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HANDBOOK FOR THE ACCIDENT RECONSTRUCTIONIST

M. J. LOFGREN

98667

INSTITUTE OF POLICE TRAFFIC MANAGEMENT
UNIVERSITY OF NORTH FLORIDA

U.S. Department of Justice
National Institute of Justice

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ABOUT THE AUTHOR

Myron J. Lofgren

Myron J. Lofgren was born November 14, 1931. A native of Minnesota, he served 21 years in the Minnesota State Patrol. In 1974, he was named Minnesota Trooper of the Year and in 1979, National Police Officer of the Year.

Now in private practice, Mr. Lofgren is a consultant to various law enforcement agencies in several states as well as to insurance companies and to law firms that require accident reconstruction for use in civil litigation.

Mr. Lofgren also teaches an accident reconstruction course offered several times each year by the Institute of Police Traffic Management at the University of North Florida in Jacksonville. He formerly taught courses in accident investigation for Northwestern University Traffic Institute in Evanston, Illinois. He has conducted seminars in accident investigation and reconstruction for police officers from all over the United States and from other countries and for the Minnesota Supreme Court, the American Bar Association, other bodies of judges and attorneys, law enforcement agencies, insurance companies, Trailways, motor fleet safety organizations, and civic groups.

No classroom theorist, Mr. Lofgren has investigated over 1,000 vehicle accidents. He has conducted, and continues to conduct, tests with vehicles ranging in size from mini-compacts to large tractor-trailor rigs. These tests involve (where appropriate) skids, critical scuffs, airborne trials, vehicle dynamics, conservation of momentum, and weight shift on braking.

Drawing on his store of knowledge and practical experience, Mr. Lofgren has written a wealth of teaching materials, including a syllabus for accident investigation for use by Greyhound. He has authored articles that have appeared in police and traffic safety journals and has recently updated his widely used training text HANDBOOK FOR THE ACCIDENT RECONSTRUCTIONIST, adding in the 1983 edition a chapter dealing in part with the overturning of vehicles on curves.

In this HANDBOOK, Mr. Lofgren underscores--sometimes with a touch of irony--the importance of valid accident reconstruction procedures as a prerequisite for effective traffic and vehicle safety legislation.

Mr. Lofgren is President of the International Association of Accident Reconstruction Specialists.

HANDBOOK FOR
THE ACCIDENT RECONSTRUCTIONIST

By M. J. Lofgren

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To *Colonel James C. Crawford and Lt. Colonel Gerald R. Kittridge*, Chief and Assistant Chief of the Minnesota State Patrol, who made the original decision that a course in accident reconstruction should be taught.

To *All the State Troopers in the field* with whom I have worked on accidents and who have supplied me with exhibits to use in teaching. This includes the Troopers who have told me, "Why the hell don't you go write a book or something!"

To *J. Stannard Baker*, world-renowned Accident Reconstructionist, author, and presently consultant for Northwestern University Traffic Institute, who freely gave his time in sharing with me his vast knowledge and experience.

To the late *Dr. Joseph Kauffmann*, former Director of Research at Northwestern University Traffic Institute, who spent countless hours in conference and correspondence with me. His letters on mathematical procedure and its adaptation to motor vehicles are a source that I will always be able to draw from. His passing left me with an empty sadness, and I will always remember him.

To *Thad Aycock*, Assistant Supervisor, Unit Course Branch, Northwestern University Traffic Institute, who wrote the kitchen recipe chapter on Remedial Math and graciously allowed me to use it in this manual. To this old friend and associate I am eternally grateful.

To *my brother Lyle*, who through the years has spent many nights and holidays trying to teach me how to derive formulas and without whose assistance this project would never have been possible.

To *Diane Lindstrom, Dick Hodge, and Carl Legursky*, who have typed, edited, and illustrated for this manual. Without their splendid cooperation and durability, none of this would have appeared in print.

To *my wife Bonnie*, who has put up with long field trips, insect bites, and cigar smoke and who has patiently listened to and helped solve problems and has dodged traffic on half the roads in this country holding the dangerous end of the tape for me.

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FOREWORD

The history of the automobile is short but bloody. In 83 years the automobile, with the cooperation of the person behind the wheel, has killed more people than all our wars combined. And in just one year, more people can be injured in traffic accidents than have been wounded in all our wars combined. The only instrument as efficient in mass murder and mayhem as the automobile is the hydrogen bomb. Strangely enough, the hydrogen bomb is considered too horrible to use in war, but the automobile rolls merrily on.

People have a tendency to look on a traffic accident as an act of God. This type of blame-laying may well have made "the wrath of God" a popular phrase.

Traffic safety legislation is by and large not effectual because actual contributing factors to accidents are seldom known. This is so because the police are by and large not proficient in even basic accident reporting. The police are not proficient because very few departments really want to bother with accidents. Everyone knows that the police are supposed to catch real criminals like bank robbers, murderers, rapists, and peeping toms. They are supposed to ignore the licensed American driver who, after all, is only trying to get from point A to point B and has no intention of hurting anyone.

The fact is that this "well-intentioned" fiend controls a hurtling projectile that disembowels, decapitates, and generally violates human beings to such a magnitude that by comparison Jack the Ripper seems a bumbling amateur.

It has long been recognized that accident reconstruction requires the application of scientific techniques. The objective of this manual is to consolidate these techniques and apply them to the specific areas where they are effective. The objective is also to present these techniques in a manner that can be easily absorbed by a select group of police officers.

It must be pointed out that this manual is set up for reference purposes in accident reconstruction. The manual should be taught to a select group of police, and it is imperative that there be control over the individual once he puts the techniques to practical use in the field. Control is needed to prevent abuses and to keep performance evaluation current.

It is not my intention to attempt to teach a course in physics nor to discourage anyone from getting a formal education in that field. Nor do I apologize for the many areas of mathematical computations that have been conveniently bypassed or ignored. I have simply picked out the equations that related to accident

reconstruction and these are the only ones we will deal with.

I hope this course will put to rest two long-standing myths: first, that no police officer can handle math problems beyond a fourth grade level; and second, that auto accidents cannot be satisfactorily reconstructed because no two are alike. The first is simply no longer true and I'm not sure it ever was. In regard to the second, there are phenomena that recur in accidents with such regularity that many collision results are completely predictable.

The success of the individual police officer in this course rests heavily on his desire to excel and his willingness to research problem areas that may arise. If this desire is not present, we are beating a dead horse, and his having a doctorate from Harvard would not produce the desired results.

The purpose of teaching accident reconstruction to a select group of police officers is to improve performance in three areas -- criminal prosecution, civil action, and statistical information. Aside from accomplishing this, such instruction upgrades the prestige of the department and puts its reputation in proper perspective -- that of a highly trained, professional police organization.

It is impossible, and may well be undesirable, to train everyone in accident reconstruction. We can, however, train a few, and these few can assist other police officers where and when the need arises.

It has been said that, due to the energy situation, this is a late date to expand operations in the field of accident reconstruction. Petroleum is, after all, in short supply, and because of this, the automobile will go flimmering into oblivion.

The automobile, however, will be with us until the end. I have faith in humanity, and I firmly believe that if the world runs out of petroleum, the very last drop will vanish down the throat of a four barrel carburetor on I-94 near the junction of Snelling Avenue. While the driver looks at his gas gauge he will coast into the last multi-car accident we will ever have. Then and only then will the final silence descend upon us all.

M. LOFGREN

REFRESHER MATHEMATICS

Appendix A of this manual is devoted to Refresher Mathematics. It is set up to be self-explanatory. It is necessary for you to go through Appendix A carefully before starting this course even though you have probably had this material at some point in your formal education. It is also important for you to work the problems, as this level of math will be used extensively.

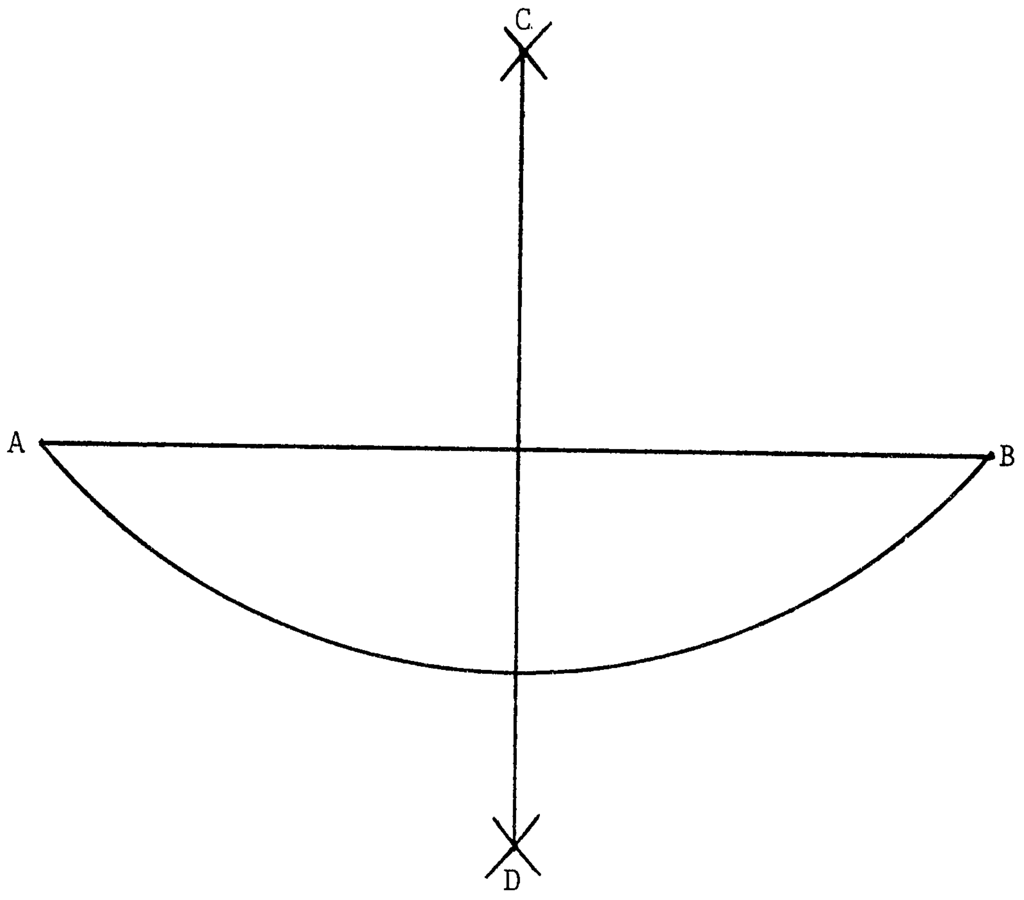
The material in Appendix A was written by T. L. Aycock, formerly a Georgia State Trooper and presently a staff member of Northwestern University Traffic Institute. It is reproduced here with his explicit permission.

CHAPTER I

GEOMETRIC CONSTRUCTION

This chapter shows a few simple techniques for dividing lines and arcs and for drawing parallel lines. You should become familiar with these techniques and practice using them because they will serve you well in the vector diagramming part of conservation of linear momentum and will assist you in some scale diagrams.

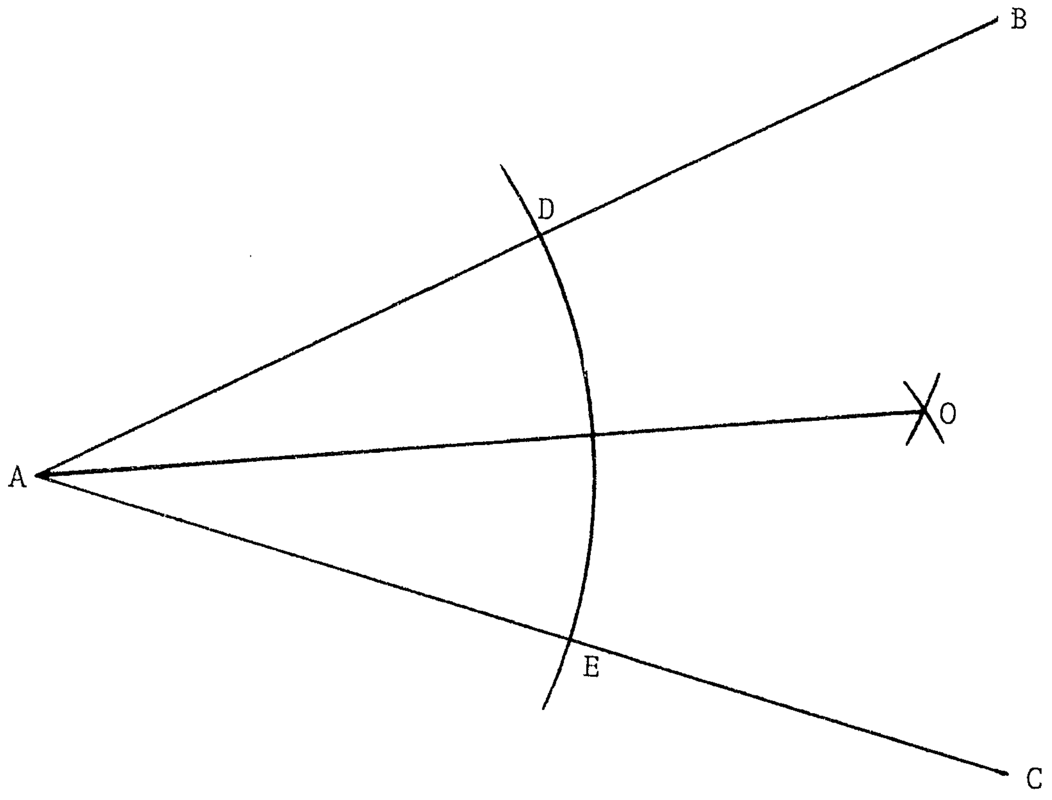
Bisecting a Line or Arc



1. Given line or arc "AB":
2. Set compass at any radius <u>greater</u> than one half "AB".
3. Using "A" and "B" as centers, draw arcs at "C" and "D".
4. Where line "CD" crosses "AB" is the exact center of "AB".

Bisecting an Angle

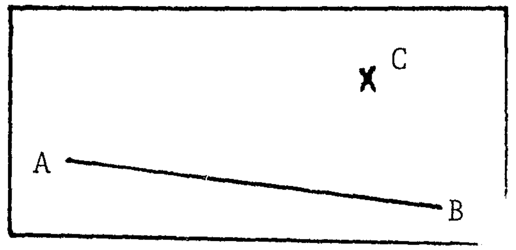
1. Given angle "BAC":
2. With "A" as center draw an arc of any radius cutting line "AB" at "D" and line "AC" at "E".
3. Set compass at a radius greater than ½ "DE".

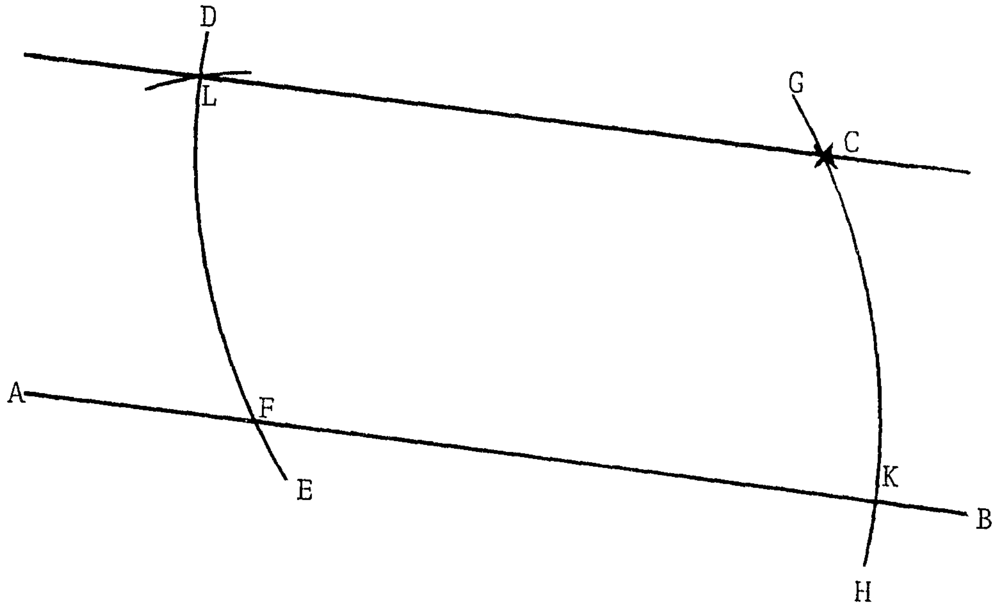


4. Using "D" and "E" as centers, draw arcs at "O".
5. Draw a line from "O" through "A". This line bisects the angle.

Drawing a Straight Line through a Point Parallel to Another Straight Line

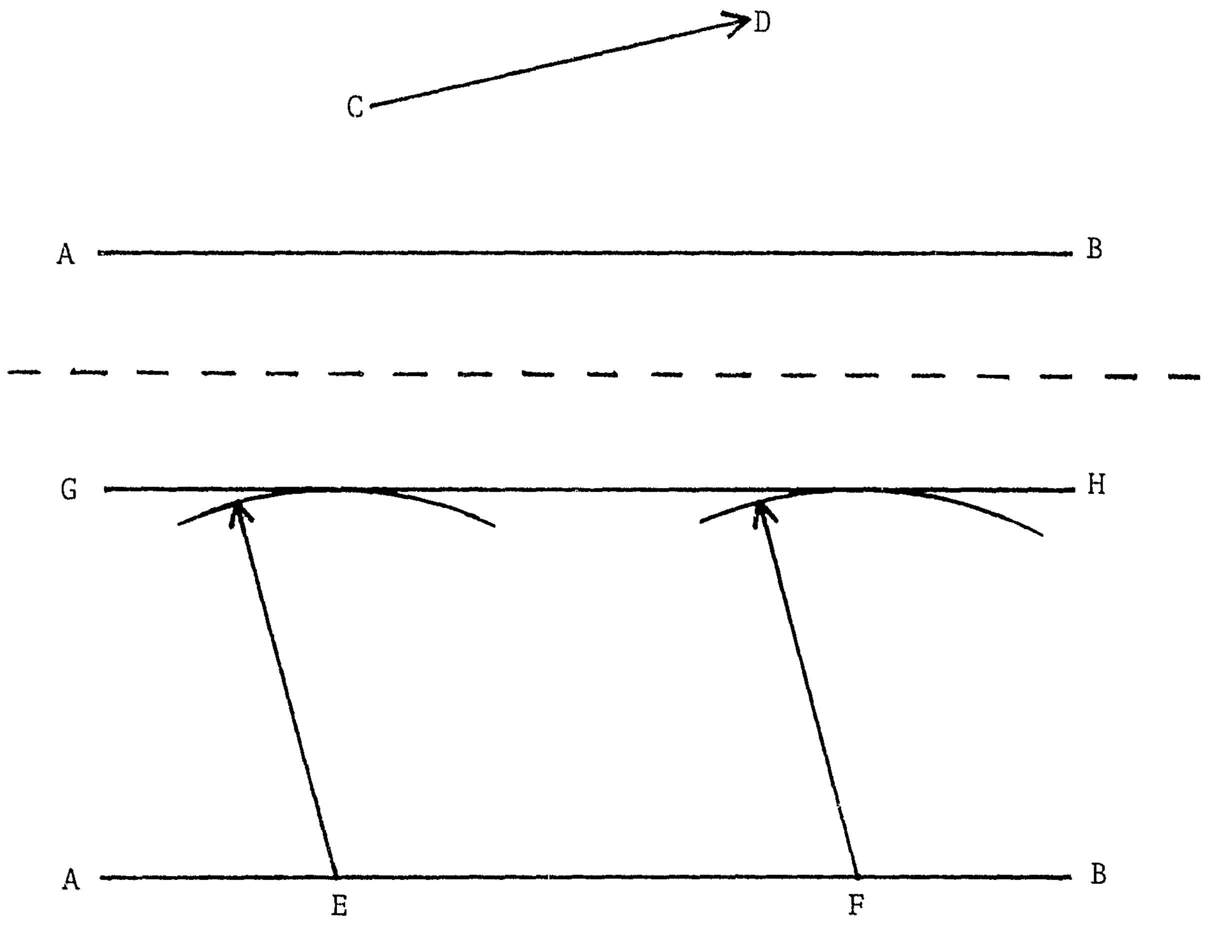
1. Given line "AB" and required point "C":
2. With "C" as center draw arc "DE" at any radius.





3. With "F" as center and using the same radius, draw arc "GH".
4. Using "CK" as radius and "F" as center, strike an arc intersecting arc "DE" at "L".
5. Draw a straight line through "L" and "C".

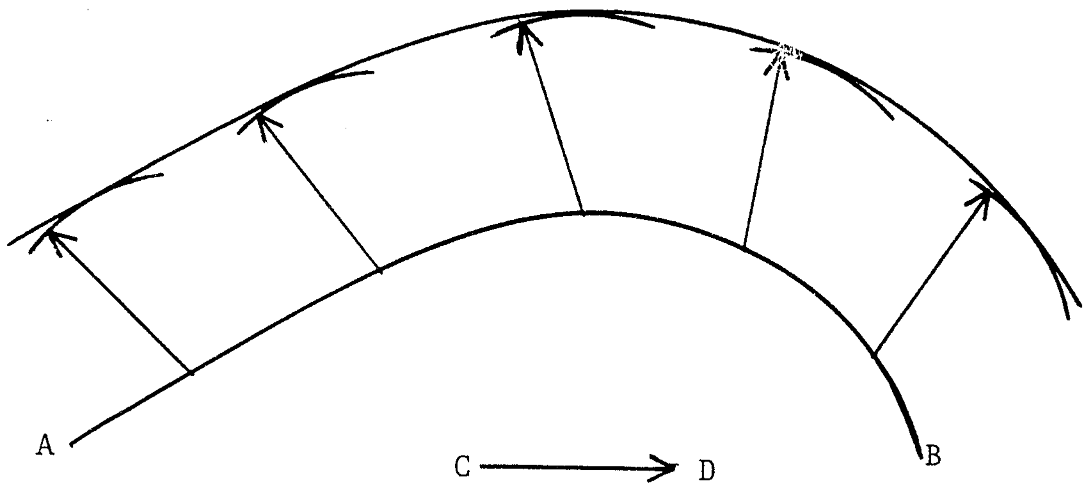
Drawing a Line Parallel to Another Line at a Given Distance



1. "AB" is the given line and "CD" the given distance.
2. Using "CD" as the radius and any points "E" and "F", draw two arcs.
3. Draw line "GH" tangent to the two arcs.

Method of Drawing Curved Parallel Lines

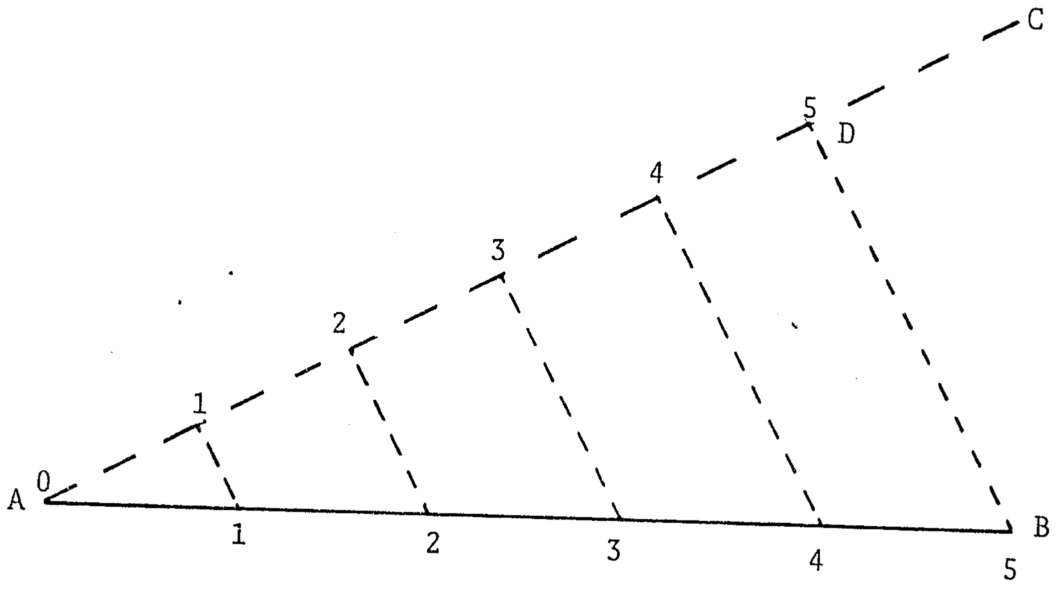
1. "AB" is a curved line. "CD" is the given distance.
2. Draw a series of arcs from "AB" with a radius equal to "CD".



3. With a flexicurve, draw a curved line tangent to these arcs.

Dividing a Line into Equal Parts

1. Line "AB" is to be divided into five equal parts.
2. Draw line "AC" at any angle.



3. Start at "A" and lay off five equal spaces on line "AC".
4. From the end of the fifth space draw a line from "D" to "B".
5. From each point on line "AC" draw a line parallel to "DB".
6. The five division points are found where these parallel lines intersect line "AB".

Drawing an Arc through Three Given Points

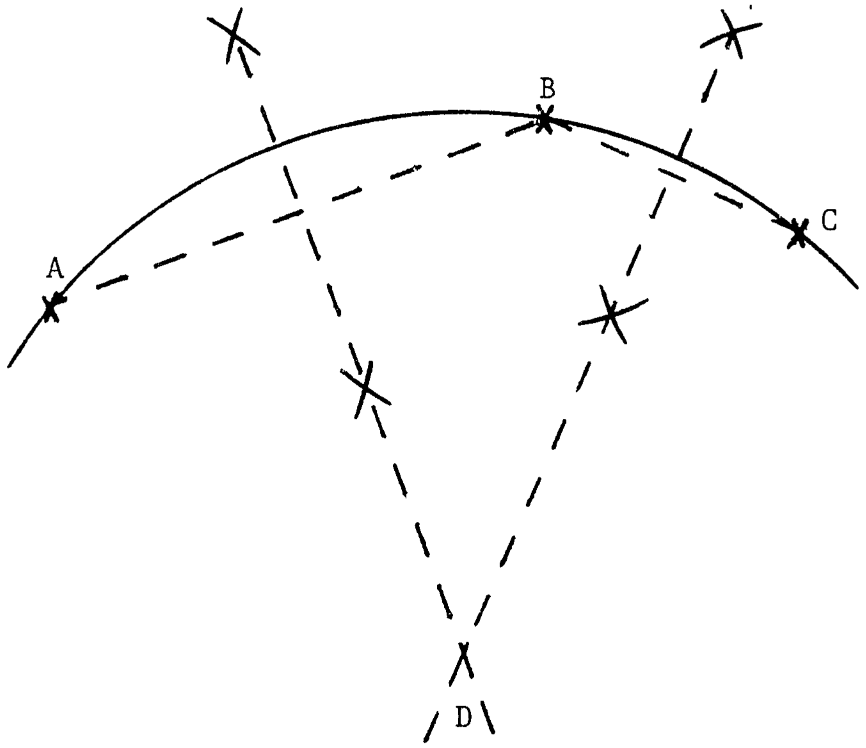
1. Given points "ABC":

B
x

A
x

C
x

2. Draw lines connecting "AB" and "BC".



3. Refer to page 2 and bisect lines "AB" and "BC".
4. Where these lines cross at "D" is the center of the arc.
5. Set compass radius to the distance "AD" and draw the arc.

