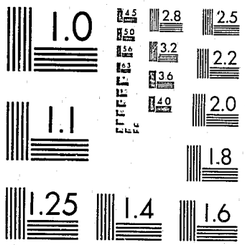

A MODEL TO ALLOCATE STATE POLICE MANPOWER TO DISTRICTS; REVISED VERSION

Illinois Department of Law Enforcement
Division of Administration

Richard A. Raub

93168

MODEL TO ALLOCATE STATE POLICE MANPOWER TO DISTRICTS: REVISED VERSION

Illinois Department of Law Enforcement
Division of Administration
Deputy Director Alex Ferguson

Richard A. Raub
February 1984

ABSTRACT

Since the first version of a model to allocate State Police officers in Illinois was prepared, numerous revisions and refinements have been made. The more important changes comprise a reduction in the number of variables used in the computations and accounting for time of officers assigned to calls for service but not engaged in the activity. This latter change reduces the tendency of the model to weight rural operations more heavily than urban activity.

A brief history and description is included. The revised list of variables and parameters also is presented along with revised formulas. The revisions attempt to retain the original variable names; however, changes have been made to improve clarity.

Part of the work on the revision was done with the assistance of a micro computer. As a result, this led to a complete programming of the model in BASIC programming language. While it does not operate as rapidly or have as many plans as the model designed for the larger computer, it can produce allocations based on the same methodology.

AKNOWLEDGEMENTS

A large portion of the credit and work toward producing the first version of the model is due George L. Sweat, Assistant Director, Division of Criminal Investigation. Director Sweat also was co-author of several of the initial reports. Deputy Superintendent William Pierce of the Division of State Police has used the results of the model and made many useful recommendations. Helping with the interpretation of the numbers and with the many runs has been Sergeant Monica Ice also of the Division of State Police. The many revisions and runs have been made quickly and efficiently by Ken Loyd. Without his help, the current version would not have been possible. Finally, acknowledgement is given to Rose Krause for suffering through the many typed revisions and in particular, typing and retyping the formulas.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

MODEL TO ALLOCATE STATE POLICE MANPOWER TO DISTRICTS: REVISED VERSION

I. BACKGROUND AND DESCRIPTION OF THE MODEL

BACKGROUND

Since 1982, the Illinois Department of Law Enforcement (DLE) has been using a mathematical model for allocating state police (uniformed) officers throughout the state of Illinois. Personnel are allocated to districts, the principal level of command in the field. Subsequent allocation to individual patrols remains the prerogative of district command. The methodology described in this paper is a revision of the original concepts. It remains a tool for planning for the allocation of police personnel.

The model first was decribed in a report by Raub and Sweat in June of 1981.[1] Although the methodology drew from such persons as Chaiken, Larsen, and LeGrande, it was unique in that an attempt was made to distribute personnel over a large (56,000 square mile) area. Further, this area had counties with large variations in population, from more than seven million in Cook County to less than 10,000 in many rural counties. Yet, there had to be a balance of service to all persons regardless of density.

In early 1982, the model was transferred to the main computer of the Department. It has been run as a planning tool many times and used for allocating graduating cadets to districts, examining how service would be changed as a result of decreases in the number of uniformed officers, and determining what strength is required to meet desired levels of service. During this time, revisions have

occurred. They have been reported both at a conference,[2] and most recently by the International Association of Chiefs of Police.[3]

The frequent operation of the model has pointed to areas which needed strengthening and those which could be eliminated. Consequently, the mathematics of the model have been revised and simplified. Fewer variables are used. This paper describes the model in its entirety using these revised mathematics. Where possible, an attempt has been made to use the same symbols as were used previously. Changes have been made to variable names when required for clarity. Subsequent Tables summarize these symbols. Also, as part of this effort, the model has been programmed for the APPLE II microcomputer in BASIC.

DESCRIPTION OF THE MODEL

The reports by Raub and Sweat and other papers referenced above have described the historic development and philosophies in detail. Summarized in this section are the premises on which the model is based. The objective of the model is to allocate uniformed personnel to districts such that their presence equitably serves the entire state. The model is divided into three sections: administrative support, response to calls for service, and general patrolling called "policing and patrolling".

Administrative support represents those persons needed to administer the organization both centrally and at the district level. Such can not be described according to mathematical formulas. Rather, the number of persons to be assigned to each district and to central office is based upon managerial needs. Of course, there must be balance between expressed needs and the number of officers

available for allocation. Assigning too many to adminsitrative support will defeat the rationale behind the model. Those allocated to administrative support are not considered available for patrol or response except in unusual circumstances. Therefore, administrative support is subtracted from the number of persons to be allocated before other computations are made.

The second section of the model yields the number of positions needed in each district to respond to requests for police services. For this model, the term "positions" refers to one officer on duty. Because the police operate every day, more than one officer is required to handle one position.

Two categories of response are used: accidents and criminal complaints. The number of positions needed to handle a call for service during any period depends upon three factors: average time taken to handle the incident, the average number of calls expected during that period, and the proportion of calls that are not handled immediately (either by delaying response or requesting assistance from some other department). This latter situation is considered as a "queue" which is serviced in some fashion other than by immediate response. Given these three factors, the number of positions required can be determined from a Poisson distribution.

However, there are many counties in the state where the volume of accidents and criminal complaints will require no assignment. Occurrences are too infrequent, but uniformed personnel still must be available for response. The model, therefore, also computes a minimum number of positions needed to satisfy a maximum response time. If this number is greater than the number of positions needed to handle calls for service, it is used as the base.

Finally, those officers not needed to answer calls for service provide general policing and patrolling. Governing the allocation of personnel are the miles of highway and volumes on those highways, and the number of local law enforcement personnel. This portion of the allocation deals primarily with rural patrol. Such patrol is limited to all rural areas and to incorporated municipalities with 1,500 or fewer persons. There are three bases upon which patrol is allocated: four-lane highways, all other rural roads, and support for local law enforcement.

The reason for dividing patrol between the four-lane (mostly Interstate or limited access roads) stems from how the highways are patrolled. All Interstate mileage, except within the City of Chicago, primarily is the jurisdiction of the Illinois State Police (ISP). Patrol on these highways is more frequent than on other types. On two-lane roads, the ISP generally do not patrol portions of highways inside municipalities with populations above 1,500 persons, therefore such mileage is excluded. There is a further distinction in patrol between low volume roads, those less than 1,500 vehicles per day (Average Daily Traffic -ADT) and those with a higher volume. Patrol on these low volume roads is very infrequent. It represents a small percentage (in relation to mileage available) of the ISP patrols.

Additionally, the State Police are called upon by sheriffs and small municipal departments for assistance for many reasons. The model allocates a portion of ISP personnel depending upon the sufficiency of local enforcement personnel. Such sufficiency is determined from the number of persons living in rural areas given an average rate of police officers per one thousand persons.

All of the computations for the model use data on a county-wide basis. A smaller jurisdiction could be used, but many of the data only exist at the county

level. Because the activity varies during the day and because the police operate in shifts, the model builds the allocation on a shift-by-shift basis. A uniform three-shift sequence is used for the model: first - 11 p.m. to 7 a.m., second - 7 a.m. to 3 p.m., and third - 3 p.m. to 11 p.m. Minor revisions could allow the model to use different timing of shifts such as hourly or 24 hours. The actual times for the shift are irrelevant. The eight-hour shift generally is the most logical division. Further, the model incorporates a coverage factor which, district by district, indicates that a shift is manned (1) or unmanned (0). This coverage factor determines whether the model will allocate positions to a specific shift in a county.

