A REVIEW OF FINGERPRINT INDIVIDUALITY MODELS

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INTRODUCTION.

Once the value of fingerprints for personal identification was recognized, the degree of individuality present in a fingerprint pattern naturally became of interest. Attempts to provide a probabilistic estimate of fingerprint individuality began with Galton's investingations in 1892, and continue to the present day. There have been seven distinct appoaches: Galton (1892), Henry/Balthazard (1900/1911), Roxburgh (1933), Amy (1946), Trauring (1963), Kingston (1964), and Osterburg et al. (1977). Minor modifications of the Henry/Balthazard approach have been made by Bose (1917), by Wentworth & Wilder (1918), and by Cummins & Midlo (1943). Osterburg's method has been extended by Sclove (1979, 1980).

Reviews and critiscms of the above approaches have been few, and none have been comprehensive. Wentworth & Wilder (1918) briefly discussed the methods of Balthazard and Galton. Roxburgh (1933a) and Pearson (1930) have reviewed Galton's model in more detail, and Amy (1946, 1947, 1948) has reviewed Balthazard's model. Kingston (1964, 1965) reviewed the methods of Galton, Balthazard, Wentworth & Wilder, and Cummins & Midlo, but erroneously claimed these to be the only important previous attempts to quantify fingerprint individuality. Roxburgh, Amy, and Trauring each have distinct and important contributions to the problem. Osterburg (1977) critiqued Kingston's method and compared Kingston's results to his own.

The lack of a comprehensive review of the existing

fingerprint probability models justifies their detailed consideration.

GALTON'S MODEL (1892).

Galton made the first attempt to quantify fingerprint individuality. His basic idea was to divide a fingerprint into small regions, such that the ridge detail within each region could be treated as an independent variable.

Galton worked with photographic enlargements of fingerprints. The enlargements were placed on the floor, and paper squares of various sizes were allowed to fall randomly on the enlarged fingerprint. Galton then attempted to reconstruct the ridge detail which was masked by the paper squares, given the surrounding ridge flow. Based on his experiance, Galton chose reconstructions with a natural appearance, such as would be expected to occur given these surrounding ridges. He sought the size of square region where he could successfully predict the actual ridge detail with a frequency of 1/2. Galton found that for a square region "six ridge-intervals" on a side he was able to predict the hidden detail correctly with a frequency of 1/3. He concluded that a square region with five ridge-intervals on a side was very nearly the size he was seeking.

To ensure that any errors would overestimate the chance of fingerprint duplication, Galton used a 6 ridge-interval square region, and assumed a probability of 1/2 for finding the existing minutia configuration, given the surrounding ridges. A complete fingerprint was estimated to consist of 24 such square regions. Assuming independence among these regions, Galton calculated the probability of a specific fingerprint configuration, given the surrounding ridges, as:

$$\left(\frac{1}{2}\right)^{24} \text{ or } 5.96 \times 10^{-8}$$

Next Galton estimated the chance that a particular configuration of surrounding ridges would occur. Two factors were considered: the occurrence of the general fingerprint pattern type, and the occurrence of the correct number of ridges entering and exiting each of the twenty-four regions. Galton estimated the probability for coincidence of pattern type (b) as 1/16, and the probability that the correct number of ridges would enter and exit each region (c) as 1/256. (The latter estimate was largely arbitrary, and both were presented as grossly over-estimating the true probabilities.)

Combining the frequencies of finding the necessary ridge pattern outside the six ridge-interval regions with the frequencies of finding all necessary ridge detail within the regions, Galton predicted the frequency of finding any given fingerprint as:

$$\left(\frac{1}{16}\right)\left(\frac{1}{256}\right)\left(\frac{1}{2}\right)^{24} \text{ or } 1.45 \times 10^{-11}$$

Galton concluded that since the total number of human fingers is about 16 billion, the odds of finding a person anywhere in the world with a pattern identical to a given fingerprint would be approximately 1/4.

Discussion of Galton's Model.

Galton's model has been criticised by Pearson (1930), by Roxburgh (1933), and by Kingston (1964, 1965). Most of this criticism has focused on Galton's basic assumption that, given the surrounding ridges, there is probability of 1/2 for the occurrence of any particular ridge configuration in one of his six ridge-interval regions.

Pearson considered this assumpton "drastic" and suggested an alternative approach for determining the probability of a particular configuraton. Assuming that the position of a minutia may be resolved to within one square ridge-interval, there would be 36 possible minutia locations within one of Galton's regions. An assumption of one minutia in each of 24 independent regions gives a probability of

$$\left(\frac{1}{36}\right)^{24} \text{ or } 4.45 \times 10^{-38}$$

for a particular configuration of minutiae, given the surrounding ridges. If we combine this figure with Galton's factors "b" and "c", the probability of a particular fingerprint becomes:

$$\left(\frac{1}{16}\right)\left(\frac{1}{256}\right)\left(\frac{1}{36}\right)^{24} \text{ or } 1.09 \times 10^{-41}$$

Pearson noted that the actual figure would be smaller for two reasons. First, because minutiae are not uniformly restricted to one in each Galton region, and secondly because of variability in minutia type.

Roxburgh's criticism is more fundamental. He notes that

Galton has investigated only variation within single fingerprints, whereas his conclusions concern variation among entire fingerprints. This is a basic confusion of "within-group" and "between-group" variation. Roxburgh presents a series of illustrations showing that these two levels of variation need have no relationship with one another. Roxburgh concedes that Galton has calculated the probability that, given any particular print, he can reconstruct it wholly in square regions, six ridge-intervals on a side. The probability of 1/2 for a correct guess, however, is determined by the size of the region relative to the ridge characteristics, rather than by the variation or distribution of the characteristics themselves. If a one ridge-interval square region were used we could always guess correctly. One could reconstruct any particular print, given the ridges surrounding the squares, yet one could say nothing about variation between fingerprints.

Roxburgh points out that Gallon's analysis proceeds as if he had surveyed a number of fingerprints, comparing square regions in corresponding positions within the prints. If Galton had done this, Roxburgh would agree with the analysis. The actual experiments, however, were quite different, and as a result Roxburgh dismisses Galton's model.

Kingston makes somewhat the same point, noting that Galton's ability to guess the content of a square region is not an indication of the variation in actual fingerprint patterns. If Galton had shown that, given the surrounding ridges, his region could contain only two configuratons, Kingston would see some merit to his calculations. Seeing no evidence to support this contention, Kingston rejects Galton's model.

The above criticisms are only partially valid. Galton intended his factors of "b" and "c" to summarize much of the variation among fingerprints. His factor "b" accounted for variation in general pattern type, and his factor "c" accounted for variation in the number of ridges entering and leaving each square region. Clearly the values of "c" would change radically were the size of the region to vary. In particular, for the limiting cases where the ability to guess the content of the region approached certainty, the factor "c" would become very small. Unfortunately, Galton did not consider these factors in any detail. Instead he chose arbitrary and excessively large estimates for both factors.

If we accept the concept of Galton's factors "b" and "c", the question becomes whether or not Galton's experiments reasonably approximate a survey of corresponding regions in different fingerprints. It is clear that Galton had this in mind when he wrote (page 107):

> "When the reconstructed squares were wrong, they had none the less a natural appearance.... Being so familiar with the run of these ridges in fingerprints, I can speak with confidence on this. My assumption is that any one of these reconstructions represents lineations that might have occurred in Nature, in association with the conditions outside the square, just as well as the lineations of the actual print."

Galton makes a further assumption (page 108):

"... when the surrounding conditions alone are taken

> into account, the ridges within their limits may either run in the observed way or in a different way, the chance of these two contrasted events being taken (for safety's sake) as approximately equal."

The weakness of Galton's model lies in the magnitude of this approximation and in the arbitrary value chosen for "c". We may justly criticize his final figure as a gross underestimate of fingerprint variability. Pearson's calculations of the variability in one of Galton's regions may be closer to the mark, but both his hypothesis and Galton's remain untested.

THE HENRY/BALTHAZARD MODELS (1900-1943).

The Henry/Balthazard approach is used in five closely related, simple models for fingerprint individuality. Each employs a fixed probability "P" for the occurrence of one minutia. Assuming independence of these occurrences, the probability of a particular configuration of N minutiae is given by:

$$(P)^N$$

Henry (1900) was the first to use this approach, and Balthazard (1911) made the most extensive analysis. Minor variations are encountered in the works of Bose (1917), Wentworth & Wilder (1918), and Cummins & Midlo (1943).

Henry.

Henry (1900, pp. 57-58) chose an arbitrary probability of 1/4 for the occurrence of each minutia, as well as for the general pattern type, and the core-to-delta ridge count. Using his method one would count the number of minutiae, and if the pattern type were visible, add two minutia-equivalents. This value would be used as N, with 1/4 as P.