CHAPTER II

MINIMUM SPEED FROM SKIDMARKS

Formula for Finding Minimum Speed from Skidmarks $S = \sqrt{30\ Df}$

Formula for Finding Drag Factor $f = \frac{S^2}{30\ D}$

Formula for Finding Distance $D = \frac{S^2}{30\ f}$

In this chapter we will deal with the derivation of the minimum speed formula, the drag factor formula, and the distance formula. All three are derived from the formula for kinetic energy, which is $Ke = \frac{1}{2} MV^2$.

You are able to use the minimum speed formula, so no time will be devoted to this.

Sooner or later you will have to defend the origin of the speed formula in court. The following pages show step for step how $Ke = \frac{1}{2} MV^2$ becomes $S = \sqrt{30\ Df}$. I have used the simplest method I know and one that I have used in court and thus know is acceptable.

It is not necessary to know any derivation by heart, but it is necessary to know what you are doing. You may refer to notes in court, but remember that the defense attorney has the right to take those notes and look at them. If you do everything correctly, no defense attorney in the world can discredit you. But if you screw up and the defense attorney catches it, he will turn you inside out and hang you out to dry.

The following legend identifies the characters used in this chapter:

Ke = Kinetic energy or energy of motion
g = Acceleration of gravity or 32.2 feet per second squared (32.2 FPS^2)
W = Weight in lbs.
S = Speed in miles per hour (MPH)
V = Velocity in feet per second (FPS)
f = Drag factor (coefficient of friction)
D = Distance in feet
M = Mass, which is the same as the weight of an object divided by the acceleration of gravity or $\frac{W}{g}$
30 = A constant that arises in the process of the derivation
1.466 = A conversion factor: 5280/3600

First, you <u>must</u> take note of a conversion factor that we will use in nearly every formula derivation. That is the number 1.466. This number is found by dividing 5280 feet in a mile by 3600 seconds in an hour.

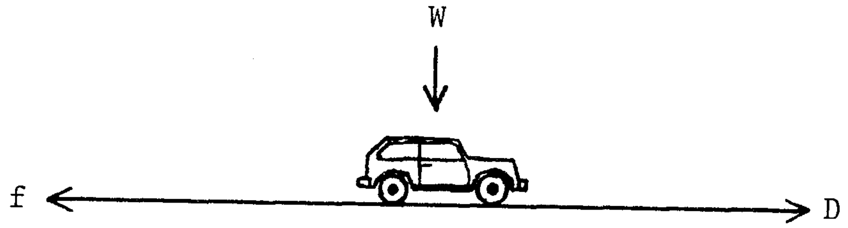
Actually this gives 1.4666666, which we round off to 1.466. This number is used to convert FPS into MPH or vice versa. As an example: 60 MPH times 1.466 equals 88 FPS or 88 FPS divided by 1.466 equals 60 MPH. In general you will see Velocity (V), which is in FPS, replaced by (S x 1.466), which simply changes FPS to MPH and does not alter the equations. If you don't remember another thing from this course, it is imperative that you remember this paragraph. If you don't understand it, contact someone who does at once.

$Ke = \frac{1}{2} MV^2$ is the formula for energy of motion

Assume that a vehicle slides to a stop on a road surface. Assume further you know the speed of the vehicle and the drag factor of the road surface.

While the vehicle is moving it has kinetic energy equal to $\frac{1}{2} MV^2$. When the wheels start to slide, the kinetic energy of the vehicle must be equal to the work it can do while it slides to a stop.

The vehicle's work energy is found by multiplying the weight of the vehicle times the drag factor of the road surface times the distance it will slide in stopping. This can be expressed as simply (WfD).



So in a skidding vehicle

(1) $WfD = \frac{1}{2} MV^2$

Change (M) to $(\frac{W}{g})$ and divide out (W).

$$\frac{\cancel{W}fD}{\cancel{W}} = \frac{\frac{1}{2}\frac{\cancel{W}}{g}V^2}{\cancel{W}} \text{ which leaves}$$

(2) $fD = \frac{1}{2g} V^2$

To get (V^2) alone, multiply both sides of the equal sign by (2g).

$$\frac{2g}{1} fD = \frac{\cancel{1}}{\cancel{2g}} V^2 \frac{\cancel{2g}}{\cancel{1}} \text{ which leaves}$$

(3) $V^2 = 2g\ fD$

In equation (3) we will change (g) to 32.2 and multiply it by the 2, which changes (2g) into 64.4.

(4) $V^2 = 64.4\ fD$

Next (V^2) is changed to MPH.

(5) $(S \times 1.466)^2 = 64.4\ fD$

Next square 1.466 and divide out 2.15 to get (S^2) alone.

$$\frac{S^2 x\ \cancel{2.15}}{\cancel{2.15}} = \frac{64.4\ fD}{2.15} \text{ which leaves}$$

(6) $S^2 = 30\ fD$

Now extract the square root from each side of the equal sign.

$$\sqrt{S^2} = \sqrt{30\ fD} \qquad \text{which becomes}$$

(7) $S = \sqrt{30\ fD}$

If there is a % grade uphill, you must add it to (f) and if it is downhill, you must subtract it from (f). Always remember that if test skids are run in the same place and direction as the accident car, no correction for grade is necessary.

Now return to equation (6) above.

$$S^2 = 30\ fD$$

Divide both sides of the equal sign by (30 D) to get (f) alone.

$$\frac{S^2}{30\ D} = \frac{\cancel{30}\ f\cancel{D}}{\cancel{30}\ \cancel{D}} \qquad \text{which leaves}$$

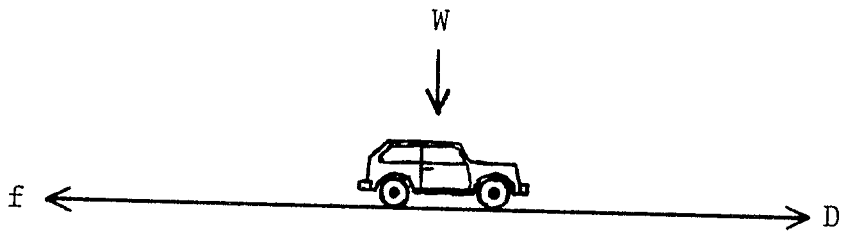
(8) $f = \frac{S^2}{30\ D}$ (Drag factor formula)

Using the same process to get (D) alone, you end up with

(9) $D = \frac{S^2}{30\ f}$ (Distance formula)

If you think that wasn't bad, you are right. But gird your loins, my friend, because the derivations get much worse.

The vehicle's work energy is found by multiplying the weight of the vehicle times the drag factor of the road surface times the distance it will slide in stopping. This can be expressed as simply (WfD).



So in a skidding vehicle

(1) $WfD = \frac{1}{2} MV^2$

Change (M) to $(\frac{W}{g})$ and divide out (W).

$$\frac{\cancel{W}fD}{\cancel{W}} = \frac{\frac{1}{2}\frac{\cancel{W}}{g}V^2}{\cancel{W}} \text{ which leaves}$$

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To get (V^2) alone, multiply both sides of the equal sign by (2g).

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Next (V^2) is changed to MPH.

(5) $(S \text{ x } 1.466)^2 = 64.4\ fD$

Next square 1.466 and divide out 2.15 to get (S^2) alone.

$$\frac{S^2 \text{x } \cancel{2.15}}{\cancel{2.15}} = \frac{64.4\ fD}{2.15} \text{ which leaves}$$

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Now extract the square root from each side of the equal sign.

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If there is a % grade uphill, you must add it to (f) and if it is downhill, you must subtract it from (f). Always remember that if test skids are run in the same place and direction as the accident car, no correction for grade is necessary.

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Divide both sides of the equal sign by (30 D) to get (f) alone.

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Using the same process to get (D) alone, you end up with

(9) $D = \frac{S^2}{30\ f}$ (Distance formula)

If you think that wasn't bad, you are right. But gird your loins, my friend, because the derivations get much worse.

A final comment regarding drag factor -- There is a popular concept that the drag factor decreases with the speed of the vehicle. Tests show that this is absolutely not true. The drag factor normally remains constant at all speeds. The only change in drag factor occurs on some, but not all, long skids. On long skids the drag factor sometimes increases slightly.

CHAPTER III

FALL SPEED

$$S = 2.74 \text{ x } \frac{D}{\sqrt{h}} \qquad \text{for level take-off}$$

$$S = 2.74 \text{ x } \frac{D}{\sqrt{h \pm (Dm)}}$$

(+) if take-off is up

(-) if take-off is down

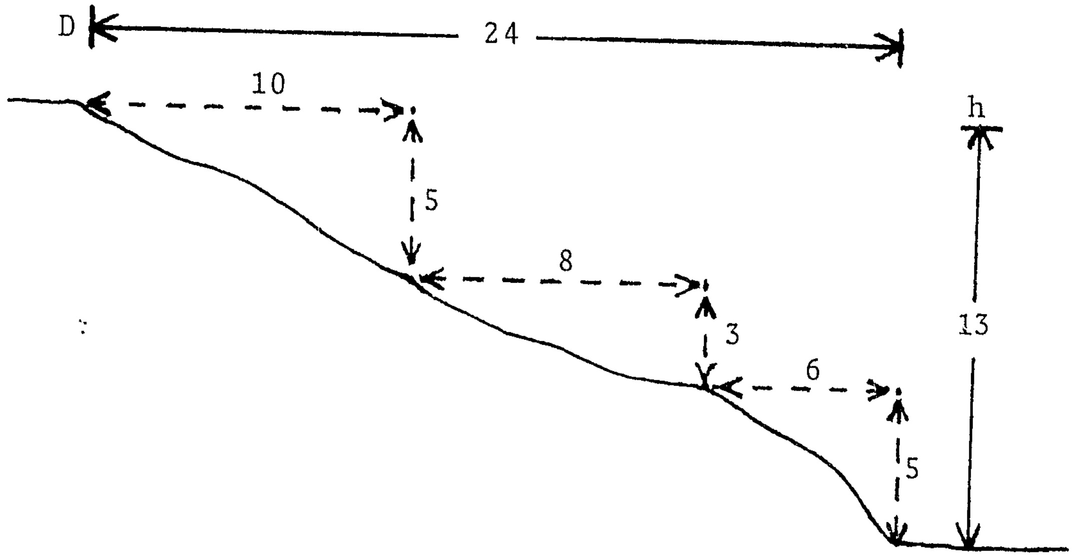
This chapter deals with the fall formula. A large number of accidents result in one vehicle or more running off the road into a ditch. If the speed is great enough, the vehicle will go airborne for a distance before it comes in contact with the ground again. If you can measure the horizontal distance (D) and the height of fall (h), you can determine the speed of the vehicle when it left the road.

The easiest method to measure this fall distance for normal distances is to anchor a snap line at the shoulder edge where the approximate center of the vehicle left the road and stand over the first mark left in the dirt where the car landed. Keeping the line tight and against your body, use a line level to insure you have the line held level. Then measure the vertical distance from the string to the ground where the first mark was made. Measure to the bottom of the hole or depression made by the landing vehicle. This will keep your speed computation conservative. A rule of thumb for the fall formula is never measure (D) too long and never measure (h) too short. You must adjust for the distance the center of mass of the vehicle traveled. As a rule of thumb, you can assume the center of mass is at one-third the total height of the vehicle measured from the ground, and at a point where an "X" is formed if you connect the diagonally opposite wheels with straight lines.

You must also determine the % grade of take-off plus or minus. If the shoulder slopes down, the take-off is minus. Nevertheless, try to treat a minus grade take-off as a level take-off. If this gives you a high enough speed, you are home free, since you will have a minimum speed, which is nice. Any take-off grade that is plus must be considered and used in the equation. As a general rule you will find that a take-off up to +2% will not change the speed much and that a surface that looks level can be considered level.

The fall formula is valid only for take-offs up to 10%. When the take-off is more than 10%, you must use the vault formula from the next chapter.

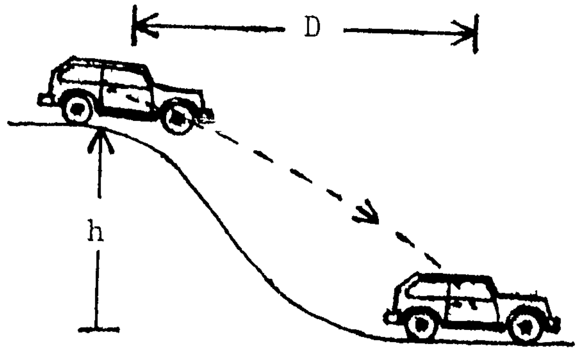
If a car goes off a steep embankment, this method can be used to measure (D) and (h):



In this case, D = 24 feet and h = 13 feet.

Derivation of

$$S = 2.74 \times \frac{D}{\sqrt{h}}$$



Remember that velocities and forces that are at right angles to each other are independent of each other, so the force of gravity, which accelerates the car to its final resting place, has absolutely no effect on the car's horizontal velocity providing the car was traveling on level ground. The question is: How long in seconds was the car in the air? Since air friction is negligible, the car keeps going straight ahead at constant velocity while it is in the air and travels a measureable distance (D).

The formula for distance traveled at a constant velocity is

(1) $D = V\ T$ (T) is time in seconds.

The formula for distance traveled by a falling body during a given time (T) is

(2) $h = ½\ gT^2$ (h) is height of fall in feet.

Since we can measure and know (h) but not (T), we rewrite (2) to solve for (T).

(3) $T^2 = \frac{2h}{g}$ or $T = \frac{\sqrt{2h}}{\sqrt{g}}$

This (T) is the same (T) as in equation (1), so we can combine equation (1) and (3) to get

(4) $D = V \times \frac{\sqrt{2h}}{\sqrt{g}}$

But we want to find (V), so we must solve for it.

(5) $V = \frac{D}{\frac{\sqrt{2h}}{\sqrt{g}}}$ or $V = \frac{D}{\frac{\sqrt{2}}{\sqrt{g}}\ \sqrt{h}}$

The only reason I split up $\frac{\sqrt{2h}}{\sqrt{g}}$ is because $\frac{\sqrt{2}}{\sqrt{g}}$ can be calculated ahead of time while (h) varies from case to case. By using the rule for dividing fractions (invert and multiply), we change equation (5) to

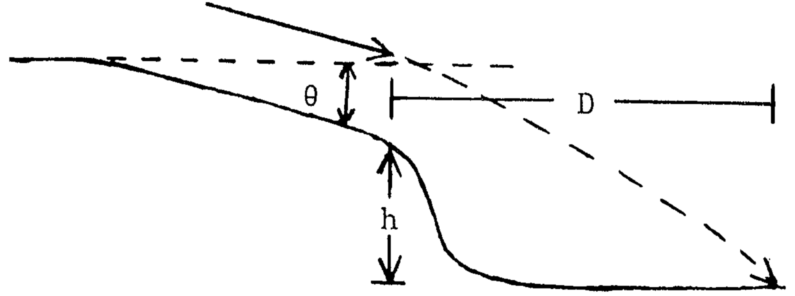
(6) $V = \frac{\sqrt{g}}{\sqrt{2}} \times \frac{D}{\sqrt{h}} = \sqrt{16.1} \times \frac{D}{\sqrt{h}} = 4.01 \times \frac{D}{\sqrt{h}}$

Now convert FPS to MPH and divide out the constant 1.466 (1.466 must be used here instead of 1.47).

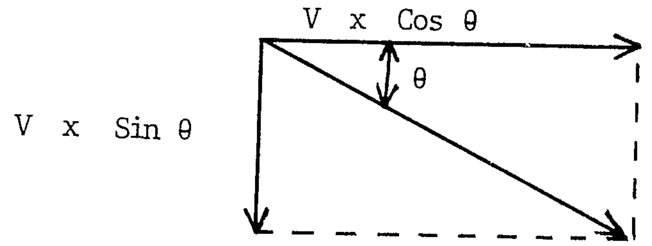
$$\frac{S \times \cancel{1.466}}{\cancel{1.466}} = \frac{4.01}{1.466} \times \frac{D}{\sqrt{h}}$$

(7) $S = 2.74 \times \frac{D}{\sqrt{h}}$

Now if the car is going down a grade when it goes airborne:



Break up (V) into two components - one in a downward direction and one in a horizontal direction.



So what was horizontal (V) in the first case is now (V x Cos θ), and instead of starting to fall from rest, the car now has an initial velocity (V_0) of (V x Sin θ). Note that (V_0) stands for initial velocity in feet per second.

The formula for distance traveled by a falling body which starts with an initial velocity (V_0) is

$$(8) \quad h = V_0 \times T + \tfrac{1}{2} gT^2$$

Or plugging in the value for (V_0)

$$(9) \quad h = (V \times \text{Sin } \theta \times T) + (\tfrac{1}{2} gT^2)$$

Now instead of equation (1) we have

$$(10) \quad D = \underbrace{V \times \text{Cos } \theta}_{\text{horizontal velocity}} \times T$$

The easiest way to combine equations (9) and (10) is to solve

equation (10) for (T) and plug that into equation (9), so rewrite equation (10) to solve for (T).

$$(11) \quad T = \frac{D}{V \times \text{Cos } \theta} \quad \text{and of course}$$

$$(12) \quad T^2 = \frac{D^2}{V^2 \times (\text{Cos } \theta)^2}$$

Now take equation (9) and put equation (11) in place of (T) and equation (12) in place of (T^2).

$$h = (V \times \text{Sin } \theta \times \frac{D}{V \times \text{Cos } \theta}) + (\tfrac{1}{2} g \times \frac{D^2}{V^2 \times (\text{Cos } \theta)^2})$$

On the left side of the + sign, (V) will cancel out leaving

$$(13) \quad h = (\text{Sin } \theta \times \frac{D}{\text{Cos } \theta}) + (\tfrac{1}{2} g \times \frac{D^2}{V^2 \times (\text{Cos } \theta)^2})$$

Now all we have to do is get (V) by itself. Start by moving the first right side term across the equal sign and putting the terms in a more workable form.

$$(14) \quad h - D \times \frac{\text{Sin } \theta}{\text{Cos } \theta} = \tfrac{1}{2} g \times \frac{D^2}{V^2 \times (\text{Cos } \theta)^2}$$

From trigonometry we know that

$$\frac{\text{Sin } \theta}{\text{Cos } \theta} = \text{Tan } \theta \qquad \text{so}$$

$$(15) \quad h - D \times \text{Tan } \theta = \frac{g}{2} \times \frac{D^2}{V^2 \times (\text{Cos } \theta)^2}$$

Multiply both sides by (V^2).

$$V^2 \times (h - D \times \text{Tan } \theta) = \frac{g}{2} \times \frac{D^2}{\cancel{V^2} \times (\text{Cos } \theta)^2} \times \cancel{V^2}$$

Now divide both sides by (h - D x Tan θ).

$$\frac{V^2 \times (\cancel{h} - \cancel{D} \times \cancel{\text{Tan } \theta})}{\cancel{h} - \cancel{D} \times \cancel{\text{Tan } \theta}} = \frac{\frac{g}{2} \quad \frac{D^2}{(\text{Cos } \theta)^2}}{h - D \times \text{Tan } \theta}$$

Which leaves

$$(16) \quad V^2 = \frac{g}{2} \times \frac{1}{(\text{Cos } \theta)^2} \times \frac{D^2}{h - D \times \text{Tan } \theta}$$

Or taking the square root

$$(17) \quad V = \frac{\sqrt{g}}{\sqrt{2}} \times \frac{1}{\text{Cos } \theta} \times \frac{D}{\sqrt{h - D \times \text{Tan } \theta}}$$

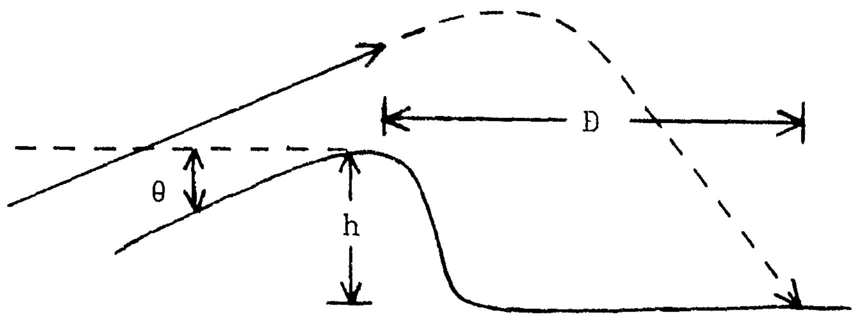
Equation (17) is the exact solution and works no matter what the angle θ. The Cos of small angles is approximately 1, so

the Cos can be removed from equation (17); and the Tan of all angles is equal to % grade, so (Tan θ) can be replaced by (m), which stands for % grade. Now convert MPH.

$$(18) \quad \frac{S \times \cancel{1.466}}{\cancel{1.466}} = \frac{4.01}{1.466} \times \frac{D}{\sqrt{h - (Dm)}}$$

$$(19) \quad S = 2.74 \times \frac{D}{\sqrt{h - (Dm)}}$$

Now the take-off is uphill rather than downhill.



V_0 is now of the opposite sign from the force of gravity, so equation (9) becomes

$$(20) \quad h = -V \times \text{Sin } \theta \times T + \tfrac{1}{2} gT^2$$

Equations (10), (11), and (12) stay the same (straight ahead is still a plus direction), but equation (13) becomes

$$h = -(\text{Sin } \theta \times \frac{D}{\text{Cos } \theta}) + \left[\tfrac{1}{2} g \times \frac{D^2}{V^2 \times (\text{Cos } \theta)^2}\right]$$

So equation (15) becomes

$$(21)\quad h + D \times \mathrm{Tan}\ \theta + \frac{g}{2} \times \frac{D^2}{V^2 \times (\mathrm{Cos}\ \theta)^2}$$

So, of course, equation (17) becomes

$$(22)\quad V = \frac{\sqrt{g}}{\sqrt{2}} \times \frac{1}{\mathrm{Cos}\ \theta} \times \frac{D}{\sqrt{h + D \times \mathrm{Tan}\ \theta}}$$

And using the procedure to change FPS to MPH, we change equation (19) to

$$(23)\quad S = 2.74 \times \frac{D}{\sqrt{h + (Dm)}}$$

So much for Chapter III, and it only took 23 steps. If at this point you neither smoke, drink, or swear, you are too pure to be a police officer.

CHAPTER IV

DERIVATION OF THE VAULT FORMULA

$$V_0{}^2 = \frac{g}{2} \cdot \frac{D^2}{D \cdot \mathrm{Sin}\ \theta \cdot \mathrm{Cos}\ \theta \pm (h \cdot \mathrm{Cos}^2\ \theta)}$$

V_0 = Velocity in FPS (initial)

g = Acceleration of gravity (32.2 FPS^2)

D = Distance in feet

h = Height difference in feet as the landing relates to take-off (Note: If the landing is lower than take-off, (h) is plus. If the landing is higher than take-off, (h) is minus.)

$\mathrm{Sin}\ \theta$ = Sine of the take-off angle (Get this from a table.)

$\mathrm{Cos}\ \theta$ = Cosine of the take-off angle (Get this from a table.)

X = Horizontal component of velocity

Y = Vertical component of velocity

T = Time in seconds

This is the original granddaddy of all airborne speed formulas. If you can establish the take-off angle, it will give you the speed of anything from a half-crazed Mongolian broad jumper to a howitzer shell. I do not suggest you experiment. It is not hard to get the Mongolian, but your neighbors will complain if you fire a howitzer, and you would have to do it in a vacuum anyway.

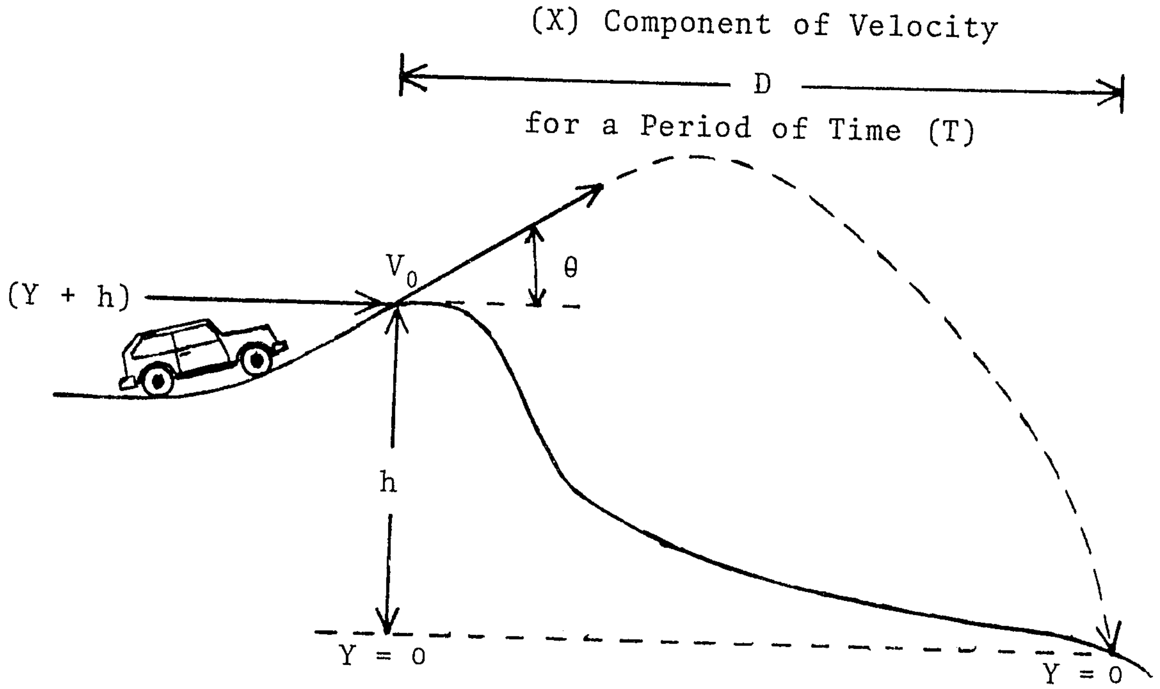
In a great many accidents a vehicle goes into a ditch and hits a driveway or some kind of approach. If the speed is great enough, the vehicle will be launched into the air for a measureable distance to where it lands. The vehicle usually scoops out a portion of the approach and thus leaves an inclined plane behind which gives you the angle the vehicle was launched upward. You can easily measure this departure angle. Then from the highest point of take-off, measure the horizontal distance to the first landing point, and using a snap line and line level, determine if the landing is higher or lower than take-off and how much this is in feet. This measurement is important, and if you cannot get it with a snap line (like if it is 150 feet away in the middle of a briar patch behind a big rock), you may have to get someone with a transit to do it for you.