As noted, the model allocates positions. The number of officers is found by multiplying these positions times the number of officers required to staff one position during one shift. This factor is found by dividing 2,920 (the number of hours in one 8-hour shift, 365 days a year) by the number of annual man-hours worked by the police. A figure of 1,760 man-hours per year has been used by the Department.

DATA

The model distinguishes between two types of values called parameters and variables. Parameters are defined as those values that control the operation of the model as a whole. They can be changed before any run to simulate different set of conditions. Most parameters, such as the average time taken to handle an accident, or the division of ADT among the three shifts, apply to all districts. There are two parameters that apply individually to each district. One, already

noted, is a staffing parameter in which 1 denotes each shift that is staffed and 0 describes each shift not staffed. The second district parameter is the number of persons (not positions) assigned to administrative support. Included in this is the number of officers assigned to central office which includes security for elected officials.

Variables generally refer to data used by the model as the base for computations. Most of the values, such as number of accidents, are part of a data base which is updated annually. The remainder are computed, generally for each shift and county. After computations, all results are aggregated by three shifts into the individual counties, then into districts before they are printed.

Table 1 gives a summary of the parameters used in the model. This includes the notation used in the mathematical treatment that follows. Throughout the mathematical treatment, with the exception of persons to be allocated which is handled like a parameter, all notation for parameters is given in lower case letters. Table 2 gives the names and notation of the variables used. Table 3 shows the subscript notation, and Table 4 provides a list of the information used as the data base. As noted, all information from the data base, except for current strength, is given for each county.

TABLE 1

PARAMETERS USED
AND THEIR NOTATION

Parameter Number	Notation	Description
Accidents (a)		
1.	p_a	Proportional factor - used to increase or decrease historic data.
2.	t_a	Average time taken to handle one accident.
(proportion of responses delayed by shift)		
3.	q_{a1}	Proportion during shift 1 - 2300-0659
4.	q_{a2}	Proportion during shift 2 - 0700-1459
5.	q_{a3}	Proportion during shift 3 - 1500-2259
Criminal Complaints (b)		
6.	p_b	Proportional factor - used to increase or decrease historic data.
7.	t_b	Average time taken to handle one complaint.
(proportion of responses delayed by shift)		
8.	q_{b1}	Proportion during shift 1 - 2300-0659
9.	q_{b2}	Proportion during shift 2 - 0700-1459
10.	q_{b3}	Proportion during shift 3 - 1500-2259
Maximum Response Time (by shift)		
11.	d'_1	Maximum response time, shift 1
12.	d'_2	Maximum response time, shift 2
13.	d'_3	Maximum response time, shift 3

TABLE 1 (continued)

Parameter Number	Notation	Description
Proportion of Policing and Patrolling Assigned		
14.	p_f	Interstate (four-lane highway) patrol
15.	p_o	Other highway patrol
16.	p_r	Rural law enforcement support
	Note: $p_f + p_o + p_r = 1.0$	
Miles of Patrol		
17.	h_j *	Four-lane (Interstate) highways (excluding Chicago)
18.	h_k *	Two-lane, high volume (ADT greater than 1,500 vehicles) highways outside incorporated areas with populations greater than 1,500 persons.
19.	h_l	Two-lane highways (ADT 1,500 vehicles or less) and all other rural roads outside incorporated areas with populations greater than 1,500 persons.

*These are zero if the model solves for a given allocation. They are greater than zero when the model is used to compute a minimum strength.

Proportion of ADT (Average Daily Traffic) per Shift		
20.	w_1	Proportion of ADT during shift 1
21.	w_2	Proportion of ADT during shift 2
22.	w_3	Proportion of ADT during shift 3
	Note: $w_1 + w_2 + w_3 = 1.0$	
Other State-wide Parameters		
23.	t_{ann}	Annual manhours worked per person
24.	t_{enf}	Average number of hours required to assist a motorist and make a traffic enforcement stop.

TABLE 1 (continued)

Parameter Number	Notation	Description
25.	T	Number of persons to be allocated. (Note: this is equal to zero when a maximum strength is to be computed.)
26.	n	Number of counties
27.	r_{min}	Minimum number of state police positions assigned in any county to assist local law enforcement personnel.
28.	r_{max}	Maximum number of positions assigned to any county to assist local law enforcement personnel.
29.	z	Number of districts
30.	s	Shift staffing factor (computed)
31.	h_r	Persons per rural police patrol
32.	d_s	Maximum driving distance required to meet maximum response time during any shift, s (computed)
District-wide Parameters (one for each of the z districts)		
33.	c_1	Shift coverage factor for shift 1, either is a 0 or a 1.
34.	c_2	Shift coverage factor for shift 2.
35.	c_3	Shift coverage factor for shift 3.
36.	o	Administrative support for each of z districts. There is also a value for central office (which includes Executive Security) noted as o_{50}.

Alphabetic Summary

Notation	Description
c_1, c_2, c_3	- Shift coverage factor (0 or 1) for three shifts (by district).
d'_1, d'_2, d'_3	- Maximum response time in minutes for each of three shifts.

TABLE 1 (continued)

Notation	Description
d_s	- Maximum driving distance required to meet maximum response time during any shift, s (computed).
h_j, h_k, h_l	- Miles of patrol for four-lane, two-lane, and other highways (h_j and h_k normally are computed).
h_r	- Persons per rural police patrol.
n	- Number of counties (102 in Illinois).
o	- Administrative support for each district; o_{50} is administrative support for central office.
p_a, p_b	- Proportional factor used to increase or decrease historic accidents, a, or criminal complaints, b.
p_f, p_o, p_r	- Proportion of patrolling positions to be assigned to four-lane (Interstate) highway, other highway, and in support of local law enforcement: $p_f + p_o + p_r = 1.0$.
q_{as}, q_{bs}	- Proportion of calls for service for each shift, s, which are not answered immediately (placed in queue) for accidents, a, or criminal complaints, b.
r_{min}, r_{max}	- Minimum and maximum state police positions per shift to assist local law enforcement personnel.
s	- Shift staffing factor (computed).
t_a, t_b	- Average time, in hours, required to handle an accident, a, and criminal complaint, b.
t_{ann}	- Annual man-hours worked per person.
t_{enf}	- Average time, in hours, required to complete an enforcement stop and motorist assist.
T	- Total strength to be allocated (if zero, a maximum strength required is computed).
w_1, w_2, w_3	- Proportion of Average Daily Traffic (ADT) assigned to each of the three shifts: $w_1 + w_2 + w_3 = 1.0$.
z	- Number of districts. (currently 21 in Illinois)

TABLE 2

VARIABLES USED AND THEIR NOTATION
(by county unless noted otherwise)

Notation	Name of Variable	Source*
A	Accidents handled (annual)	TIPS
B	Case Data Reports (annual)**	TIPS
C	Officers allocated to calls for service and is the maximum of those needed for calls, C', or to minimize response, C"	Computed
D	Officers allocated to policing and patrolling	Computed
	D_f - Allocated to four-lane patrol	
	D_o - Allocated to other highway patrol	
	D_r - Allocated to rural assistance	
D_e	Equivalent total officers allocated to policing and patrolling	Computed
D'	Number of positions for policing and patrolling	Computed
	D'_f - Four-lane patrol	
	D'_o - Two-lane patrol	
	D'_r - Rural assistance	
D'_e	Equivalent total positions available for policing and patrolling	Computed
L	Rural law enforcement officers	
	L' - Sheriffs patrols	ISP (district commanders' reports)
	L" - Municipal police (serving incorporated areas of less than 1,500 persons)	UCR - Crime in Illinois