Balthazard.

Balthazard's method (1911) is important particularly because it is the historical basis for widely accepted rules regarding fingerprint individuality. While noting that other minutiae types exist, Balthazard assumed that for each minutia there were four possible events:

1) fork directed to the right
2) fork directed to the left
3) ending ridge directed to the right
4) ending ridge directed to the left

Assigning equal probability to each of these events, Balthazard took P as 1/4, and N as the number of minutiae. He concluded that to observe N coincidentally corresponding minutiae it would be necessary to examine 4^N fingerprints.

Balthazard went on to calculate the number of minutiae needed for conclusive identification. Using an estimate of the world population of 1.5 billion there would be 15 billion fingers. According to his model, seventeen corresponding minutiae would be found with a frequency of only one in 17 billion. Balthazard concluded that 17 minutiae should be used to avoid error when the world population was considered. A lesser number of corresponding minutiae, such as 11 or 12, was considered to be sufficient if one could be certain that the fingerprint donor was restricted to a particular geographical area (e.g. North America, California).

Bose.

Bose (1917) assumed the same value of 1/4 for P, but arrived at this value using a different rationale. He reasoned that there were at least four possiblities at each square ridge-interval location in a fingerprint:

1) an island
2) a fork
3) an ending ridge
4) a continuous ridge

Wentworth & Wilder.

Wentworth & Wilder (1918) felt that Balthazard's value of 1/4 for P was absurdly high, and proposed without justification a value of 1/50.

Cummins & Midlo.

Cummins & Midlo (1943) adopted the value of 1/50 suggested by Wentworth & Wilder, but they also introduced a "pattern factor" similar to Galton's. For the most common fingerprint pattern they used a probability of 1/31. This estimate was for an ulnar loop, and included the core-to-delta ridge count. Cummins & Midlo's calculation for N corresponding minutiae and a corresponding pattern was therefor:

$$\left(\frac{1}{31}\right)\left(\frac{1}{50}\right)^N$$

Discussion of the Henry/Balthazard Models.

The Henry/Balthazard models have been justly criticised as arbitrary over-simplifications. Henry's method is purely arbitrary, as is Wentworth & Wilder's. Balthazard's choice of P was based on the number of possible minutia events. He has been criticised for allowing only four possible events (Wentworth & Wilder, p. 321; Kingston, 1965, p. 67), and for failing to included a "pattern factor" (Amy, 1948, p. 96). Amy's experiments (1946) have shown that Balthazard's events are not equally probable.

Bose's model does not consider the possible events for each minutia, but rather possible events at each ridge-interval location. Thus one of the allowed events is "a continuous ridge", i.e., no minutia at all. Bose's assumption of equal probability for his four events is grossly in error, as pointed out by Roxburgh (1933, p. 62). A continuous ridge is by far the most common event, and islands are much less common than either forks or ending ridges.

In spite of the simplicity of the Henry/Balthazard models, they may be useful as a measure of fingerprint individuality. Roxburgh has criticized the value of 1/4 for P because this grossly underestimates the individuality of fingerprints. Wentworth & Wilder's value of 1/50 could be closer to reality. The primary weakness of these models is the absence of experimental verification. There may be some empirically chosen value of P, however for which the model is adequate. This possibility remains unexplored.

Most of the remaining models partition fingerprint indvduality into three categories: variation in overall ridge pattern, variation in minutia location, and variation in minutia type. With the exception of Cummins & Midlo's "pattern factor", the Henry/Balthazard models assign variation soley to the minutia type. Variation due to minutia location is not explicitly considered. Balthazard's two possible minutia types, with two possible orientations, will be seen incorporated into the models of Roxburgh, Amy, and Trauring. Bose's concept of minutia locations will also be

seen in Osterburg et al.'s model. Note that Bose ignores minutia orientation, allows a wider variety of minutia types, and includes "continuous ridges" as one of these types.

ROXBURGH'S MODEL (1933).

Roxburgh based his model on a polar coordinate system. Concentric circles spaced one ridge-interval apart represent the fingerprint pattern. An axis is drawn extending upward from the origin, intersecting the concentric "ridges." From this initial position, the axis is rotated clockwise about the origin. As the angle from the initial position increases, minutiae are encountered. For each minutia, the ridge count from the origin is noted, along with the type of minutia. The full rotation of the axis about the concentric pattern results in an ordered list of the minutia types together with their ridge counts from the origin. (Whereas the angle of rotation itself might be used for positioning, Roxburgh elected to simply order the minutiae. This avoids defining angular resolution along each of the concentric circles.) After defining this system of minutia coding, Roxburgh calculated the total variability which could occur. This represents the upper bound for variation in his model.

Calculation of Total Variability.

Under the model, alternative ridge counts are equally likely, as are each of the minutia types. Assuming n minutiae, M_L concentric ridges, and M_t minutia types, the number of possible fingerprint varieties is:

$$(M_L \times M_t)^n$$

An additional pattern factor "P" is included to estimate the probability of encountering the particular fingerprint pattern and core type. Thus the final formula is:

$$P \times (M_L \times M_t)^n$$

To estimate the total variability in a fingerprint Roxburgh takes P = 1000, n = 35, M_L = 10, and M_t = 4. The allowable minutia types are identical to Balthazard's: a minutia may be either a fork or an ending ridge and may be oriented in one of two (opposite) directions. Substitution of these values results in a prediction of 1.18 X 10^{59} possible fingerprint types as an upper bound for this model.

Correction for Correlation.

Roxburgh next considered the question of correlation of successive ridge counts and successive minutia types. Using a series of 271 fingerprints he recorded the first four minutiae in each print. The data were classified according to the sequence of ridge numbers and sequence of minutia types. Without statistical analysis, Roxburgh noted that there appeared to be roughly even distribution with respect to the ridge number sequences, and also with respect to types of minutiae on each particular ridge. There was, however, an excess of minutiae which cause production of ridges, compared to minutiae which cause loss of ridges. Roxburgh attributed this to the clockwise rotation of his axis, and a tendency for ridges to diverge as one proceeds from the vertical. The largest group of fingerprints showing the same sequence of minutia types had eight members. Roxburgh therefore estimated that the value (271/8) could be used as a conservative estimate for the variability of four minutiae with respect to type, and that for one minutia the value would the fourth root of this (2.412). This corrected value of M_t was proposed to adjust for the observed correlation.

Correction for Connective Ambiguity.

Up to this point only ideal fingerprints had been considered. Roxburgh made a further modification to allow for poorly defined or poorly recorded prints. Due to variation in recording of fingerprints, a true fork may appear as an ending ridge, either above or below the ridge bearing the fork. Similarly, a true ending ridge may appear as a fork, joining either the ridge above or the ridge below. Apart from recording difficulties, the nature of some minutiae is uncertain on the skin surface. The term "connective ambiguity" is useful to describe the general phenomenon where one is uncertain of the minutia type. In the extreme, connective ambiguity allows two additional configurations for each minutia. There is not only opportunity for change in minutia type, but a potential change in ridge count as well. Depending on the quality of the fingerprints being compared, one may select a factor "Q" varying from 1, in an ideal print, to 3 in a print where complete connective ambiguity must be allowed. Roxburgh estimates Q as 3/2 for a good average

print, 2 for a poor average print, and 3 for a poor print. The factor Q for connective ambiguity decreases the number of distinguishable fingerprint configurations by a factor of $(1/Q)^n$.

Correction for Uncertainty in Positioning of the Whole Configuration.

Roxburgh made one additional correction for circumstances where the fingerprint pattern is insufficiently clear to allow proper determination of the ridge count from the core. The relative positions of the minutiae are not affected, but there is some uncertainty about the position of the whole configuration relative to the core. The factor "C" is the number of possible positionings for the configuration, and the number of distinguishable patterns (P) must be divided by C. In the extreme, where the pattern is not at all apparent, the factor P must be dropped altogether.

Result.

Roxburgh's final formula for calculating the chance for duplication of a given configuration is one chance in:

$$\left(\frac{P}{C}\right) \times \left(\frac{M_L \times M_t}{Q}\right)^n$$

where n = number of minutiae; M_t = number of minutia types possible; M_L = number of possible ridge counts from the core; P = number of distinguishable pattern types; C = number of possible positionings of the configuration within the pattern; and Q = quality factor for the print. For a good average fingerprint showing the pattern type and 35 minutiae, the values for the variables are = 2.412, = 10, n = 35; P = 1000, C = 1, and Q = 3/2. This gives an estimated chance of duplication of

$$1 \text{ in } 1.67 \times 10^{45} \text{ or } 5.98 \times 10^{-46}$$

For any specific case circumstances, Roxburgh recommends estimating the number of individuals with access to the fingerprint location (be it the entire population of a country, city, or whatever). The chance for a duplication of a particular configuration of minutiae in this population may then be considered, and the number of minutiae needed for the desired confidence level may be determined. Roxburgh suggests a chance of duplication of 1 in 50,000 as an appropriate confidence level, and presents a table with the number of corresponding minutiae needed for various populations and fingerprint qualities.