The same rule of thumb applies here as with the fall formula: Do not measure (D) too long or (h) too short. The vault formula is used where you can measure the angle of take-off, and usually this angle is great enough that the car travels a considerable distance in the air. Because of this the car has time to rotate and usually lands nose first. This can cause some problems in measuring the actual distance of travel of the center of mass.

From countless testing we have found that subtracting one half the wheelbase of the vehicle from your measured distance will compensate adequately and insure a conservative speed estimate.

You will also find that a vehicle will not travel very far once it lands. The larger the angle of take-off the shorter distance the car will roll or slide when it comes back to earth. The greater the angle of take-off the higher into the air the vehicle will go. The higher into the air it goes the harder it will hit the ground and the deeper it will try to bury itself. A car launched into the air at high speed will have the drag factor of a 6-bottom John Deere plow when it lands. This very high drag factor has a tendency to cancel the tickets of any occupants of the vehicle.

Please note in this derivation there is no (x) used to mean times. You will see a dot (•) in place of the times sign. This is because we must use (x) for other purposes. Anywhere in an equation where you see a dot that is higher than period level in a sentence, the dot means times; and anywhere you see (X), the (X) refers to the horizontal direction of travel. Also, the final answer will be in feet per second, and you will have to divide it by 1.466 to convert it to miles per hour.



First we will deal with the (X) direction, which is the horizontal component of velocity. In this direction we must use the cosine value of the take-off angle.

(1) $X = V_0 \cdot \text{Cos}\ \theta \cdot T$

Now for the (Y) or vertical component of velocity. In this direction we must use the sine value of the take-off angle.

(2) $Y = V_0 \cdot \text{Sin}\ \theta \cdot T - \frac{1}{2} gT^2 + h$

Next take equation (1) and solve for (T). Divide both sides of the equal sign by ($V_0 \cdot \text{Cos}\ \theta$).

(3) $T = \dfrac{X}{V_0 \cdot \text{Cos}\ \theta}$

Now plug equation (3) into equation (2) where (T) and $(T)^2$ show up.

$$Y = V_0 \cdot \text{Sin}\ \theta \cdot \frac{X}{V_0 \cdot \text{Cos}\ \theta} - \frac{1}{2} g \frac{X^2}{V_0{}^2 \cdot \text{Cos}^2\ \theta} + h$$

I will repeat the above equation showing (V_0) being cancelled.

$$Y = \cancel{V_0} \cdot \text{Sin}\ \theta \cdot \frac{X}{\cancel{V_0} \cdot \text{Cos}\ \theta} - \frac{1}{2} g \frac{X^2}{V_0{}^2 \cdot \text{Cos}^2\ \theta} + h$$

(4) $Y = \dfrac{\text{Sin}\ \theta \cdot X}{\text{Cos}\ \theta} - \frac{1}{2} g \dfrac{X^2}{V_0{}^2 \cdot \text{Cos}^2\ \theta} + h$

Now because (Y = 0) at impact (see diagram) and (X) is really the horizontal distance the vehicle travels, we replace (Y) with (0) and (X) with (D). This makes equation (4) look like this:

(5) $0 = \dfrac{\text{Sin}\ \theta \cdot D}{\text{Cos}\ \theta} - \frac{1}{2} g \dfrac{D^2}{V_c{}^2 \cdot \text{Cos}^2\ \theta} + h$

Now we want to isolate ($V_0{}^2$), so we start by moving the negative term across the equal sign, conveniently changing it to plus.

(6) $\frac{1}{2} g \dfrac{D^2}{V_0{}^2 \cdot \text{Cos}^2\ \theta} = \dfrac{\text{Sin}\ \theta \cdot D}{\text{Cos}\ \theta} + h$

Now regroup slightly and multiply both sides by ($V_0{}^2$).

$$\cancel{V_0{}^2} \cdot \left(\frac{g}{2} \cdot \frac{D^2}{\cancel{V_0{}^2} \cdot \text{Cos}^2\ \theta}\right) = \left(\frac{\text{Sin}\ \theta \cdot D}{\text{Cos}\ \theta} + h\right) \cdot V_0{}^2$$

This leaves

(7) $\dfrac{g}{2} \cdot \dfrac{D^2}{\text{Cos}^2\ \theta} = V_0{}^2 \cdot \left(\dfrac{\text{Sin}\ \theta \cdot D}{\text{Cos}\ \theta} + h\right)$

Now isolate ($V_0{}^2$).

$$\frac{\dfrac{g \quad D^2}{2\ \mathrm{Cos}^2\ \theta}}{\dfrac{\mathrm{Sin}\ \theta \cdot D}{\mathrm{Cos}\ \theta} + h} = \frac{V_0{}^2 \cdot \left(\dfrac{\cancel{\mathrm{Sin}\ \theta} \cdot \cancel{D}}{\cancel{\mathrm{Cos}\ \theta}} + \cancel{h}\right)}{\dfrac{\cancel{\mathrm{Sin}\ \theta} \cdot \cancel{D}}{\cancel{\mathrm{Cos}\ \theta}} + \cancel{h}}$$

$$(8) \quad V_0{}^2 = \frac{g}{2} \cdot \left[\frac{\dfrac{D^2}{\mathrm{Cos}^2\ \theta}}{\dfrac{\mathrm{Sin}\ \theta \cdot D}{\mathrm{Cos}\ \theta} + h}\right]$$

Now the term in brackets in equation (8) is one huge fraction. The rule for dividing fractions says you invert and multiply, so we get this:

$$\frac{D^2}{\mathrm{Cos}^2\ \theta} \cdot \frac{1}{\dfrac{\mathrm{Sin}\ \theta \cdot D}{\mathrm{Cos}\ \theta} + h}$$

Now ($D^2 \cdot 1 = D^2$). And ($\mathrm{Cos}^2\ \theta$) must be multiplied by both ($\mathrm{Cos}\ \theta$) and (h) because there is a plus sign.

$$\frac{D^2}{\mathrm{Cos}^{\cancel{2}}\ \theta \cdot \dfrac{\mathrm{Sin}\ \theta \cdot D}{\cancel{\mathrm{Cos}\ \theta}} + \mathrm{Cos}^2\ \theta \cdot h}$$

This reduces this term to

$$\frac{D^2}{D \cdot \mathrm{Sin}\ \theta \cdot \mathrm{Cos}\ \theta + h \cdot \mathrm{Cos}^2\ \theta}$$

Now put this term back into the bracketed part of equation (8) and we have the finished product.

$$(9) \quad V_0{}^2 = \frac{g}{2} \cdot \frac{D^2}{D \cdot \mathrm{Sin}\ \theta \cdot \mathrm{Cos}\ \theta + (h \cdot \mathrm{Cos}^2\ \theta)}$$

Remember there is a plus because landing was lower than take-off. If landing had been higher than take-off, there would be a minus.

$+ \quad (h \cdot \mathrm{Cos}^2\ \theta)$ vehicle lands below take-off

$- \quad (h \cdot \mathrm{Cos}^2\ \theta)$ vehicle lands above take-off

Remember also your answer is velocity in feet per second (FPS), so you <u>must</u> divide your answer by 1.466 to convert it to miles per hour (MPH).

EXAMPLE: A car hits a driveway approach and is launched upward at 20 degrees and lands 80 feet away 2 feet lower than the point of take-off. What was its speed at the take-off point?

20^0 angle: Sin = .342 and Cos = .940

D = 80 feet h = 2 feet

(h) will be plus because the car landed lower than take-off. Substituting values in equation (9) we get

$$V_0{}^2 = \frac{32.2}{2} \cdot \frac{80^2}{80 \text{ times } .342 \text{ times } .940 + (2 \text{ times } .940^2)}$$

$$V_0{}^2 = 16.1 \cdot \frac{6400}{25.71 + 1.76}$$

$$V_0^2 = 16.1 \cdot 232.98$$

$$\sqrt{V_0^2} = \sqrt{3750.97}$$

$$V_0 = 61.24 \text{ FPS}$$

$$\frac{S \cdot \cancel{1.466}}{\cancel{1.466}} = \frac{61.24}{1.466}$$

$$S = 41.6 \text{ MPH}$$

If you cannot measure the take-off angle you can assume it to be 45^0 and use this short MPH equation:

$$(10) \quad S = 3.87 \cdot \frac{D}{\sqrt{D \pm h}}$$

This has long been considered a minimum speed formula in that we assumed 45^0 take-off insured the lowest possible speed. This is not the case. The formula gives minimum speed only where take-off and landing are at the same level.

$$\text{Then} \quad S = 3.87 \cdot \sqrt{D}$$

In most cases the error is so small as to be of no consequence; however, when (h) and (D) become close to the same number, the error is enough to warrant concern.

The formula that shows you the angle that will give you minimum speed is covered in the miscellaneous formulas chapter.

Remember that equation (10) is only for situations where you <u>cannot</u> measure the take-off angle and must assume 45^0.

So much for this chapter. It has been said that if you work with this stuff long enough, you find you can do anything with numbers and letters. Once you realize this, you go slightly insane and become a mathematician. If you hear sounds in the basement and the corner of your eye has started to twitch, take two aspirin tablets and lie down. It will pass.

CHAPTER V

DERIVATION OF THE CRITICAL SPEED FORMULA

$$S = 3.86\sqrt{R\,(f \pm m)}$$

The derivation of this formula, while it is correct, has two trivial points you must be aware of: the use of the term "centripetal" and the assumption that the tangent of an angle equals the sine of an angle for small angles. Note: The derivation on the following pages shows that sine equals tangent only up to about 6 degrees. However, this constraint is not a problem because sine divided by cosine equals tangent, and tangent always equals percent grade or superelevation. As an example, the tangent of a 6 degree angle is .105. From this you know that a 6 degree angle has a 10.5% grade.

Another point to remember about this equation is that it deals with the radius of a curve made by the scuff marks of a vehicle when it was in a yawing situation. This radius must be adjusted to the radius made by the center of mass of the vehicle. In most cases you need only subtract one half the car width from the radius of the scuff made by the outside wheels. However, in cases of severe yawing action, you <u>must</u> draw all the scuff marks to scale, position a scaled vehicle over these marks, and from several positions plot on paper the curve made by the center of mass.

In spite of what you may have read or have been told, you can use a full drag factor in computing critical speed from scuff marks. You must, however, use care where you measure the radius. Do not include the very first traces of the scuff mark when you measure your chord. Rather move into the scuff 10 to 15 feet or to a point where the curve is smooth and well defined. If both outside wheels leave scuff marks, you should measure your chord in the area where the rear wheel is tracking outside the front wheel. The rear wheel will usually move to about a foot outside the front wheel and stabilize there. If you measure a chord and middle ordinate of the scuff made by the front wheel in that area, you can have confidence that your speed estimate will be very close to correct. Again, remember to adjust your radius for the center of mass of the vehicle. The difference in speed will be very small, usually less than one half MPH, but all formulas deal with the path traveled by the center of mass, and the critical speed formula is no exception.

Anyone who wants to be sure of getting this equation into court should run a few tests at known speeds and be able to testify in court that he knows the equation to be correct because he has personally tested it.

You must also remember that this formula gives close to exact speed and is not minimum speed. Therefore, you must measure carefully. The drag factor you get from test skids must be corrected to the % superelevation (m). Superelevation is the same as grade, but it is measured across the road rather than along it.

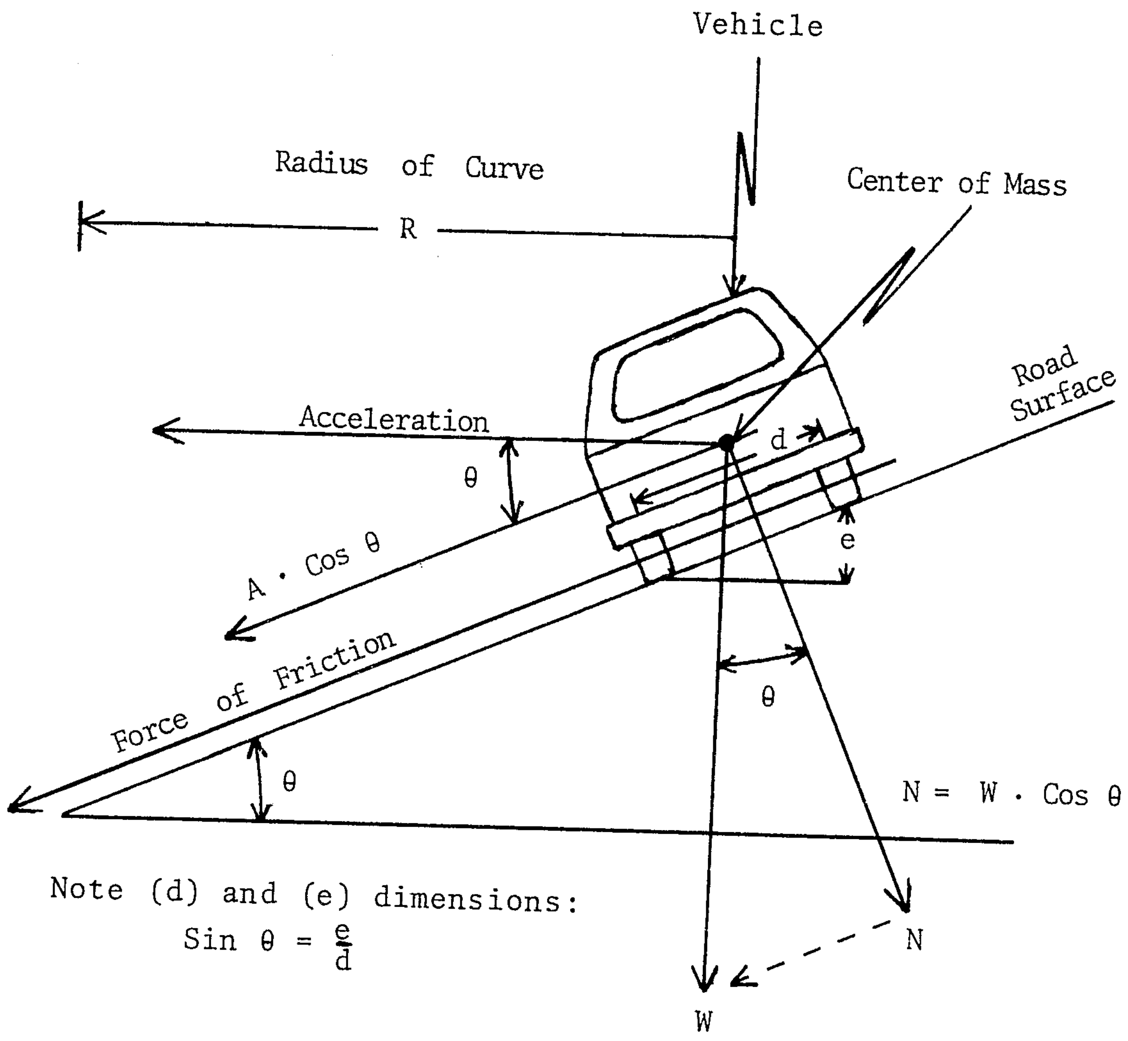
When the front wheels of a car are turned, two things happen. The center of mass of the car tries to keep going straight while the frictional force between the tires and the road surface tries to change the direction of the center of mass. And the centripetal acceleration tries to pull the car into the center of the circle. The forces compromise, and the car starts to turn.

If the car is going fast enough, the force trying to keep it going in a straight line overcomes the drag factor, and the car starts to side slip. You will note centrifugal force is not mentioned in the derivation, nor should it be. Centrifugal force is an unfortunate term, and most physics books avoid it or skirt around it. In this derivation we consider the force acting upon a rotating body to be centripetal acceleration or, if you will, centripetal force.

The desire of the center of mass to go in a straight line once it is set in motion is called "inertia." This follows Newton's Laws of Motion. Old Sir Isaac said, "All you centers of mass gotta go straight!" and after that they always did and still do. Until, that is, they get nailed by an outside unbalanced force like that of a Mack truck. It can be said that the curved scuff mark on a road surface is the path of travel of a vehicle where inertia and centripetal acceleration are in balance.

One more point to remember in the use of the critical speed formula -- you must establish that there was no braking in the scuff marks you measure. This you can easily do by observing the scuff marks themselves, but it is something you must remember.

The following drawing represents the rear view of a car going around a banked curve. Various symbols have been placed in the drawing so that when you see them in the derivation, you will know what part of the operation we are talking about.



Note (d) and (e) dimensions:
Sin θ = $\frac{e}{d}$

Weight Force parallel to
Force of Friction = W · Sin θ

Force of friction = f · N

f = Drag factor

N = Weight component normal to the road surface

W = Weight force parallel to frictional force or (W · Sin θ)

Σ = The sum of... (new symbol)

From Newton's Law: The sum of forces acting upon a body is equal to the mass ($M = \frac{W}{g}$) of the body times its acceleration or

(1) Σ forces = M · A

A rotating body has a centripetal acceleration.

(2) $A = \frac{V^2}{R}$ V = Velocity in FPS
R = Radius of the curve

Now rewrite equation (1) substituting equation (2) for (A).

(3) Σ forces = $M \cdot \frac{V^2}{R}$

Next find the forces involved along the road surface.

Frictional force = f · N

N = W · Cos θ so

<u>Frictional force = f · W · Cos θ</u>

The component of weight in the same direction as frictional force is

<u>Weight force = W · Sin θ</u>

So the forces along the road surface become the above two underlined terms added together.

(4) Σ forces = f · W · Cos θ + W · Sin θ

To get the centripetal acceleration component along the road surface multiply the right hand side of equation (2) by the cosine of the angle at which the road surface is banked.

$$(5) \quad A_r = \frac{V^2}{R} \cdot \text{Cos}\ \theta$$

Now substitute equation (4) and equation (5) into equation (3).

$$f \cdot W \cdot \text{Cos}\ \theta + W\ \text{Sin}\ \theta = M \cdot \frac{V^2}{R} \cdot \text{Cos}\ \theta$$

Change (M) to $(\frac{W}{g})$ and divide out (W), rearrange, and divide both sides by (Cos θ). This is shown in 3 steps.

<u>Step 1</u> - Change (M) to $(\frac{W}{g})$ and divide out (W).

$$\frac{f \cdot \cancel{W} \cdot \text{Cos}\ \theta}{\cancel{W}} + \frac{\cancel{W}\ \text{Sin}\ \theta}{\cancel{W}} = \frac{\frac{\cancel{W}}{g} \cdot \frac{V^2}{R} \cdot \text{Cos}\ \theta}{\cancel{W}}$$

<u>Step 2</u> - Take the term (f · Cos θ) and move it right of the equal sign (remember to change the sign).

$$\text{Sin}\ \theta = \frac{V^2}{gR} \cdot \text{Cos}\ \theta - f \cdot \text{Cos}\ \theta$$

<u>Step 3</u> - Divide both sides of the equal sign by (Cos θ).

$$\frac{\text{Sin}\ \theta}{\text{Cos}\ \theta} = \frac{\frac{V^2}{gR} \cdot \cancel{\text{Cos}\ \theta}}{\cancel{\text{Cos}\ \theta}} - \frac{f \cdot \cancel{\text{Cos}\ \theta}}{\cancel{\text{Cos}\ \theta}}$$

This leaves

$$\frac{\text{Sin}\ \theta}{\text{Cos}\ \theta} = \frac{V^2}{gR} - f$$

Remember $\frac{\text{Sin}\ \theta}{\text{Cos}\ \theta} = \text{Tan}\ \theta$, so the above becomes

$$(6) \quad \frac{V^2}{gR} - f = \text{Tan}\ \theta$$

For small angles up to at least 6 degrees, (Tan θ) equals (Sin θ) or does so at least approximately.

Now from the diagram we find that $(\text{Sin}\ \theta = \frac{e}{d})$. $(\frac{e}{d})$ is the slope or superelevation (m) of the road. Because of this you can say in reference to equation (6) that (Tan θ) is the same as (Sin θ), which is the same as $(\frac{e}{d})$, which is superelevation. So as if by magic (Tan θ) becomes (m) or superelevation.

I don't know about you, but I think this paragraph is enough to knock your hat in the creek.

Now in equation (6) substitute (m) for (Tan θ), rearrange, and multiply both sides by (gR) to get (V^2) alone.

<u>Step 1</u> - Substitute (m) for Tan θ.

$$\frac{V^2}{gR} - f = m$$

<u>Step 2</u> - Move (-f) to the right of the equal sign and change its sign.

$$\frac{V^2}{gR} = (f + m)$$

Step 3 - Multiply both sides by (gR).

$$\cancel{gR} \cdot \frac{V^2}{\cancel{gR}} = gR\ (f + m)$$

This leaves

$$(7)\quad V^2 = gR\ (f + m)$$

Now take the square root of both sides to get (V) alone.

$$\sqrt{V^2} = \sqrt{g} \cdot \sqrt{R\ (f + m)}$$

$$(8)\quad V = 5.67 \cdot \sqrt{R\ (f + m)}$$

Now change to MPH

$$\frac{S \cdot \cancel{1.466}}{\cancel{1.466}} = \frac{5.67}{1.466} \sqrt{R\ (f + m)}$$

$$(9)\quad S = 3.86 \sqrt{R\ (f + m)}$$

If the road banks downward (f + m) will of course become (f - m). Fortunately, there are not too many curves constructed this way.

One last point on the critical speed formula. If you want to be really fussy, divide your answer by the square root of one minus the product of drag factor times superelevation. This gives you the exact critical speed. It also gives you a slightly higher number.

The exact critical speed formula is

$$(10)\quad S = 3.86 \times \frac{\sqrt{R\ (f \pm m)}}{\sqrt{1 - (f \times m)}}$$

CHAPTER VI

KINETIC ENERGY EQUATION

$$Ke = \tfrac{1}{2} MV^2$$

COMBINED SPEED EQUATION

$$S_C = \sqrt{S_1^2 + S_2^2}$$

This chapter contains the derivation of the kinetic energy equation for a skidding vehicle and the derivation of the combined speed equation.

As far as I know, there is no court in the country that will not allow the kinetic energy equation admitted as evidence. However, the combined speed equation may have to be proven in some courts. And since the combined speed equation utilizes the kinetic energy equation, you should know the basic steps in the derivation of the kinetic energy equation. There may well be people who believe there is no such thing as kinetic energy. They also believe the world is flat and Galileo and Newton were garage mechanics in Hoboken, New Jersey.

Kinetic Energy Equation

First we will deal with the kinetic energy of a skidding vehicle:

(1) $Energy = F \cdot D$ (Force times Distance)

(2) $Force = M \cdot A$ (Mass times Acceleration)

(3) $Energy = M \cdot A \cdot D$ (Mass times Acceleration times Distance)

Next we can say

$D = V \cdot T$ (Distance = Velocity times Time)

But average distance is

(4) $D = V_0 \cdot T + \tfrac{1}{2} AT^2$

In this derivation remember (V_0) is initial velocity, and where you see only (V), it means final velocity. Since ($A = \frac{V - V_0}{T}$), when you plug this into equation (4) you get

$$D = V_0 \cdot T + \tfrac{1}{2} \cdot \frac{V - V_0}{\cancel{T}} \cdot T^{\cancel{2}}$$

Now the equation looks like this:

(5) $D = V_0 \cdot T + \tfrac{1}{2} \cdot (V - V_0) \cdot T$

And without changing anything we can write

(6) $D = V_0 \cdot T + (\tfrac{1}{2}V - \tfrac{1}{2}V_0) \cdot T$

Now combine the first and third terms in equation (6), giving $V_0 \cdot T - \frac{1}{2} V_0 \cdot T$, which will leave $\frac{1}{2} \cdot V_0 \cdot T$. Now equation (6) becomes

$$(7) \quad D = \tfrac{1}{2} V_0 \cdot T + \tfrac{1}{2} V \cdot T$$

Now since (T) is multiplied by both sides of the plus sign, we simply bracket the whole works and put ($\frac{1}{2}$) on the outside of the parentheses.

$$(8) \quad D = T \cdot \left[\tfrac{1}{2} \cdot (V_0 + V)\right]$$

Remember we are dealing with the kinetic energy of a skidding vehicle. So using the terms we have come up with so far, we can write the equation (Energy = M • A • D) as

$$\text{Kinetic energy} = M \cdot \frac{V - V_0}{\cancel{T}} \cdot \cancel{T} \cdot \left[\tfrac{1}{2} (V_0 + V)\right]$$

Note (T) cancels out, leaving

$$(9) \quad \text{Kinetic energy} = M \cdot (V - V_0) \cdot \left[\tfrac{1}{2} (V_0 + V)\right]$$

Now we can multiply $(V - V_0)$ by $(V_0 + V)$.

$$\begin{array}{rl} & V - V_0 \\ \text{times} & V + V_0 \\ \hline & V^2 - V \cdot V_0 \\ & \quad + V \cdot V_0 - V_0{}^2 \\ \hline & V^2 + (\text{cancelled}) - V_0{}^2 \end{array}$$

The above has become $(V^2 - V_0{}^2)$.