TABLE 2 (continued)

Notation	Name of Variable	Source*
M	Miles of highway	Illinois DOT
	M_j - Miles of Interstate and four-lane highway. All Interstate except toll road and in City of Chicago. All four-lane highway outside incorporated areas of 1,500 and greater.	
	M_k - Miles of two-lane, state or federally marked, where average daily travel (ADT) exceeds 1,500, and outside of incorporated areas of more than 1,500 persons.	
	M_l - Miles of other two-lane highway including state or federally marked with ADT less than 1,500 and all other highways outside incorporated areas greater than 1,500 persons.	
O	Total administrative support $O = \sum o_m + o_{50}$	ISP
P(x)	Probability of occurrence of x, an event, generally computed by shift and by county	Computed
Q	Area	Illinois Blue Book
R	Rural population including persons in incorporated areas of 1,500 and less	1980 Census
T'	Manpower less administrative support. $T' = T - O$ (where T is a parameter)	Computed
V	Average daily travel (ADT)	Illinois DOT
W	Vehicle miles in 1,000's	Illinois DOT

*Source refers to those relevant to the Illinois Department of Law Enforcement. TIPS stands for Traffic Information and Planning System, the data collection portion.

**Case Data Reports are a measure of criminally related calls for service. This soon will be replaced by a direct count of calls for service.

TABLE 3

SUBSCRIPTS

a - accidents

b - calls for service, criminally related (Case Data Reports)

d - minimized response

f - Interstate highway allocation

i - any county

j - Interstate/four-lane highways

k - two-lane highways

l - other highways

m - any district

o - other highway allocation

r - rural patrol allocation

s - any shift where s = 1 (first - 2300-0659)
s = 2 (second - 0700-1459)
s = 3 (third - 1500-2259)

TABLE 4

REQUIRED DATA FOR MANPOWER ALLOCATION

Item	Title and Variable Name		Description
By County			
1	County Number	-	DLE county numbers
2	District Cross Reference	-	ISP District number
3	Population (000's) (R)	-	Rural (includes towns of 1,500 and less)
4	Local Police (L')	-	Sheriffs' Patrols
5	Local Police (L")	-	Municipal Police (includes those in towns 1,500 and less)
6	Area (Q)	-	Area in square miles
7	Highway Miles (M_j)	-	4-Lane and Interstate highways (excluding toll road and City of Chicago)
8	Highway Miles (M_k)	-	2-Lane (ADT greater than 1,500)
9	Highway Miles (M_l)	-	All other (ADT less than 1,500)
10	Vehicle Miles (000's) (W_j)	-	4-Lane and Interstate highways (excluding toll road and City of Chicago)
11	Vehicle Miles (000's) (W_k)	-	2-Lane (ADT) greater than 1,500)
12	Vehicle Miles (000's) (W_l)	-	All other (ADT less than 1,500)
13	ISP Investigated Accidents (A_1)	-	Shift 1 - 2300-0659
14	ISP Investigated Accidents (A_2)	-	Shift 2 - 0700-1459
15	ISP Investigated Accidents (A_3)	-	Shift 3 - 1500-2259
16	Field (Case Data) Reports (B_1)	-	Shift 1 - 2300-0659
17	Field (Case Data) Reports (B_2)	-	Shift 2 - 0700-1459
18	Field (Case Data) Reports (B_3)	-	Shift 3 - 1500-2259

TABLE 4 (continued)

Item			
By District			
19	ISP Strength	-	Troopers
20	ISP Strength	-	Sergeants
21	ISP Strength	-	Command

II. MATHEMATICAL TREATMENT

MATHEMATICS

1. Preliminary Computations

The mathematics of the model are separated into two sections: the preliminary computations and computations for the various components of the model. The initial steps combine sheriff patrols and municipal police into one variable, local law enforcement, compute average daily travel from vehicle miles, and compute the shift staffing factor. Additionally, though not described at this point, could be the computation of congestion factors and reduction in patrolling speed resulting from enforcement and motorist assists.

A. Local Police (per county i) (L)

$$L_i = L'_i + L''_i \qquad (1.1)$$

where:

L'_i - Sheriffs patrols

L''_i - Municipal police

B. Average Daily Traffic (ADT per county i) (V)

for V_j - Four-lane/Interstate highways

V_k - Two-lane highways (ADT 1,500)

$$V_{ji} = 1000\ W_{ji} / M_{ji} \qquad (1.2)$$

where:

W_{ji} - Volume in thousands of vehicle miles for four-lane roads (j) in county i

M_{ji} - Miles of road for four-lane roads (j) in county i

Note: substitute k for j and recompute (1.2) which yields V_{ki}

C. Shift Staffing (one value) (s)

$$s = 2920 / t_{ann} \qquad (1.3)$$

where:

2920 - Number of hours in one eight-hour shift, 365 days per year

t_{ann} - Annual man-hours worked per person

2. Administrative Support

Administrative support represents the officers needed to administer the organization. Because of the nature of their work, they are not expected to patrol or answer calls for service except under special circumstances. The number of persons needed for this aspect is established by the executive. It is based on perceived needs both at the field level and in central office. For the Illinois State Police, central office staff also covers executive security.

After computation, the number of officers are subtracted from those to be allocated. The amounts for each district and central office are shown on the summary sheets. (See Appendix A for sample summary sheets.)

$$O = \sum o_m + o_{50} \qquad (2.1)$$

where:

O - Total administrative support

o_m - Administrative support in district m

o_{50} - Central office staff (sworn and executive security)

$$T' = T - O \tag{2.2}$$

where:

T' - Strength available for allocation after deducting administrative support

T - Total strength to be allocated

Only those persons not assigned to administrative support are allocated according to needs. Because the model is capable of computing a total strength, T may be computed as an end product. Establishing T equal to 0.0 as a parameter should serve as the signal for a computer program to compute T as in equation (2.3) below. As used in this equation, T' then is computed from values already given and shown elsewhere in the description of the formulas.

$$T = T' + O \tag{2.3}$$

3. Calls for Service

The response to requests for service is divided into two categories: response to accidents and response to criminal complaints. The two categories are

considered to represent most of the ISP responses. Assignment is performed by positions required. At the end, it is changed to officers by multiplying the shift staffing factor, s. The number of positions required to service each category is derived from a Poisson distribution where the number of requests answered is that x where the sum from P(o) to P(x) equals or just exceeds the proportion of calls to be answered. However, the number of positions required for response will not be less than the number required to minimize response time. This latter is a function of distance and congestion.

