C
      DO 830 K=1,NC
C
C  LOOP THROUGH ALL PAIRS OF REGIONS
C
      DO 820 I=1,N
      DO 815 J=1,N
C
C  MULTIPLY TIMES PROB CAR K IS IN J TIMES EXPECTED TRAVEL FROM J TO I
C  TRAVEL IS 1.3 TIMES GREATER FOR EXPEDITE THAN IMMEDIATE
C
      ETRAV = ESTA(L,K,J)*TTMN(J,I)*5/TESTA(L,K)
      IF (L .NE. 2) ETRAV = ETRAV * 1.3
      RESPON(I,K,L)=RESPON(I,K,L)+ETRAV
C
C*DEBUG      WRITE (6,8015) ETRAV
8015  FORMAT (' ETRAV= ',F5.3)
C*DEBUG      WRITE (6, 8016) K, I, TTMN(J,I)
8016  FORMAT (' TTMN(',I1,',',I1,')= ',F5.3)
C
C  CALCULATE CUMULATIVE PROB THAT RESPONSE TIME TO REGION I
C  IS LESS THAN OR EQUAL TO ICT BETWEEN 3 AND 27 MINUTES
C
      IF (L.NE.2) GO TO 815
C
      DO 814 ICT=1,9
      IT=ICT*3
      CALL CTRAV(A(J,I),KP(J,I),IT,CDV)
      CDF(I,ICT)=CDF(I,ICT)+ESTA(2,K,J)*COVI(K,I)*CDV/TESTA(2,K)
C*DEBUG      WRITE (6,8141) I,J,CDV
8141  FORMAT (' CDV(',I2,',',I2,')= ',F6.3)
814   CONTINUE
C
815   CONTINUE
820   CONTINUE
830   CONTINUE
840   CONTINUE
C
C  CALCULATE EXPECTED TIME AN EXPEDITE OR UNFOUNDED CALL IS QUEUED
C
C  FIND DENOM FOR PROBS OF CALLS COMING FROM REGION I
C
      TXLAM1=0.0
      TXLAM2=0.0
      TXLAM3=0.0
      DO 841 I=1,N
      TXLAM1=TXLAM1+XLAM(I,1)
      TXLAM2=TXLAM2+XLAM(I,2)
      TXLAM3=TXLAM3+XLAM(I,3)
841   CONTINUE
C
C  CALCULATE PROB OF ALL CARS BUSY WITH IMMEDIATE CALLS
C
      ABUS=1.0
      DO 8412 K=1,NC
```

```
      ABUS=ABUS*BUSI(K)
8412  CONTINUE
C
C  CALCULATE PROB OF CAR HANDLING CALL BUSY WITH EXPEDITE AND ALL OTHER
C  CARS WITH RESPONSIBILITY FOR REGION I BUSY WITH ANY TYPE CALL
C
      DO 8417  I=1,N
      RBUS(I)=0.0
C
      DO 8416 K=1,NC
      IF (COVE(K,I).EQ.O) GOTO 8416
      BUS=BUSY(K)-BUSI(K)
C
      DO 8415 KK=1,NC
      IF (COVE(KK,I).EQ.O) GOTO 8415
      IF (KK.EQ.K) GOTO 8415
      BUS=BUS*BUSY(KK)
8415  CONTINUE
C
      RBUS(I)=RBUS(I)+COVE(K,I)*BUS
C
C*DEBUG      WRITE (6,8418) I, RBUS(I)
8418  FORMAT (' RBUS(',I2,')= ',F6.3)
C
8416  CONTINUE
C
8417  CONTINUE
C
C  EXPECTED QUEUING TIME GIVE CALL COMES FROM I
C  EQUALS P(CALL IS QUEUED BEHIND A TYPE 1 OR 2 CALL)
C  TIMES EXPECTED QUEUE TIME (SERVICE TIME)
C  TYPE 2 CALLS CAN ONLY BE QUEUED BEHIND TYPE 2 CALLS
C
      DO 8411 I=1,N
      DO 8411 L=1,3
      Q(I,L)=0
8411  CONTINUE
C
      DO 842 I=1,N
      Q(I,1)=(RBUS(I)*SRVMN(I,1) + ABUS*SRVMN(I,2))*5
      Q(I,2)=ABUS*SRVMN(I,2)*5
      Q(I,3)=Q(I,1)
C*DEBUG      WRITE (6,8420) (Q(I,L), L=1,3)
8420  FORMAT (' Q1= ',F5.3,' Q2= ',F5.3,' Q3= ',F5.3)
842   CONTINUE
C
C
C  CALCULATE EXPECTED RESPONSE TIMES TO EACH REGION
C
      DO 8430 I=1,N
      DO 8430 L=1,3
      ERESP(I,L)=0.0
8430  CONTINUE
C
      DO 8431 I=1,N
      DO 8431 L=1,3
      DO 8431 K=1,NC
      IF (L.EQ.2) ECOV=COVI(K,I)
      IF (L.NE.2) ECOV=COVE(K,I)
      ERESP(I,L)=ERESP(I,L)+ECOV*RESPON(I,K,L)
```

```
8431  CONTINUE
C
C  ADD QUEUING TIME TO EXPECTED RESPONSE TIMES
C
      DO 843 I=1,N
      DO 843 L=1,3