Discussion of Roxburgh's Model.

Roxburgh's model is both novel and conceptually advanced. There are a number of noteworthy aspects which warrent discussion:

- use of the polor coordinate system to assess the individuality of minutia position;
- adjustment for correlation among neighboring minutiae;
- adjustment for fingerprint quality because of connective ambiguity;
- consideration of uncertainty in the position of the entire minutia configuration, relative to the pattern core.

Roxburgh introduced most of these concepts for the first time, and repeatedly drew upon his experimental observations. His work must be considered revolutionary in these respects. It is remarkable that Roxburgh's model has escaped the attention of all subsequent investigators. No review or citation of Roxburgh's work has been found.

Before Roxburgh, the contribution of minutia position to individuality had only been considered briefly by Pearson in his discussion of Galton's work. Pearson had proposed simply that each square ridge-interval was a distinguishable minutia position. Roxburgh defined minutia position using a polar coordinate system, with "ridge count" for the radial measure, and simple ordering of minutiae with increasing angular measure. Polar coordinates are a natural choice for whorl patterns with radial symmetry, and for fingertips, where ridges are semi-circular and nearly concentric. The model is not directly applicable where ridges form loops, triradii, or patternless, parallel ridges. Broader application results if the origin is allowed to move along a reference ridge. An axis may thus sweep up one side of a loop and down the other, or across a series of parallel ridges:

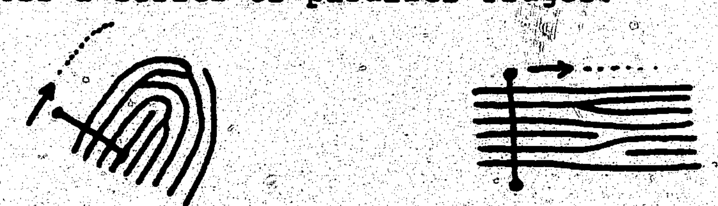

Figure 1.

Roxburgh briefly considered a second model, similar to Pearson's, which used rectangular coordinates to define minutia positon. Each minutia was assumed to occupy 2.5 square ridge-units, and minutia density was estimated as one per 25 units. Assuming minutiae are evenly distributed, this allows for ten possible positions per minutia. If each position is equally likely, then the probability for occupancy for a given minutia position is estimated at 1/10. For more accuracy, Roxburgh suggested that resolution of minutia positions be treated differently along ridges than across them. Across ridges we may easily distinguish a one ridge-unit interval, whereas along ridges Roxburgh suggested an average resolution of 3.5 ridge intervals. Here Roxburgh points out the convenience of his polor-coordinate model, where the question of resolution need not be considered.

Although Roxburgh's model avoids this definition, resolution is clearly a fundamental aspect of fingerprint individuality. Computational excercises designed to demonstrate individuality might justifiably omit consideration of resolution, but such omission is a major defect in a fingerprint model. Roxburgh argues that, in practice, it is relative distances between minutiae which we compare, and that our criteria for correspondence among minutia positions varies with the distance between them. Minutiae on neighboring ridges will show comparatively less variation in relative position than will minutiae separated by several ridges. These observations help characterize the complexity

of the problem, but do not diminish its fundamental importance. By side-stepping the issue of resolution Roxburgh weakens his model.

The imperfections we note in the definition of minutia position are overshadowed by the zeal with which Roxburgh refines his model. He first considers correlation of minutiae. Even though he "eyeballs" the lack of correlation among successive ridge counts and among minutia types, his observations have an exprimental basis and are distinguished as the first (and nearly only) consideration of correlation among minutiae. Roxburgh does find a correlation among minutia orientations, attributable to the generally observed divergece of ridges at the fingertips. A somewhat crude over-correction is made for this correlation, as Roxburgh assumes it to be the maximum he observes.

Next Roxburgh considers the effect of print quality on connective ambiguity. Galton discussed connective ambiguity (1892, p. 91-92) and undoubtedly made allowances for it when he judged his ability to guess ridge structures. Roxburgh, however, was the first to make specific allowance for connective ambiguity, and to link the allowance to print quality. Print quality is very important in defining how much connective ambiguity is allowable. Even in excellent prints an occasional minutia will exhibit variability in recording. The presence of more than a few would warrant suspicion of non-identity. In very poorly recorded prints, however, one must allow this variation in virtually all minutiae. In this extreme we know only that a new ridge appears in a given location. The three possible minutiae which could produce the ridge account for Roxburgh's correction factor Q = 3.

Roxburgh's last refinement of his model is an assessment of the uncertainty of the position of an entire minutia configuraton within the overall pattern. Roxburgh observed that when one does not have a clearly defined reference point, such as a pattern core, one may make several positionings in an attempt to find a corresponding minutia configuration. The absence of a reference point thus increases the possibility of chance correspondence by a factor equal to the number of possible positionings. With hindsight this point is obvious and amounts simply to an observation that there are several opportunities for a particular event to occur. Of the remaining fingerprint models, only Amy's and Osterburg's incorporate this important feature.

AMY'S MODEL (1946-48).

Amy defined two general contributions to fingerprint individuality: variability in minutia type (facteur d'alternance), and variability in the number and positioning of minutiae (facteur topologique).

Variability in Minutia Type.

Amy assumed the same possible minutia types as did both Balthazard and Roxburgh: minutiae can be either forks or ending ridges, and can have one of two (opposite) orientations. Using a database of 100 fingerprints, Amy determined that the relative frequencies of forks and ending ridges were 0.40 and 0.60, respectively. He also noted that divergence or convergence of ridges was very common, and that when this occurs there is an excess of minutiae with one orientation. Amy estimated a frequency of 0.75 for minutiae with one orientation and a frequency of 0.25 for minutiae with the opposite orientation.

If one has "f_1" forks and "e_1" ending ridges in one direction, and "f_2" forks and "e_2" ending ridges in the other, Amy calculates the probability of a particular ordering (A_1) as:

$$A_1 = [(.75)(.40)]^{f_1}\,[(.25)(.40)]^{f_2}\,[(.75)(.60)]^{e_1}\,[(.25)(.60)]^{e_2}$$

which reduces to:

$$A_1 = (.3)^{f_1}(.1)^{f_2}(.45)^{e_1}(.15)^{e_2}$$

In the general case we do not know the absolute orientation of the minutia configuration. Therefore Amy also considered the probability of the ordering of where the relative orientation is opposite (A_2):

$$A_2 = (3)^{f_2}(.1)^{f_1}(.45)^{e_2}(.15)^{e_1}$$

The complete probability is given by:

$$A = A_1 + A_2 = (.1)^{f_1+f_2}(.15)^{e_1+e_2}(3^{f_1+e_1} + 3^{f_2+e_2})$$

Variation in Number and Position of Minutiae.

Amy considerd a square ridge patch, n ridge-interval units on a side. Let L be the probability that there will be p minutiae in a patch of area n^2. Let N be the total number of arrangements of the p minutiae, and let N_t be the number of these arrangements which are indistinguishable from the particular arrangement of minutiae at issue. The probability that we will have p minutiae forming a design of type t in a patch of area n^2 is given by:

$$T = \frac{L_n N_t}{N}$$

The patch size (n) is variable because the boarders of the patch are not precisely defined. We must therefore sum over the possible values of n:

$$T = \frac{\Sigma(L_n N_t / N)}{\Sigma L_n}$$

Assuming a minimum distance between two minutiae of one ridge interval, we have n^2 positions in which to place p minutiae. Using an estimate of average minutia density of one minutia per 22.5 square ridge units (one per square 4.7 ridge units on a side), the binomial theorem yeilds values for L_n and N:

$$L_n = \frac{n^2!}{p!(n^2-p)!}(.04444)^p(.95555)^{n^2-p}$$

$$N = \frac{n^2!}{p!(n^2-p)!}$$

Making these replacements in the equation for T we have:

$$T = p! \frac{\Sigma(.95555)^{n^2}(N_t)}{\Sigma(n^2!/(n^2-p)!)(.95555)^{n^2}}$$

It remains to calculate N_t, the number of the total possible minutia arrangements which are of the particular type "t" (i.e. indistinguishable from one given arrangement). Amy notes that relative, rather than absolute, positioning is of concern, and proposes that the event necessary for positional identity between two fingerprints is only that the same number of minutiae appear on corresponding ridges of the fingerprints. This means that variation due to absolute positioning of minutiae along the ridge is disregarded. One minutia on a ridge has n possible positions, two minutiae have [n(n - 1)/2] possible positions, three minutiae have [n(n - 1)(n - 2)/3!] possible positions, etc.