A. Accidents

The average number of accidents occurring during the time taken to handle one accident for any shift, s, is shown in equation (3.1). In all succeeding equations, unless otherwise noted, the computations are performed for each county i. The subscript, i, has been left out of the formulas.

$$m_{as} = A_s P_a t_a \; / \; 2920 \tag{3.1}$$

where:

m_{as} - Expected number of accidents on one shift

A_s - Accidents that have occurred for a 12-month period during shift s

P_a - Proportional factor to increase or decrease historic data

t_a - Average time taken to handle one accident

2920 - Number of annual hours in one, 8-hour shift

The number of positions needed to handle m_{as} accidents per shift is X_{as} which is found when the sum of P(x) in the Poisson distribution, from o to x just exceeds some given proportion of calls to be answered. For each shift, P(x) is determined by

$$P(x) = e^{-m_{as}} \; m_{as}^{\;x} \; / \; x! \qquad (3.2)$$

where:

P(x) - Probability of zero or more events occurring during time, t_a, in shift, s

e - Natural log

m_{as} - From equation (3.1) for shift, s

x - 0, 1, . . .

and X_{as} is that x which satisfies the equation:

$$P(x) \geq (1 - q_{as}) \qquad (3.3)$$

where:

q_{as} - Proportion of requests for assistance at accidents not answered immediately

B. Criminal Complaints

The routines used for accidents are repeated for criminal complaints.

$$m_{bs} = B_s \; P_b \; t_b \; / \; 2920 \qquad (3.4)$$

where:

m_{bs} - Expected number of criminal complaints during one shift

B_s - Calls for service, criminally related matters

P_b - Proportional factor for criminal complaints to adjust historic data

t_b - Time taken to handle one criminal complaint

The number of positions needed to handle m_{bs} criminal complaints per shift is X_{as} which is found when the sum of P(x) from the Poisson distribution, from o to x just exceeds some given proportion of calls to be answered. For each shift, P(x) is determined by

$$P(x) = e^{-m_{bs}} \quad m_{bs}^{\;x} \; / \; x! \qquad (3.5)$$

and X_{bs} is that x which satisfies the equation

$$P(x) \geq (1 - q_{bs}) \qquad (3.6)$$

where:

q_{bs} - Proportion of requests for assistance at criminal complaints not answered immediately

C. Minimized Response

The number of positions required in a specific area to answer calls for service also must be sufficient to provide response in a maximum allowable time.

Therefore, in addition to solving the number of positions needed to answer calls, another equation must be used to solve positions needed to minimize response time. Response depends upon driving speed (assumed for ease of computation to be 60 mph on four-lane roads and 45 mph on two-lane roads) and the amount of congestion on the highways. The latter is a function of volume. The adjustment is solved only for multi-lane and two-lane roads on the assumption that emergency response rarely uses local roads. Although the i for individual county is not shown, each equation is solved for county, i and within the county for shift, s.

$$f_{js} = V_j\, w \;/\; 8000 \qquad (3.7a)$$

for $V_j\, w_s \leq 10300$

$$f_{js} = 3.88 + (V_j\, w_s - 10300) \;/\; 1300 \qquad (3.7b)$$

for all other $V_j\, w_s$

where:

f_{js} - Reduction in speed in mph on multi-lane roads during any shift for county i

V_j - ADT, from equation (1.2)

w_s - Percent of ADT during shift s

The factor for two-lane roads is found similarly, substituting k (two-lane) for j (four-lane):

$$f_{ks} = V_k\, w_s/2000 \qquad (3.8a)$$

for $V_k\, w_s \leq 3400$

$$f_{ks} = 5.15 + (V_k\, w_s - 3400) \;/\; 70 \qquad (3.8b)$$

for all other $V_k\, w_s$

Next solved is the adjusted miles that can be driven during the maximum response time for any shift s in any county i. The subscript i is not shown; however, the d_s in (3.9) is found for every county; thus, it would be defined as d_{is}.

$$d_s = d'_s \left[(60 - f_{js})\, M_j + (45 - f_{ks})\, M_k \right] / (60\, M_j + 45\, M_k) \qquad (3.9)$$

where:

d_s - Adjusted distance driven during maximum response time in any county i

d'_s - Maximum response time for any one shift

f_{js}, f_{ks} - Congestion factors for four-lane and two-lane roads, from (3.7) and 3.8)

M_j, M_k - Miles of highway, four-lane and two-lane roads

60, 45 - 60 mph response speed assumed on four-lane roads and 45 mph on two-lane roads

$$X_{ds} = Q \;/\; 2\, d_s^2 \qquad (3.10)$$

where:

X_{ds} - Number of positions required to meet maximum response time during shift s in county i

Q - Area in square miles

d_s - From equation (3.9)

D. Number of Officers Required

The number of positions required to answer calls for service then is the maximum of the number required for calls or to meet the maximum response time.

$$C'_s = \max \left[X_{as} + X_{bs},\ X_{ds} \right] c_s \qquad (3.11)$$

where:

C'_s - Number of positions required to handle either calls for service or respond during any shift s in county i

X_{as} - Number of positions needed to handle accidents, from (3.3)

X_{bs} - Number of positions needed to handle criminal complaints, from (3.6)

X_{ds} - Number of positions needed to maintain maximum response time, from (3.10)

c_s - Shift coverage factor, either a 1 for shift covered or a 0 for no coverage in each district, m, which includes one or more counties

Finally, the number of persons needed to fill these positions is found by multiplying the positions times the shift staffing factor. This latter was computed from equation (1.3).

$$C_{si} = s\, C'_s \qquad (3.12)$$

where:

C_{si} - Number of officers required to handle calls for service in any county during any shift in county i

C'_s - Number of positions required to handle calls for service in any county i during any shift, from (3.11)

s - Shift staffing factor as shown in equation (1.3)

4. Policing and Patrolling

Those officers not required to answer calls for service patrol the highways and rural areas. These assignments depend upon the miles of highway to be patrolled, the traffic volume on those highways, the size of the rural population, and available local police. In addition, when those officers assigned to calls for service are not engaged, they also patrol. This non-obligated time must be included in the allocation of officers for the third part, policing and patrolling. First is computed the remaining officers after subtracting administrative support and calls for service.

$$D = T' - \sum_s \sum_i C_{si} \qquad (4.1)$$

where:

D - Number of officers not assigned to administrative support or calls for service

T' - Number of officers available for allocation, from (2.2)

C_{si} - Officers assigned to calls for service for each shift, s in each county, from (3.12)

This D is increased by the time of those officers not handling calls for service. This adjusted amount is called "equivalent officers for policing and patrolling". Because allocations are based on positions, the "equivalent officers" subsequently is converted to "equivalent positions". This latter value is found by dividing the equivalent officers by the shift staffing factor, s.

$$D_e = D + \sum_i \Big(t_{ann} \sum_s C_s - p_a t_a \sum_s (1 - q_{as}) A_s - p_b t_b \sum_s (1 - q_{bs}) B_s \Big) / t_{ann} \qquad (4.2a)$$

$$D'_e = D_e / s \qquad (4.2b)$$

where:

D_e - Equivalent number of officers available for policing and patrolling

D - Number of officers remaining for policing and patrolling, from (4.1)

t_{ann} - Annual man-hours of work per officer

p_a, p_b - Proportional factors to adjust historic data for accidents, a, and criminal complaints, b

C_s - Number of officers assigned to handle calls for service, from (3.12)

t_a, t_b - Time taken to handle an accident or criminal complaint

q_{as}, q_{bs} - Proportion of calls for accidents or criminal complaints not answered immediately during shift s

A_s, B_s - Accidents and criminal complaints

D'_e - Equivalent number of positions for policing and patrolling

s - Shift staffing factor, from (1.3)