      ERESP(I,L)=ERESP(I,L)+Q(I,L)
843   CONTINUE
C
C
C  WRITE OUT EXPECTED RESPONSE TIMES
C
      GOTO 8900
C
      WRITE (6,849)
      WRITE (2,849)
849   FORMAT (72X)
      WRITE (6, 850)
      WRITE (2, 850)
850   FORMAT (' EXPECTED TRAVEL TIMES')
      DO 890 K=1,NC
      WRITE (6,849)
      WRITE (2,849)
      WRITE (6,851) K
      WRITE (2,851) K
851   FORMAT (' CAR',I2,'  .          CALL TYPES')
      WRITE (6,852)
      WRITE (2,852)
852   FORMAT (' REGION   EXPEDITE    IMMEDIATE   UNFOUNDED')
C
      DO 890 I=1,N
      WRITE (6,853) INAME(I), (RESPON(INAME(I),K,L), L=1,3)
      WRITE (2,853) INAME(I), (RESPON(INAME(I),K,L), L=1,3)
853   FORMAT (2X,A4,5X,F6.3,6X,F6.3,6X,F6.3)
C
890   CONTINUE
C
8900  CONTINUE
C
      WRITE (6,849)
      WRITE (2,849)
      WRITE (6,849)
      WRITE (2,849)
      WRITE (6,8490)
      WRITE (2,8490)
8490  FORMAT (' AVERAGE RESPONSE TIME TO EACH REGION (MINUTES)')
      WRITE (6,852)
      WRITE (2,852)
C
      DO 8495 I=1,N
      WRITE (6,853) INAME(I), (ERESP(I,L), L=1,3)
      WRITE (2,853) INAME(I), (ERESP(I,L), L=1,3)
8495  CONTINUE
C
C  PRINT OUT AVERAGE QUEUE TIMES
C
      WRITE(6,849)
      WRITE(2,849)
      WRITE(6,849)
      WRITE(6,849)
```

```
      WRITE(6,8501)
      WRITE(2,8501)
 8501 FORMAT(' AVERAGE TIME IN QUEUE (MINUTES)')
      WRITE(6,852)
      WRITE(2,852)
C
      DO 8502 I=1,N
      WRITE(6,853) INAME(I),(Q(I,L),L=1,3)
      WRITE(2,853) INAME(I),(Q(I,L),L=1,3)
 8502 CONTINUE
C
C  PRINT OUT CUMULATIVE DISTRIBUTION OF TRAVEL TIMES TO IMMED CALLS
C
      WRITE(6,849)
      WRITE(6,849)
      WRITE(2,849)
      WRITE(2,849)
      WRITE(6,8505)
      WRITE(2,8505)
 8505 FORMAT(' PROBABILITY TRAVEL TIME TO IMMEDIATE CALLS IS LESS THAN OR EQUAL TO')
      WRITE(6,8506)
      WRITE(2,8506)
 8506 FORMAT('                           MINUTES')
      WRITE(6,8507)
      WRITE(2,8507)
 8507 FORMAT(' REGION    3     6     9    12    15    18    21    24    27')
C
      DO 8509 I=1,N
      WRITE(6,8508) INAME(I),(CDF(I,J),J=1,9)
      WRITE(2,8508) INAME(I),(CDF(I,J),J=1,9)
 8508 FORMAT(2X,A4,1X,9F6.3)
 8509 CONTINUE
C
C
C  END OF RESPON
C
      RETURN
      END
C*******************************************************************
C
C     THIS SUBROUTINE CHANGES OLD ACCORDING TO NEW SPECIFICATIONS
C
C*******************************************************************
      SUBROUTINE CHNGDT
      DIMENSION XLAM(20,3),XCHNG(20,20,5),TTMN(20,20),SRVMN(20,2)
      DIMENSION P(5,160,160),PI(160),TLAM(3,5)
      DIMENSION COVP(5,20), ESTA(3,5,20), INAME(20), A(20,20)
      INTEGER DATA(4), KP(20,20)
      DATA IY/'Y   '/
      DATA MC1/'RATE'/,MC2/'TRAV'/,MC3/'SERV'/,MC4/'SWIT'/,MC5/'COVE'/,
     1  MD1/'EXPE'/,MD3/'UNFO'/,MD2/'IMME'/,IZERO/'0   '/
      COMMON /X1/XLAM,XCHNG,TTMN,SRVMN,P,PI,TLAM,DATA,N,COVP,NC,
     1 BUSY,BUSI,COVE,COVI,ESTA,TLAMP,TTLAMP,NX,M,PROB
      COMMON /X2/INAME,A,KP
      WRITE(6,1)