Now put this into equation (9).

$$Ke = \tfrac{1}{2} M(V^2 - V_0{}^2)$$

And write this equation as

$$(10) \quad Ke = \tfrac{1}{2} MV^2 - \tfrac{1}{2} MV_0{}^2$$

This is the kinetic energy equation for a stopping vehicle. You may never need this, but who knows? Some day your mother-in-law may say, "I wonder where that old kinetic energy comes from?" You can then whip this out and lay it on her.

Aside from impressing your mother-in-law, you can apply the kinetic energy equation to head-on collisions. For example:

If a vehicle weighing 4000 pounds hits a wall head-on going 30 MPH (44 FPS), how does that compare with the same vehicle hitting a wall head-on at 60 MPH (88 FPS)?

First compare kinetic energies for each:

$$Ke = \tfrac{1}{2} MV^2 \qquad M = \frac{W}{g}$$

V = 44 FPS (30 MPH)

$$Ke = \tfrac{1}{2} \cdot \frac{4000}{32.2} \cdot 44^2$$

Ke = 120248.44 (vehicle hits the wall at 30 MPH)

$$Ke = \frac{1}{2} \cdot \frac{4000}{32.2} \cdot 88^2$$

Ke = 480993.77 (vehicle hits the wall at 60 MPH)

Now you can see that at 60 MPH a vehicle has 4 times the kinetic energy it had at 30 MPH. 120248.44 times 4 = 480993.77.

The statement is often made that two cars of equal weight having a head-on collision at 30 MPH each are equal to one car of that same weight hitting a wall head-on at 60 MPH. This is completely wrong, as is obvious here.

If the kinetic energy for one car at 30 MPH is 120248.44, then this number times 2 will be the total kinetic energy used up in a head-on collision of two cars at 30 MPH each. 2 Ke = 120248.44 times 2 = 240496.88. Now this total kinetic energy is only half the kinetic energy of one car at 60 MPH. So a single 4000 pound car hitting a wall head-on at 60 MPH has twice the total kinetic energy of two 4000 lb. cars hitting head-on at 30 MPH each.

A correct statement would be that one 4000 lb. car hitting a wall at 42.4 MPH would produce the same kinetic energy as two similar cars having a head-on collision at 30 MPH each. This

is how you find that out. Let

Ke_{sv} = Kinetic energy single vehicle into wall

Ke_{ho} = Kinetic energy head-on collision for two cars

Then we start with this equation:

$$Ke_{sv} = 2 \cdot Ke_{ho} \quad \text{or}$$

$$\frac{1}{2} MV^2{}_{sv} = 2 \cdot \frac{1}{2} MV^2{}_{ho}$$

$$\frac{1}{2} \frac{W}{g} V^2{}_{sv} = 2 \cdot \frac{1}{2} \frac{W}{g} V^2{}_{ho}$$

$$\frac{\frac{\cancel{W}}{2g} V^2{}_{sv}}{\cancel{W}} = \frac{\frac{\cancel{W}}{g} V^2{}_{ho}}{\cancel{W}}$$

$$\cancel{2g} \cdot \frac{V^2}{\cancel{2g}}_{sv} = \frac{V^2}{\cancel{g}}_{ho} \cdot 2\cancel{g}$$

$$\sqrt{V^2{}_{sv}} = \sqrt{V^2{}_{ho}} \cdot \sqrt{2}$$

$$V_{sv} = V_{ho} \cdot 1.414$$

Back to the question: What speed of a single vehicle (sv) hitting a wall produces Ke equal to the Ke of two cars having a head-on (ho) collision at a speed of 30 MPH (44 FPS) each? You take 44 times 1.414 and get 62 FPS, which divided by 1.466 is 42.4 MPH.

Combined Speed Equation

You will remember from previous training that when a vehicle skids across two surfaces, you must combine these speeds and not add them. The combined speed equation is

$$S_c = \sqrt{S_1{}^2 + S_2{}^2}$$

This is its derivation:

Remember from earlier in this chapter that for a stopping vehicle

$$(1) \quad Ke = ½\,MV^2 - ½\,MV_0{}^2$$

And remember (Ke = W · f · D). Here it is a minus factor because the vehicle is decelerating.

$$(2) \quad -WfD = ½\,MV^2 - ½\,MV_0{}^2$$

Now exchange the minus terms (each one becomes plus) and divide out (½ M).

$$\frac{\cancel{½}\,\cancel{M}V_0{}^2}{\cancel{½}\,\cancel{M}} = \frac{\cancel{½}\,\cancel{M}V^2}{\cancel{½}\,\cancel{M}} + \frac{WfD}{½\,M}$$

A point must be made at this time. Mass (M) is equal to weight divided by acceleration of gravity ($\frac{W}{g}$), and if ($M = \frac{W}{g}$), then ($W = M \cdot g$), so the (W) in the above equation can be replaced by M · g). Now the above equation will look like this:

$$(3) \quad V_0{}^2 = V^2 + \frac{\cancel{M} \cdot g \cdot f \cdot D}{½\,\cancel{M}}$$

Now (g · f · D divided by the fraction ½) is the same as (g · f · D times $\frac{2}{1}$), which equals (2 gfD), and equation (3) becomes

$$(4) \quad V_0{}^2 = V^2 + 2\,gfD$$

Now change to MPH.

$$(S_0 \cdot 1.466)^2 = (S \cdot 1.466)^2 + 2gfD$$

$$\frac{S_0{}^2 \cdot \cancel{2.15}}{\cancel{2.15}} = \frac{S^2 \cdot \cancel{2.15}}{\cancel{2.15}} + \frac{2gfD}{2.15}$$

Remember 2g = 2 times 32.2 = 64.4.

$$(5) \quad S_0{}^2 = S^2 + \frac{64.4\ fD}{2.15}$$

$$(6) \quad \sqrt{S_0{}^2} = \sqrt{S^2 + 30\ fD}$$

Note that at this point (30 fD) can simply be replaced by (S^2). The derivation of the combined speed equation has partially overlapped with the derivation of the minimum speed equation.

So the final equation becomes

$$(7) \quad S_0 = \sqrt{S_1{}^2 + S_2{}^2}$$

The above (S_0) is referred to as (S_c) in the combined speed equation. (S_c) simply means "speed combined," but actually it is initial speed.

CHAPTER VII

DERIVATION OF THE RADIUS EQUATION

$$R = \frac{C^2}{8M} + \frac{M}{2}$$

and

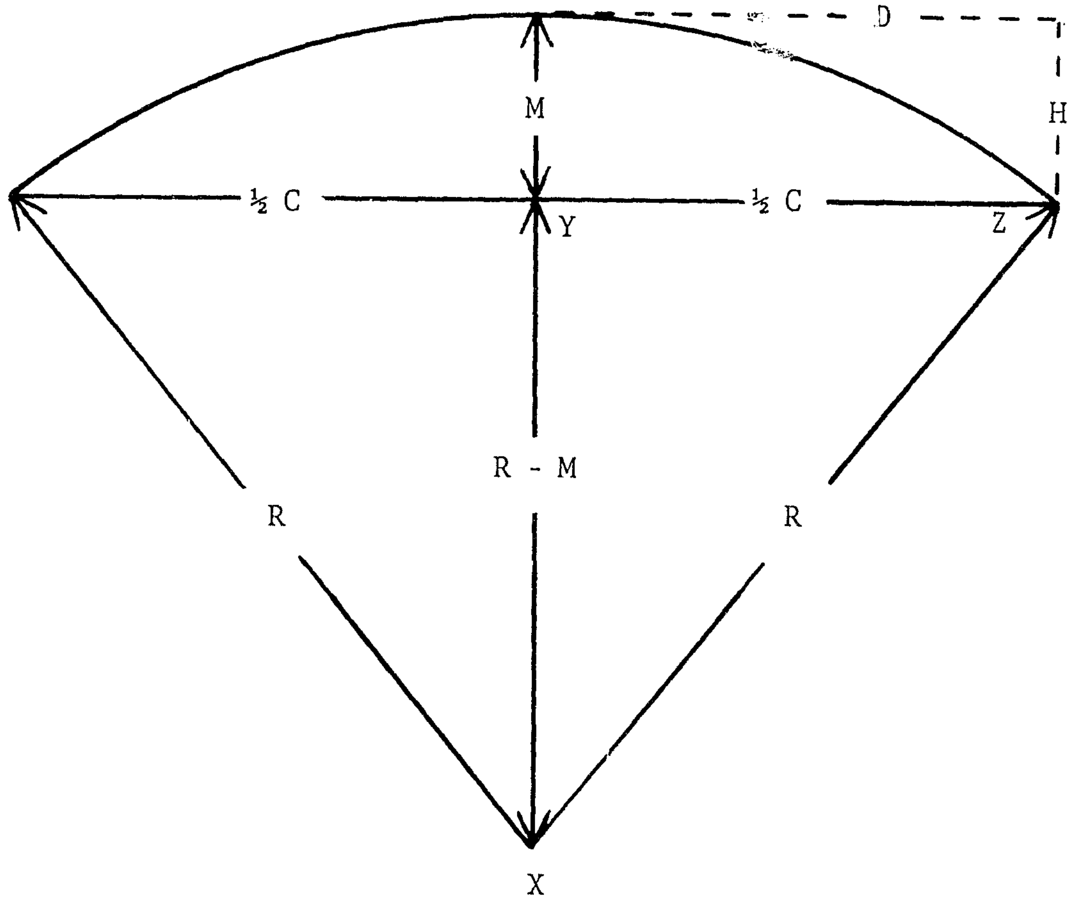
DERIVATION OF THE TANGENT OFFSET EQUATIONS

$$H = \frac{D^2}{2R} \qquad \text{and} \qquad H = R - \sqrt{R^2 - D^2}$$

(approximate) (exact)

Radius Equation

The methods used to measure a chord and middle ordinate of a curve have been covered in previous courses as has the use of the radius equation, so there is no need to go over this old ground. The attached diagram and explanation are what you need to prove that the equation used is correct and has scientific foundation.



R = Radius of the curve

½C = One half of the chord

M = Middle ordinate

R - M = The radius of the circle minus the length of the middle ordinate

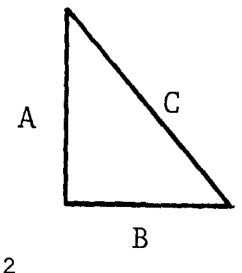
X - Y - Z = The 3 corners of a triangle formed inside the circle ("X" is also the center of the circle.)

D = A distance to be used in the tangent offset equation

H = A distance to be used in the tangent offset equation

First, consider the triangle X - Y - Z, which is a right triangle.

The Pythagorean theorem says: "In any right triangle the square of the hypotenuse is equal to the sum of the squares of the arms."



$$C^2 = A^2 + B^2$$

$$C = \sqrt{A^2 + B^2}$$

$$B = \sqrt{C^2 - A^2}$$

$$A = \sqrt{C^2 - B^2}$$

Pythagoras was a Greek who ran a secret society in Italy around 580 BC. There was a snitch in the society who let the above secret out, and because of this, you can derive the radius equation.

Apply the Pythagorean theorem to our triangle X - Y - Z.

$$(1)\quad (\tfrac{1}{2}C)^2 + (R - M)^2 = R^2$$

Square (R - M)

$$\begin{array}{rr} & R - M \\ \text{times} & \underline{R - M} \\ & R^2 - RM \qquad \\ & \underline{\quad - RM + M^2} \\ & R^2 - 2RM + M^2 \end{array}$$

Square ($\tfrac{1}{2}$C)

$$(\tfrac{1}{2}C)^2 = \frac{C^2}{1} \cdot \frac{1}{4} \text{ or } \frac{C^2}{4}$$

Rewrite equation (1) and cancel out (R^2).

$$(2)\quad \frac{C^2}{4} + \cancel{R}^2 - 2RM + M^2 = \cancel{R}^2$$

Next take the (-2RM) and move it right of the equal sign, which makes it a (+2RM).

$$(3)\quad \frac{C^2}{4} + M^2 = 2RM$$

To get (R) alone divide (2M) through equation (3).

$$\frac{\frac{C^2}{4}}{2M} + \frac{M^{\cancel{2}}}{2\cancel{M}} = \frac{\cancel{2}\ R\cancel{M}}{\cancel{2}\cancel{M}}$$

You then are left with

$$(4)\quad R = \frac{\frac{C^2}{4}}{2M} + \frac{M}{2}$$

Now invert and multiply the first fraction.

$$\frac{C^2}{4} \text{ divided by } \frac{2M}{1} = \frac{C^2}{4} \text{ times } \frac{1}{2M}$$

This becomes

$$\frac{C^2}{8M}$$

From that, of course, you get the finished product.

$$(5)\quad R = \frac{C^2}{8M} + \frac{M}{2}$$

Accidents will quite often occur on large curves in the road or will be closely related to curves. Making a scale diagram of a curve usually presents a problem.

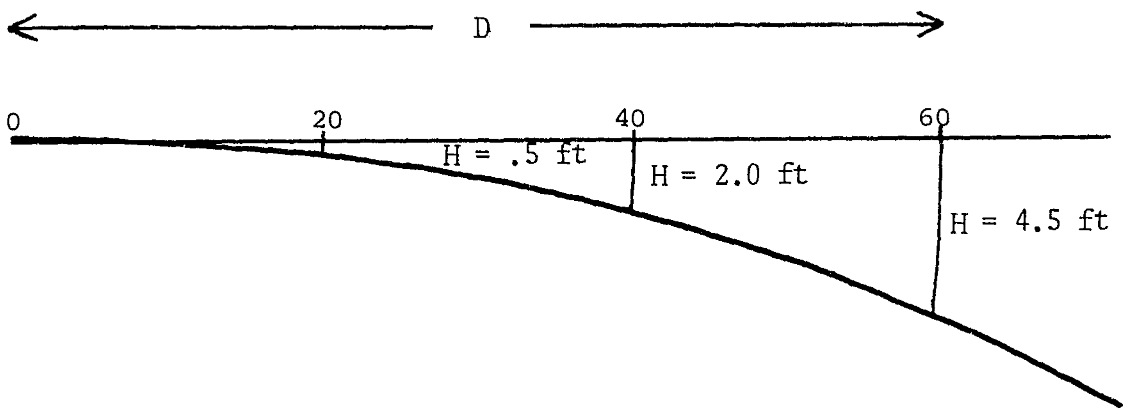
Most curves in the open country will have a radius of 800 or more feet. Such a radius is too large to be drawn with an ordinary compass, even to a 1:240 scale. The radius equation is fine for finding out what the radius is but does nothing to put it on paper for you.

For this purpose we have the tangent offset equations.

Tangent Offset Equations

There are two equations, the approximate equation and the exact equation, as shown on the first page in this chapter. If you have a calculator with a square root function, by all means use the exact equation. The approximate equation is quite close to being accurate, but once you get beyond 150 running feet of curve, the discrepancy gets noticeable. At 200 feet you can be off by as much as 10 feet if the curve has a radius of 300 feet. In open country, however, the larger curves produce much less error. If a curve has a radius of 1000 feet, the difference at 200 feet is about 2 inches and at 400 feet is only about 3 feet. Anyway, to be safe use the exact formula if you can.

We will assume you are to draw a curve with a radius of 400 feet. First, draw a straight line on graph paper and mark off the line in 20-foot increments. Use a scale of one inch to 10 feet. These increments will show the value of (D), and the value of (R) will be 400. The answer you get will be in feet measured at a right angle from the points on the line. When you connect these points by use of a flexicurve, you will reproduce a curve with a radius of 400 feet.



$$H = R - \sqrt{R^2 - D^2}$$

$$(D = 20) \quad H = 400 - \sqrt{400^2 - 20^2} = .5 \text{ ft}$$

$$(D = 40) \quad H = 400 - \sqrt{400^2 - 40^2} = 2.0 \text{ ft}$$

$$(D = 60) \quad H = 400 - \sqrt{400^2 - 60^2} = 4.5 \text{ ft}$$

If you have a very long curve, you can use **the zero** point as the center of your curve and work both directions. In that way each measurement can be used twice.

Now for the derivation:

Go back to the radius drawing at the beginning of this chapter. From the drawing you will see that $(H = M)$ and $(D = \frac{1}{2}C)$. If $(D = \frac{1}{2}C)$, then (C) has to equal $(2D)$.

The radius formula is

$$(1) \quad R = \frac{C^2}{8M} + \frac{M}{2}$$

Now substitute (H) for (M) and $(2D)$ for (C). You get

$$(2) \quad R = \frac{(2D)^2}{8H} + \frac{H}{2}$$

Note that (D) is usually much greater than (H), so certainly (D^2) is even greater. Under these conditions $(\frac{H}{2})$ can be neglected, and equation (2) reduces to

$$R = \frac{4D^2}{8H} \quad \text{or}$$

$$(3) \quad H = \frac{D^2}{2R}$$

This is the approximation of the tangent offset equation. By ignoring $(\frac{H}{2})$ you inject an error into the equation. That is why you should avoid using the approximate equation if possible.

As (H) becomes larger the error in the equation becomes greater, and obviously there must be a more accurate way of doing things.

Return to equation (2) and solve for (H).

$$R = \frac{(2D)^2}{8H} + \frac{H}{2}$$

First multiply through by (2H).

$$2HR = \frac{(2D)^2 \cdot \cancel{2H}}{\underset{4}{\cancel{8H}}} + \frac{\cancel{2}HH}{\cancel{2}}$$

$$2HR = \frac{\cancel{4}D^2}{\cancel{4}} + H^2$$

$$(4) \quad 2HR = D^2 + H^2$$

Interchange (D^2) and (H^2) and move (2HR) over the equal sign, which leaves

$$(5) \quad H^2 - 2RH + D^2 = 0$$

Now apply the general equation for solving quadratic equations.

$$H = \frac{2R \pm \sqrt{4R^2 - 4D^2}}{2}$$

$$H = \frac{2R \pm \sqrt{4\ (R^2 - D^2)}}{2}$$

$$H = \frac{\cancel{2}R \pm \cancel{2}\sqrt{R^2 - D^2}}{\cancel{2}}$$

$$(6) \quad H = R \pm \sqrt{R^2 - D^2}$$

Now physically (H) can never be greater than (R), so

$$(7) \quad H = R - \sqrt{R^2 - D^2}$$ (Exact tangent offset equation)

CHAPTER VIII

QUADRATIC EQUATIONS

On rare occasions you will run into a situation where you will have (X) and (X^2) in an equation and have to find the value of (X).

The only way you can do this is by solving a quadratic equation.

I will not devote any space to the source or origin of the quadratics. If you have a burning desire to learn more about these monsters, you can find almost any number of high school and college math books that gleefully devote chapters to the subject.

Instead I will set up a problem where you will need to use a quadratic equation, and hopefully you will be able to relate this exercise to any combination of situations you may run into.

We will use two basic equations:

The standard quadratic equation form

$$(1) \quad AX^2 + BX + C = 0$$

where (A) is not equal to zero, and

the general formula for solving a quadratic equation

$$(2) \quad X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

In Chapter VII we saw a quadratic equation used in the derivation of the tangent offset equation. A more job-related use of a quadratic equation might involve skidmarks.

Following is a hypothetical accident situation illustrating such use.

Assume you have an intersection accident where one of the vehicles left 200 feet of skidmarks leading up to the point of impact. Assume the drag factor for the road surface is .80. Further assume you establish the speed at the start of the visible skidmarks at 132.3 FPS (90 MPH). In a time-distance reconstruction you must find out how long in seconds (T) it took for the vehicle to skid those 200 feet to the point of impact.

D = Distance of skid (200 feet)
f = Drag factor (.80)
V_0 = Initial velocity (132.3 FPS)
T = Time in seconds (unknown)

First consider a distance formula that tells you how far a vehicle will skid from a known speed on a known drag factor in any period of time.

$$(3) \quad D = V_0 \cdot T - 16.1 \cdot f \cdot T^2$$

In our problem we know distance and all that other garbage, but we don't know the time. Logically, then, we should be able to solve equation (3) for (T) and be all set. However, both (T) and (T^2) are in the equation, and therein lies a difficulty.

You now have two options. You can say "damn" and take your sister-in-law out for an ice cream cone. Defense attorneys will love you, and the only witness stand you will be on is in divorce court. If your sister-in-law is ugly, or if you are afraid of your wife (or both), you can solve equation (3) for (T), in which case you must solve a quadratic equation.

You must rearrange equation (3) to make it equal zero. In other words, move all the terms to the same side of the equal sign. Remember when you move a term across the equal sign, you must change its sign: plus to minus and minus to plus.

Equation (3) is repeated.

$$D = V_0 \cdot T - 16.1 \cdot f \cdot T^2$$

Move all terms to the right side.

$$0 = -16.1 \cdot f \cdot T^2 + V_0 \cdot T - D \quad \text{or}$$

$$(4) \quad -16.1 \cdot f \cdot T^2 + V_0 \cdot T - D = 0$$

Equation (1), the standard quadratic equation form, is repeated and relabeled equation (5).

$$(5) \quad AX^2 + BX + C = 0$$

Equation (5) must be related to equation (4):

Letters in equation (5)		What these letters relate to in equation (4)
A	=	Everything multiplied by the squared term (T^2), or ($-16.1 \cdot f$)
X^2	=	The squared term (T^2)
B	=	Everything multiplied by the companion term (T), or (V_0)
X	=	The companion term (T)
C	=	Everything not multiplied by the squared term or its companion term. [In equation (4) it is only a minus D (-D).]

Next supply the values from our hypothetical accident situation.

A = -16.1 times .80 = -12.88

X^2 = T^2 (the unknown)

B = 90 MPH or 132.3 FPS

X = T

C = -200 feet

Equation (2), the general formula for solving a quadratic equation, is repeated and relabeled equation (6).

$$(6) \quad X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Now put the above values into equation (6).

Step 1

$$T = \frac{-132.3 \pm \sqrt{132.3^2 - [4 \cdot (-12.88) \cdot (-200)]}}{2 \cdot -12.88}$$

Step 2

$$T = \frac{-132.3 \pm \sqrt{17503.29 - [+10304]}}{-25.76}$$

Note under the square root sign a minus outside the bracket and a plus inside. This means a minus times a plus, which leaves a minus number.

Step 3

$$T = \frac{-132.3 \pm \sqrt{17503.29 - 10304}}{-25.76}$$

Step 4

$$T = \frac{-132.3 \pm \sqrt{7199.29}}{-25.76}$$

Step 5

$$(7) \quad T = \frac{-132.3 \pm 84.84}{-25.76}$$

Now consider the plus and minus between terms in equation (7). The presence of both signs means you will have two answers, but only one answer will be logical.

Try first

$$T = \frac{-132.3 - 84.84}{-25.76} = \frac{-217.14}{-25.76}$$

(8) T = 8.4 seconds

Try next

$$T = \frac{-132.3 + 84.84}{-25.76} = \frac{-47.46}{-25.76}$$

(9) T = 1.8 seconds

Obviously (9) is the correct answer. He started skidding at a velocity of 132.3 FPS, so on a .80 drag factor his slide-to-stop distance (without hitting anything) would be only 337 feet. At 132.3 FPS (90 MPH) his time used to skid to a stop (without hitting anything) would be about 5 seconds.

This makes the 8.4 second time ridiculous. In 8 seconds he would have had time to come to a complete stop, get out, shout obscenities at the other driver, and roar away.

CHAPTER IX

TIME - DISTANCE EQUATIONS

In a great many intersection or right-of-way related accidents, the following questions are raised:

1. Was the car with the right-of-way exceeding the speed limit?
2. Did the car stop for the stop sign?
3. If the car started from a stopped position, what speed could be reached in a specific time or distance?
4. How long does it take for a car to move a specific distance
    a. at an average speed?
    b. while accelerating?
    c. while decelerating?
5. If two cars are on a collision course, what are their relative positions at any point in time?
6. What was the speed of the car at any point in the skidding distance?
7. What was the speed of the car at any time in the skid?

8. How far does a car travel in any given time when an average or constant speed is combined with a preceding or following slow-down or acceleration?

This chapter will be devoted to the various time-distance equations that are used to answer these questions.

There will be no derivations. These formulas have all been in use for hundreds of years and appear in all college physics textbooks.

A question can be raised as to whether there are sufficient data, or whether the data are sufficiently accurate, to make the use of any of these equations appropriate. A question should never be raised or allowed as to the validity of any of these equations.

This is not to say that an attorney will never question any of these equations. I have seen attorneys who would question the properties of electricity while being strapped into the electric chair.

These equations are of extreme importance as they often provide a back door approach to an accident where normal data, such as the length of skidmarks, are inaccurate or unavailable.

They also provide a cross-check in finding any possible error you might have made in the conventional methods of determining speed.

In past chapters we have covered methods of finding the speed of a vehicle from skidmarks. We will now cover methods of finding the speed that a vehicle can accelerate to in a known distance or time. We will be dealing with the acceleration factor, and a point must be made here. When you run tests to determine a drag factor, you can use any vehicle that has good brakes and an accurate speedometer. But when you run tests to determine an acceleration factor, you must use a vehicle that has approximately the same weight, horsepower, and rear-end ratio as the vehicle for which you are finding the acceleration factor. These need not be exactly the same, only nearly so.

The acceleration factor varies from car to car. A VW and a Shelby GT 500 will skid to a stop in the same distance from the same speed, but in accelerating from a stop, the GT will suck the VW through its carburetors quicker than you can say "Deutschland über alles!"

If you need to find the acceleration factor of a vehicle in a particular location, you must run tests with a similar vehicle in that location and in the same direction as that in which the vehicle for which you are finding the acceleration factor was traveling.

Acceleration for Uniform Motion

In this case you must accelerate the test vehicle to a speed you know to be correct and record the required time with a stop watch.

(1) $A = \frac{V}{T}$

A = Rate of acceleration
V = Velocity in FPS
T = Time in seconds

Assume the vehicle goes from 0 to 30 MPH (44 FPS) in 8 seconds. What is the rate of acceleration?

$$A = \frac{44}{8} = 5.5 \text{ feet per second per second}$$

Rate of acceleration is 5.5 feet per second squared.

(2) $f = \frac{\text{Rate of acceleration}}{g}$

f = Acceleration factor
g = 32.2 feet per second squared

$$f = \frac{5.5}{32.2}$$

$$\underline{f = .17}$$

Rate of acceleration (A) = $32.2 \cdot f$

$$V = A\ T$$

$$V = 32.2 \cdot f \cdot T$$

(3) $f = \frac{V}{32.2 \cdot T}$

You accelerate to 44 FPS in 8 seconds.

$$f = \frac{44}{32.2 \cdot 8} = \frac{44}{257.6}$$

$$\underline{f = .17}$$

The same result is obtained from equation (3) as from equation (2) and is obtained more easily.