This D'_e is then distributed among the patrolling functions. Three are used: patrol of four-lane roads, Interstate, and other multi-lane highways; patrol of two-lane roads; and rural patrol which is additional policing support for local law enforcement personnel. The proportion of the available manpower to be assigned each function is based on the perceived needs and is an executive function.

$$D'_e = p_f D'_e + p_o D'_e + p_r D'_e \qquad (4.3)$$

Note: $p_f + p_o + p_r = 1.0$

where:

D'_e - Equivalent positions available for policing and patrolling, from (4.2b)

p_f - Proportion of positions to be assigned to four-lane roads

p_o - Proportion of positions to be assigned to two-lane roads

p_r - Proportion of positions to be assigned to rural policing

A. Four-Lane Roads

One portion of positions will be assigned to patrol of four-lane roads. It is described by D'_f as computed in (4.4). This amount is allocated throughout the state.

$$D'_f = p_f D'_e \qquad (4.4)$$

where:

D'_f - Positions assigned to four-lane patrol

D'_e - Equivalent positions for patrol, from (4.2b)

p_f - Proportion of positions to be assigned to four-lane roads

Used as the basis for assignment is the miles of four-lane highway outside of municipalities with 1,500 or more persons and all Interstate mileage except that in Chicago. The miles which can be patrolled is inversely related to volume of traffic on those highways. As volume increases, patrolling speeds decrease because of congestion and because the police initiate more actions (motorist assists and traffic enforcement). The adjustment factor for congestion already has been shown in (3.7) for four-lane roads. Decreases in patrolling speed have the same effect as increasing miles of patrol. If patrolling speed is assumed to be 50 mph, its relative reduction from congestion is shown in (4.5).

$$u_{fs} = 50 \; / \; (50 - f_{js}) \qquad (4.5)$$

where:

u_{fs} - Factor which increases the miles of highway resulting from reduced patrolling speed for each county, i

f_{js} - Reduction in patrolling speed resulting from congestion, as shown in (3.7), for each county, i

50 - Patrolling speed in miles per hour

In addition to the reduction in patrolling speed because of congestion, the number of stops also increase with increasing volume. These too reduce speed.

$$Y_{js} = 6 \, W_j \, w_s \, c_s \; / \; 2920 \qquad (4.6)$$

where:

Y_{js} - Number of actions per hour during any eight-hour shift, s, for each county i

W_j - Traffic volume on four-lane roads in thousands of vehicle miles for each county i

w_s - Proportion of ADT during each shift, s

c_s - Shift coverage factor, a 0 or 1 for each shift, s, for each of the m districts as applied uniformly to each county in those districts

2920 - Number of man-hours in one annual eight-hour shift

In order to allocate officers to highways based on the miles of highway in each county, the average number of miles to be patrolled per position, state-wide, must be computed. This is done as an average for the three shifts for all counties combined.

$$h_j = \sum_i M_j \sum_s c_s u_{fs} \; / \; (D'_f - t_{enf} \sum_s \sum_i Y_{js} u_{fs}) \qquad (4.7)$$

where:

h_j - Average length of patrol, in miles, of four-lane roads

M_j - Miles of four-lane roads in each county i

c_s - Shift coverage factor, 0 or 1, for each shift, s, for each of the m districts applied uniformly to all counties in those districts

u_{fs} - Factor which increases length of patrol as a result of reduced patrolling speeds, from (4.5)

t_{enf} - Average number of hours required to complete a traffic action for traffic enforcement and motorist assists

Y_{js} - Number of traffic actions based on volume, from (4.6)

The shift-by-shift coverage for each county then can be found by substituting the value for h_j in (4.8) below. The result is a number of positions per shift per county required to meet the requisite patrol of four-lane roads. Areas with higher traffic volumes will require more positions than areas with lower volumes for the same length of highway. (Note: the steps below are performed for each county even though the individual county, i, is not specifically identified as a subscript.)

$$X'_{fs} = M_j c_s u_{fs} \; / \; h_j + t_{enf} Y_{js} u_{fs} \qquad (4.8)$$

where:

X'_{fs} - Positions required to patrol a four-lane highway during shift, s

h_j - Miles of four-lane patrol, from (4.7)

t_{enf} - Average number of hours required to complete a traffic action

Y_{js} - Number of traffic actions based on volume, from (4.6)

M_j - Miles of highway in county i

c_s - Shift coverage factor, either a 0 or a 1

u_{fs} - Factor which increases length of patrol, from (4.5)

B. Two-Lane Roads

Patrol of two-lane roads is computed in a manner similar to that used for four-lane roads. The difference is the separation of two-lane roads into two types, those heavily traveled and other local roads. The dividing line selected for Illinois is an average daily traffic (ADT) of 1,500 vehicles. All highway mileage used for the lies outside corporate boundaries except in those towns with fewer than 1,500 persons. Similar to four-lane roads, there is an adjustement in speed for congestion and traffic enforcement. D'_o is the number of positions available for two-lane patrol.

$$D'_o = p_o D'_e \qquad (4.9)$$

where:

D'_o - Equivalent number of positions for two-lane patrol

p_o - Proportion of positions assigned to two-lane patrol

D'_e - Equivalent number of positions for policing and patrolling, from (4.2b)

$$u_{ks} = 50 / (50 - f_{ks}) \qquad (4.10)$$

where:

u_{ks} - Factor which increases miles of highway resulting from a reduced patrolling speed on two-lane state and federally numbered roads with ADT of more than 1,500 vehicles

f_{ks} - Reduction in patrolling speed, from (3.8)

$$Y_{ks} = 8.7\ W_k\ w_s\ c_s\ /\ 2920 + 1.8\ W_j\ w_s\ c_s\ /\ 2920 \qquad (4.11)$$

where:

Y_{ks} - Reduction in speed for traffic enforcement and motorist assists on two-lane roads

Note: All other variables and parameters have been described in (4.6). The subscript, k, stands for two-lane, heavily traveled roads and the subscript, l, for all other two-lane roads.

$$h_k = \sum M_k \sum c_s\ u_{ks}\ /\ (D'_o - \sum_i M_l \sum_s c_s\ /\ h_l - t_{enf} \sum_s \sum_i Y_{ks}\ u_{ks}) \qquad (4.12)$$

where:

h_k - Average miles of patrol on two-lane roads with volumes greater than 1,500 vehicles

M_k, M_l - Miles of patrol on the higher volume two-lane roads, k, and all other two-lane roads, l

c_s - Shift coverage factor, a 0 or 1 for each district, m, applied equally to each county

h_l - Miles of patrol on two-lane, low volume roads, already shown as a parameter

u_{ks} - Factor which increases miles of patrol, from (4.10)

Y_{ks} - Reduction in speed resulting from traffic actions, from (4.11)

t_{enf} - Hours required to complete a traffic action, for traffic enforcement and motorist assists.

Substituting h_k in equation (4.13) below then gives the number of positions for each shift and for each county. As was done with four-lane patrol in equation (4.8), the subscript for counties is not shown.

$$X'_{os} = M_k\ c_k\ u_{ks}\ /\ h_k + M_l\ c_s\ /\ h_l + t_{enf}\ Y_{ks}\ u_{ks} \qquad (4.13)$$

where:

X'_{os} - Number of position required to patrol two-lane roads for shift, s

h_k - Miles of patrol of two-lane, high-volume roads, from equation (4.10)

Note: All other variables and parameters are given in (4.12) and apply to the county.