    1 FORMAT(' WHAT FILE DO YOU WANT ?')
      READ(5,2) DATA
    2 FORMAT(4A4)
      CALL FTNCMD('ASSIGN 2=?',10,DATA)
C
```

```
C READ IN THE NAME
C
      READ(2,302) DATA
302   FORMAT(1X,4A4)
      WRITE(6,3) DATA
3     FORMAT(' FILE =',4A4)
C
C  READ IN THE NUMBER OF REGIONS
C
      READ(2,303) N
303   FORMAT(/I3)
C
C   READ NUMBER OF CARS
C
      READ(2,303) NC
C
C READ IN TRAVEL ALPHA VALUES
C
      READ (2, 3031) IDUM
3031  FORMAT (A4)
C
      DO 305 I=1,N
      READ(2,304) (A(I,J),J=1,N)
304   FORMAT (12F6.2)
305   CONTINUE
C
C  READ IN TRAVEL K VALUES
C
      READ(2,3031) IDUM
C
      DO 3051 I=1,N
      READ(2,3052) (KP(I,J),J=1,N)
3051  CONTINUE
3052  FORMAT (12I6)
C
C READ IN CALL RATES
C
      READ (2, 306) IDUM
306   FORMAT (/ A4)
C
      DO 315 I=1,N
      READ(2,307) IDUM, (XLAM(I,J),J=1,3)
307   FORMAT (2X,A4,3X,F5.3,5X,F5.3,5X,F5.3)
315   CONTINUE
C
C READ IN SERVICE MEANS
C
      READ (2, 306) IDUM
C
      DO 320 I=1,N
      READ(2,308) IDUM, (SRVMN(I,J),J=1,2)
308   FORMAT (2X,A4,3X,F5.2,5X,F5.2,5X,F5.2)
320   CONTINUE
C
C READ IN SWITCH PROBABLITIES
C
      READ (2, 3031) IDUM
      DO 330 K=1,NC
      READ (2, 3031) IDUM
      DO 330 I=1,N
```

```
      READ(2,311) (XCHNG(I,J,K),J=1,N)
311   FORMAT (12F6.3)
330   CONTINUE
C
C
C   READ COVERAGE
C
      READ (2, 306) IDUM
C
      DO 340 K=1,NC
      READ(2,312) IDUM, (COVP(K,J),J=1,N)
312   FORMAT (I3,12F6.3)
340   CONTINUE
C
C  READ REGION NAMES
C
      READ (2,3031) IDUM
C
      DO 350 I=1,N
      READ (2,3410) IDUM, INAME(I)
3410  FORMAT (I3,5X,A4)
350   CONTINUE
C
C END OF DATA RECOVERY
C
C-----------------------------------------------------------------
C
C
C   SELECT THE AREA TO CHANGE
C
C
9     CONTINUE
      WRITE(6,10)
10    FORMAT(' WHAT WOULD YOU LIKE TO CHANGE ?',/,
     1 '(RATES,TRAVEL,SERVICE,SWITCH,COVERAGE,STOP)')
      WRITE (6, 12)
12    FORMAT (' ENTERING O WILL STOP LEVEL OF QUERY')
      READ(5,11) JCH
11    FORMAT(A4)
      IF(JCH.EQ.MC1) GO TO 20
      IF(JCH.EQ.MC2) GO TO 30
      IF(JCH.EQ.MC3) GO TO 40
      IF(JCH.EQ.MC4) GO TO 50
      IF(JCH.EQ.MC5) GO TO 60
      RETURN
C
C HERE YOU RETURN TO THE MAIN PROGRAM
C
C
C --------------------------------------------------
C
C
C    CHANGE THE CALL RATES
C
20      CONTINUE
      WRITE(6,21)
21     FORMAT(' WHICH TYPE OF RATE WOULD YOU LIKE TO CHANGE ?',/,
     1 ' (EXPEDITE,IMMEDIATE,OR UNFOUNDED)')
      READ(5,22) JCH
      IF (JCH .EQ. IZERO) GOTO 9
```

```
22    FORMAT(A4)
      IF(JCH.EQ.MD1) IL=1
      IF(JCH.EQ.MD2) IL=2
      IF(JCH.EQ.MD3) IL=3
C
C NEXT SELECT THE REGION
C
28    CONTINUE
      WRITE(6,23)
23    FORMAT(' WHAT REGION? (I2)')
      READ(5,24) I
      IF (I .EQ. 0) GOTO 20
24    FORMAT(I2)
      XL=XLAM(I,IL)
204   CONTINUE
      WRITE(6,25) JCH, XL
25    FORMAT(' THE CURRENT HOURLY' / A4,' RATE IS',F8.4,/,
     1 ' WHAT WOULD LIKE TO CHANGE IT TO ?(F5.3)')
      READ(5,26) XL
26    FORMAT(F5.3)
      XLAM(I,IL)=XL
      GO TO 28
C
C  CHANGE THE TRAVEL MEANS
C
30     CONTINUE
      WRITE(6,31)
31     FORMAT(' WHAT ARE THE REGIONS THAT YOU WANT TO CHANGE THE ',/,
     1 ' TRAVEL MEANS FOR ? (2I2)')