A second type of variation to be disregarded arises from ridges without minutiae at the fingerprint boarder. If there are q such featureless ridges at the upper boarder, then arrangements with (q - 1) featureless ridges at the upper boarder and one featureless ridge at the bottom boarder would be indistinguishable. Generally, for q featureless ridges at the boarders there would be (q + 1) possible arrangements of these ridges, each resulting in an indistinguishable fingerprint pattern.

Based on the above, N (the number of indistinguishable minutia arrangements of type t) may be calculated given n, p, and the number of ridges with 0, 1, 2, etc. minutiae. If there is only one minutia per ridge and no interior lines without ridges, then q = (n - p), and:

$$N_t = n^p (n-p+1)$$

If there are now z internal ridges without minutiae the formula becomes:

$$N_t = n^p (n-p+1-z)$$

with two minutiae on one ridge, and one on each of the others:

$$N_t = \frac{n(n-1)}{2}\, n^{(p-2)} (n-p+2-z)$$

with two ridges with two minutiae, and one on each of the others:

$$N_t = \frac{n^2(n-1)^2}{4}\, n^{(p-4)} (n-p+3-z)$$

and the formula generalizes easily.

Correction for Minutia Clusters on One Ridge.

Amy noted that the forgoing theory failed when clusters of minutiae appeared on one ridge. A problem of definition results: when does one ridge become two ridges? Amy defines "groups" as an interconnected cluster of minutiae which is treated as if it were a single ridge. Within one of these groups not all relative positions of minutiae may be possible, and some new positions may be created. Consider two ridge endings. We predict 4 possible arrangements (each ending has two possible orientations). However, if two ridge endings appear on the same ridge, they must point in opposite directions; if they point in the same direction then there are two ridges:

one ridge:

two ridges:

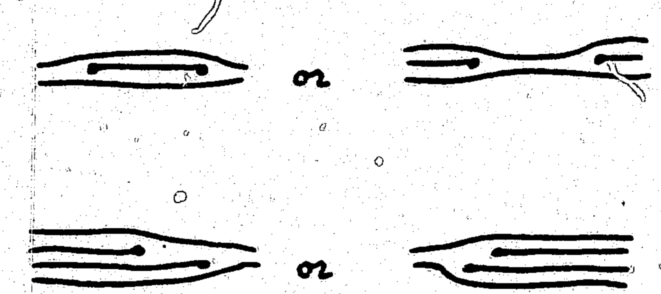

Figure 2.

Where patterns of multiple forks appear, there is not a loss of possible orientations, but an increase. In essence, multiple forks create a compound ridge wherein there is greater potential variation. Consider two forks. We predict four possible arrangements as with the ridge endings. In fact there are eight:

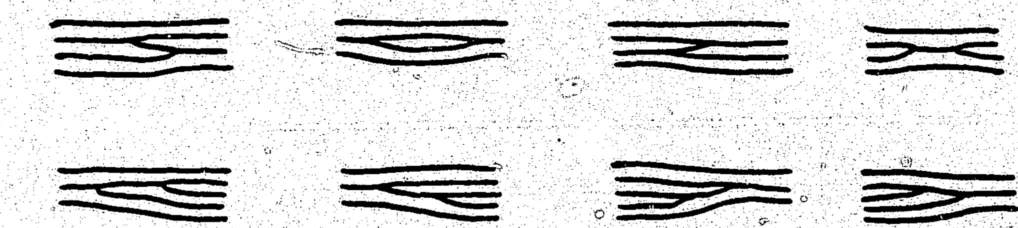

Figure 3.

Amy introduced a factor "G" to correct for these failings in the model. G is the ratio of the possibilities predicted by the model to the actually observed possibilities. Amy calculated G for clusters of two to six minutiae.

Frequency of Occurrence of a Minutia Configuration.

Combining the factors A, T, and G, the frequency of a particular minutia combination is given by:

$$\varphi = (A)(T)(G)$$

Chances of False Association.

Amy noted that the chance of a false association depends on the number of comparisons one makes. Thus if the frequency of a particular configuration is φ, the chance that a given area is not of this configuration is $[1 - \varphi]$, and for ρ comparisons the chance of association by random is given by:

$$\pi = 1 - (1-\varphi)^{\rho}$$

The Taylor expansion for $(1-\varphi)^{\rho}$ yields:

$$\pi = 1 - \left[1 - \varphi\rho + \frac{\varphi(\varphi+1)}{2!}\rho^2 + \ldots\right]$$

which for small φ and small $\varphi\rho$ reduces to:

$$\pi = \varphi\rho$$

The number of comparisons (ρ) may be calculated as follows. Consider a print left at a crime scene which fills a square region of area n^2, n ridge-intervals on a side. This print is to be compared to an area of ridges of size N^2, N ridge-intervals on a side. The number of horizontal positions which the smaller crime scene print may occupy in the larger print is given by:

$$(N-n+1)$$

There are an equal number of vertical positions, one for each ridge. Therefor the total number of positions for comparison is:

$$(N-n+1)^2$$

The values of N^2 (in square ridge-interval units) are estimated as 90 for each thumb, 70 for each of the other fingers, and 1000 for palms. The total number of positions for one person is therefore:

$$r = 2(\sqrt{90}-n+1)^2 + 8(\sqrt{70}-n+1)^2 + 2(\sqrt{1000}-n+1)^2$$

which reduces to:

$$r = 12n^2 - 256n + 1986$$

When the palms are not examined the value drops to:

$$r = 10n^2 - 192n + 923$$

We have thus far considered patternless fingerprint traces and patternless ridges on the hands. Where loops, whorls and triradii exist, the number of positions for comparison is less, regardless of whether the fingerprint trace contains these points. If the fingerprint trace is patternless, then for each pattern singularity n positions are excluded. Letting s equal the number of pattern singularities, we have:

$$\rho = r - n^2 s$$

Result.

Amy's final equation for the chance of a random association of a particular fingerprint trace is given by combining the factors derived above:

$$\Pi = 1 - (1 - AT\sigma)^{(r - n^2 s)}$$

Amy concludes by calculating the chances of random association for a series of examples with different numbers of minutiae and different group arrangements. In a subsequent paper (1947), Amy presents tables which considerably simplify the calculation of an upper bound for the chance of random association. In a final paper (1948), Amy calculates the number of minutiae needed to limit the chance of random association to one in a billion. He also notes that when fingerprint files are searched as a means for developing a

suspect, the factor ρ increases, and the criteria for identification becomes more stringent.

Discussion of Amy's Model.

Amy's model, like Roxburgh's, has neither been reviewed nor cited in the English literature. Amy himself was apparently unaware of Roxburgh's work, and possibly even of Galton's: only Balthazard is cited by Amy.

Amy's model is comparable to Roxburgh's in complexity, innovation, and general approach. The two investigators recognize many of the same issues and their responses are understandably closely related. Both models begin by dividing fingerprint individuality into two parts: variability of minutia type, and variability of minutia position.

Amy's consideration of minutia type is more sophisticated than Roxburgh's in two respects. First, Amy experimentally determines the relative frequencies of forks and ending ridges, instead of assuming the two types are equally likely. Secondly, Amy makes an estimate of the non-independence of minutiae orientation, based on his observations in 100 fingerprints. The non-independence occurs because ridges converge and diverge as they flow around pattern areas. Roxburgh had observed this, and corrected for it by introducing a factor based on the greatest degree of correlation he had observed. Amy's approach is more realistic, since he incorporates probabilities for the alternative orientations directly into his calculation.

Amy's treatment of minutia positional variation is also more sophisticated that Roxburgh's. Amy treats both the number of minutiae and the area of the fingerprint as variables. He uses the binomial theorem and an estimate of minutia density to calculate both the probability of a given number of minutiae and the probability of any particular positional arrangement. These calculations require definition of the possible minutia positions within a fingerprint. Amy assumed the minimum distance between two minutiae was one ridge interval. The number of possible minutia positions was thus equal to the area in square ridge-intervals. Note that the issue here is not our ability to resolve minutia positions, but rather to determine the number of possible minutia positions. Amy considers the problem of minutia resolution by another, more questionable, process.