C. Rural Patrol

A final portion of policing and patrolling assigns state police to assist local law enforcement personnel. Because this patrol is designed to be supplementary, upper and lower limits are placed on the allocations. For the Illinois State Police, the maximum, max is one position per shift. On the other hand, even though there are sufficient local police, State Police strength is not decreased. Therefore, the minimum, r_{min} is 0.0.

The controlling factor is rural population. Because of the constraints placed on the assignment of positions, the allocation among the counties can not be solved in a single step as performed for patrolling mileage. It is solved iteratively. First is computed an expected population coverage, h_r. This value is substituted and all assignments are made, within the bounds of r_{min} and r_{max}. The sum of these assignments then is compared to number allocated. If the sum is greater than this number, then h_r must increase. If it is smaller, h_r must decrease.

$$D'_r = p_r D'_e \tag{4.14}$$

$$h_r = 1000 \sum_i R_i / (D'_r + \sum_i L_i / 3) \tag{4.15}$$

where:

D'_r - Positions assigned to rural patrol

D'_e - Equivalent number of positions for patrolling, from (4.2b)

p_r - Proportion of positions to be assigned to rural patrol

h_r - Population coverage factor per police position

R_i - Rural population in thousands for county, i (includes incorporated areas with less than 1,500 persons)

L_i - Number of local law enforcement personnel from in county i, from (1.1)

This population coverage factor is inserted into the equation (4.16). The value of X'_{rs} is solved for each county, i, and shift, s, within that county,

then summed and compared to D'_r. Depending upon the direction of the difference, h_r is adjusted and X'_{rs} again solved.

$$X'_{rs} = 1000 \, R \, c_s / h_r - L / 3 \tag{4.16}$$

such that

$$r_{min} \leq X'_r \leq r_{max} \tag{4.17}$$

and satisfies the equality

$$X'_{rs} = D'_r \pm 0.01 \, D'_r \tag{4.18}$$

where:

X'_{rs} - Number of positions per shift for rural policing in county i

R - Rural population in thousands in any county i

c_s - Shift coverage factor, 0 or 1 during shift, s, in county i

h_r - Population coverage factor, initially from (4.15)

L - Number of local law enforcement officers, from (1.1)

r_{min}, r_{max} - Minimum and maximum state police coverage per shift

D'_r - Total positions assigned to rural policing, from (4.14)

D. Adjustment for Non-Assigned Time

The positions established in steps A through C above, defined as X'_{fs}, X'_{os}, and X'_{rs} must be reduced to eliminate the additional available patrolling

time that was added from calls for service. This is computed by decreasing the number of positions by the proportion of free time added. Equations 4.19 through 4.23 show the computations required to be performed for each county. (In the following series the subscript i is used to help differentiate between variables applying to individual counties and to the state as a whole.)

$$D_{ei} = s \sum_{s} (X'_{fs} + X'_{os} + X'_{rs})_i \qquad (4.19)$$

where:

D_{ei} - Equivalent officers assigned for all three shifts in any county

X'_{fs} - Number of positions for four-lane highway patrol for county i, from (4.8)

X'_{os} - Number of positions for other highway patrol for county i, from (4.13)

X'_{rs} - Number of positions for rural patrol for county i, from (4.16)

s - Shift staffing factor, from (1.3)

The total manpower then assigned to policing and patrolling is adjusted by the percentage of the equivalent positions.

$$f_d = D / \sum_{i} (\sum_{s} C_{si} + D_{ei}) \qquad (4.20)$$

where:

f_d - Statewide adjustment factor

D - Officers available for policing and patrolling, after deleting calls for service and administrative support, from (4.1)

C_{si} - Persons allocated to calls for service for each shift, s, in county i, from (3.12)

D'_{ei} - Number of equivalent persons for patrol in county i for all three shifts, from (4.19)

$$u = f_d (\quad C_{si} + D'_{ei}) / D'_{ei} \qquad (4.21)$$

where:

u - County adjustment factor for any shift in each county i

f_d - Statewide adjustment factor, from (4.

D'_{ei}, C_{si} - Noted in (4.20)

All three allocations of positions: to four-lane (f), two-lane (o), and rural patrol (r) are then adjusted by the factor, u. This is done for each shift and for each county individually. The results are cumulated to districts. From X'_{fs}, X'_{os}, and X'_{rs} which represent positions, derive adjusted number of officers required for each portion of policing and patrolling.

$$X_{fs} = u\ s\ X'_{fs} \qquad (4.22)$$

where:

X_{fs} - Number of officers assigned during any shift, s, in county, i

X'_{fs} - Equivalent number of positions assigned to four-lane highway patrol in county i, from (4.8)

u - County adjustment factor, from (4.21)

s - Shift staffing factor, from (1.3)

Note: Equation 4.22 is used to solve X_{os} and X_{rs} by substituting X'_{os} or X'_{rs} for X'_{fs} and recomputing.

III. OUTPUT AND MICROCOMPUTER PROCESSING

OUTPUT

The mathematics have been converted to a program that is run on the departmental main-frame. In addition to performing all calculations, the program allows the user to change parameters and rerun the model. Before printing, the output can be examined. The user also can specify a range in parameters and print the results of each change. By doing so, the user can test different philosophies of allocation and determine what effects changes in these philosophies have on the distribution of officers. Sample reports are shown in Appendix A. Because this paper is being used to revise the current operation of the program, copies of actual reports are not included.

There are two additions to the operation of this allocation on a computer which enhances its use. First, the user can change the parameters at each run. There is a base set of parameters which may be changed permanently. Most of the time, however, changes are made only for a particular run. To facilitate making these changes, all parameters at the start of a run have a starting point, a maximum value, and an incremental value.

The second option is the ability to compute the required number of officers rather than allocate a fixed staff. All formulae shown, except (4.7), (4.12), and (4.15) are used. The total number of officers to be allocated also is set to zero initially. The parameters for miles of patrol on four-lane and two-lane, high volume roads, h_j and h_k, and for population coverage, h_r, must be greater than zero. The calls for service are solved first and available patrolling time

substituted as shown in the equations. All remaining computations are performed yielding required coverage for all activities. These are summed and the total becomes the strength needed to satisfy the parameters used.

APPLICATION TO MICRO COMPUTERS

A computer program for an Apple II Plus micro computer, written in BASIC, performs all computations described in the previous section. It is not as flexible or fast as the program written for the departmental main frame. However, it does the computations for all shifts and for all counties. Each of the shifts as well as total assignments are cumulated by district and shown along with the allocation of administrative support. Appendix B repeats all formulae as used in preparing the computer program.

SUMMARY

The model has been run successfully for two years. During this time numerous adjustments have been made to its operation. This paper represents the culmination of these changes. In all cases, the changes have been directed toward simplifying the computations and to improving the usefulness of the information. Further, the model has been used to allocate new officers, review the current allocations of officers, examine what changes in service will occur if cuts are made in strength, and to examine what strength would be needed to accomplish different goals. The programming of the operation for a micro computer, while it is not as flexible or extensive, allows users with smaller operations to benefit. Overall, the model has proven a valuable tool.

NOTES

1. Richard A. Raub and George L. Sweat, A Method For Allocating State Police Officers In Illinois, Illinois Department of Law Enforcement, Springfield, Illinois, June 1981.