      READ(5,32) I,J
      IF (I .EQ. 0) GOTO 9

32    FORMAT(2I2)
      TT=TTMN(I,J)
      WRITE(6,33) I,J,TT
33    FORMAT(' THE CURRENT MEAN TIME FROM',I4,' TO ',I4,' IS ',F6.3)
      WRITE(6,34)
34    FORMAT(' WHAT NEW VALUE DO YOU WANT ? (F6.3)')
      READ(5,35) TT
35    FORMAT(F6.3)
      TTMN(I,J)=TT
      TTMN(J,I)=TT
      GO TO 30
C
C CHANGE SERVICE MEANS
C
40    CONTINUE
      WRITE(6,41)
41     FORMAT(' WHAT TYPE OF SERVICE TIME WOULD YOU LIKE TO CHANGE?',/,
     1 '(EXPEDITE OR IMMEDIATE)')
      READ(5,42) JCH
      IF (JCH .EQ. IZERO) GOTO 9
42    FORMAT(A4)
      IF(JCH.EQ.MD1) IL=1
      IF(JCH.EQ.MD2) IL=2
C
C CHOOSE REGION
C
48    CONTINUE
      WRITE(6,43)
```

```
 43   FORMAT(' WHAT REGION? (I2)')
      READ(5,44) I
      IF (I .EQ. 0) GOTO 40
44     FORMAT(I2)
      SS=SRVMN(I,IL)
       WRITE(6,45) SS
45     FORMAT(' THE CURRENT VALUE IS',F8.4)
      WRITE(6,46)
46    FORMAT(' NEW MEAN (IN MINS)? (F8.4)')
      READ(5,47) SS
47    FORMAT(F8.4)
      SRVMN(I,IL)=SS
      GO TO 48
C
C   CHOOSE NEW SWITCH PROBABILITIES
C
50     CONTINUE
      WRITE (6, 56)
56    FORMAT (' FOR WHICH CAR DO YOU WANT TO CHANGE PATROL SWITCH PROBS? (I2)')
      READ (5, 24) K
      IF (K .EQ. 0) GOTO 9
57    CONTINUE
      WRITE(6,51)
51    FORMAT(' WHAT PATROL SWITCH PAIR (I,J) DO YOU WANT TO CHANGE? (2I2)')
      READ(5,52) I,J
      IF (I .EQ. 0) GOTO 50
52    FORMAT(2I2)
      XS=XCHNG(I,J,K)
      WRITE(6,53) K,I,J,XS
53    FORMAT(' THE OLD PROB FOR CAR',I4,'  FROM',I4,' TO ',I4,' WAS ',F8.4)
      WRITE(6,54)
54    FORMAT(' WHAT NEW PROB DO YOU WANT? (F5.3)')
      READ(5,55) XS
55    FORMAT(F5.3)
      XCHNG(I,J,K)=XS
      GO TO 57
C
C
C     HERE YOU CHANGE THE COVERAGE FUNCTION
C
60    CONTINUE
      WRITE(6,61)
61    FORMAT(' WHAT CAR ? (I2)')
      READ(5,62) K
      IF (K .EQ. 0) GOTO 9
62    FORMAT(I2)
66    CONTINUE
      WRITE (6, 63)
63    FORMAT (' COVERAGE FOR WHAT REGION? (I2)')
      READ (5, 24) I
      IF (I .EQ. 0) GOTO 60
      WRITE (6, 64) K, I, COVP(K,I)
64    FORMAT (' COVERAGE FOR CAR ',I2,' IN REGION ',I2,' IS ',F5.3)
      WRITE (6, 65)
65    FORMAT (' NEW COVERAGE? (F5.3)')
      READ (5, 55) CV
      COVP(K,I) = CV
      GOTO 66
C
C END OF CHANGER
```

```
C
      END
C*****************************************************************
C
C  THIS SUBROUTINE CALCULATES CUMULATIVE PROBABILITY VALUES FOR A ERLANG
C  DISTRIBUTION.  INPUT PARAMETERS ARE ALPHA (AP), K (KP), AND T
C  THE PROB THAT T(I) LE T IS RETURNED IN THE VARIABLE CDJ
C
C*****************************************************************
C
      SUBROUTINE ERLANG(AP,KP,TT,CDJ)
C
      IF (KP.GE.10) KP=10
C
      SUM=0
C
      DO 10 K=1,KP
      SUM=SUM+(((AP*TT)**(K-1))*EXP(-(AP*TT)))/IFACT(K-1)
10    CONTINUE
C
      CDJ=1-SUM
C
      RETURN
      END
C
C*****************************************************************
C
C  THIS FUNCTION COMPUTES THE FACTORIAL OF THE INTEGER K
C
C*****************************************************************
C
      FUNCTION IFACT(K)
C
      IFACT=1
      IF (K.LT.2) GOTO 7
      DO 6 I=2,K
      IFACT=IFACT*I
6     CONTINUE
7     CONTINUE
C
      RETURN