You accelerate and record the speed you reach in a distance you can measure.

(4) $f = \frac{S^2}{30D}$

A car accelerates to 30 MPH in 176 feet.

$$f = \frac{30^2}{30 \cdot 176} = .17$$

Note: (S) is in MPH.

If a car goes from a stop to a speed of 30 MPH (44 FPS) with an acceleration factor of .17, how long in seconds will it take?

$T = \frac{V}{A}$ $\quad$ $(A = 32.2 \cdot f)$

(5) $T = \frac{V}{32.2 \cdot f}$

$$T = \frac{44}{32.2 \cdot .17} = 8 \text{ seconds}$$

Equation (5) also gives the time required to skid to a stop from a known speed.

A vehicle has brake lock-up at 60 MPH (88 FPS) and slides to a stop on a drag factor of .70. How long does it take in seconds?

$$T = \frac{88}{32.2 \cdot .70} = 3.9 \text{ seconds}$$

If a vehicle accelerates for 8 seconds with an acceleration factor of .17, how much speed will it gain?

$V = A\ T$ $\quad$ $(A = 32.2 \cdot f)$

(6) $V = 32.2 \cdot f \cdot T$

$V = 32.2 \cdot .17 \cdot 8$

$V = 43.792$ FPS = approximately 30 MPH

Equation (6) also works for a skid.

A vehicle's brakes are locked up and the vehicle skids for 4 seconds, how much speed will it lose if the drag factor is .60?

$V = 32.2 \cdot .60 \cdot 4$

$V = 77$ FPS = 52 MPH

A driver says he was going 70 MPH when he saw a hazard and applied his brakes. He says he had slowed to 30 MPH when he hit the other car. If (f) =.70, how long had he been braking?

(7) $T = \frac{V_0 - V}{32.2 \cdot f}$ $\quad$ $V_0 = 70$ MPH = 102.9 FPS

$V = 30$ MPH = 44.1 FPS

$$T = \frac{102.9 - 44.1}{32.2 \cdot .70} = 2.6 \text{ seconds}$$

The same result is obtained from equation (3) as from equation (2) and is obtained more easily.

You accelerate and record the speed you reach in a distance you can measure.

$$(4) \quad f = \frac{S^2}{30D}$$

A car accelerates to 30 MPH in 176 feet.

$$f = \frac{30^2}{30 \cdot 176} = .17$$

Note: (S) is in MPH.

If a car goes from a stop to a speed of 30 MPH (44 FPS) with an acceleration factor of .17, how long in seconds will it take?

$$T = \frac{V}{A} \qquad (A = 32.2 \cdot f)$$

$$(5) \quad T = \frac{V}{32.2 \cdot f}$$

$$T = \frac{44}{32.2 \cdot .17} = 8 \text{ seconds}$$

Equation (5) also gives the time required to skid to a stop from a known speed.

A vehicle has brake lock-up at 60 MPH (88 FPS) and slides to a stop on a drag factor of .70. How long does it take in seconds?

$$T = \frac{88}{32.2 \cdot .70} = 3.9 \text{ seconds}$$

If a vehicle accelerates for 8 seconds with an acceleration factor of .17, how much speed will it gain?

$$V = A\ T \qquad (A = 32.2 \cdot f)$$

$$(6) \quad V = 32.2 \cdot f \cdot T$$

$$V = 32.2 \cdot .17 \cdot 8$$

$$V = 43.792 \text{ FPS} = \text{approximately } 30 \text{ MPH}$$

Equation (6) also works for a skid.

A vehicle's brakes are locked up and the vehicle skids for 4 seconds, how much speed will it lose if the drag factor is .60?

$$V = 32.2 \cdot .60 \cdot 4$$

$$V = 77 \text{ FPS} = 52 \text{ MPH}$$

A driver says he was going 70 MPH when he saw a hazard and applied his brakes. He says he had slowed to 30 MPH when he hit the other car. If (f) =.70, how long had he been braking?

$$(7) \quad T = \frac{V_0 - V}{32.2 \cdot f} \qquad V_0 = 70 \text{ MPH} = 102.9 \text{ FPS}$$

$$V = 30 \text{ MPH} = 44.1 \text{ FPS}$$

$$T = \frac{102.9 - 44.1}{32.2 \cdot .70} = 2.6 \text{ seconds}$$

A car was going 30 MPH when its driver hit the brakes and skidded into another car turning out of a driveway. The driver claims he had stopped at a stop sign 70 feet back from the place he started braking. His type of car has an acceleration factor of .18. Is he telling the truth about stopping at the stop sign?

$$(8)\quad D = \frac{S^2}{30f} = \frac{30^2}{30 \cdot .18}$$

$$D = 166 \text{ feet}$$

It would take him 166 feet to accelerate to 30 MPH, so he is telling a baldfaced lie.

A driver says he started up from a stop sign and got 80 feet into the intersection when he was hit. You know his acceleration factor is .17. How long in seconds was it from the time the driver started up until he was hit?

$$(9)\quad T = .25\,\frac{\sqrt{D}}{\sqrt{f}}$$

$$T = .25\,\frac{\sqrt{80}}{\sqrt{.17}}$$

$$T = 5.4 \text{ seconds}$$

This means that when the driver started up, the car that hit him was 5.4 seconds away. If this oncoming car was going 60 MPH (88 FPS), it would have been 475 feet away.

Equation (9) also works for a skid.

If a car skids to a stop for 200 feet on a drag factor of .70, what is the time for this?

$$T = .25\,\frac{\sqrt{200}}{\sqrt{.70}}$$

$$T = 4.2 \text{ seconds}$$

This will give you a basis to estimate how long in seconds the driver had been aware of the hazard.

Remember equation (9) is for start from <u>stop</u> or slide to <u>stop</u>.

If you want to know the time in seconds to skid from one point along a skid to another point, you must use the quadratic equation.

$$D = V_0 \cdot T - 16.1 \cdot f \cdot T^2$$

$$-16.1 \cdot f \cdot T^2 + V_0 \cdot T - D = 0$$

$$(\text{Remember } AX^2 + BX + C = 0)$$

$$(10)\quad T = \frac{-V_0 \pm \sqrt{V_0{}^2 - \left[(4) \cdot (-16.1) \cdot (f) \cdot (-D)\right]}}{(-32.2 \cdot (f)}$$

If a car is going 50 MPH (73.5 FPS) when the brakes are applied, how far will the car skid in .75 seconds if (f) = .80?

(11) $D = V_0 \cdot T - \frac{1}{2} A T^2$

$f = .80$

$A = 32.2 \cdot f$

$T = .75$ seconds

$V_0 = 73.5$ FPS

$D = (73.5 \cdot .75) - (\frac{1}{2} \cdot 32.2 \cdot .80 \cdot .75^2)$

$D = 47.8$ feet

If a driver has been going 80 MPH (117.6 FPS) for 6 seconds when he slams on his brakes and starts to skid, how far will he travel in a <u>total</u> of 8 seconds?

Here the equation is the same as (11), but (T) = Total time and (T^2) = Slowing time squared.

$f = .80$

$T = 6 + 2 = 8$ seconds

$T^2 =$ 2 seconds squared

$D = (117.6 \cdot 8) - (\frac{1}{2} \cdot 32.2 \cdot .80 \cdot 2^2)$

$D = 889$ feet

You can calculate the speed of a vehicle at any time in a skid (seconds into the skid).

A car's brakes are locked up at 60 MPH (88 FPS). What is its speed after 3 seconds of sliding on a drag factor of .75?

(12) $V = V_0 - A T$

$A = 32.2 \cdot f$

$f = .75$

$T = 3$ seconds

$V_0 = 88$ FPS

$V = 88 - 32.2 \cdot .75 \cdot 3$

$V = 15.55$ FPS $\div 1.466$ so

$S = 10.6$ MPH

Note: If equation (12) were used for a car accelerating, it would have a plus sign before (AT). (f) would be the acceleration factor and (V_0) would be whatever speed the car was going at the start of acceleration.

You can calculate the speed of a vehicle at any distance in a skid.

A car starts to skid from 70 MPH (102.62 FPS). What is his speed at the 100 foot mark in his skid?

(13) $V^2 = V_0^2 - 2 AD$

$A = 32.2 \cdot f$

$f = .80$

$D = 100$ feet

$V_0^2 = 102.9^2$

$V^2 = (102.62)^2 - (2 \cdot 32.2 \cdot .80 \cdot 100)$

$$V^2 = 10530.86 - 5152$$

$$\sqrt{V^2} = \sqrt{5378.86}$$

$$V = 73.34 \text{ FPS} \div 1.466 \text{ so}$$

$$S = 50 \text{ MPH}$$

For an acceleration problem, equation (13) would have a plus sign before (2AD) and (V_0) would be whatever speed the car was going at the start of acceleration.

The following equations are listed for your information only. You may see them in court. The other side may try to confuse you with them.

These are the basic equations and are for average values only.

(14) $V = \frac{D}{T}$

(15) $T = \frac{D}{V}$

(16) $D = V \cdot T$

If you wish to convert the average value (V) from FPS to MPH:

$$V = \frac{D}{T} = \frac{\text{Distance ft.}}{\text{Time sec.}}$$

$$S = \frac{\text{miles}}{\text{hour}} = \frac{\frac{\text{ft.}}{5280}}{\frac{\text{sec.}}{3600}}$$

$$S = \frac{\text{ft.}}{5280} \cdot \frac{3600}{\text{sec.}} = \frac{3600}{5280} \cdot \frac{\text{ft.}}{\text{sec.}}$$

So you can say

(17) S (in miles per hour) = $.682 \cdot \frac{D}{T}$

(18) T (in seconds) = $.682 \cdot \frac{D}{S}$

Now solve equation (18) for (D).

$$S \cdot T = .682 \cdot \frac{D}{\cancel{S}} \cdot \cancel{S}$$

$$\frac{S \cdot T}{.682} = \frac{\cancel{.682} \cdot D}{\cancel{.682}}$$

$$D = \frac{S \cdot T}{.682}$$

$$D = \frac{1}{.682} \cdot S \cdot T$$

(19) $D = 1.466 \cdot S \cdot T$

Remember the above equations are for average values only.

Following are more conversions to MPH. Remember (V) gives you FPS and (S) gives you MPH.

$$V = V_0 + A\,T \qquad (A = 32.2 \cdot f)$$

$$V - V_0 = A\,T$$

$$V - V_0 = 32.2 \cdot f \cdot T$$

$$\frac{\cancel{1.466} \cdot S}{\cancel{1.466}} = \frac{32.2}{1.466} \cdot f \cdot T$$

$$(20) \quad S = 22 \cdot f \cdot T$$

Now solve equation (20) for (f).

$$f = \frac{1}{22} \cdot \frac{S}{T}$$

$$(21) \quad f = .045 \cdot \frac{S}{T} \qquad \text{and}$$

$$(22) \quad T = .045 \cdot \frac{S}{f}$$

In equation (20) we see

$$S = 22 \cdot f \cdot T$$

And from equation (4) we see

$$S = \sqrt{30} \cdot \sqrt{Df} = 5.5\sqrt{Df} \qquad \text{so}$$

$$22 \cdot f \cdot T = 5.5\sqrt{D \cdot f} \qquad \text{or}$$

$$(22)^2 \cdot f^{\cancel{2}} \cdot T^2 = 30 \cdot D \cdot \cancel{f} \qquad \text{or}$$

$$f = \frac{30}{(22)^2} \cdot \frac{D}{T^2}$$

$$f = .062 \cdot \frac{D}{T^2}$$

$$\sqrt{T^2} = \sqrt{.062\,\frac{D}{f}}$$

$$(23) \quad T = .25 \cdot \sqrt{\frac{D}{f}}$$

Here is the last time-distance explanation. Return to

$$f = .062 \cdot \frac{D}{T^2} \qquad \text{and solve for (D).}$$

$$T^2 \cdot f = .062 \cdot \frac{D}{\cancel{T^2}} \cdot \cancel{T^2}$$

$$\frac{T^2 \cdot f}{.062} = \frac{\cancel{.062} \cdot D}{\cancel{.062}}$$

$$D = \frac{1}{.062} \cdot f \cdot T^2$$

$$(24) \quad D = 16.1 \cdot f \cdot T^2$$

Well, so much for Chapter IX. The next set of skidmarks you examine should tell you everything but what the driver had for breakfast. If an autopsy is performed on the driver, you may even find that out.

CHAPTER X

PERSPECTIVE GRID PHOTOGRAPHY

This chapter will be short, as the course we teach on grid photography or photogrammetry deals mostly with techniques for using the camera and constructing a grid on a photograph from an object of known size that appears in the photograph.

There is a booklet by J. Stannard Baker published by NUTI that covers this subject in detail.

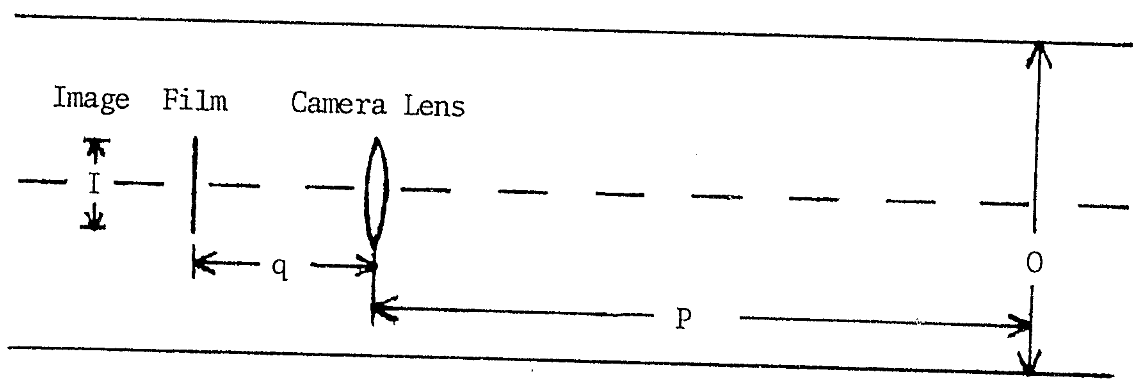
This chapter will deal with an equation that shows the relationship between the size of an image on a negative and the true size of the object photographed. You will learn how a vanishing point is established mathematically. The equation will be of little use to you, however, unless the perspective grid is challenged in court.

When you look down a road or railroad track, it narrows as it vanishes in the distance. Of course it does not actually narrow but only seems to do so. The place where the edges of the road or the rails of the track seem to touch is called the vanishing point.

The same phenomenon occurs in the lens of a camera so that a vanishing point is projected onto the negative. This vanishing point is used to construct a grid on the photograph from which you can establish measurements.

A camera with normal lens must be used for the greatest accuracy. A normal lens is one where the distance from the lens to the face of the negative is equal to the diagonal measurement of the negative. In a 35mm camera that distance must be 50mm to meet this requirement.

In the drawing below, the various distances have been given symbols. When these symbols are used in an equation, you can see what distances are being represented.



I = Image size, or length of the image on the negative (This will be the unknown.)

q = Image distance, or the distance from the lens to to the face of the negative (With a 50mm lens this distance is 50mm or 1.968 inches or .164 feet.)

P = Object distance, or the distance from the lens to the object (in feet)

O = Object length, or the actual size of the object you are photographing

$$\frac{\text{Image length}}{\text{Object length}} = \frac{\text{Image distance}}{\text{Object distance}} \quad \text{or}$$

$$\frac{I}{O} = \frac{q}{P}$$

$$(1) \qquad I = \frac{q \cdot O}{P}$$

Convert 50mm to 1.968 inches this way:

2.54 cm = 1 inch

5 cm = 50mm

50mm = 5cm ÷ 2.54 cm = 1.968 inches

To convert 1.968 inches to feet, simply divide by 12.

1.968 ÷ 12 = .164 feet

To put the above mess to a practical application, assume you have taken a picture down a road and are concerned with one lane which is 10 feet wide. If you pick a spot on the lane 25 feet from the camera lens, you know how wide the lane is at that spot, but how wide will the lane be on the negative in a 35mm camera with a 50mm lens?

I = Unknown

q = .164 feet

O = 10 feet

P = 25 feet

Now return to equation (1):

$$I = \frac{q \cdot O}{P}$$

$$I = \frac{.164 \cdot 10}{25} = \frac{1.64}{25}$$

I = .0656 feet or .7872 inches

(.0656 feet · 12 = .7872 inches)

Now ask yourself, "How wide will the 10 foot lane be on the negative if you pick a spot 50 feet from the camera lens?"

$$I = \frac{.164 \cdot 10}{50} = \frac{1.64}{50}$$

I = .0328 feet or .3936 inches

Note when the distance was doubled the size of the image was cut in half.

If you now double 50 to make (P) = 100, (I) will be .1968 inches. Each time you double (P), (I) is cut in half.

P = 200 I = .0984 inches

P = 400 I = .0492 inches

This progresses in an orderly manner until at infinity (I) = zero.

CHAPTER XI

CONSERVATION OF LINEAR MOMENTUM

This chapter will deal with the law of conservation of momentum and how it can be used to determine the speeds of vehicles at impact.

The law of conservation of momentum states: "For any collision, the vector sum of the momentums of the colliding bodies after collision is equal to the vector sum of their momentums before collision." This is just a fancy way of saying, "If you can determine momentum after impact, you can determine momentum before impact." We shall see later in the chapter that if you can do this, you can also determine velocity at impact.

The idea that momentum is conserved throughout the entire universe was first thought up by a Franciscan friar in about 1347. In those days you had to be sure to give God credit for all things great and small, so the friar was very quick to say that the Almighty was doing all the conserving. You did this for self protection because if you failed to give God the credit, a man came around with a big axe and chopped your head off.

The conservation law was handed down with minor changes until the 17th Century, when Galileo, Newton, and other physicists put it in the form used today. Sir Isaac Newton sat around under apple trees getting concussions until he came to the conclusion that momentum was equal to mass times vector velocity, and so it stands today.

The conservation laws are especially useful because there is no deviation from them whatsoever. They are absolute in every detail. Momentum is conserved in a closed system. It does not matter if that closed system is the universe, our solar system, or a stretch of U.S. 53 south of International Falls where two Detroit spawned cocoons collide on a Thursday afternoon.

In using this law to determine speed at impact, we completely ignore the amount of damage done through impact. This is possible because we are dealing with momentum and not energy.

From the outset I will kid no one about the track record for the use in court of conservation of momentum in connection with auto accidents. You will have to fight tooth and nail to keep conservation of momentum from being thrown out of a criminal trial. Past court cases show about a 50/50 acceptance/rejection ratio. It is important to note that when the use of momentum has been denied by the courts, it has generally been because of lack of any marks on the road to show angles of approach or departure. Its use has also been rejected on occasion because original investigators have failed to get enough evidence at the accident scene to support the information that is so vital to the equation. In the past there have been experts who have tried to use momentum on very little factual information and who in many cases have never visited the scene of the accident nor seen the cars involved. In other words, they have tried to bluff in court -- something even a stoop-shouldered, knuckle-dragging, club-footed rookie knows enough not to do.

Attorneys will tell you that you will never get momentum into any court in the country. This is especially true of prosecutors who are so afraid of a trial they won't prosecute a first-degree murder charge unless the deed was committed in front of a Lutheran congregation by a guy going over Minnehaha Falls on a coal barge. Even then they will probably plea bargain down to overtime parking with a recommendation for probation.

Do not try to get conservation of momentum into court unless you can establish these points:

1. <u>Path of approach for each vehicle</u>

   This can be found from:

   (a) damage that shows force lines, and/or

   (b) skidmarks leading up to the point of impact.

2. Path of departure for each vehicle

   This is usually found from marks on the road that lead from the point of impact to the point of rest.

3. Speed of each vehicle after impact

   This can be found by calculating:

   (a) the minimum speed after impact, using whatever drag factor you can establish for metal or rubber or rolling on wheels.

   (b) the airborne speed of a car going into a ditch or vaulting into the air.

   (c) any portion of a load launched into the air by the force of impact.

4. The approximate weight of each vehicle plus its load

   This need not be exact, but if you can weigh the vehicle -- great! Used car books give you measurements and weights of vehicles that are usually pretty close. The conservation of momentum equation we deal with contains a weight ratio, so the source of your weight should be the same for each vehicle. Also, if there is a great difference in the weights of passengers, this difference should be compensated for.

Be very careful of head-on collisions, as the approach angles

become very critical. If one or both vehicles collide with some other object as they go from impact to final position, you should avoid using momentum unless this second impact is slight or unless you have some way to adjust or compensate for the resultant speed loss. If there is steering after impact, make sure that the departure angle you use is one caused by the collision and not steering. If the vehicle is under power after impact, do not use momentum. (Example: An individual has a collision and then drives down the street to park.) Special care must be used when a semi is involved because of the jackknifing properties of a vehicle with a fifth wheel.

If you use momentum correctly, you will be successful in court most of the time. If you abuse it, I hope you don't mind nail holes because you are certain to be crucified.

Momentum involves the weight and speed of objects. We will now define the terms used to discuss momentum.

Velocity

Velocity is a quantity which specifies distance divided by time (or distance per unit of time) and combines speed and direction.

Example:

Car #1 is moving at 50 MPH. This is speed only.

Car #1 is moving 50 MPH due north. This is velocity. Car #2 is moving at 50 MPH due south. Car #2 has the same speed as Car #1 but in the opposite direction. Therefore, the velocities of the two cars are not equal even though their speeds are.

(Please note that in this chapter velocity is in MPH as well as in FPS. We treat MPH quantities as velocity here rather than speed because in the equations direction is important.)

Mass

Mass is a basic property of matter which involves weight and gravitational attraction or pull. Mass can also be defined as the quantity of matter an object contains. The mass of a particular object is constant, but the weight of the object varies depending on gravitational attraction. An object weighs less on the moon since the gravitational attraction is less, but the object has the same mass.

Mass is equal to the weight of an object divided by the acceleration of gravity. Acceleration of gravity is a measure of the gravitational attraction of one body to another. The earth pulls on an object so that when the object is dropped, the object will have an acceleration

or change in velocity of 32.2 feet per second per second. This 32.2 is essentially the acceleration of gravity on the earth's surface.

Numerically, mass is equal to weight divided by 32.2.

Example:

A man weighs himself on the earth's surface and finds he weighs 161 lbs., so his mass = 161/32.2 = 5 units mass. Mass remains constant until speeds approaching the speed of light are attained. The speed of light is 186,000 miles per second. If you find a car that will go that fast, write down the license number, and if the driver has an accident, do not use momentum to figure speed.

Momentum

Momentum is defined as mass times velocity. The momentum of an object can be increased by increasing either the mass (weight) or the velocity or by increasing both. An object with a small mass but high velocity can have the same momentum as an object with a large mass but small velocity. So momentum means mass times velocity, with mass involving the object's weight.

A barge loaded with sand and moving approximately 4 MPH can do tremendous damage if it hits a bridge, while a rifle bullet going hundreds of MPH will barely chip the cement.

An M-16 rifle slug will be deflected by a twig, while a shotgun slug will hardly vary. The shotgun slug has the greater momentum.

Autos weighing thousands of pounds and developing high speeds have tremendous momentum. In a head-on collision, the vehicle with the most momentum will drive the other vehicle backwards.

If the momentum of two vehicles colliding head-on is equal, both vehicles will stop dead in their tracks.

Say a truck weighing 30,000 lbs. hits head-on with a car weighing 2000 lbs. Each has a velocity of 40 MPH. But because of the momentum difference, the truck will push the car backwards at approximately 35 MPH after impact. If the 2000 lb. auto were going 600 MPH, both vehicles would stop over the point of impact because they would have the same momentum. It can be said that in a head-on collision a bumble bee could stop a freight train if he could only beat his wings hard enough.

From Newton's Laws

Every body continues in a state of rest or uniform motion in a straight line unless acted upon by unbalanced forces.

A book resting on a table stays there because forces up and down on it are the same.

If a car locks up all four wheels on a level surface and the drag factor is equal on all wheels, the car will slide in a straight line unless acted upon by an unbalanced force. An example of an effective unbalanced force acting upon the motion of another force would be a Mack truck loaded with dynamite nailing a skidding car directly in the side. This is an unbalanced force guaranteed to alter the car's straight line skid.

It must be remembered that momentum always has numerical value and direction. If you face north and consider this direction positive, then south is negative. East, or to your right, is positive and west, or to your left, is negative. This means a car sliding north or east will have a plus value in the equation and a car sliding south or west will have a minus value.

In angle collisions the direction of momentum will enable you to make a reasonable estimate as to which car had the most weight or speed or both.

Do not confuse momentum with kinetic energy or energy of motion, which is an object's ability to do work, or in accidents, a vehicle's ability to do damage.

Momentum and energy are like horses and houses. They look alike in print but are different in form and function. Imagine a bullet striking a block of wood. Momentum will determine how hard the bullet hits the object, and kinetic energy will determine how deep it penetrates the block of wood.

Momentum equations are not energy equations, and their use allows us to ignore the amount of damage done in the accident. Two cars can have equal momentum but vastly different energy. Assume you have two cars. Car #1 weighs 4000 lbs. and is going 29.32 FPS, and Car #2 weighs 2000 lbs. and is going 58.64 FPS.

Momentum = Mass x Velocity

Car #1 = $\frac{4000}{32.2} \times 29.32 = 3642.236025$

Car #2 = $\frac{2000}{32.2} \times 58.64 = 3642.236025$

You can see that the momentum is the same for each vehicle. Now consider kinetic energy for each.