2. Richard A. Raub, Paper, 2nd Annual Conference on Operations Research, Sangamon State University, Springfield, Illinois, April 1983.

3. Richard A. Raub and George L. Sweat, "Manpower Allocation, Rationale For Methods In State Police Districts," The Police Chief, 50:6, June 1983, pp. 62-65.

APPENDIX A

SAMPLE OUTPUT REPORTS

PART I

CALLS FOR SERVICE
ALLOCATION

Manpower to be Allocated xxxxx
Available Manpower: xxxx.x*

District	Accidents Handled Number Handled	Positions/Shift 1	2	3	Criminal Complaints Number Handled	Positions/Shift 1	2	3	Minimum Response Positions/Shift 1	2	3	Total Officers Allocated
01	xxx,xxx	xx	xx	xx	xx,xxx	xx	xx	xx	xx.x	xx.x	xx.x	xxxx.x
02												
.												
.												
.												
20												
Totals	xxx,xxx	xx	xx	xx	xxx,xxx	xx	xx	xx	xx.x	xx.x	xx.x	xxxx.x

Note: Shift 1: 2300 - 0659 Shift 2: 0700 - 1459 Shift 3: 1500 - 2259

*(NOTE: Derived from total manpower less administrative support.)
(This note is not included)

CALLS FOR SERVICE
PARAMETERS

			(source)*
Manhours Per Year	xxxx	Hours	t_{ann}
Shift Staffing Rate	x.xxx		$2920/t_{ann}$
Accidents Handled			
Time to Handle	xx.xx	Hours	t_a
Multiplier	xxx.x	Percent	$p_a \times 100$
Percent Handled			
Shift 1	xx.xx	%	$(1 - q_{a1}) \times 100$
Shift 2	xx.xx	%	$(1 - q_{a2}) \times 100$
Shift 3	xx.xx	%	$(1 - q_{a3}) \times 100$
Criminal Complaints			
Time to Handle	xx.xx	Hours	t_b
Multiplier	xxx.x	Percent	$p_b \times 100$
Percent Handled			
Shift 1	xx.xx	%	$(1 - q_{b1}) \times 100$
Shift 2	xx.xx	%	$(1 - q_{b2}) \times 100$
Shift 3	xx.xx	%	$(1 - q_{b3}) \times 100$
Maximum Response Time			
Shift 1	xxx	Minutes	d'_1
Shift 2	xxx	Minutes	d'_2
Shift 3	xxx	Minutes	d'_3

Note: Shift 1: 2300 - 0659 Shift 2: 0700 - 1459 Shift 3: 1500 - 2259

*(NOTE: This indicates where the information applies.)
(This column and note is not included in the listing.)

PART 2

POLICING AND PATROLLING
ALLOCATION

Manpower to be Allocated xxxxx
Available Manpower: xxxx.x*

District	Miles of Highway 4-Lane	2-Lane	Other	Rural Population (000's)	Police	Highway Patrol 4-Lane	2-Lane	Rural Patrol	Patrolling Positions Positions/Shift 1	2	3	Total Officers Allocated
01	xxx,xxx	xxx,xxx	xxx,xxx	x,xxx.x	xxx	xxx.x	xxx.x	xxx.x	xxx.x	xxx.x	xxx.x	xxxx.x
02												
.												
.												
.												
20												
Totals	xxx,xxx	xxx,xxx	xxx,xxx	x,xxx.x	xxxx	xxx.x	xxx.x	xxx.x	xxx.x	xxx.x	xxx.x	xxx.x

Note: Shift 1: 2300 - 0659 Shift 2: 0700 - 1459 Shift 3: 1500 - 2259

*(NOTE: Derived from total manpower less calls for service and administrative support.)
(This note is not included)

PART 2

POLICING AND PATROLLING PARAMETERS

			(source)*
Four-Lane Patrol			
Percent of Positions Assigned	xxx.x	Percent	$p_f \times 100$
Miles of Patrol per Position	xx,xxx.x	Miles	h_j
Two-Lane Patrol			
Percent of Positions Assigned	xxx.x	Percent	$p_o \times 100$
Miles of Patrol per Position			
2-Lane Roads	xx,xxx.x	Miles	h_k
Other Roads	xx,xxx.x	Miles	h_l
Rural Patrol			
Percent Assigned	xxx.x	Percent	$p_r \times 100$
Persons per Rural Police Patrol	xx,xxx.x	Persons	h_r
Limits on State Police Assignment			
Minimum per County	xx.x	Positions	r_{min}
Maximum per County	xx.x	Positions	r_{max}
Percent ADT Assigned per Shift			
Shift 1	xxx.x	%	$w_1 \times 100$
Shift 2	xxx.x	%	$w_2 \times 100$
Shift 3	xxx.x	%	$w_3 \times 100$
Average Time for Enforcement Stop	x.xxx	Hours	t_{enf}

Note: Shift 1: 2300 - 0659 Shift 2: 0700 - 1459 Shift 3: 1500 - 2259

*(NOTE: This indicates where the information applies.)
(This column and note is not included in the listing.)

REPORT 3

SUMMARY OF ALLOCATION

District	Administrative Support	Calls For Service	Policing and Patrolling	Total Allocation	Current DSP Deployment Total	Patrol	Command
01	xxx.x	xxxx.x	xxxx.x	xxxx.x	xxxx	xxxx	xxx
02							
.							
.							
.							
20							
Totals	xxx.x	xxxx.x	xxxx.x	xxxx.x	xxxx	xxxx	xxx

40

PART 3
SUMMARY OF ALLOCATION
PARAMETERS

			(source)*
Administrative Support	xxx.x	Persons	O
Manhours per Year	xxxx	Hours	t_{ann}
Shift Staffing Rate	x.xxx		$2920/t_{ann}$
Average Time per Enforcement	x.xxx	Hours	t_{enf}
Percent ADT per shift			
shift 1	xxx.x	%	$w_1 \times 100$
shift 2	xxx.x	%	$w_2 \times 100$
shift 3	xxx.x	%	$w_3 \times 100$
Maximum Response Time per shift			
shift 1	xxx	Minutes	d'_1
shift 2	xxx	Minutes	d'_2
shift 3	xxx	Minutes	d'_3
Percent Positions Assigned			
Four-Lane Highway Patrol	xxx.x	%	$p_f \times 100$
Other Highway Patrol	xxx.x	%	$p_o \times 100$
Rural Law Enforcement	xxx.x	%	$p_r \times 100$
Miles of Patrol per Position			
Four-Lane Roads	xx,xxx.x	Miles	h_j
Two-Lane Roads	xx,xxx.x	Miles	h_k
Other Roads	xx,xxx.x	Miles	h_l
Persons per Rural Police Patrol	xx,xxx.x	Persons	h_r
State Police Rural Patrol			
Minimum per County	xx.x	Positions	r_{min}
Maximum per County	xx.x	Positions	r_{max}

PART 3
SUMMARY OF ALLOCATION
PARAMETERS (continued)

			(source)
Accidents			
Average Time to Handle	xx.xx	Hours	t_a
Multiplier	xxx.x	Percent	$p_a \times 100$
Percent to Be Queued			
shift 1	xxx.x	%	$q_{a1} \times 100$
shift 2	xxx.x	%	$q_{a2} \times 100$
shift 3	xxx.x	%	$q_{a3} \times 100$
Criminal Complaints			
Average Time to Handle	xx.x	Hours	t_b
Multiplier	xxx.x	Percent	$p_b \times 100$
Percent to Be Queued			
shift 1	xx.x	%	$q_{b1} \times 100$
shift 2	xx.x	%	$q_{b2} \times 100$
shift 3	xx.x	%	$q_{b3} \times 100$

Shift Coverage Factors

District	Shift 1	Shift 2	Shift 3
01	X	X	X
02			
..			
.			
20	X	X	X

Note: Shift 1: 2300-0659 Shift 2: 0700-1359 Shift 3: 1400-2259

*(NOTE: This indicates where the information applies.)
(This column and rate are not included in the listing.)