When comparing fingerprints we are unable to distinguish among all the possible minutia configurations. Roxburgh recognized this and was content to use a resolution of one ridge interval across ridges, and to merely order the minutiae along ridges. This essentially avoids the issue. Amy's treatment is more complex, but he makes a functionally equivalent approximation. Amy assumes that any positional arrangement which has the same number of minutiae on each ridge will be indistinguishable. This means that, in fact, only minutia ordering along the ridges is considered. This assumption is a serious flaw in Amy's model, even more so than in Roxburgh's. In both models the assumption is unrealistic, since our ability to distinguish minutia

configurations is far greater than to merely note their sequence along a ridge. In Amy's model the approximation is also particularly difficult to apply. Roxburgh only required a ridge count as a radial measure -- the continuity along a particular ridge was of no concern. Amy strives to preserve the concept of individual ridges, while still allowing multiple minutiae on a ridge. Amy must thus introduce the concept of "groups" and define interconnected ridge systems as a single compound ridge. The complexity introduced by Amy's group-correction factor is awkward enough, but more importantly, Amy's model can in no way account for connective ambiguities among ridges. Connective ambiguities prevent the definition of discrete, interconnected ridge systems. Amy totally ignores this issue, and provides no consideration of, or correction for, connective ambiguities. Inasmuch as connective ambiguity is an unavoidable feature of fingerprint comparison (see, e.g., Cowger 1983, p. 174; Battley 1932, p. 9), Amy's model is not a realistic assessment of fingerprint individuality.

This deficiency aside, Amy continues his innovative and sophisticated approach. Next he introduces a correction for featureless boarder ridges. Amy notes that it does not matter whether such ridges appear above or below the central, minutia- bearing ridges: the central ridges alone contain the features which determine distinguishability among minutia configurations. Configurations which possess the same central ridges and the same total number of featureless boarder ridges are thus indistinguishable and contribute to his value of N_c.

Amy concludes his work with a calculation of the chances of false association. Given the size of the ridge configuration, Amy estimates the number of possible positionings for these ridges on a person's hands. The number of positions varies depending on the size of the ridge configuration, on whether both palms and fingers are to be considered, and on the presence of pattern elements in either the fingerprint trace or on the person's hands. The purpose of calculating the possible positionings is to estimate the number of trials one has in which to find an indistinguishable ridge configuration. At each possible positioning one makes a comparison and there is a chance of false association.

Obviously, the more possible positionings for a fingerprint trace on an individual, the greater the chance of false association. This concept was not new. Galton and Balthazard recognized one positioning for each of a person's ten fingers. Roxburgh introduced his factor,"C", the number of possible positionings which a configuration could have relative to the pattern core. Amy, however, extends the concept to include all the fingers and the palms. Furthermore, Amy makes the critical observation that the more people you compare a fingerprint to, the less is the significance of any resulting association. Each person represents a set of trials, and each trial carries with it a chance of false

association. Only Amy treats this issue properly. Others either assume one compares each fingerprint against an entire population, or that there is only one comparison. Amy alone appreciates that the actual number of trials is the relevant quantity.

TRAURING'S MODEL (1963).

Trauring estimated the chances of false association by fingerprints in connection with a proposed automatic identification system. The system is based on prior selection of three reference minutiae on a finger, and the recording of a number of test minutiae. Relative coordinates derived from the reference minutiae are used to describe the positions of the test minutiae. As proposed, the test minutiae appear within the triangular region described by the reference minutiae, and the approximate positions of the reference minutiae on the finger are known.

Trauring makes the following assumptions:

1) minutiae are distributed randomly;
2) there are two minutia types: forks and ending ridges;
3) the two minutia types are equally likely to occur;
4) the two possible orientations of minutiae are equally likely to occur;
5) minutia position and orientation are independent variables; and
6) for repeated registration of one individual's finger the uncertainty in the position of the test minutiae relative to the reference minutiae does not exceed 1.5 ridge-intervals.

Correspondence of a test minutia requires its presence within a circular region of radius 1.5 ridge-intervals (area = 7.07 square ridge-intervals). The chance of a minutia appearing in this region is equal to the minutia density (s)

multiplied by the area. The minutia may be one of two equally likely types, and have one of two equally likely orientations. The probability of a corresponding test minutia, given acceptable reference minutiae is thus:

$$\frac{.707}{4s} = .177s$$

If the chance of acceptable reference minutiae on one finger is "r," then a person would have [10 X r] chances of registering an acceptable set of reference minutiae. If the number of test minutiae is "N," then the chance of random correspondence of a finger from one individual with a previously defined print is:

$$10r\,(.177s)^N$$

Based on a series of twenty prints, Trauring found a maximum value of 0.11 for minutia density. He also estimated that the probability of correspondence of three randomly corresponding reference minutiae could be conservatively taken as 1/100. Substituting these values for s and r, the formula becomes:

$$10\left(\frac{1}{100}\right)\left[(.177)(.11)\right]^N = \frac{(.1944)^N}{10}$$

where N is the number of test minutiae.

Discussion of Trauring's Model.

Trauring's perspective differs from the other investigators; he is not interested in fingerprint individuality as such, but rather in the ability of computerized optics to identify a particular finger. Our purpose here is to evaluate Trauring's model as it applies to fingerprint individuality, a function for which it was not actually proposed. The computerized system works more easily with independent rectangular coordinates than with the actual ridge system. When fingerprints are compared manually, actual distances between minutiae are not compared; instead, ridge counts are made across ridges, and relative distances are compared along ridges. It is understandable that for computerized recording and comparison of fingerprints, the ridge count might be dispensed with, but in so doing one departs from reality. Ridge count is an essential part of the actual identification criteria.

Trauring's model is similar to the Henry/Balthazard models, although better thought out. Trauring's first five assumptions are identical to Balthazard's, and the result fits the Henry/Balthazard format ($p = 0.4641$ for the three reference minutiae and thereafter $p = 0.1944$). Trauring, however, lays a better foundation for his model. His derivation is based on consideration of minutia density, estimates of error in minutia positioning, and the concept of reference and test minutiae. Trauring shares some of the faults of the simple models: he assumes minutia types and orientations to be equally probable, and considers neither connective ambiguity nor correlation among minutiae.

The most important feature of Trauring's model is his concept of reference minutiae. Trauring uses the locations of

three reference minutiae to bring a finger into register. Positions of the remaining "test" minutiae are determined relative to the reference minutiae. In actual fingerprint identification a similar process is followed. A characteristic group of minutiae or a ridge pattern such as a loop or delta is used as a reference point. Comparison with other prints begins by searching for this reference point. If an corresponding reference point is found, the remaining minutiae are used to test the comparison. Ridge count from the reference point, relative lateral position, orientation and minutia type are compared. Each minutia sought is a test of the hypothesis that the prints are from the same individual.

Although no other fingerprint model explicitly distinguishes between reference and test minutiae, the issue has arisen in different forms. Galton, Henry, Cummins & Midlo and Roxburgh used "pattern factors" to estimate the chances of encountering a particular fingerprint pattern type. Pattern cores and delta regions, and even diverging ridges in arch patterns, provide good reference points. Roxburgh also allows for uncertianty in position of minutiae with his factor "C" -- the number of possible positionings of the minutia configurtion relative to the pattern core. Amy considered the number of possible positionings of a fingerprint trace on the whole of the palmar surface. The "reference points" of loops, triradii and whorls eliminate some of the possible positionings, increasing his factor "n".

Apart from these indirect treatments, the concept of test and reference minutiae remains undeveloped in its application to conventional fingerprint comparison.

KINGSTON'S MODEL (1964).

Kingston divides his model for fingerprint individuality into three probability calculations, much like Amy did:

1) the probability of finding the observed number of minutiae in a fingerprint of the observed size,
2) the probability that the particular positions of the minutiae would be observed given the above, and
3) the probability that minutiae of the observed type would occupy the positions.

Probability of the Observed Number of Minutiae.

Kingston estimated the probability of a particular number of minutiae from the minutia density, using a Poisson distribution. (Kingston demonstrated that this distribution approximated experimental observations for the core area of ulnar loops.) Minutia density was measured for the specific type of fingerprint pattern, and the specific location within this pattern. Graphs were constructed of expected minutia number vs. size of the sample region. For a region of given size, the expected minutia number (λ) was read, and the probability of the observed number was calculated:

$$P(N) = e^{-\lambda} \frac{\lambda^N}{N!}$$

Probability of the Observed Positioning of Minutiae.

Kingston assumed that each minutia occupied a square region, 0.286 mm on a side. This size region was chosen from experimental observations of minutia clustering. Within this region other minutiae are excluded. The center positions of other minutiae are thus excluded from a square region which is 0.571 mm on a side.

An uncertainty in measurement of minutia position of 0.286 mm along each axis was also assumed, based on repeated coordinate readings from a single fingerprint.

Consider N minutiae occurring in a region "S" square mm in area. The number of distinguishable minutia locations within the region is: L = (S)/(0.082). One minutia position is used for reference, and located at any position with a probability of unity. Subsequent minutiae are located with equal probability over the remaining unoccupied area. The excluded, occupied area is generally equal to:

$$(i-1) \times (0.571)^2$$

for the "ith" minutia. The probability of the particular set of positionings is therefor calculated as:

$$\prod_{i=1}^{N} \left(\frac{S}{.082} - (i-1)(.571)^2 \right)$$

This value is used except were the minutiae are sufficiently close so that the excluded regions about the minutiae overlap. Where there is overlap, the total area excluded to the next minutia is less than the given value. Presence of the overlap also causes the order in which the minutiae are taken to affect the calculation. Kingston cautions against ignoring the overlap, but does not consider the magnitude of the error

this would cause. Should significant error occur, Kingston recommends taking an average over all possible orderings of the minutiae.