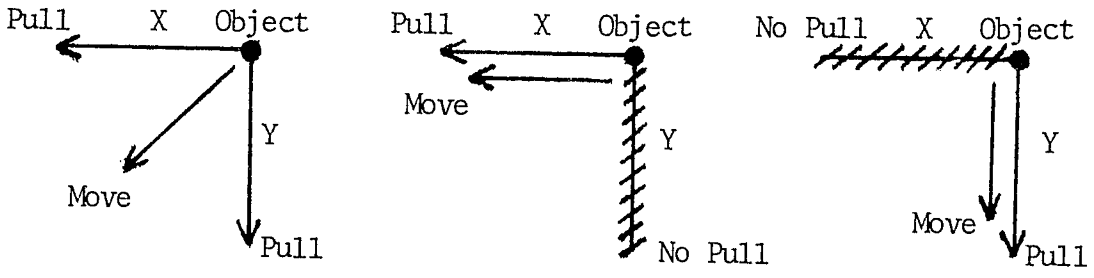
$Ke = \frac{1}{2} MV^2$

Car #1 $Ke = \frac{1}{2} \times \frac{4000}{32.2} \times 29.32^2 = 53395.18012$

Car #2 $Ke = \frac{1}{2} \times \frac{2000}{32.2} \times 58.64^2 = 106790.3602$

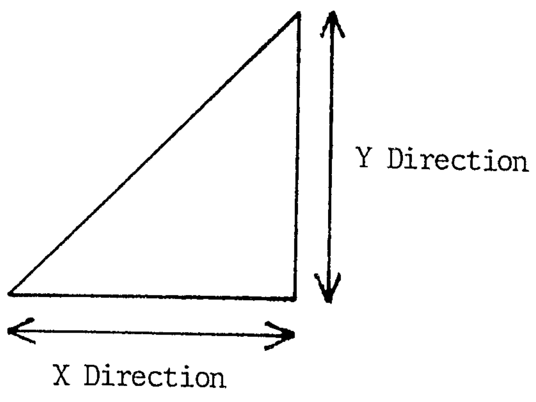
You can see that even though the momentum for each car is the same, the kinetic energy differs by a factor of 2. Bear in mind that we will deal with momentum, not energy, and will disregard any and all damage. This is not to imply that damage is unimportant. Damage is one reason we investigate accidents. We are only saying that we can disregard damage in figuring velocity at the point of impact.

In the diagrams below, two strings have been attached to an object so that the direction of pull of the one string is at a right angle to the direction of pull of the other. It should be understood that increasing or decreasing the pull on the one does not increase or decrease the pull on the other. Note the resulting direction of movement of the object.



Assume that two vehicles are traveling down roads at a right angle to each other. Each vehicle has momentum only in the direction it is traveling, and the momentum of each vehicle is completely independent of the momentum of the other. The vehicles collide where the roads intersect.

As always, construct a right triangle of the accident and consider the momentum of each vehicle in both the (X) and (Y) directions, sometimes referred to as components or axes.



Above all else, remember to consider the momentum of each vehicle in the (X) and (Y) directions before and after impact. Also remember that while the momentum of each vehicle in the (X) and (Y) directions may change after impact, total momentum -- that is, the sum of the momentums of <u>all</u> vehicles involved in the collision -- is nevertheless conserved in the (X) and (Y) directions. This follows from the law of conservation of momentum.

For example, when two vehicles collide, conservation of momentum requires that total momentum before impact be equal to total momentum after impact. This statement is an equation involving mass and velocity in the same way on both sides of the equal sign.

$$(1) \quad M_1 V_1 + M_2 V_2 = M_1 V_3 + M_2 V_4$$

Since $M = \frac{W}{g}$ at the earth's surface, we can substitute $(\frac{W}{g})$ into equation (1).

$$\frac{W_1}{g} V_1 + \frac{W_2}{g} V_2 = \frac{W_1}{g} V_3 + \frac{W_2}{g} V_4$$

And multiplying all terms by (g), we can everywhere eliminate acceleration of gravity so that the equation will involve only weight and velocity.

$$(2) \quad W_1 V_1 + W_2 V_2 = W_1 V_3 + W_2 V_4$$

To illustrate the simplest form of conservation of momentum let us assume that a vehicle (W_1) rear-ends another vehicle (W_2) while (W_2) is stopped at a traffic light. Use the following values:

W_1 = 4000 lbs.

V_1 = Velocity for W_1 (unknown)

W_2 = 5000 lbs.

V_2 = Zero (W_2 was standing still.)

V_3 = 58 MPH

Both vehicles stick together after impact and, with all wheels locked, slide 160 feet on a surface with (f) = .70, which makes (V_3) = 58 MPH. Now the equation will look like this:

$$(3) \quad W_1 V_1 + W_2 V_2 = (W_1 + W_2) V_3$$

Since (V_2) is zero, the whole term ($W_2 V_2$) is zero. So after we divide each side by (W_1), our equation will look like this:

$$\frac{\cancel{W_1} V_1}{\cancel{W_1}} = \frac{(W_1 + W_2)}{W_1} V_3$$

Now put in numbers.

$$V_1 = \frac{(4000 + 5000)}{4000} \cdot 58$$

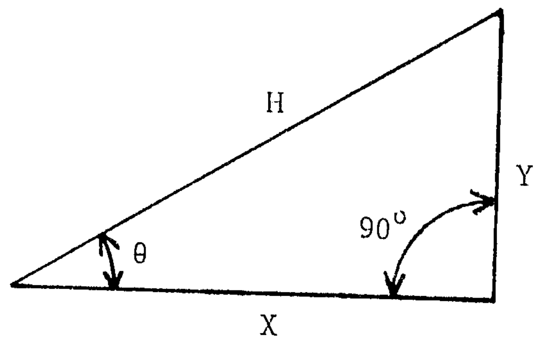
$$V_1 = 130.5 \text{ MPH}$$

(V_1) is the speed of (W_1) at the instant of impact.

We mentioned earlier in this chapter that in angle collisions each vehicle has momentum in the direction it is traveling and that total momentum is conserved along the (X) and (Y) axes. Now consider conservation of momentum in a right angle collision.

If two vehicles meet at a right angle intersection and are at right angles to each other, you must determine the angle of departure for each vehicle. You must then find the sine and cosine for each such angle and use those values in the equation.

In a collision you can measure your angles of departure to be less than 90 degrees, so bear in mind we are speaking of angles of 90 degrees or less. Every angle has a sine (Sin) and cosine (Cos), which are numbers less than or equal to one. Further, $(\text{Sin})^2 + (\text{Cos})^2$ is always equal to one. You can best get these numbers from a table, but if a table is not available, you can find the numbers using equation (4) below.



Cos θ = Adjacent side to angle (θ) over Hypotenuse

Sin θ = Opposite side to angle (θ) over Hypotenuse

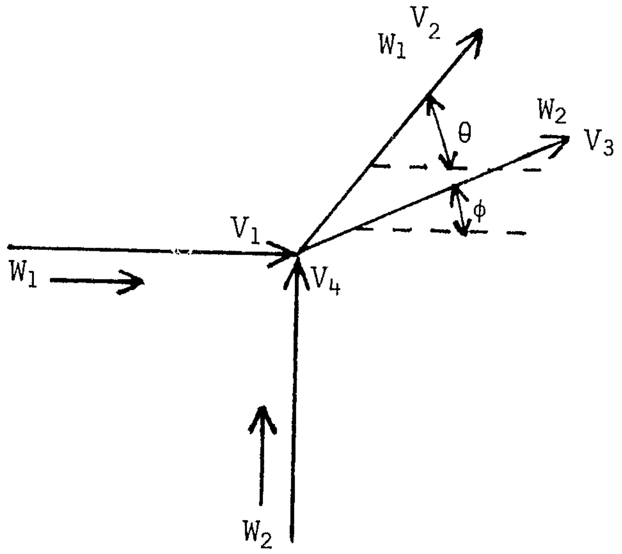
or

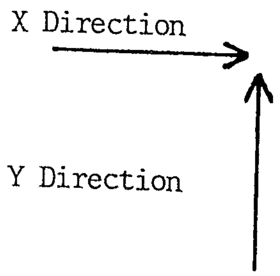
$$(4) \quad \text{Cos } \theta = \frac{X}{H} \quad \text{and} \quad \text{Sin } \theta = \frac{Y}{H}$$

These trigonometric functions give numerical value to the right triangles you draw on your scale diagram and must be used in momentum equations when collision vehicles are at angles to each other.

There is sometimes confusion in the minds of students as to why (Sin) and (Cos) must be used. If you consider equation (4) above, you will see that (Y) can be found by multiplying (Sin θ and H) and (X) can be found by multiplying (Cos θ and H). If two vehicles collide and deflect off at angle (θ), then the (Y) component can be returned to its original direction by multiplying it by (Sin θ), and the (X) component can be returned to its original direction by multiplying it by (Cos θ).

I realize this is a hen house explanation and may not make much sense. I hope the following diagram of a right angle collision will at least show how to set up the equation.





First, solve for (V_1) in the (X) direction.

<u>Before Impact</u> <u>After Impact</u>

$$W_1\ V_1 + W_2\ V_4 \quad = \quad W_1\ V_2 \cos\theta + W_2\ V_3 \cos\Phi$$

Note $(W_2\ V_4)$ is zero because (W_2) has no momentum in the (X) direction before impact. The equation will look like this with $(W_2\ V_4)$ gone:

$$W_1\ V_1 = W_1\ V_2 \cos\theta + W_2\ V_3 \cos\Phi$$

and finally becomes

$$(5)\quad V_1 = V_2 \cos\theta + \frac{W_2}{W_1} V_3 \cos\Phi$$

(V_1) is the velocity in MPH of (W_1) at impact.

Now solve for (V_4) in the (Y) direction.

<u>Before Impact</u> <u>After Impact</u>

$$W_2\ V_4 + W_1\ V_1 \quad = \quad W_1\ V_2 \sin\theta + W_2\ V_3 \sin\Phi$$

Note in this case $(W_1\ V_1)$ is zero because (W_1) has no momentum in the (Y) direction prior to impact.

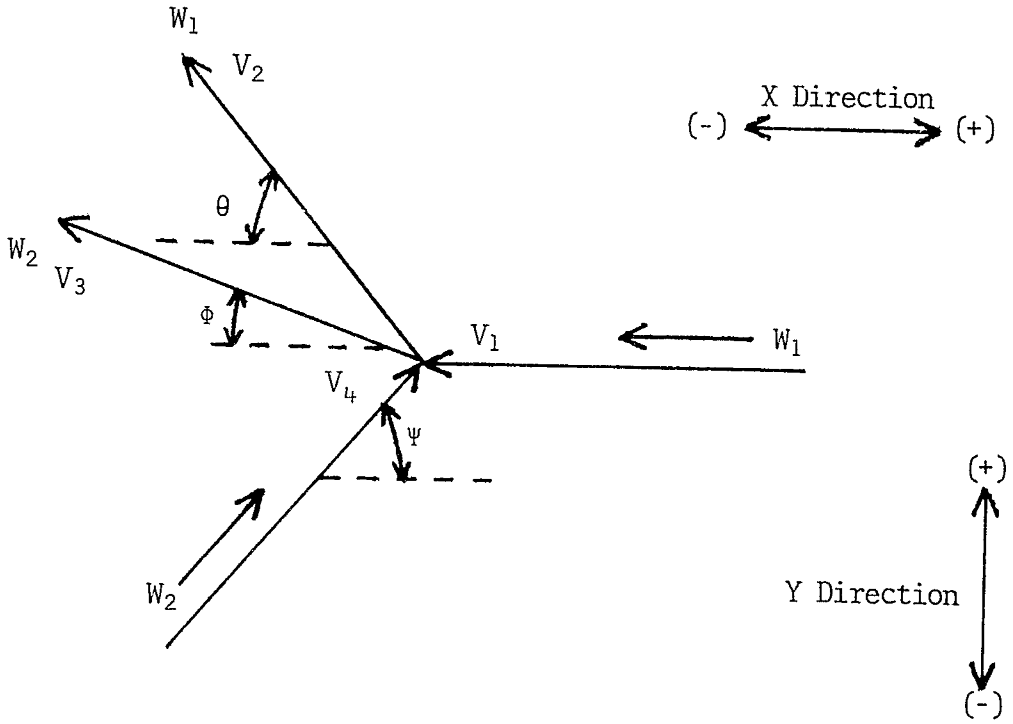
$$W_2\ V_4 = W_1\ V_2 \sin\theta + W_2\ V_3 \sin\Phi$$

And finally we get

$$(6)\quad V_4 = \frac{W_1}{W_2} V_2 \sin\theta + V_3 \sin\Phi$$

(V_4) is the velocity in MPH of (W_2) at impact.

Always consider momentum before and after impact, and remember that total momentum before impact must equal total momentum after impact because momentum is conserved.



The above diagram shows two vehicles in a partial head-on collision. You must solve for the (Y) direction first. Use the sine value.

$$W_2 V_4 \text{ Sin } \Psi = W_1 V_2 \text{ Sin } \theta + W_2 V_3 \text{ Sin } \Phi$$

$$(7) \quad V_4 = \frac{\frac{W_1}{W_2} \quad V_2 \text{ Sin } \theta + V_3 \text{ Sin } \Phi}{\text{Sin } \Psi}$$

(V_4) is the velocity of (W_2) at impact.

Now solve for the (X) direction. Use the cosine value.

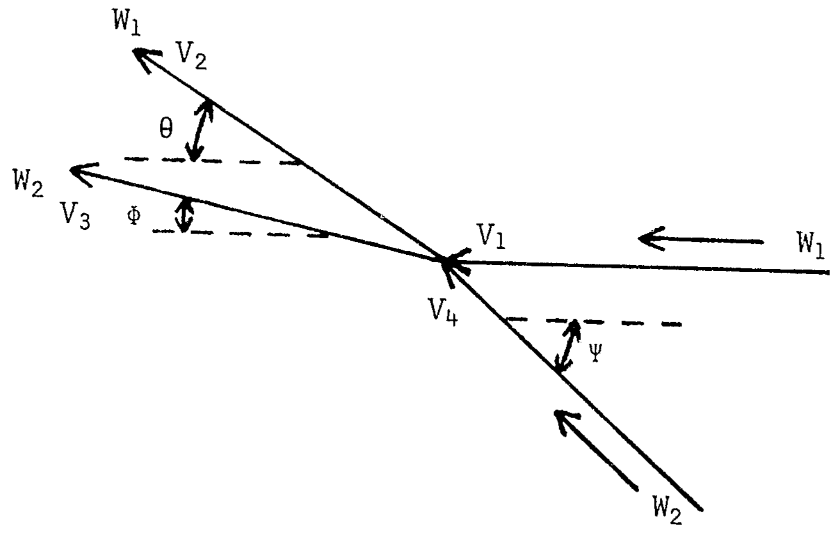
$$W_2 V_4 \text{ Cos } \Psi - W_1 V_1 = -W_1 V_2 \text{ Cos } \theta - W_2 V_3 \text{ Cos } \Phi \text{ or}$$

$$-W_1 V_1 = -W_1 V_2 \text{ Cos } \theta - W_2 V_3 \text{ Cos } \Phi - W_2 V_4 \text{ Cos } \Psi$$

Remember when you divide through by $(-W_1)$ everything becomes a plus, so

$$(8) \quad V_1 = V_2 \text{ Cos } \theta + \frac{W_2}{W_1} V_3 \text{ Cos } \Phi + \frac{W_2}{W_1} V_4 \text{ Cos } \Psi$$

The diagram below shows two vehicles colliding at an acute angle.



Again you must solve for the (Y) direction first.

$$W_2 V_4 \text{ Sin } \Psi = W_1 V_2 \text{ Sin } \theta + W_2 V_3 \text{ Sin } \Phi$$

$$(9) \quad V_4 = \frac{\frac{W_1}{W_2} V_2 \text{ Sin } \theta + V_3 \text{ Sin } \Phi}{\text{Sin } \Psi}$$

(V_4) is the velocity of (W_2) at impact.

Now solve for the (X) direction.

$$-W_1 V_1 - W_2 V_4 \text{ Cos } \Psi = -W_1 V_2 \text{ Cos } \theta - W_2 V_3 \text{ Cos } \Phi$$

$$-W_1 V_1 = -W_1 V_2 \text{ Cos } \theta - W_2 V_3 \text{ Cos } \Phi + W_2 V_4 \text{ Cos } \Psi$$

$$V_1 = V_2 \text{ Cos } \theta + \frac{W_2}{W_1} V_3 \text{ Cos } \Phi - \frac{W_2}{W_1} V_4 \text{ Cos } \Psi$$

(V_1) is the velocity of (W_1) at impact.

There is a method of figuring the above equations by use of a vector diagram. The last page of this chapter has a vector diagram labeled to go with the next few paragraphs. Please refer to that diagram when reading them.

Consider once more: Momentum = M x V
This is a vectorial quantity having both magnitude and direction. For a two-car collision we can write

$$M_1 V_1 + M_2 V_2 = M_1 V_3 + M_2 V_4$$

[Note that (V_4) is an after-impact speed here.]

Eliminating the acceleration of gravity we get

$$W_1 V_1 + W_2 V_2 = W_1 V_3 + W_2 V_4$$

This is still a vector equation and includes both the (X) and (Y) directions. If you calculate $(W_1 V_1, W_2 V_2, W_1 V_3, W_2 V_4)$ or any (WV) you have, you will usually know all angles and weights but not all speeds.

You must draw a vector diagram noting the point of impact and you must draw in the appropriate angles using long lines. Assume you can determine the after-impact speeds (V_3) and (V_4) from skidmarks, so you can find $(W_1 V_3)$ and $(W_2 V_4)$. You of course know the angles (θ) and (Φ) because you have measured them on your paper.

Determine a convenient scale for your diagram from the quantities $(W_1 V_3)$ and $(W_2 V_4)$. Measure off $(W_1 V_3)$ on the (Φ) line and $(W_2 V_4)$ on the (θ) line. These now indicate the true momentum vectors after collision.

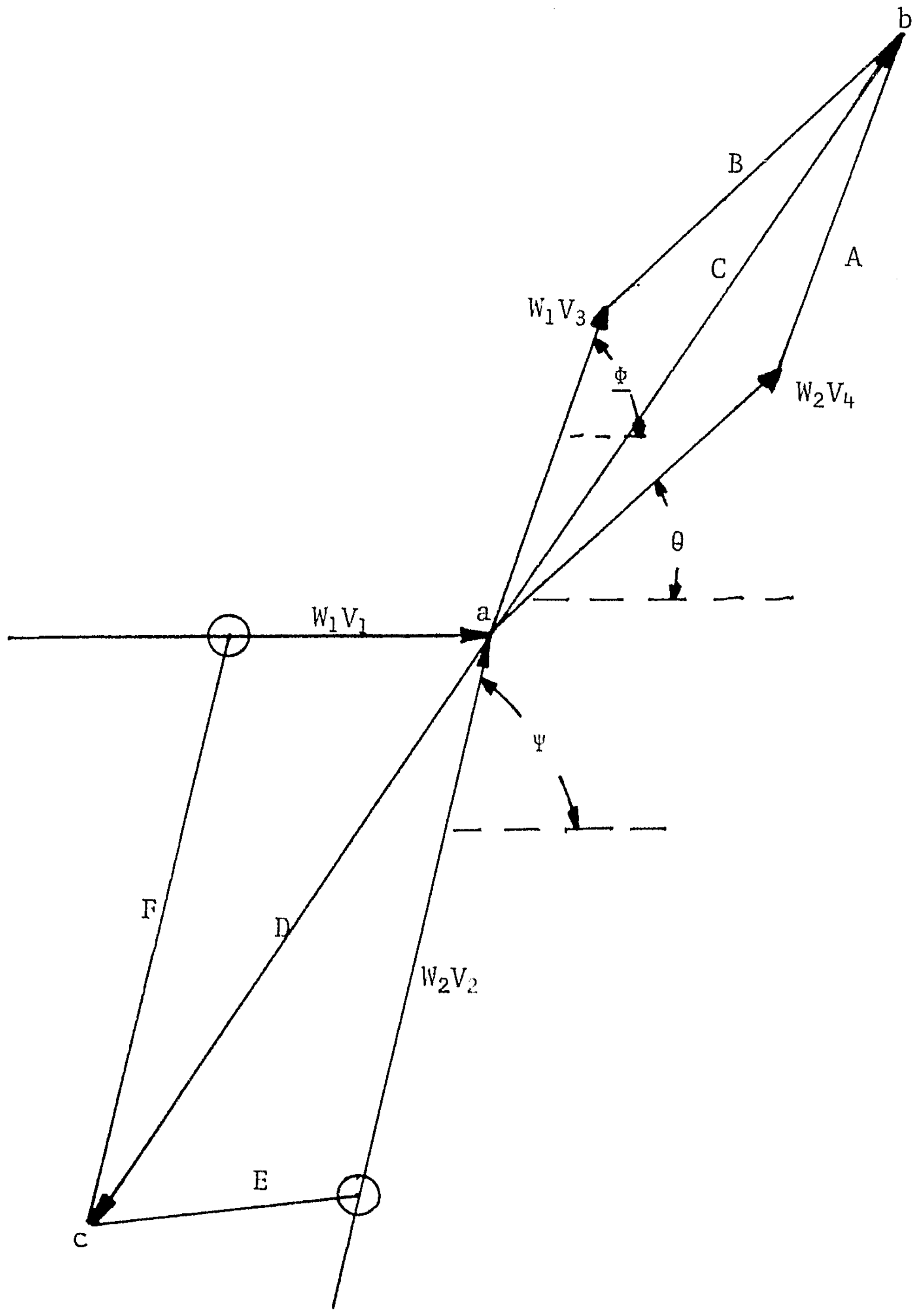
From the end of $(W_2 V_4)$ draw line "A" parallel to $(W_1 V_3)$. From the end of $(W_1 V_3)$ draw line "B" parallel to $(W_2 V_4)$. "A" and "B" intersect at point "b". Draw line "C" from "a" to "b". This is the vector addition of $(W_1 V_3 + W_2 V_4)$ or the right hand side of the momentum equation. Extend line "C" in the opposite direction from "a" and measure the distance of "a" to "b". Lay off the distance from "a" to "b" on the extended line to give you point "c" and vector "D". "D" is now $(W_1 V_1 + W_2 V_2)$ or the left side of the momentum equation. Note that $(W_1 V_1 + W_2 V_2) = (W_1 V_3 + W_2 V_4)$ is the result.

Now draw a line "E" from point "c" to the (Ψ) line parallel to $(W_1 V_1)$. The distance from point "a" to line "E" is the vector $(W_2 V_2)$. You find the magnitude of (V_2) by measuring this vector and dividing by the known (W_2).

Then draw a line "F" from point "c" to the $(W_1 V_1)$ line parallel to $(W_2 V_2)$. This gives you the length of $(W_1 V_1)$, and that length divided by (W_1) gives you the magnitude of (V_1).

If you use a sharp pencil and are careful, this method is very accurate; but if you have a calculator, the equations are quicker.

You may find it easier to use a vector diagram in court, as the jury may understand it better. Of course there is also the chance that the jury will find you stark-raving mad and fit you with a white jacket that laces up the back and has very long sleeves. If you keep your calculator charged up and your hours of sick leave at maximum, you should be in good shape whatever happens.



VECTOR DIAGRAM

CHAPTER XII

WEIGHT SHIFT

This chapter will deal with special problems you must consider when you calculate speed from skidmarks for a vehicle that has no brakes or inoperative brakes on some of its wheels.

I had hoped to avoid this chapter, as the speed equation using weight shift is extremely complicated and in the end the MPH difference is small. However, an unaccountable error of even 2 MPH can be disastrous. A good defense attorney will clasp those 2 MPH to his bosom as a mother clasps her only child. He will nurture, expand, and in spine-tingling oratory expound on those 2 MPH until the destruction of Sodom and Gomorrah will pale by comparison.

Since the best way to keep from dying is to defend ourselves against all manner of things that may kill us, let us gird our loins and with clear eye, tight smile, and traffic template held at portarms, forge onward and upward.

First we will deal with the area where you are on firm ground

if you completely ignore weight shift.

If a vehicle has brakes on all its wheels and if the brakes retard equally, you can ignore weight shift. This does not mean that all wheels must leave skidmarks. It does mean that visible skidmarks must be straight. If the skids are straight and the surface is level, all wheels are either skidding or about to skid. At any rate, to have straight skids you must have equal retardation on all wheels. If you have equal retardation, the amount of weight on each wheel becomes irrelevant, as it will not affect the skidding distance. Note, I have not said stopping distance but skidding distance. Please also note that you can get straight skids if you have brakes on the front wheels only. This, however, is rare and will be obvious, as you will have characteristic front wheel skidmarks only. And an examination of the vehicle will, of course, show no brakes on the rear wheels.

What all this means is that if you have brakes on all wheels or brakes on the front wheels only, you need not concern yourself with weight shift. All wheel braking will give you minimum speed from skidmarks with use of a full drag factor. If two front wheels skid on a 4-wheel vehicle, 50 percent of the full drag factor will give you a minimum speed. In fact, the speed in this case will be lower than minimum by from 2 to 6 MPH. Now for the braking situations where you must consider weight shift.

First we will consider the most common type vehicles you will deal with on which some weight-bearing wheels will have no brakes. Trucks manufactured before 1975 with three axles were required to have brakes on only two of those three axles. Brakes were not installed on the front or steering axle of many of these 3-axle trucks. Most common are tandem dump trucks and truck-tractors with tandem drivers not hooked to a semi-trailer. Operation of a truck-tractor without it being hooked to a semi-trailer is commonly called bob-tailing.

If Federal regulations applicable after 1975 are retained, there may be no more trucks manufactured without brakes on the front axle. However, large trucks run for millions of miles before they finally wear out. So long after we have all passed to that great intersection in the sky where we will direct traffic on the golden streets forever, there will still be accidents involving trucks with no brakes on the front steering axle.

You will notice when you apply the brakes on your car, the nose dips down. This is due to a weight shift toward the front of the car. The wheels try to stop and the center of mass tries to keep going. The suspension system that hooks the car body to the axles allows the car to pivot, and the nose dips down. When this weight shift is transferred to wheels with brakes, it makes no difference to the skidding distance. If this weight shift is to a front axle having wheels with no brakes, then the stopping ability of the rear wheels is lessened because weight is removed from them and is transferred to wheels that are free rolling and not braking. This action reduces the stopping ability of the vehicle and increases the distance for skidding to a stop from a given speed.

The height off the ground of center of mass is a factor that must be considered. The higher the center of mass, the greater the weight shift to the front end during skidding. The greater the weight that is shifted to a non-braking axle, the greater the skidding distance from a given speed. As an example, a stub-nosed truck-tractor has a higher center of gravity than a conventional or long-nosed truck-tractor. Therefore, if these two truck-tractors have no brakes on the front axle and are both going 50 MPH on a road with a drag factor of .75, the stub-nosed truck-tractor will have more weight shifted onto its front axle and will skid a longer distance in coming to a stop.