APPENDIX B

SUMMARY OF FORMULAS

SUMMARY OF COMPUTATIONS

1. Preliminary Steps

A. Local Police (per county):

$$L_l = L'_i + L''_i \tag{1.1}$$

B. ADT: 4-lane, 2-lane (per county)

$$V_{ji} = 1000\ W_{ji} / M_{ji} \tag{1.2a}$$

$$V_{ki} = 1000\ W_{ki} / M_{ki} \tag{1.2b}$$

C. Shift Staffing

$$s = 2920 / t_{ann} \tag{1.3}$$

2. Administrative Support

Total

$$O = \sum_m o_m + o_{50} \tag{2.1}$$

Officers Available for Allocation

$$T' = T - O \tag{2.2}$$

If strength computed

$$T = T' + O \text{ or } \sum_m (C_m + D_m) + O \tag{2.3}$$

3. Calls for Service

A. Accidents (per county, per shift):

Average Rate

$$m_{as} = A_s P_a t_a / 2920 \tag{3.1}$$

Poisson Distribution

$$P(x) = e^{-m_{as}} m_{as}^{x} / x! \qquad (3.2)$$

such that x_s satisfies

$$P(x) \geq (1 - q_{as}) \qquad (3.3)$$

B. Criminal Complaints (per county, per shift):

Average Rate

$$m_{bs} = B_s p_b t_b / 2920 \qquad (3.4)$$

Poisson Distribution

$$P(x) = e^{-m_{bs}} m_{bs}^{x} / x! \qquad (3.5)$$

such that x satisfies

$$P(x) \geq (1 - q_{bs}) \qquad (3.6)$$

C. Congestion Factors:

Four-lane

$$f_{js} = V_j w_s / 8000 \qquad (3.7a)$$

for $V_j w_s \leq 10300$

$$f_{js} = 3.88 + (V_j w_s - 10300) / 1300 \qquad (3.7b)$$

for all other $V_j w_s$

Two-lane

$$f_{ks} = V_k w_s / 2000 \qquad (3.8a)$$

for $V_k w_s \leq 3400$

$$f_{ks} = 5.15 + (V_k w_s - 3400) / 70 \qquad (3.8b)$$

for all other $V_k w_s$

D. Miles of Response, Adjusted

$$d_s = d'_s \left[(60 - f_{js}) M_j + (45 - f_{ks}) M_k \right] / (60 M_j + 45 M_k) \qquad (3.9)$$

E. Required Positions (per county, per shift)

$$X_{ds} = Q / 2 d_s^2 \qquad (3.10)$$

F. Positions Required for Calls for Service (per county, per shift)

$$C'_s = \max \left[X_{as} + X_{bs}, X_{ds} \right] c_{sm} \qquad (3.11)$$

G. Required Officers (per county, per shift)

$$C_s = s \; C'_s \qquad (3.12)$$

4. Policing and Patrolling

A. (Per county unless otherwise noted)

Officers Remaining

$$D = T' - \sum_s \sum_i C_{si} \qquad (4.1)$$

B. Positions Available for Patrol (i assumed)

$$D_e = D - \sum_i \left[(t_{ann} \sum_s C_s - p_a t_a \sum_s (1 - q_{as}) A_s - p_b t_b \sum_s (1 - q_{bs}) B_s \right] / t_{ann} \qquad (4.2a)$$

$$D'_e = D_e / s \qquad (4.2b)$$

Assignment of D'_e to patrolling

$$D'_e = p_f D'_e + p_o D'_e + p_r D'_e \qquad (4.3)$$

Note: $p_f + p_o + p_r = 1.0$

C. Four-Lane Roads (per county, per shift):

$$D'_f = p_f D'_e \qquad (4.4)$$

Reduced Patrolling Speed

$$u_f = 50 / (50 - f_{js}) \qquad (4.5)$$

Congestion Factor

$$Y_{js} = 6 W_j w_s c_s / 2920 \qquad (4.6)$$

Miles of Patrol

$$h_j = \sum_i M_j \sum_s c_s u_{fs} / (D'_f - t_{enf} \sum_i \sum_s Y_{js} u_{fs}) \qquad (4.7)$$

Positions Required (per county, per shift)

$$X'_{fs} = M_j c_s u_{fs} / h_j + t_{enf} Y_{js} u_{fs} \qquad (4.8)$$

D. Other Highways (per county, per shift):

$$D'_o = p_o D'_e \qquad (4.9)$$

Reduced Patrolling Speed

$$u_k = 50 / (50 - f_{ks}) \qquad (4.10)$$

Congestion Factor

$$Y_{ks} = 8.7 W_k w_s c_s / 2920 + 1.8 W_k w_s c_s / 2920 \qquad (4.11)$$

Miles of Patrol

$$h_k = \sum_i M_k \sum_s c_s u_{ks} / (D'_o - \sum_i M_l \sum_s c_s / h_l - t_{enf} \sum_s \sum_i Y_{ks} u_{ks}) \qquad (4.12)$$

Positions Required (per county, per shift):

$$X'_{os} = M_k c_s u_k / h_k + M_l c_s / h_l + t_{enf} Y_{ks} u_{ks} \qquad (4.13)$$

E. Assist Rural Law Enforcement (per county, per shift):

$$D'_r = p_r D'_e \qquad (4.14)$$

Persons Per Rural Patrol (initial computation)

$$h_r = 1000 \sum_i R_i / (D'_r + \sum_i L_i/3) \qquad (4.15)$$

Positions Required

$$X'_{rs} = 1000 R c_s / h_r - L / 3 \qquad (4.16)$$

such that

$$r_{min} \leq X'_{rs} \leq r_{max} \qquad (4.17)$$

and the sum of the positions satisfies

$$X'_{rs} = D'_r \pm 0.01 D'_r \qquad (4.18)$$

SUMMARY OF COMPUTATIONS (continued)

Adjustment For Patrolling Time

Persons Per County

$$D'_{ei} = s \sum_{s} (X'_{fs} + X'_{os} + X'_{rs})_i \qquad (4.19)$$

Adjustment Factor

$$f_d = D / \sum_{i} (\sum_{s} C_{si} + D'_{ei}) \qquad (4.20)$$

$$u = f_d (\sum_{s} C_{si} + D'_{ei}) / D'_{ei} \qquad (4.21)$$

Persons Per Shift

$$X_{fs} = s\, u\, X'_{fs} \qquad (4.22)$$

(repeat for X_{os} and X_{rs})

COMPUTING STRENGTH GIVEN MILEAGE PARAMETERS

1. Use equation 2.3 instead of 2.2 at the end of the computations and solve for T.

2. Substitute h_j, h_k, h_i, and h_r as parameters which are greater than zero and skip equations (4.8), (4.12), and (4.15).

END