Probability of Minutiae of the Observed Types.

Kingston estimated the frequencies of minutia types using a survey of 2464 minutiae in 100 ulnar loops. The results were:

Ending Ridge	.459
Fork	.341
Dot	.083
Enclosure	.032
Bridge	.019
Tri-radii	.017
Other	.031

The probability of the observed minutia types filling the observed positions is given by:

$$\prod_{i=1}^{N} P_i \left[\frac{S}{.082} - (i-1)(.571)^2 \right]$$

Result.

Combining the three probabilities above yields the overall probability of a given minutia configuration:

$$P = e^{-\lambda} \frac{\lambda^N}{N!} \prod_{i=1}^{N} P_i \left[S/.082 - (i-1)(.571)^2 \right]$$

No additional factor for pattern was included; rather, the pattern singularities are included as minutiae. Tri-radii were explicitly considered as minutiae above, and an arbitrary frequency of 0.25 was assigned to recurving ridges.

Probability of False Association.

Kingston calculated the probability of false association using the Poisson distribution. Suppose there exist K persons with the given minutia configuration. The probability that we have the correct person, given this configuration is (1/K). For small probabilities of occurrence, K takes on a Poisson distribution with parameter $\lambda = np$, where n is the relevant population and p is the probability of the event. The expectation of 1/K is thus:

$$E\left(\frac{1}{K}\right) = \frac{\sum_1^{\infty} \frac{1}{K} \frac{\lambda^K}{K!}}{\sum_1^{\infty} \frac{\lambda^K}{K!}}$$

Substitution for the demoninator yields:

$$E\left(\frac{1}{K}\right) = \frac{1}{e^{\lambda}-1} \sum_1^{\infty} \frac{\lambda^K}{K \cdot K!}$$

Since (1/K) is the probability that the correct person is found, 1 - (1/K) gives the probability that an error has occurred. This value turns out to be very close to [λ /4] for λ in the range of 1 to 10^{-7}.

Discussion of Kingston's Model.

The general similarity between Kingston's model and Amy's has been mentioned previously. Both models first determine the probability of a particular number of minutiae in a

region of given size; secondly, they compute the possible permutations of minutia positions; and thirdly, they consider variation in minutia type. The principle difference between the two models is that Amy describes minutia position within the ridge structure, whereas Kingston uses only the coordinates of minutia positions.

For calculating the probability of a particular number of minutiae, Kingston uses the Poisson Distribution. Amy had used the Binomial Distribution. These two probability distributions are closely related: the Poisson is merely a special case of the Binomial. Both distributions describe the probability of a particular number of statistically independent events, given a number of trials. In our case, events are the occurrence of minutiae and the probability is that a particular number of minutiae will occur in a region of a given size. Two parameters are necessary to describe the Binomial Distribution: the probability of the event, and the number of trials. Thus Amy uses a probability of 0.0444 for a minutia to occur within a unit area and takes the number of trials as total area of the region. When the number of trials is high and the probability of the event is low, the Binomial probabilities are accurately given by the Poisson Distribution. The Poisson Distribution is described using only a single parameter: the expected number of events. Although this expectation is dependent on the number of trials and the probability of the event, these two parameters need not be individually determined. Kingston uses average minutia counts in different sized regions to obtain an empirically derived Poisson parameter.

Kingston used an empiricle approach because he had observed variations in minutia density among different sized regions and among different locations within the fingerprint. Amy had simply assumed a uniform minutia density. Kingston's data established that increased minutia densities occur near deltas and near loop cores. As one proceeds outward from these locations, the density falls off, creating a lower overall density as the size of the region increases. Kingston adjusts for these phenomena by restricting his consideration to circular regions about the core of loops and empirically determining the expected number of minutiae for regions of different size.

Kingston's observations of variation in minutia density are noteworthy, but his method of handling this variation is open to criticism. If we accept that the density of minutiae decreases as we move outward from the core, then the probability of minutia occurrence clearly depends on the distance from the core. We may not assume, therefore, that within a circular region about the core there is a uniform probability of minutia occurrence. Such an assumption is inherent in the use of the Poisson (or Binomial) Distribution, and thus there is an inconsistency in the model. We may infer that for small regions Kingston assumes that the density may be taken as constant. This assumption might well be valid in areas of the fingerprint where density

is reasonably constant. Unfortunately, Kingston has chosen an area where the density varies dramatically. His own data reflects that the density falls off nearly 50% as the radius of his region changes from three ridge-intervals to six.

Kingston makes this same inconsistent assumption when he considers variation in minutia position. His method is to sequentially add minutiae to a region. The probability that each successive minutia will occupy any particular position is determined by the ratio of minutia size to the remaining unoccupied space. No provision is made for the proximity of the minutia to the core, or for any variations in minutia density.

An interesting consequence of Kingston's sequential introduction of minutiae is that clustering of minutiae is slightly disfavored. When minutiae are close together, the spaces excluded to subsequent minutiae may overlap, producing a lower overall excluded area. Arrangements where the overlap occurs will be slightly less probable because subsequent minutiae will have more available space. This feature of Kingston's model is inaccurate: clustering of minutiae is actually more probable, as demonstrated by Sclove (1979).

Further difficulty with Kingston's modeling of minutia position is encountered with his definitions of minutia size and resolution. Kingston assumes that each minutia occupies a square region 0.286 mm on a side. This is equivalent to 0.333 square ridge intervals. Amy had used a full square ridge interval region, thus Kingston allows three times more minutia positions than Amy did. This difference obviously has a profound effect on number of possible minutia arrangements. Kingston's minutia size is unrealistic when evaluated within the actual ridge structure. Minutiae on adjacent ridges can be no closer than one ridge interval. Along a ridge, the question becomes one of definition. When do two minutiae which are very close become one event? Kinston does not describe his criteria for determining minutia type, but he does classify "spurs" and "double bifurcations" as simple bifurcations (forks). "Dots", "enclosures" and "bridges" are given separate categories. By allowing this variety in minutia type, Kingston, in effect, redefines any two minutiae which become close to one another. Two opposing forks are redefined as an enclosure; a fork with a quickly terminating branch is redefined as a spur and included as a simple fork; a very short segment of a ridge is re-defined as a dot. This redefinition of events prevents minutiae from getting closer than one or two ridge intervals from one another.

Kingston also uses an inappropriately small value for minutia resolution. He accepts correspondence only if a minutia is found in an area of 0.082 square mm about the expected position. Trauring used an area of 1.73 squre mm, even after correcting for fingerprint distortion using reference minutiae. Kingston's value was determined by noting the error in repeated measurements on a single fingerprint, in contrast to measurements of different prints from the same

finger. This is a fundamental error. We are not interested in how many distinguishable patterns we may measure, but in how we may distinguish among prints from different fingers. Kingston makes no allowance for the minor printing variations which are present even under ideal conditions.

Kingston's modeling of minutia positional variation, therefore, has three serious flaws: the inconsistent assumption of uniform density, excessivly small minutia size, and excessively high minutia resolution. An additional flaw is Kingston's failure to consider positioning of the minutia configuration as a whole. Since his model is restricted to the core areas of loops, this omission is not serious: the loop pattern allows positioning of the configuration on the finger. Furthermore, Kingston allows arbitrary positioning of one minutia as a starting point.

Kingston's approach to variation in minutia type differs fundamentally from the previous models. Kingston allows a much greater variety of minutiae and assigns probabilities based on their relative frequencies. Orientation of minutiae, however, is not incorporated within the model. It would be difficult to utilize orientation in a meaningful way without reference to the ridge structure. Since Kingston's model is independent of this structure, the omission is not surprising.

When minutia types other than forks and ending ridges are defined, three issues are highlighted. First, one notes that the new minutia types are compound forms of forks, ending ridges, or both. This is necessarily so, for there are only two fundamental operations which may occur to produce a new ridge. Secondly, one notes that there is a continuum between the compound forms and distinct fundamental forms. That is, for example, if we have two opposing forks close to one another they are defined as an "enclosure". As the distance between the forks is increased, one finds a continuum between what is defined as an enclosure and what is defined as two distinct forks. Definition of the compound forms must therefor include a (somewhat arbitrary) judgement of when the compound character is lost. The third issue which one notes is that the frequencies of the compound forms are much lower than those of the fundamental forms, and that there is substantial variation among the frequencies of compound forms.