Determining how much farther one truck skids than the other, and how much farther either of them will skid than a vehicle with brakes on all wheels, requires a complicated equation which we will plunge into presently. The information needed to work this equation is:

1. Wheelbase in inches

   The wheelbase of a 3-axle rig is measured from the center of the front axle to a point halfway between the rear tandems.
2. Height of the center of mass in inches off the ground
3. Location of the center of mass horizontally between the front and rear axles
4. Weight on the rear tandems
5. Weight on the front wheels
6. Total weight of the vehicle
7. Drag factor of the road surface

Since the weight shift equation uses center of mass (center of gravity), we will first consider how to locate the center of mass of a vehicle in relation to its wheelbase.

$$\text{Center of mass in inches} = \frac{\text{Wheelbase in inches times front axle weight in lbs.}}{\text{Total vehicle weight in lbs.}}$$

The answer to the above equation will be in inches and will be measured from the center of the rear axle toward the front.

This is the longitudinal center of mass. The vertical center of mass is the distance above the ground in inches at which the longitudinal center of mass is located. It is found by using a

rather complicated procedure requiring this equation:

$$H = h_2 + \frac{(W_g - W_F) \cdot L_2 \text{ X } \sqrt{(L_2)^2 - (h_1 - h_2)^2}}{W \cdot (h_1 - h_2)}$$

H = Height of the center of mass (inches)

h_2 = Front tire radius (to outside of tire)

W_g = Weight on the front wheels with the rear end raised

The rear end must be raised 40 inches or until the front bumper nearly touches the ground. The rear end must not be raised by the rear bumper. Rather, the weight must be supported by the rear wheels or rear axle so that the suspension system is compressed. The front end is then weighed.

h_1 = Height in inches from the center of the rear wheels to the ground while the rear end is raised as described in the previous paragraph

W_F = Weight on the front wheels when the rear end is resting on the ground in a normal level position

L_2 = Length of the wheelbase in inches

W = Total weight of the vehicle

The above information is difficult to obtain, especially if the vehicle is a large truck. However, if the case is important enough, you can find equipment to do the job no matter how large the truck is.

The exact location of the center of mass is usually not critical to the weight shift formula. Raising or lowering the center of mass of a large truck by 12 inches will change your answer by only about 2 MPH. If the vehicle is a front engine American-made car, the center of mass will be at about one-half the distance between the front and rear wheels and about one-third the total height of the car measured from the ground. The manufacturers of large trucks have information on the location of the center of mass of every model truck they ever made. However, in light of the fact that everyone and his uncle tries to sue large truck manufacturers for everything from unsafe paint to lumpy mattresses in the sleeper, a truck manufacturer is reluctant to give out this kind of information.

A bob-tailed stub-nosed truck-tractor will have the center of mass about 40 to 50 inches off the ground. Some truck-tractors will have the center of mass 60 inches off the ground, but Federal regulations do not allow it to be higher than 75 inches off the ground.

It may seem futile to wallow about in this quagmire when all you

have to do is guess at the location of the center of mass. However, if some day you are sitting in the stifling, silent heat of a July courtroom when with a sinister glare the cross-examining attorney tries to raise the specter of weight shift from the coffin, you can slam the lid on his fingers by calmly saying you have calculated the effect of weight shift and have incorporated it into your conclusions.

Now if you haven't gone on your annual vacation and haven't given up reading this dreary chapter, we will finally address ourselves to the weight shift equation.

First you should have a look at the finished equation. Remember, this equation is only for a car, a straight truck, or a truck-tractor <u>not</u> hooked to a semi-trailer.

$$\Delta W = \frac{\frac{h}{L}\left[W_F \mu_F + W_R \mu_R\right]}{\left[1 - \frac{h}{L}\mu_F + \frac{h}{L}\mu_R\right]}$$

ΔW = Delta weight in lbs. (weight shift or weight change)

The (Δ) symbol simply means there is a change in the term it is connected to. In this case (ΔW) will be the weight shifted from the rear wheels to the front wheels.

h = Height of the center of mass in inches

L = Length of the wheelbase in inches

μ = Coefficient of friction or drag factor

Mu, pronounced "mew," is the 12th letter in the Greek alphabet.

W_F = Weight on front wheels in lbs.

W_R = Weight on rear wheels in lbs., or in the case of a bob-tailed tandem axle truck-tractor or a tandem axle straight truck, weight on the rear tandems

μ_F = Drag factor for the front wheels

μ_R = Drag factor for the rear wheels

A full drag factor is used for (μ_F and μ_R). So if you get a drag factor of .75 from testing a road surface, (μ_F) would equal .75 and (μ_R) would also equal .75, and you would use these values in the weight shift equation. Now for the derivation of the equation:

The basic weight shift equation is

$$(1) \qquad \Delta W = \frac{F \cdot h}{L}$$

F = Force

h = Height of the center of mass

L = Wheelbase

(F) in equation (1) would be equal to the right side of the following equation. [The subscript number after (W) indicates which of the four wheels the weight is on.]

$$F = \left[(W_1 + W_2) \cdot \mu_F\right] + \left[\Delta W \cdot \mu_F\right] + \left[(W_3 + W_4) \cdot \mu_R\right] - \left[\Delta W \cdot \mu_R\right]$$

Now let

$$(W_1 + W_2) = W_F \text{ and } (W_3 + W_4) = W_R \text{ so}$$

$$(2) \qquad F = \left[(W_F + \Delta W) \cdot \mu_F\right] + \left[(W_R - \Delta W) \cdot \mu_R\right]$$

Now plug equation (2) into equation (1):

$$(\Delta W = \frac{F \cdot h}{L} = \frac{h}{L} \cdot F) \qquad \text{to get}$$

(3)

$$\Delta W = \frac{h}{L}\left[(W_F \cdot \mu_F) + (\Delta W \cdot \mu_F) + (W_R \cdot \mu_R) - (\Delta W \cdot \mu_R)\right]$$

Now multiply ($\frac{h}{L}$) by every term inside the brackets.

(4)

$$\Delta W = \left[(\frac{h}{L} \cdot W_F \cdot \mu_F) + (\frac{h}{L} \cdot \Delta W \cdot \mu_F)\right] + \left[(\frac{h}{L} \cdot W_R \cdot \mu_R) - (\frac{h}{L} \cdot \Delta W \cdot \mu_R)\right]$$

Now move all (ΔW) terms to the left of the equal sign and change the signs.

(5)

$$\left[\Delta W - \frac{h}{L} \cdot \Delta W \cdot \mu_F + \frac{h}{L} \cdot \Delta W \cdot \mu_R\right] = \left[\frac{h}{L} \cdot W_F \cdot \mu_F + \frac{h}{L} \cdot W_R \cdot \mu_R\right]$$

Now move (Δ W) outside the brackets on the left side of the equal sign and move ($\frac{h}{L}$) outside the brackets on the right side of the equal sign. Note when the (Δ W) which is farthest left in equation (5) is pulled out, you have to put the number one in its place. This is done because of the minus sign which would otherwise sit there like a wart on the nose of a dairy princess.

(6)

$$\Delta W \left[1 - \frac{h}{L} \cdot \mu_F + \frac{h}{L} \cdot \mu_R \right] = \frac{h}{L} \left[W_F \cdot \mu_F + W_R \cdot \mu_R \right]$$

Now divide both sides of the equal sign by

$\left[1 - \frac{h}{L} \cdot \mu_F + \frac{h}{L} \cdot \mu_R \right]$ and you are left with

$$(7) \quad \Delta W = \frac{\frac{h}{L}\left[W_F \cdot \mu_F + W_R \cdot \mu_R \right]}{\left[1 - \frac{h}{L} \cdot \mu_F + \frac{h}{L} \cdot \mu_R \right]}$$

Equation (7) gives you the weight in lbs. shifted to the front end of a vehicle during braking. If there are no brakes on the front wheels, (μ_F) will equal zero. If there are no brakes on the rear wheels, (μ_R) will equal zero. Remember, if you have a truck with tandem rear axles, the wheel base will be measured from a point halfway between the rear tandems to the center of the front axle, and (W_R) will equal the total weight on the rear tandems.

Now for the speed equation using weight shift.

If you will recall from the derivation of the minimum speed formula that at one point you can have

$$W \cdot V^2 = 2g \cdot W \cdot F \cdot D$$

If you do not eliminate the (W) you can have

$$V^2 = \frac{2g}{W} \cdot W \cdot F \cdot D$$

The weight shift speed equation is similar to this last equation but allows for a weight shift, or (Δ W), and looks like this:

(8)

$$V^2 = \frac{2g}{\underset{(\text{total})}{W}} \cdot \left[\mu_F \cdot (d_1 + d_2) \cdot \left(\frac{W_F + \Delta W}{2}\right) + \mu_R \cdot (d_3 + d_4) \cdot \left(\frac{W_R - \Delta W}{2}\right) \right]$$

V^2 = Velocity in feet per second

$2g$ = 2 x 32.2 or 64.4

W (total) = Total weight of the vehicle

μ_F = Drag factor for the front wheels

d_1 = Length of skidmark in feet from one front wheel

d_2 = Length of skidmark in feet from the other front wheel

W_F = Total weight on the front wheels

ΔW = Weight shifted to the front end during braking

The number you put into the equation in place of (W) is of course found from equation (7).

$(W_F + \Delta W)$ is divided by 2 because there are two wheels on the front axle.

μ_R = Drag factor on the rear wheels

d_3 = Length of skidmark in feet from one rear wheel

d_4 = Length of skidmark in feet from the other rear wheel

W_R = Total weight on the rear wheels

ΔW = Weight shifted forward from the rear wheels in braking

Since weight is transferred from rear to front, (ΔW) is subtracted from the rear axle weight. It is divided by 2 because there are two wheels on the rear axle.

The following is a list of data to show how these equations work:

Height of the center of mass = 24 inches
Wheelbase = 120 inches
Drag factor = .80
Skid distance = 200 feet
Weight of the vehicle = 4000 lbs. (1000 lbs. on each wheel)

Now match these numbers to the terms used in the equation:

$h = 24$ inches $\quad d_1 = 200 \quad W_1 = 1000$ lbs.
$L = 120$ inches $\quad d_2 = 200 \quad W_2 - 1000$ lbs.
$\mu_F = .80 \quad d_3 = 200 \quad W_3 = 1000$ lbs.
$\mu_R = .80 \quad d_4 = 200 \quad W_4 = 1000$ lbs.

Assume there are no brakes on the front wheels. So

$\mu_F = 0$
$d_1 = 0$
$d_2 = 0$

Now look at equation (7). The letters (terms) have been replaced with their corresponding numbers.

$$\Delta W = \frac{\frac{24}{120} \cdot \left[(2000 \cdot 0) + (2000 \cdot .80) \right]}{\left[(1 - \frac{24}{120} \cdot 0) + (\frac{24}{120} \cdot .80) \right]}$$

$$\Delta W = \frac{.20 \cdot 1600}{1 + .16} = \frac{320}{1.16}$$

$$\Delta W = 275 \text{ lbs.}$$

Next, put (ΔW) into equation (8) to find the speed of a vehicle that skids 200 feet with brakes on the rear wheels but no brakes on the front wheels. Use these values in equation (8):

V^2 = Velocity in feet per second

2g = 2 x 32.2 or 64.4

W (total) = 4000 lbs.

$\mu_F = 0$

$\mu_R = .80$

W_F = 2000 lbs.

W_R = 2000 lbs.

ΔW = 275 lbs.

$d_1 = 0$

$d_2 = 0$

$d_3 = 200$

$d_4 = 200$

$$V^2 = \frac{2g}{W_{\text{total}}} \cdot \left[\mu_F \cdot (d_1 + d_2) \cdot \left(\frac{W_F + \Delta W}{2}\right) + \mu_R \cdot (d_3 + d_4) \cdot \left(\frac{W_R - \Delta W}{2}\right) \right]$$

$$V^2 = \frac{64.4}{4000} \cdot \left[0 \cdot (0 + 0) \cdot \left(\frac{2000 + 275}{2}\right) + .80 \cdot (200 + 200) \cdot \left(\frac{2000 - 275}{2}\right) \right]$$

$$V^2 = .0161 \cdot \left[0 + .80 \cdot 400 \cdot \frac{1725}{2} \right]$$

$$V^2 = .0161 \cdot 276000$$

$$V^2 = 4443.6$$

$$V = 66.66 \text{ FPS} \quad \text{So}$$

$$S = 45 \text{ MPH}$$

If we had used 50% of the full drag factor of .80, so that (f = .40) while (d = 200), then the speed would have been 49 MPH.

You can see that the minimum speed formula no longer gives minimum speed for a vehicle with no brakes on the front wheels. It gives you 4 MPH more than the vehicle was actually going. If there were brakes on the front wheels only, the difference would be the other way around. The minimum speed formula would give you a speed below the minimum speed you get from the speed equation using the weight shift.

There is practical application for weight shift equations. A 3-axle truck with brakes on the rear tandems only can skid for long distances and the driver can keep the vehicle from swapping ends by skillful steering. Given this situation you must use the weight shift equations. If the truck is in an accident and if the rear wheels lock up from damage at impact, the impact is usually severe enough so that the driver no longer has control of the truck afterwards. If the wheels lock up and no one can steer to keep the truck headed straight, the truck will rotate 180 degrees and the rear wheels, which had been leaving rear wheel skids, will become the wheels that now leave leading or front wheel skids. If part of the after-impact skids are rear wheel skids and part are front wheel skids, the results will even out, and 50% of the full drag factor will give you very close to a correct speed. Every situation will be different and each case must be decided on its own merits.

We are not dealing only with trucks, of course. In a great many accidents some of the wheels of a car will lock up from collision damage, and if the wheels skid a substantial distance from the point of impact to the final position, you may well have to consider using weight shift equations to determine the velocity after impact, so vital to conservation of momentum.

As if all this were not enough, we must now consider weight shift problems where a semi-trailer is hooked to a truck-tractor that has no brakes on its front wheels.

If you have a truck-tractor semi-trailer combination and if all wheels have operable brakes, then basically this rig will skid the same distance as a car with 4-wheel brakes, provided of course they are both going the same speed on the same surface. Remember, I again said skidding distance not stopping distance. However, many 5-axle semis have no brakes on the steering axle or have a limiting switch that controls front axle braking. If you are dealing with an accident involving a semi with a limiting switch for the front axle, it is important to find out how the switch was set at the time of the accident. The wording may vary, but often the switch will have settings for "wet" or "dry". If the switch is set for a wet road, the front wheels will have reduced braking. If it is set for a normal or dry road, there will be full braking on the front wheels. It is also possible to have a semi equipped with limiting switches that reduce the front wheel braking to nearly zero. In this case we are back to a truck-tractor with no brakes on the front wheels, and weight shift must be considered.

It is common to have a semi with good brakes on all its wheels but with wheels on the front axle that do not slide. This is because the front wheels, which already have a slightly smaller braking system, are overloaded during braking. From testing it has been found that under these conditions you can take from 80% to 90% of a full drag factor and be reasonably close to correct. Buses traveling over the road will sometimes develop only 75% of a full drag factor because of this weight shifting problem.

In a bob-tailing operation you can get a drastically reduced drag factor if the front wheels have no brakes and if the driver is skillful enough to keep the tractor headed straight. If a semi-trailer is hooked to this type tractor, you will not have as large a reduction in drag factor, but the amount of reduction is difficult to determine. The reason for the difficulty is that weight is shifted from the trailer forward onto the fifth wheel, which is resting on wheels equipped with brakes. If this was as far as the weight shifted, there would be no problem. However, some of the weight gets shifted onto the front axle, and weight from the tractor also shifts forward. All of this weight shifting is further complicated by the fact that the center of gravity for the tractor and the center of gravity for its loaded trailer are in two different places and are not at the same height from the ground.

At this point it would appear that the problems here are insurmountable. In fact, just thinking how to find the height of the center of mass for half a load of skim milk in a tank with no baffles is enough to make your stomach feel as if you had just eaten a pound of boiled cabbage and drunk a quart of beer.

Hope springs eternal, however, and we can side step the insurmountable problems and snatch triumph from the jaws of despair. I have been involved in test skidding truck-tractor semi-trailer combinations through the years, and a general rule has remained constant. Whether the trailer is loaded or not, it appears you can take 60% to 75% of a full drag factor and use this in computing minimum speed from skidmarks. Remember this is for a 5-axle semi with no brakes on the steering axle.

For example, if you get a drag factor of .80 by using a car to run test skids, you can then multiply .80 by .60 and .80 by .75 and use a range of .48 to .60 for the drag factor to find minimum speed from skidmarks. Of course if you run test skids with the same or a similar truck, you won't need to adjust the drag factor you get. It is, however, seldom that you will find someone willing to test skid $80,000.00 worth of equipment.

There is a mathematical equation that can be used to compute minimum speed from skidmarks for a truck-tractor semi-trailer combination. As a guide in your initial investigation, however, you can use the minimum speed formula, taking 60% to 75% of a full drag factor. If the initial investigation reveals a speed high enough to warrant court action, you may then wish to base your speed estimate on the test skidding of a similar truck or on the use of the weight shift equation.

The derivation of a speed equation of this magnitude was way beyond my abilities, so I dumped the problem on my brother. He carefully studied the physical dimensions of a truck-tractor semi-trailer combination. He evaluated the potential variables and refused to touch it. Only after direct threats to expose childhood crimes to our mother did he reconsider and with the energy of a go-go dancer tackled the job.

It took him months. He lost half his friends, all of his religion, and his pet boa constrictor went into nervous convulsions and constantly shed her skin.

I must confess I had feelings of guilt for putting him to this task. However, thoughts of my fame and fortune from his equation completely smothered those feelings and I let him sweat it out.

He finally made it. His wife recorded his expression on film moments after he completed the last notation. This photo, for posterity, appears below.

Oddly enough, his position and expression have not changed since he completed the derivation. Ah well, c'est la vie! and on to the equation.

On the next page there is a diagram of a truck. Its only purpose is to show where the terms used in the equation come from.

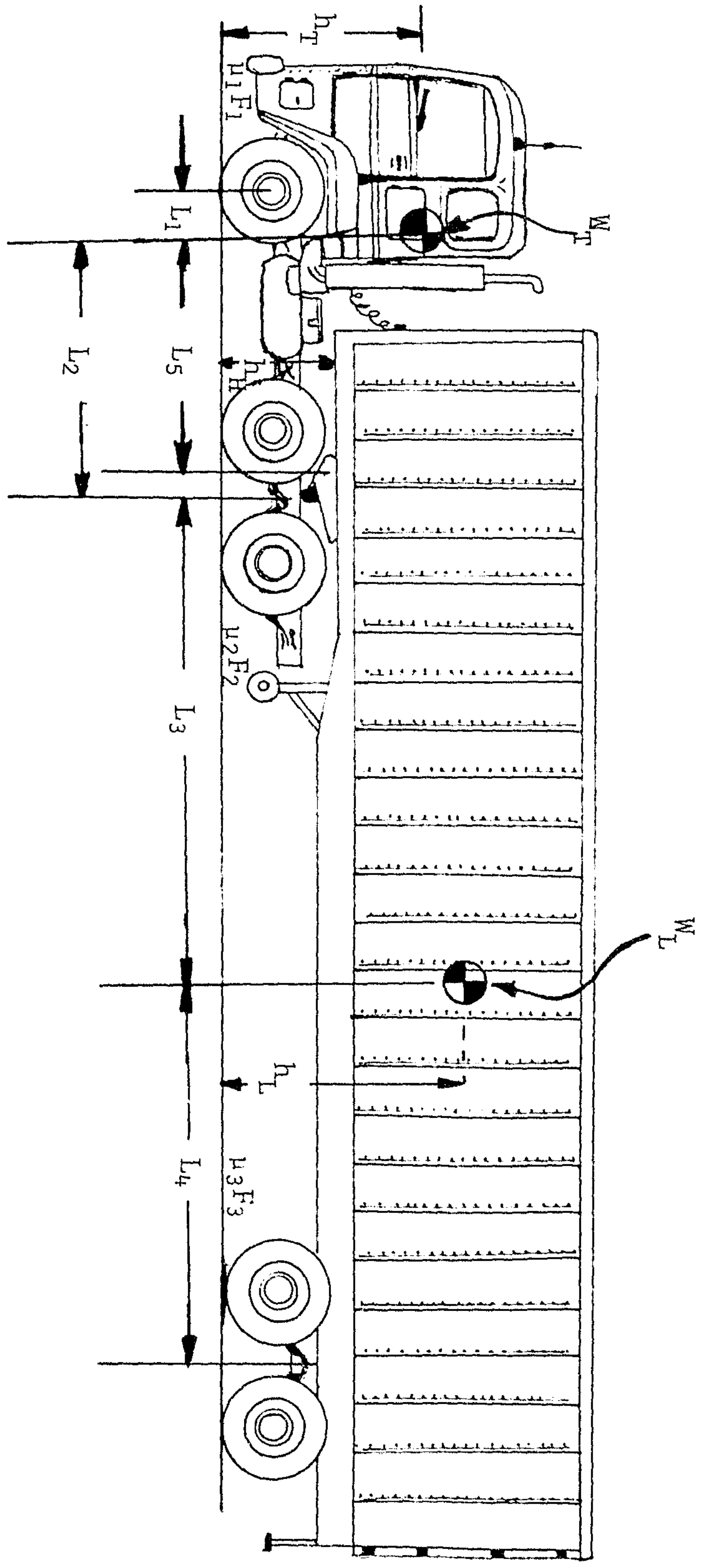


Diagram of a Truck-Tractor Semi-Trailer

The next step is to identify the terms:

W_T = Weight of the tractor

W_L = Weight of the load

$$\left.\begin{matrix}\mu_1\\ \mu_2\\ \mu_3\end{matrix}\right\} = \text{Drag factor}$$

L_1 = Distance from the center of the front wheel to the center of mass of the tractor

L_2 = Distance from the center of mass of the tractor to the center of the bogeys or tandem drive axles

L_3 = Distance from center of the drive axles to the center of mass of the trailer

L_4 = Distance from the center of mass of the trailer to the center of the tandem trailer axles

L_5 = Distance from the center of mass of the tractor to the hitch or 5th wheel pin

h_T = Height of the center of mass of the tractor from the ground

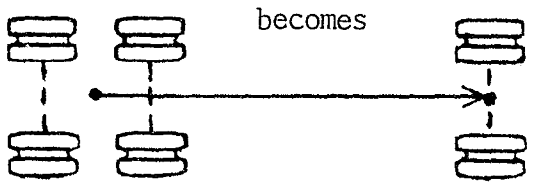
h_H = Height of the hitch or height of the 5th wheel from the ground

h_L = Height of the center of mass of the trailer or height of the center of mass of the load from the ground

$$\left.\begin{matrix}F_1\\ F_2\\ F_3\end{matrix}\right\} = \text{Drag force per wheel}$$

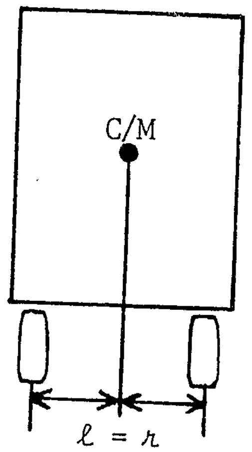
You should be able to get from the manufacturer information pertaining to the location of the center of mass of the tractor and trailer. Chapter XIII, Miscellaneous Formulas, explains the concepts "center of mass" and "load combination."

Because tandem axles share the load equally, they can be replaced in the equation with a single axle located midway between the tandem axles. Thus



In reality the semi has 5 axles, but in the equation it will be treated as though it had 3 axles.

The load is centered in the lateral (sideways) direction, and the skid is straight ahead (or nearly so). There is no appreciable lateral weight shift, and in the lateral direction the drag factor is equal for each wheel.



Now (W_1, W_2 and W_3) are static weights <u>per wheel</u>, and (F_1, F_2 and F_3) are drag forces <u>per wheel</u>. If dual tires are used, assume they behave like a single tire.

Thus ($2W_1 + 2W_2 + 2W_3 = W_T + W_L$) when the truck is <u>standing still</u>.

Next (W_T, W_L, h_T, h_H, h_L, L_1, L_2, L_3, L_4 and L_5) are values you must measure, weigh, and estimate because they are knowns in the equation. The skid distance (D) is also a known. The only unknown will be the velocity (V), which the equation will find for you.

The complete derivation would take about 20 pages and would be of little value, as the end result is what is important. It is based on the derivation of the original minimum speed equation, so I will mention only several assumptions for its use.

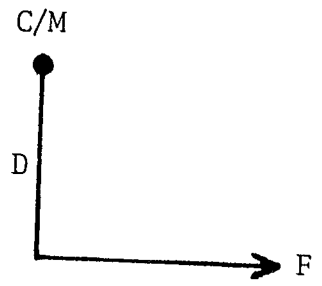
We assume the truck skids on the road surface and does not rise or fall. That is, it does not ascend into the heavens on the wings of a dove or plunge into an abyss on the belly of a serpent. So the sum (Σ) of the forces in the (Y) direction is zero.

$$\Sigma F_Y = 0$$

We also assume that the truck does not rotate about its center of mass like a pin wheel. If a body does not rotate, the sum of the torque (L) about its center of mass must equal zero.

$$\Sigma L = 0$$

Torque equals the force times the right angle distance between the force and the center of mass. Torque in a clockwise direction is positive; in a counterclockwise direction it is negative.



$$L = FD$$

So finally the kinetic energy equation takes the form

$$\frac{1}{2} \cdot \frac{W}{g} \cdot (V_0^2 - V_f^2) = F \cdot D$$

where (D) = skidding distance, (V_0) = initial velocity, and (V_f) = final velocity.

This, then, is the basis for the weight shift speed equation. The values for the tractor and trailer are considered separately and are then combined in the equation.

Remember that the equation must be strung out when you use it. Pay strict attention to brackets and parentheses.