      END
```

APPENDIX F: A SEMI-MARKOV MODEL

The Markov process model presented in Section 4 assumes that travel times are exponentially distributed. This assumption does not alter the steady state probabilities, but the model may not be valid for transient analysis if the assumption does not hold. In this case, a *semi-Markov process* can be used. A semi-Markov process differs from a Markov process in that the rate of transition from state i to state j depends on the time spent in state i in addition to the states i and j. A Markov process is then a special case of a semi-Markov process in which all transition time distributions are exponential. (For a more detailed account, see Cinlar [14].)

The travel times found for Washtenaw County appeared to have a non-exponential Erlang-type distribution (see Appendix B). The main goal of the Markov model, however, was to find steady state probabilities and to be easily implementable. Since the Markov model converged fairly quickly, it has been developed in this report. In the future, however, it may prove beneficial to attain a higher level of detail in defining transient effects by employing the semi-Markov model presented here.

We will present a rudimentary $2N$ state model analagous to the $3N$ state model of section 4.1. In this model, a patrol state $p(i)$ and a service state $s(i)$ exist for each region i. We could add states for different types of service and queueing as in the Markov model, but we will use the simpler model for ease of exposition. The basic difference between this model and the Markov model is that no travel state is required. Instead, the time of travel is reflected in a non-exponential distribution for travel between two states.

p_{ij} = probability of going from $p(i)$ to $s(j)$ in the next transition ,

r_{ij} = probability of going from $p(i)$ to $p(j)$ in the next transition.

The matrix of these probabilities is the transition probability matrix P.

We use arrival rates, λ_i, switching rates, x_{ij}, and mean service times, μ_j, as in the Markov model. We assume that travel from region i to region j is Erlang distributed with

$$\text{mean} = \frac{n_{ij}}{\beta_{ij}} \text{ and}$$

$$\text{variance} = \frac{\sqrt{n_{ij}}}{\beta_{ij}} .$$

We then have that

$$p_{ij} = \frac{\lambda_j}{\sum_{j=1}^{N} \lambda_j + \sum_{\substack{j=1 \\ j \neq i}}^{N} x_{ij}} \text{, and} \tag{1}$$

$$r_{ij} = \frac{x_{ij}}{\sum_{j=1}^{N} \lambda_j + \sum_{\substack{j=1 \\ j \neq i}}^{N} x_{ij}} . \tag{2}$$

Semi-Markov processes are governed by a *semi-Markov kernel* Q which is the product of the probability of a transition from i to j and the distribution of that transition time. In this model, we define three types of kernels. The first two are:

$$Q_p(i,j,t) = P\left\{X_{n+1} = s(j),\ T_{n+1} - T_n \le t \mid X_n = p(i)\right\}$$

$$= p_{ij}\, f(i,j,t), \text{ and}$$

$$Q_r(i,j,t) = P\left\{X_{n+1} = p(j),\ T_{n+1} - T_n \le t \mid X_n = p(i)\right\},$$

$$= r_{ij}\, g(i,j,t),$$

where T_n is the time of the n^{th} transition and X_n is the state of the system immediately after the n^{th} transition.