If one is to use the compound forms, it is appropriate to assign weights based on their frequency of occurrence. This principle has been applied subjectively for some time in fingerprint comparison (see, e.g., Locard 1930, p. 221), but Osterburg's survey (Osterburg, 1964) demonstrated that there was no concensus among fingerprint examiners regarding these frequencies. Amy (1946) assigned variable weights to minutiae, but only considered the fundamental types. Santamaria (1955) was the first to propose specific weighting of compound minutiae. His method was simply to assign a weight equal to the number of fundamental minutiae which were required to produce the compound one. Kingston was the first

to include frequencies of compound minutiae in a model for fingerprint individuality.

Two problems arise from Kingston's use of compound minutiae. The first, as alluded to above, is a problem of definition. When are two fundamental minutiae sufficiently close to form a compound minutia? Kingston does not state his own criteria, but does observe that differences in minutia classification account for variation between his own frequencies and those determined by other investigators. The second problem is that no provision is made for connective ambiguity. Just as no allowance was made for positional variation among prints from one finger, here no allowance is made for type variation. This affects not only the comparison of minutiae, but also the frequencies which are assigned. A connective ambiguity at one end of an enclosure (frequency 0.032) would result in classification as either a spur, which Kingston includes with forks (0.341), or as a combination of a fork and an ending ridge (0.459 X 0.341 = 0.157). Re-classification as two minutiae would also markedly affect the calculations for individuality of both number and position of minutiae.

Kingston concludes his model with a calculation of the chances of false association, assuming a partial fingerprint with a given incidence. We have seen a variety of approaches to this issue. Galton and Balthazard compared the incidence of the fingerprint to the world population, and considered an identification absolute when the expectation within the population was less than one. Roxburgh accepted an identification when the incidence was below 1 in 50,000. Amy took the actual number of comparisons into account. His chance of false association was the probability of occurrence, multiplied by the number of comparisons. Kingston's method is analogous to Galton's and Balthazard's, although his techniques are much more refined.

Using the Poisson Distribution, Kingston calculates the probability that among the world population there would be N individuals with a fingerprint identical to the given one. N must be greater than or equal to one because the existance of the print is known. The Poisson probabilities are multiplied by 1/N which is the chance of randomly selecting any particular one of these individuals. If there is only one individual in the world with identical fingerprints, then the identification is valid; if there are two individuals, the chance is 1/2 that the identification is valid; if three individuals, the chance is 1/3; etc.

Kingston's calculation answers the following question:

> Given a fingerprint from an unknown source, and assuming an individual is selected randomly from among all possible sources of the print, what is the probability that this individual made the fingerprint?

The situation is analogous to having all N persons who can make the print present in a closed room. One of the persons inside is selected randomly, and we ask for the probability

that this person is the actual source of the print. We can but note that the person is one of the possible sources and that the probability is 1/N that we have the correct person. Fingerprint probabilities enter by determining the magnitude of N, the number of persons in the room.

Contrast this situation with one where the individuals within the room are selected randomly with respect to fingerprint type and where we test the individuals to determine if they could have actually made the evidence print. "N" now represents a population of suspects to be tested using the evidence print. If fingerprints of an individual in this suspect group match the evidence print, what is the significance of this finding? This question parallels the practice of fingerprint comparison, whereas Kingston's does not. Kingston has assumed that his suspect has been selected solely on the basis of correspondence with the fingerprint. Rarely would this be the case. Most often identification by a partial fingerprint would be a nearly independent event. The comparison would be used to test a few possible suspects, rather than to define the suspect group. When many suspects are screened using the fingerprint, the chances of false association rise, as Amy has pointed out. Only in the hypothetically absurd extreme, where the entire population of the world is screened, is Kingston's calculation valid.

To answer the appropriate question which we posed above, one must compare the chance of the evidence occurring under two hypotheses:

H1: that the individual in the suspect group made the print.

H2: that another (random) individual made the print.

Under H1 it is certain that the print would match the individual. Under H2 the probability is the frequency of incidence multiplied by the number of attempts we have made to compare the print. A likelihood ratio of these two probabilities gives the relative support of the evidence for the two competing hypotheses (see Evett 1983).

OSTERBURG'S MODEL (1977).

Osterburg, Parthasarathy, Raghavan & Sclove (1977) used a 1mm grid to divide fingerprints into discrete cells. Within each cell one of thirteen events was allowed. These events are listed below with their frequencies of occurrence in 39 fingerprints (8,591 cells).

Event	Frequency	Probability	(Sclove)
Empty Cell	6,584	.766	---
Ending Ridge	715	.0832	.497
Fork	328	.0382	.159
Island	152	.0177	.103
Dot	130	.0151	.102
Broken Ridge	119	.0139	---
Bridge	105	.0122	.0558
Spur	64	.00745	.0350
Enclosure	55	.00640	.0263
Delta	17	.00198	.0135
Double Fork	12	.00140	.00637
Trifurcation	5	.000582	.00279
Multiple Events	305	.0355	---

Under the assumption of cell independence, the probability of a given cell configuration was taken as the product of the probabilities of the individual cell types:

$$\prod_{i=1}^{n} P_i$$

Osterburg et al. noted that according to general practice the weakest identification which is considered absolute is an identification based on twelve ending ridges. Their model gave an estimate of 10^{-20} for this configuration, given an average print area of 72 square mm. They proposed that any fingerprint correspondence with a cell configuration probability this low should be accepted as absolute, regardless of the actual number and type of minutiae.

Correction for Positionings.

When a fingerprint of unknown origin is compared to all ten fingers of a suspect there is a reduction in the probability because of the repeated comparisons. Suppose the unknown partial fingerprint occupies a rectangular region "w" mm wide and "l" mm long. An average full fingerprint measures about 15mm by 20mm. Therefore the number of possible positionings of the unknown partial fingerprint on ten full fingerprints may be calculated as:

$$10(15-w+l)(20-l+1)$$

The random frequency of a pattern must be multiplied by the number of positionings to get the probability of random association.

Chance of False Association.

Proceeding with analysis identical to Kingston's, Osterburg et al. used a Poisson distribution to compute the

probability of false identification. The essential feature of the approach is estimating the number of possible sources for the fingerprint (N), and equating the probability of false association to (1 - 1/N).

Sclove's Modifications.

Sclove (1979, 1980) modified Osterburg's model to account for experimentally observed non-independence of cells and for multiple occurrences within one cell. Sclove found that minutiae tend to cluster. The probability that any one cell is occupied increases regularly with the number of occupied neighboring cells. To model this dependency, Sclove assumed a one-sided Markov-type process. That is, the assumption was made that the probability that a cell is occupied depends only on the outcomes of the four preceeding cells. Thus the occupancy of the cells "X" determines the dependency of the cell "Y" upon its neighbors:

X X X
X Y O
O O O

By weighting the four possible orientations of the "X" cells, estimates were made of the conditional probability of occupancy of "Y," given the number of occupied X cells. For boarder cells, where information regarding the occupancy of adjacent X cells is incomplete, estimates of occupancy of the Y cell may be made using the partial information. For these cases we have a minimum and a maximum number for adjacent cell occupancy.

Sclove also proposed a different treatment for multiple occurrences within one cell. Osterburg et al.'s method included a cell category of "multiple events." Based on a within-cell data analysis, Sclove justified an assumption of a Poisson distribution for the number of characteristics per cell. The mean number of occurrences used in this distribution is, in turn, affected by the number of occupied adjacent cells.

Sclove notes that his method avoids the need to define minutia density variations according to locations within the fingerprint patterns. Local differences in density are accounted for by the dependent probabilities of cell occupancy.

Calculation of the probability of a given cell configuration proceeds as follows. The number of occurrences in each cell is noted, along with the number of adjacent occupied cells. The appropriate conditional mean is selected from a table based on the number of occupied cells and the Poisson probability of the observed number of occurrences within the cell is calculated. This probability is multiplied by the relative frequencies of any occurrences which appear in the cell. These latter frequencies were determined from Osterburg et al.'s data, and are presented in the table above.

Discussion of Osterburg's Model.

Osterburg's model is appealing because it is simple to

apply and is statistically sophisticated. It is particularly useful for the comparison of individuality among different fingerprints. If we define some standard configuration of minutiae, the model provides a means to compare other minutia configurations to the standard. The feature which allows this comparison is simply the weighting of compound minutiae by their frequencies of occurrence. Both Santamaria (1955) and Kingston (1963, 1964) had used this concept, but Osterburg's treatment is far more rigorous and perceptive. He has been the only investigator to consider the errors in minutia frequencies. Santamaria's method amounted to mere suggestion that compound minutiae be weighted according to the number of fundamental minutiae which compose them. Kingston used actual frequencies of occurrence of the compound minutiae, but did not consider errors in these frequencies and did not give his criteria for classification of compound minutiae types. As a result, one does not know when two closely spaced minutiae should be considered as a compound form. Osterburg defines his compound minutiae precisely, and Sclove provides definite treatment for other closely spaced minutiae.

Positioning of minutiae is also treated well for comparing the individuality of different fingerprints. Osterburg defined position using a millimeter grid which divided the fingerprint into discrete cells. Discrete cells allowed extensive treatment of correlaton by Sclove, making the model robust to local variations in density. These variations had been a major problem in Kingston's model.

Positioning within the cells is ignored, but the cells are small, equivalent to about two ridges on a side. Furthermore, Sclove's treatment of multiple events provides flexibility within the cell structure.

Cells which are empty contribute to individuality within Osterburg's model. This is an important issue which has been overlooked in many of the fingerprint models. Bose (1917) was the only other investigator to directly consider the value of featureless ridges. Bose's rudimentry model allowed four equally likely events at each square ridge interval, one of which was a continuous ridge. The model grossly exaggerates the value of a continuous ridge: a single ridge extending for five ridge intervals would be assigned a frequency of less than one in a thousand. It is clear, however, that a patch of ridges without minutiae does possess some individuality. Cummins & Midlo (1943, p. 152) point out that this contribution makes their estimate of fingerprint individuality more conservative. The other Henry/Balthazard models, along with Roxburgh and Trauring, deny this contribution. Kingston and Amy indirectly address the issue. Each includes a separate calculation of the probability of finding the observed number of minutiae, given the area of the fingerprint. Galton allows a factor of 0.5 for a six ridge-interval square region, regardless of content. A featureless region of this size would be assigned a frequency of 0.0908 by Osterburg.

We have been making a distinction between the use of Osterburg's model for comparing the individuality in

different prints, and its use for determining the significance of a fingerprint comparison. This distinction is important. Comparison of individuality among prints amounts to determining the information content of a fingerprint pattern. We are not particularly concerned with the different ways in which the pattern may be expressed, or with the details of the pattern. Precise ridge counts would not be expected to affect the information content very much. Two prints differing only in the placement of one or two minutiae would have nearly the same identificaton value. Connective ambiguities and deformation of the fingerprint affect the calculation of information content to some degree, but the problem is not serious. A typical connective ambiguity for example, would create uncertainty about whether a minutia was a fork or an ending ridge. We might assume the minutia to be one or the other, or perhaps take an average the two frequencies of occurrence. Deformation would affect the relationship of the fingerprint pattern to Osterburg's grid, but without gross distortion these changes will have little effect on the overall calculation: the number of cells containing the various features would remain practically the same. Where events are grouped differently by the deformation, the effect is also small, as demonstrated by Sclove (1980, p. 691-2).

When Osterburg's model is used to evaluate a fingerprint comparison, however, these minor irritations become major weaknesses. The most serious is Osterburg's failure to incorporate the ridge structure into his model. Sclove (1979, p. 594) conceded that a ridge-dependent metric might be more appropriate for the model, but no modification has been proposed. Departure from the ridge structure has been discussed in connection with Trauring's and Kingston's models. A model which does not recognize ridges cannot incorporate the basic features of fingerprint comparison. Relative positions of minutiae are not established by absolute distances: only in a topological sense are these positions constant. It is the ridges which serve as landmarks in fingerprint comparison, establishing relative positions through ridge-count, establishing orientation of minutiae, and correcting for distortions which may be present. Trauring at least recognized that distortions would occur and corrected for them using reference minutiae. Both Kingston and Osterburg ignore this fundamental issue. Osterburg's identification criteria is the occurrence of the same events in corresponding cells as defined by the grid. Should a print be slightly compressed or stretched there could be no such correspondence. If deformation of the grid is allowed, then we admit that not all of the possible configurations of cells are distinguishable, and the foundation of the model is seriously threatened.

Uncertainty in positioning of the grid has a similar effect. Osterburg proposes that the grid first be placed on the fingerprint of unknown origin, and that the comparison proceed by attempting positionings on known fingerprints.

Cell by cell positionings are accounted for in the model as a feature of the comparison process, but minor positionings and rotations are not. On a single print, these minor operations will create multiple descriptions under Osterburg's model. Again, this means that not all of the possible descriptions will represent distinguishable fingerprints.

The presence of a variety of descriptions within the model for a single fingerprint is reminiscent of Amy's N_x; the number of minutia arrangements indistinguishable from the one at issue. A correction of this type might be introduced if the number of possible descriptions for one fingerprint were calculated. The calculation would need to incorporate minor horizontal and vertical positionings, rotational positionings, and allowable deformations of the print. Some of the difficulties could be avoided if the grid were positioned in a definite manner relative to some landmark within the print. This amounts to the use of reference minutiae. The print core, delta regions, or characteristic groups of minutiae might be used. If widely space minutiae were used, deformations could be corrected for using Trauring's technique. The simplicity of Osterburg's model would be lost if the above corrections were introduced, and even so, the fundamental importance of ridge count would remain unrecognized.

Connective ambiguity also poses a serious problem to the use of Osterburg's model to evaluate fingerprint comparisons. For any one fingerprint there will be a variety of minutia configurations which would be identifiable. Variation in minutia type must be allowed during the comparison process. Osterburg et al. join Kingston, Amy, Trauring and the Henry/Balthazard models in failing to provide for this essential feature of fingerprint comparison.

Osterburg completes his model with a discussion of the probabilities of false association. Included is a correction for possible positionings, analogous to Amy's. The chance of false association increases with the number of possible comparison positions. The bulk of Osterburg's argument, however, is identical to Kingston's and suffers the same flaws.

REFERENCES

Amy, Lucien. 1946. Valeur de la preuve en dactyloscopie I. _Journal de la Societe de Statistique de Paris_ _87_: 80-87.

Amy, Lucien. 1947. Valeur de la preuve en dactyloscopie II. _Journal de la Societe de Statistique de Paris_ _88_: 189-95.

Amy, Lucien. 1948. Recherches sur l'identification des traces papillaires. _Annales de Medecine Legale_ _28_(2): 96-101.

Balthazard, Victor. 1911. De l'identification par les empreintes digitales. _Comptes Rendus des Academies des Sciences_ _152_: 1862-64.

Battley, Henry. 1932. _Single Finger Prints_. New Haven: Yale University Press.

Bose, Rai Hem Chandra. 1917. _Finger Prints Companion_. [as cited by Roxburgh 1933, p. 62.]

Cowger, James F. 1983. _Friction Ridge Skin_. New York: Elsevier.

Cummins, Harold & Charles Midlo. 1943. _Finger Prints, Palms and Soles_, chapter 8, pp. 143-55. Philadelphia: Blackiston.

Evett, Ian W. 1983. What is the probability that this blood came from that person? A meaningful question? _Journal of the Forensic Science Society_ _23_: 35-39.

Galton, Francis. 1892. _Finger Prints_. London: McMillan.

Henry, E.R. 1900. _Classification and Uses of Fingerprints_, pp. 54-58. London: Routledge & Sons Ltd.

Kingston, Charles. 1964. Probabilistic analysis of partial fingerprint patterns. Ph.D. dissertation, University of California, Berkeley.

Locard, Edmund. 1931. _Traite de Criminalistique_, volume I. Lyon: Desvigne.

Osterburg, James. 1964. An inquiry into the nature of proof: The identity of fingerprints. _Journal of Forensic Sciences_ _9_: 413-26.

Osterburg, James; T. Parthasarathy, T.E.S. Raghavan and Stanley L. Sclove. 1977. _Journal of the American Statistical Association_ _72_(360): 772-78.

Pearson, Karl. 1930. _The Life and Letters of Francis Galton_, volume IIIA, pp. 182-83. Cambridge: University Press.

Roxburgh, T. 1933. Galton's work on the evidential value of finger prints. _Sankhya: Indian Journal of Statistics_ _1_: 50-62.

Roxburgh, T. 1933a. On the evidential value of finger prints. _Sankhya: Indian Journal of Statistics_ _1_: 189-214.

Santamaria, Florentino. 1955. A new method for evaluating ridge characteristics. _Finger Print and Identification Magazine_ _36_(11): 3-18.

Sclove, Stanley. 1979. The occurrence of fingerprint characteristics as a two-dimensional process. _Journal of the American Statistical Association_ _74_(367): 588-95.

Sclove, Stanley. 1980. The occurrence of fingerprint characteristics as a two-dimensional Poisson process. _Communications in Statistics -- Theoretical Methods_ _A9_(7): 675-95.

Trauring, Mitchell. 1963. Automatic comparison of finger-ridge patterns. _Nature_ _197_(4871): 938-40.

Wentworth, Bert and Harris Wilder. 1918. _Personal Identification_, chapter 9, pp. 312-31. Boston: Richard G. Badger.

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