For $\lambda = \sum_{j=1}^{N} \lambda_j + \sum_{\substack{j=1 \\ j \neq i}}^{N} x_{ij}$, we find

$$f(i,j,t) = P\left\{T_{n+1} - T_n \le t \mid X_{n+1} = s(j),\ X_n = p(i)\right\} \text{ by}$$

$$f(i,j,t) = \int_0^t \int_0^{t-x} \left(\lambda\, e^{-\lambda s}\right) \frac{Bs\,\left(Bsx\right)^{n_{ij}-1} C^{-Bsx}}{(n_{ij}-1)!}\, ds\, dx , \tag{3}$$

which is the probability of a call arrival in $(0,s)$, and of travel in (s,t). (Note that the distribution of the arrival of the next call is independent of where the call occurs.) From (3), we can show that

$$\begin{aligned} f(i,j,t) = 1 - e^{-\lambda t} \left(\frac{Bs}{Bs-\lambda}\right)^{n_{ij}} \\ + \sum_{k=0}^{n_{ij}-1} \frac{Bs^{n_{ij}} t^k\, e^{-Bst}}{\left(Bs-\lambda\right)^{n_{ij}-k}} \left(k!\right) \\ - \sum_{k=0}^{n_{ij}-1} \frac{\left(Bst\right)^k}{k!}\, e^{-Bsx} . \end{aligned} \tag{4}$$

We can find $g(i,j,t)$ similarly to also be equal to (4).

For transitions from service, there is no choice of state after the next transition since patrol starts in that area. In this case, we have the third kernel type

$$Q_s(i,i,t) = 1 - e^{-\frac{t}{\mu_i}} .$$

This completes the definition of the kernel Q.

The semi-Markov process is defined as Y_t where

$$Y_t = \left\{X_n \mid T_n \le t < T_{n+1}\right\}.$$

Associated with Y is a *potential function* V which represents the expected time the process spends in some state j given a start in state i,

$$V(i,j,t) = E_i\left[\int_0^t \chi_j(Y_s)ds\right],$$

where

$$\chi_j(Y_s) = \begin{cases} 1, \text{ if } Y_s = j \\ 0, \text{ otherwise} \end{cases}.$$

In a transient analysis, we would be interested in $V(i,j,t,)$ given different starting positions i. To find $V(i,j,t)$, we define the *semi-Markov renewal kernel* R by

$$R(i,j,t) = \sum_{n=0}^{\infty} Q^n(i,j,t),$$

and we define

$$h(j,t) \equiv 1 - \sum_{k=1}^{2N} Q(j,k,t).$$

We then have

$$V(i,j,t) = \int_0^t R(i,j,ds) \int_0^{t-s} h(j,u)du. \quad (5)$$

Equation (5) is typically solved by taking the Laplace transforms of R and k and inverting their product to find V (see Cinlar [14]).

We can also find the long run average time spent in each state, $v(j)$, by

$$v(j) = \lim_{t\to\infty} \frac{1}{t} V(i,j,t) = \frac{1}{\pi \cdot m} \pi(j)\cdot m(j), \quad (6)$$

where $[\pi(i)]$ is a vector satisfying $\pi P = \pi$, $\sum_{i=1}^{2N} \pi(i) = 1$ for the transition probability matrix P, and $[m(i)]$ is a vector of the mean sojourn times in each state. By using

$$m(i) = \int_0^\infty \left\{1 - \sum_l Q(i,l,t)\right\}dt, \quad (7)$$

we can find these steady-state probabilities. The resulting $v(j)$ will be the same as we would receive from the Markov model, but the transient effects in attaining these probabilities will be different.

As an example, let us consider a two region problem where

$$\lambda_1 = 0.2$$

$$\lambda_2 = 0.3$$

$$\mu_1 = \mu_2 = 2$$

$$\beta_{11} = \beta_{22} = 60$$

$$n_{11} = n_{22} = 6$$

$$\beta_{12} = \beta_{21} = 20$$

$$n_{12} = n_{21} = 4$$

$$x_{12} = x_{21} = 0.5.$$

For this problem, the transition matrix is

$$P = \begin{array}{c|cccc|} & p(1) & s(1) & p(2) & s(2) \\ p(1) & 0 & \frac{1}{5} & \frac{1}{2} & \frac{3}{10} \\ s(1) & 1 & 0 & 0 & 0 \\ p(2) & \frac{1}{2} & \frac{1}{5} & 0 & \frac{3}{10} \\ s(2) & 0 & 0 & 1 & 0 \end{array},$$

and the resulting π such that $\pi P = \pi$ and $\sum_{j=1}^{4} \pi(j) = 1$ is

$$\pi = \left\{ \frac{14}{45}, \frac{2}{15}, \frac{6}{45}, \frac{1}{5} \right\}. \tag{8}$$

The mean sojourn times from (7) and (4) (using the definitions of Q_p, Q_r, and Q_s) are

$$m = (1.18,\ 1.17,\ 0.5,\ 0.5)\,. \tag{9}$$

We then obtain from (8) and (9)

$$v(1) = 0.39$$

$$v(2) = 0.07$$

$$v(3) = 0.44$$

$$v(4) = 0.11\,. \tag{10}$$

The transition matrix for a corresponding Markov model, with a six minute transition interval and the same mean travel times, is

$$\begin{array}{c|cccccc|} & p(1) & t(1) & s(1) & p(2) & t(2) & s(2) \\ p(1) & .90 & .05 & 0 & .05 & 0 & 0 \\ t(1) & 0 & .375 & .25 & 0 & 0 & .375 \\ s(1) & .20 & 0 & .80 & 0 & 0 & 0 \\ p(2) & .05 & 0 & 0 & .90 & .05 & 0 \\ t(2) & 0 & 0 & .29 & 0 & .29 & .43 \\ s(2) & 0 & 0 & 0 & .20 & 0 & .80 \end{array}.$$

Rounding off the results from our Markov model program, we obtain

$$v(1) = 0.36$$

$$v(2) = 0.03$$

$$v(3) = 0.08$$

$$v(4) = 0.40$$

$$v(5) = 0.03$$

$$v(6) = 0.11\,. \tag{11}$$

The time spent in patrol and travel in the Markov model should be equal to the time in patrol in the semi-Markov model. The results in (11) only differ from those in (10) in the thousandths place, which is the limit of the model accuracy.

(11)

APPENDIX G: LINEAR APPROXIMATIONS FOR PRESCRIPTIVE ANALYSIS

The emphasis of this project has been on providing descriptive analysis for use in assessing different patrol policies. It may also be possible to provide direction to guide users toward "good" policies that optimize or satisfy criteria specified by the user. The simplest optimization model would be a linear program in which changes in the steady state probabilities would be approximated by linear functions. For small changes in policy, then, this model would accurately predict the change in steady-state.

In the Markov model of Section 4, we can find a steady state π by solving

$$\pi P = \pi ,$$

and

$$\pi e = 1 , \tag{1}$$

where $e = (1,1,\cdots,1)^t$. Solving (1) is equivalent to solving a system with a new matrix $\overline{P}$ which is the identity minus P with e substituting for one column,

$$\overline{P} = \left[I_1 - P_1 \mid I_2 - P_2 \mid \cdots \mid I_{N-1} - P_{N-1} \mid e \right] . \tag{2}$$

where I_j is the j^{th} column of I and P_j is the j^{th} column of P . π is then the solution of

$$\pi \overline{P} = \left[0,0,\cdots,0,1 \right] . \tag{3}$$

Let $\overline{\pi}$ solve (3) for some matrix $\overline{P}$. We want to see how $\overline{\pi}$ changes if we change $\overline{P}$ by altering the switch probabilities x_{ij} . Suppose x_{ij} is changed to $x'_{ij} = x_{ij} + \Delta$. This implies P changes to P' where

$$P'(p(i),p(j)) = x'_{ij} = x_{ij} + \Delta ,$$

$$P'(p(i),p(i)) = P(p(i),p(i)) - \Delta . \tag{4}$$

We form a new matrix $\overline{P}'$ as in (2), where

$$\overline{P}' = \overline{P} + \begin{array}{c} \\ p(i) \\ \\ \\ \end{array} \left| \begin{array}{ccccccc} 0 & 0 & \overset{p(i)}{0} & 0 & 0 & \overset{p(j)}{0} & 0 \\ \vdots & 0 & -\Delta & 0 & \cdots 0 & +\ \Delta & \\ & \vdots & & \vdots & \vdots & & \vdots \\ 0 & 0 & & 0 & & 0 & 0 \end{array} \right| . \tag{5}$$

We let $p(i)$ be the k^{th} row of $\overline{P}$ and let $p(j)$ be the l^{th} column of $\overline{P}$. We then define the k^{th} row of $\overline{P}^{-1}$ as

$$(\overline{P}^{-1})_{k \cdot} = (\alpha_{k1}, \alpha_{k2}, \cdots, \alpha_{kn}) , \tag{6}$$

and the l^{th} row of $\overline{P}^{-1}$ as

$$(\overline{P}^{-1})_{l \cdot} = (\alpha_{l1}, \alpha_{l2}, \cdots, \alpha_{ln}) . \tag{7}$$

CONTINUED

From (5), (6) and (7), we have

$$(\tilde{P})^{-1}\,\bar{P}' = \begin{array}{c} \\ \\ k \\ \\ \\ \end{array}\left|\begin{array}{ccccc} 1 & & & & \\ 0 & 1 & & 0 & \\ \Delta(\alpha_{k1}-\alpha_{l1}) & \cdots & 1+\Delta(\alpha_{kk}-\alpha_{lk}) & \cdots & \Delta(\alpha_{kn}-\alpha_{ln}) \\ & & 1 & & \\ 0 & & & & 0 \\ & & & & 1 \end{array}\right| \quad \text{(column } k\text{)} \tag{8}$$

So, from (8), we have

$$(\bar{P}')^{-1} = (\bar{P}^{-1}) \left|\begin{array}{ccccc} 1 & & & & \\ 0 & 1 & & 0 & \\ \dfrac{-\Delta(\alpha_{k1}-\alpha_{l1})}{1+\Delta(\alpha_{kk}-\alpha_{lk})} & \cdots & \dfrac{1}{1+\Delta(\alpha_{kk}-\alpha_{lk})} & \cdots & \dfrac{-\Delta(\alpha_{kn}-\alpha_{ln})}{1+\Delta(\alpha_{kk}-\alpha_{lk})} \\ & & & 1 & 0 \\ 0 & & & & \\ & & & & 1 \end{array}\right| . \tag{9}$$

Hence, the new $\pi = \pi'$ where,

$$\pi' = [0,0,\cdots 0,1]\,(\bar{P}')^{-1} , \tag{10}$$

is

$$\pi' = \left[\pi_1 - \pi_k\left[\frac{\Delta(\alpha_{k1}-\alpha_{l1})}{1+\Delta(\alpha_{kk}-\alpha_{lk})}\right], \cdots, \right.$$
$$\pi_k\left[\frac{1}{1+\Delta(\alpha_{kk}-\alpha_{lk})}\right], \cdots,$$
$$\left.\pi_n - \pi_k\left[\frac{\Delta(\alpha_{kn}-\alpha_{ln})}{1+\Delta(\alpha_{kk}-\alpha_{lk})}\right]\right] . \tag{11}$$

Now, we look at the difference between π' and $\bar{\pi}$ to find

$$\frac{\partial \pi_q}{\partial X_{ij}} = \lim_{\Delta \to 0} \frac{\pi'_q - \bar{\pi}_q}{\Delta} ,$$
$$= \begin{cases} \pi_k\,(\alpha_{kk}-\alpha_{lk}) , & q = k \\ -\pi_k\,(\alpha_{kq}-\alpha_{lq}) , & q \neq k . \end{cases} \tag{12}$$

The result in (12) can be used in a linear optimization problem to represent decision variables d_{ij} where

$$\pi'_q = \sum_j \sum_i \left[\frac{\partial \pi_q}{\partial x_{ij}}\right] dij + \pi_q . \tag{13}$$

Before solving a linear program, a set of decision variables x_{ij} is given and the corresponding steady state probabilities are found. The aim of the linear program is to find changes in the x_{ij} to optimize any of several criteria. For example, we can use the results in Chapter 4 and (13) to find the fractions of time spent patrolling in each region, the workload of each car, or the average response times. We can then optimize these criteria (for example, minimize expected response time or use goal programming to most nearly achieve our goals, such as balancing workload). One set of constraints would limit d_{ij} so that the approximation is appropriate (for instance $|d_{ij}| \leq .1$) and others would be used to guarantee minimum directed patrol times in each region.

After solving for the optimal values of d_{ij}, we could then use the new x_{ij} to find the exact values of the new steady state probabilities. These new values may be used again as in (13) and a second linear program may be solved to find another set of d_{ij} values. This procedure may be repeated until the linear program does not lead to any improvement in the objective function criteria. Various methods (see [17]) may also be used to optimize multiple criteria chosen by the user.