Weight Shift Speed Equation

$V_0^2 - V_f^2 = 2\ g\ D$ times the entire equation below:

$$W_T \left[\frac{\mu_1 L_2 + \mu_2 L_1}{L_1 + L_2} - \frac{h_T(\mu_2 - \mu_1)\ (\mu_1 L_2 + \mu_2 L_1)}{(L_1 + L_2)\left[L_1 + L_2 + h_T\ (\mu_2 - \mu_1)\right]} \right]$$

$$+ W_L \left\{ \frac{L_4}{L_3 + L_4} \left[\frac{\mu_1 L_2 + \mu_2 L_1 + L_5(\mu_2 - \mu_1)}{L_1 + L_2} \right] + \frac{(\mu_2 - \mu_1)}{L_1 + L_2 + h_T\ (\mu_2 - \mu_1)} \right.$$

$$\left[\frac{\begin{array}{l} h_T L_4 (L_3 + L_4 + \mu_3 h_H) \left[\mu_1 (L_5 - L_2) - \mu_2 (L_1 + L_5) \right] \\ + \mu_3 L_3 (L_1 + L_2) \left[h_H (L_5 - L_2) - (h_T - h_H)\ (L_3 + L_4) - \mu_2\ h_H h_T \right] \end{array}}{(L_1 + L_2)\ (L_3 + L_4)(L_3 + L_4 + \mu_3 h_H)} \right]$$

$$\left. + \mu_3 L_3 \left[\frac{L_3 + L_4 + \mu_2 h_H}{(L_3 + L_4 + \mu_3 h_H)\ (L_3 + L_4)} \right] \right\}$$

EVERYTHING ABOVE HERE IS DIVIDED BY EVERYTHING BELOW HERE

$$W_T + W_L \left[\frac{L_3 + L_4 + (\mu_3 - \mu_2)\ h_L + \mu_2 h_H}{L_3 + L_4 + \mu_3 h_H} - \frac{(\mu_2 - \mu_1)}{L_1 + L_2 + h_T\ (\mu_2 - \mu_1)} \right.$$

$$\text{TIMES} \quad \left. \frac{\begin{array}{l} h_L\ (L_5 - L_2) + h_T\ (L_3 + L_4) + h_H\ (L_2 - L_3 - L_4 - L_5) \\ + h_L h_T\ (\mu_3 - \mu_2) - h_H\ (\mu_3 h_L - \mu_2 h_T) \end{array}}{L_3 + L_4 + \mu_3 h_H} \right]$$

That is the weight shift speed equation in all its glory. If you have no brakes on the front axle, (μ_1) becomes zero and the equation becomes somewhat shorter. There are some terms which equal or nearly equal one and probably could be left out of the equation. But if you must take the equation into court, it is better to leave all terms intact.

If you have brakes on all wheels, then this monster will cancel down to

$$V_0^2 - V_f^2 = 2 g D \mu$$

giving you, of course, the original minimum speed equation and proving that the two equations have the same origin.

The trucking industry is a very large and vital part of our existence. In general, truckers have an excellent record when it comes to miles driven per accident. The trouble is it is nearly impossible to have a minor accident with a rig that weighs 80,000 pounds. In past years truck accidents have been largely ignored from the standpoint of causation factors. The dead (and sometimes they were legion) were simply hauled away, and in a day or so the highway even got cleared off and cleaned up. It was a little like doing the supper dishes. Then the whole thing was promptly forgotten because to determine the causes was too complicated and no one knew where to start. Or worse, speed from skidmarks was sometimes computed incorrectly, so that a truck driver was accused of speeds greater than what he was actually going. Insurance companies have paid out countless millions of dollars simply because they felt long skidmarks from a truck indicated excessive speed. And such settlements indirectly affect us all.

I am not saying this equation is not complicated, because it is. If Solomon were alive, he would declare its possession an act of heresy and we would all go to the stake.

What I am trying to say is that accidents involving large trucks are often of an extremely serious nature. The woods are full of crackpots with safety gimmicks to sell to the trucking companies. Without the benefit of a proper investigation into the causes of accidents, no one can say which devices are truly safety devices and which are simply expensive trinkets to hang on a truck. Truck accidents usually warrant careful accident investigation and often warrant detailed accident reconstruction. This chapter gives you a tool to use for accident reconstruction.

CHAPTER XIII

MISCELLANEOUS FORMULAS

This chapter will deal with various formulas. Some will have more applicability than others, but you will probably use all of them at one time or another.

These formulas are the result of contributions of students and old friends who took pity on me for floundering around and showed me easier and better ways of doing things.

It is a humbling experience to be a hot shot instructor and have a student 30 years your junior explain that you can get from New York to San Francisco without going through Fairbanks. In teaching accident reconstruction I have eaten crow so many times it tastes just like lobster.

I have carefully hoarded all of these formulas and now will get them into print. I am grateful to everyone who contributed to this chapter. I will not specify every contributor's name, however, because there are quite a few people involved.

Let us, then, with reckless abandon plunge into this unfamiliar morass and, with the speed of a one-armed swimmer in a feces lake, charge for the sunset.

<u>Tip-over Formula</u>

$$V_{FPS} = \sqrt{\frac{Rg}{2} \cdot \frac{d}{h}} \quad \text{(equation for level curve)}$$

V_{FPS} = Velocity in feet per second
R = Radius of the curve in feet
d = Track width of the vehicle in feet
h = Height of the center of mass in feet

The first formula we will try involves a vehicle in a rollover. It is called the tip-over formula because it gives you the speed of a vehicle at the moment of twist, or the moment the center of mass of the vehicle starts to turn over. Remember I said starts to turn over.

Assume you have a truck going around a curve to the left. Assume further that it tips over in this curve and tips, of course, to the right. The tip-over formula gives you the speed of the vehicle the instant the weight on the left wheels goes to zero.

This is a brief explanation of what the formula does.

It is dependent on the radius of the curve a vehicle is trying to negotiate and on the location of the center of mass of the vehicle. The drag factor of the road surface is not used in the tip-over formula because the radius that a vehicle can negotiate at a particular speed is already governed by the drag factor.

As an example, if a truck is going 50 MPH on a drag factor of .70 and turns as sharp as it can, its center of mass will follow an arc with a radius of at least 239 feet. If its center of mass is 6 feet off the ground and its width is 8 feet, it will tip over at 48.8 MPH if it tries to follow a radius of 239 feet. Therefore the truck will tip over at 50 MPH.

Now change the road conditions to glare ice with a drag factor of only .10. At 50 MPH the center of mass of the truck will follow an arc with a radius of at least 1678 feet. If you put this radius into the tip-over formula, you will find that the truck will not tip over unless it is going 129 MPH. It will not tip over at 50 MPH because the drag factor is too low.

The tip-over formula assumes that the drag factor is high enough to tip a vehicle over.

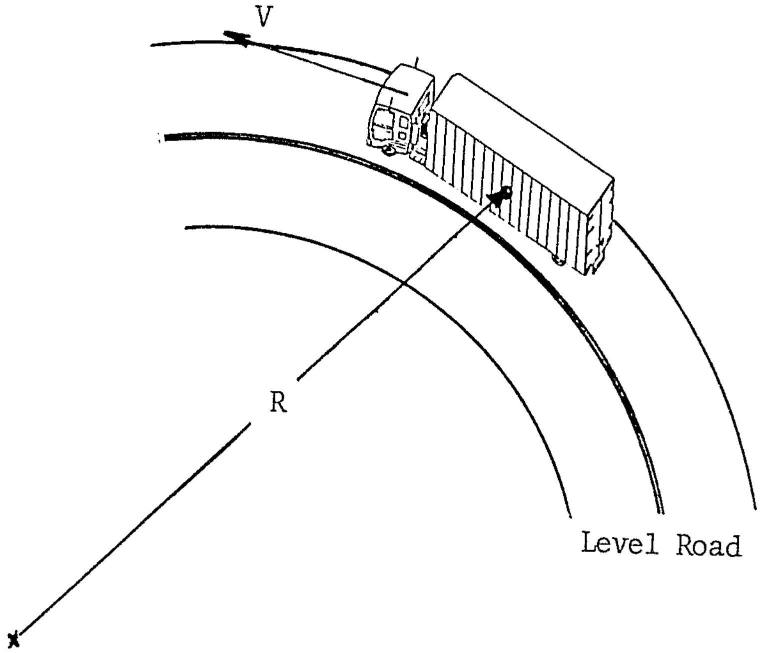
This formula is not valid if a vehicle slides sideways into something that will trip it. A prime example would be a vehicle sliding sideways into a curb or into dirt where the wheels dig in deep enough to cause the vehicle to trip and roll over. Under those circumstances even a sports car with a very low center of mass will tip over. This formula applies only to a vehicle going through a curve.

Dr. Frank Navin, Department of Civil Engineering of the University of British Columbia has graciously supplied me with a derivation of the tip-over formula (two equations) for a level and a banked curve:

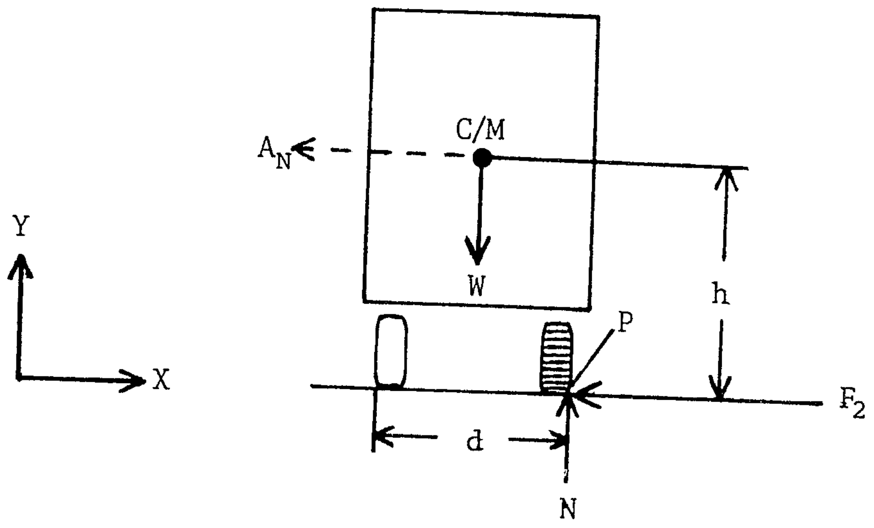
Definitions

A_N = Average normal acceleration through the center of mass in FPS^2
h = Height at the center of mass in feet
d = Track width from outside tire to outside tire in feet
W = Weight of the vehicle
M = Mass of the vehicle = $(\frac{W}{g})$
V = Velocity of the vehicle in FPS
R = Radius of the turn made by the vehicle at the center of mass in feet
F_2 = Force equal to the frictional force = MA_N
P = Point at the bottom of the outer right rear tire and on the outer edge of the tire
Mmt = Moment (Effect of a force turning around a given point)
N = Weight component normal to the road surface

Situation: A truck is going around a curve of radius "R" at velocity "V". The road is level where the truck is turning.



The truck is just about to tip over. All of the load is just off the inside wheel.



D'Alembert's principle states: "The external forces acting on a rigid body are equivalent to the effective forces of the various particles forming the body." That is, the external forces may be equated with the internal forces.

Applying d'Alembert's principle, you may write the equation

$$\Sigma F_X = \Sigma (F_X)_{EFF} \quad \text{where}$$

ΣF_X = The sum of the external forces in the (X) direction

$\Sigma (F_X)_{EFF}$ = The sum of the effective internal forces in the (X) direction

The truck is just in equilibrium. So, taking the moments about point "P" on the outer right rear tire, you get another equation from d'Alembert's principle.

$$\Sigma \text{Mnt}_P = \Sigma(\text{Mnt}_P)_{EFF} \quad \text{where}$$

ΣMnt_P = The sum of the external moments about (P)

$\Sigma (\text{Mnt}_P)_{EFF}$ = The sum of the effective internal moments about (P)

Moment (Mnt) is defined as a force (or sum of forces) multiplied by the perpendicular distance from the point around which the force turns (here P) to the so-called "line of action" (here a line straight down from the c/m of the truck in the case of the external forces, and in the case of the internal forces a line straight out from the c/m parallel to the level ground and at a right angle to the other line of action.)

Let us now return to the equation

$$\Sigma \, \mathrm{Mmt}_P = \Sigma \, (\mathrm{Mmt}_P)_{EFF}$$

We treat the sum of the moments as a single moment.

Since Mmt = F · D, we substitute F · D for Mmt.

$$\left[F \cdot D \right]_P = \left[(F \cdot D)_P \right]_{EFF}$$

Since F = M · A, we substitute MA for F.

$$\left[M \cdot A \cdot D \right]_P = \left[(M \cdot A \cdot D)_P \right]_{EFF}$$

And since $M = \frac{W}{g}$ we substitute, on the left side of the equation only, $\frac{W}{g}$ for M.

$$\left[\frac{W}{g} \cdot A \cdot D \right]_P = \left[(M \cdot A \cdot D)_P \right]_{EFF}$$

On the left A is equivalent in this instance to 32.2 or (g), and

on the right A is defined as A_N. The (g's) cancel.

$$\left[\frac{W}{\cancel{g}} \cdot \cancel{g} \cdot D \right]_P = \left[(M \cdot A_N \cdot D)_P \right]_{EFF}$$

On the left D is replaced by $(\frac{d}{2})$, and on the right D is replaced by (h).

$$W \cdot \frac{d}{2} + MA_N \cdot h$$

On the left we have defined the external moments about P, and on the right we have defined the internal effective moments about P.

Reversing the sides of the equation and dividing both sides by (h) we get

$$MA_N = \frac{Wd}{2h}$$

The equation for finding the normal acceleration for circular motion is $A_N = \frac{V^2}{R}$

So substituting $(\frac{V^2}{R})$ for (A_N) we get

$$M \frac{V^2}{R} = \frac{Wd}{2h} \quad \text{or}$$

$$V^2 = \frac{WdR}{2hM}$$

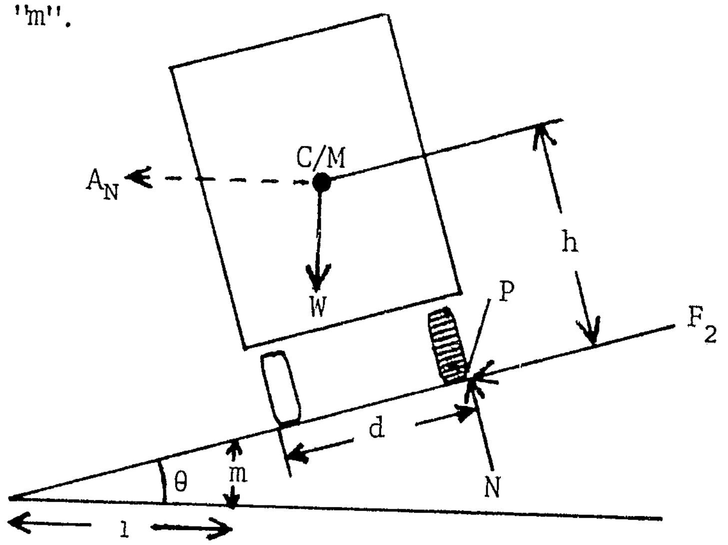
And since $M = \frac{W}{g}$ we end up with

$$(1) \quad V = \sqrt{\frac{Rg}{2}\left(\frac{d}{h}\right)}$$

Equation (1) gives you the overturning speed for a level turn.

Limit: If $\left(\frac{d}{2h}\right)$ is greater than the drag factor of the road surface, the vehicle will slide and not roll over.

Now let us assume that the truck is going around a curve with superelevation "m".



From d'Alembert's principle you can also derive

$$(2) \quad V = \sqrt{\frac{Rg\ (\frac{1}{2}\ d + hm)}{(h - \frac{1}{2}\ dm)}}$$ where

m = Superelevation = Tan θ

Equation (2) gives you the overturning speed for a banked turn.

We will first substitute the values below in equation (1) to find out at what speed the truck will tip over on a level curve.

h = 7.5 feet

d = 8.5 feet

R = 300 feet

$$(1) \quad V = \sqrt{\frac{Rg}{2}\left(\frac{d}{h}\right)}$$

$$V = \sqrt{\frac{300\ (32.2)}{2}\left(\frac{8.5}{7.5}\right)}$$

V = 74 FPS or

S = 50 MPH (Overturning speed)

We should note, however, that for a level curve the critical drag factor computed with the given values is

$$\frac{d}{2h} = \frac{8.5}{2\ (7.5)} = .57$$

This means that the drag factor of the road must be greater than .57 for the vehicle to overturn.

We will next substitute those same values in equation (2) to find out at what speed the truck will tip over on a curve with the same radius but with a superelevation of .03.

$$(2)\quad V = \sqrt{\frac{Rg\ (\tfrac{1}{2}\ d + hm)}{(h - \tfrac{1}{2}\ dm)}}$$

$$V = \sqrt{\frac{300\ (32.2)\ [\ \tfrac{1}{2}\ (8.5) + 7.5\ (.03)\ ]}{[\ 7.5 - \tfrac{1}{2}\ (8.5)\ (.03)\ ]}}$$

$V = 76$ FPS

$S = 52$ MPH (Overturning speed)

This concludes Dr. Navin's derivation of the tip-over formula and his examples.

The tip-over formula is of no use unless you can determine how far off the ground the center of mass is.

The chapter on weight shift shows how this can be done with a 4-wheel, 2-axle vehicle. But with a semi-trailer it is another story. If the trailer is empty, it can be weighed and raised just like a car. However, if it is loaded, as soon as you try to raise the front end, the load will slide to the rear end and the measurements will become invalid.

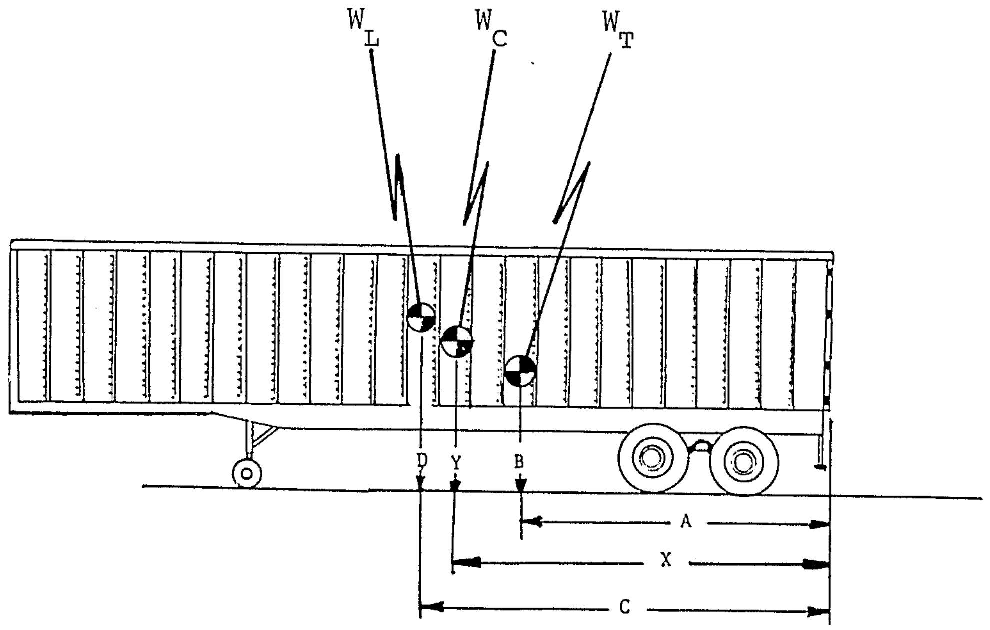
Dr. Navin also supplied me with a formula (2 equations) to find the center of mass of a semi-trailer and its load.

You need to know the center of mass of the empty trailer and the center of mass of the load.

You can probably get the center of mass of the empty trailer from the manufacturer. If you cannot, you will have to use the method described in Chapter XII, the chapter on weight shift, (See pages 114-5). Simply treat the semi-trailer as a 2-axle vehicle. The dollies will be treated as one axle, and the line midway between the tandem axles will be treated as the other axle. Then weigh the trailer. Once you have obtained the longitudinal center of mass, proceed to raise one end of the trailer. Here you must raise the end of the trailer supported by the dollies. And you must measure the distance to the ground from a point on the leg of the dolly that would correspond to the center of a truck wheel if truck wheels, rather than the small dolly wheels, were supporting that end of the trailer. Then weigh the rear end of the trailer and solve the equation for the verticle center of mass.

To find the center of mass of the load on the trailer, simply measure the outer dimensions of the load and locate the mid-point longitudinally and vertically.

To find the center of mass (c/m) of a loaded trailer when you know the center of mass of the trailer and the center of mass of the load, you must solve two equations. The following diagram shows the weights and distances whose values go into those equations.



W_L = Weight at the c/m of the load

W_T = Weight at the c/m of the trailer

W_C = Weight at the c/m of the trailer and load

Following are the equations for finding the center of mass of a loaded trailer:

$$X = \frac{\left[(W_L \cdot C) + (W_T \cdot A) \right]}{W_C}$$

$$Y = \frac{\left[(W_L \cdot D) + (W_T \cdot B) \right]}{W_C}$$

To illustrate how the equations work we will assume

W_T = 8,000 lbs.

W_L = 20,000 lbs.

W_C = 28,000 lbs.

A = 16 feet

B = 5 feet

C = 20 feet

D = 8 feet

Plugging these values into the two equations we get

$$X = \frac{\left[(20000 \cdot 20) + (8000 \cdot 16) \right]}{28000}$$

X = 18.85 feet (Longitudinal distance of the c/m of the loaded trailer from the rear end of the trailer)

$$Y = \frac{\left[(20000 \cdot 8) + (8000 \cdot 5) \right]}{28000}$$

Y = 7.14 feet (Vertical distance of the c/m of the loaded trailer from the ground)

Next we will deal with the formulas that will give you the distance necessary to avoid an object in the road in front of you.

Basically there are two formulas. The first one is

$$D = .45 \cdot V_{FPS} \cdot \frac{\sqrt{L}}{\sqrt{A_Y}} + T_R \cdot V_{FPS}$$

D = Distance in feet

V_{FPS} = Velocity in feet per second

L = Lateral distance in feet a vehicle must move over to miss the object

A_Y = Average lateral acceleration expressed in g's (Note that (A_Y) relates to drag factor.)

T_R = Reaction time

.45 = Constant

This formula gives you the distance needed to move over far enough to miss a stationary object in front of you and straighten out. In effect it gives you the distance necessary to change lanes. Obviously the unknown is the (A_Y) or lateral acceleration. A low value for (A_Y) will give you a distance comparable to that required for a comfortable normal lane change. If the turn is hard enough that you nearly leave a scuff mark, the value for (A_Y) increases to that of the full drag factor of the road. Use of the full drag factor of the road for (A_Y) will give you the shortest possible distance needed to change lanes.

The second formula is

$$D = .225 \cdot V_{FPS} \cdot \frac{\sqrt{L}}{\sqrt{A_Y}} + T_R \cdot V_{FPS}$$

The symbols are the same as for the first formula. Only the constant is changed and is .225 in the second formula.

This formula gives you the distance necessary to swerve and miss a stationary object in front of you. It does not allow for straightening out. Remember that both formulas assume the object you are trying to miss is stationary in the road.

Next we will partially discredit an old formula:

$$S = 3.87 \text{ x } \frac{D}{\sqrt{D \pm h}}$$

This is the short vault formula that assumes a take-off angle of 45 degrees. It is true that it will give you a speed for an object if the take-off is 45 degrees. It is not true that it will always give you a minimum speed. It will give you a minimum speed only if the take-off and landing are on the same level. Then, of course, the formula becomes

$$S = 3.87 \text{ x } \sqrt{D}$$

I must hasten to add that the error is small. The error is small enough to be ignored until the values of (D) and (h) start to approach one another. Nevertheless, you must remember that the short vault formula will usually not give you a minimum speed even though its structure is correct.

Anyone who has been involved in court knows the ramifications of committing an error on the witness stand. If you misrepresent a formula, you cannot expect the opposing attorney to be very understanding. If the opposing attorney is worth his salt, he will turn on you like a mad dog. And when he finally lets you go, the jury will look on you as someone Genghis Khan would have been proud to call his friend.

Now, what to do with a dilemma such as follows. Assume you have an accident where an object is launched into the air. It travels a known horizontal distance and lands a known distance lower than take-off. But you do not know the angle at which it took off. A prime example is a motorcycle hitting the side of a car. The cycle stops and the rider flies off, clearing the top of the car. You know where he lands down the street but not his take-off angle. The formula below will give you the angle of take-off. You then use this angle in the long vault formula to get the minimum speed required to launch an object into the air for a specific horizontal distance.

$$\text{Angle } \theta = \frac{\text{Arc tangent of } (\frac{D}{h})}{2}$$

This formula assumes that (h) is for landing lower than take-off. If the object lands higher than take-off, simply subtract your answer from 90 degrees and use that number as the take-off angle. For example, if D = 25 feet and h = 5 feet, the angle of take-off for the minimum speed is 39.3 degrees if the object lands lower than

take-off. If the object lands higher than take-off, the angle of take-off is (90 - 39.3) degrees or 50.7 degrees. Note that these take-off angles are not even close to 45 degrees.

If you are struck dumb at the thought of finding an arc tangent, your calculator will do it without a whimper. Hit inverse tangent or second function tangent, depending on the kind of machine you own, and there it will be slick as spit on a doorknob.

While we are involved with the vault formula, we might as well do a variation of it. If you know the velocity of an object and its angle of take-off, this formula will tell you how far the object will travel.

$$D = \frac{+\left[\frac{2V_0{}^2}{g} \cdot \text{Sin}\theta \cdot \text{Cos}\theta\right] \pm \sqrt{\left[\frac{2V_0{}^2}{g} \cdot \text{Sin}\theta \cdot \text{Cos}\theta\right]^2 - 4 \cdot 1 \cdot (-\frac{2V_0{}^2}{g}) \cdot h \cdot \text{Cos}^2\theta}}{2 \cdot 1}$$

Note that this is a quadratic equation and will give you two answers, only one of which will be logical. The above equation also assumes the object lands lower than take-off. If it lands higher than take-off, you simply change $(-\frac{2V_0{}^2}{g})$ to $(+\frac{2V_0{}^2}{g})$.

If a vehicle is launched into the air and you want to compute how high the vehicle is at the point in its arc directly opposite any given point along its horizontal distance of travel, you use the following formula.

$$h = D \cdot \mathrm{Tan}\ \theta - \tfrac{1}{2}\ g \frac{D^2}{V_0{}^2 \cdot \mathrm{Cos}^2\theta}$$

If the vehicle lands lower than take-off, (h) will become negative once the point representing the position of the vehicle in its arc falls below the level of take-off. If the total horizontal distance of travel is plugged into this equation, (h) will be a negative number equal to the distance from the level of take-off down to the lower level of landing.

If, on the other hand, the vehicle lands higher than take-off and if you put the entire horizontal distance into the equation, (h) will be a positive number equal to the distance from the level of take-off up to the higher level of landing.

If an object is launched into the air and you want to find the height of the highest point the object will reach, you can use the equation

$$h\ \mathrm{max} = \frac{V_0{}^2 \cdot \mathrm{Sin}^2\ \theta}{2g}$$

If you want to find the distance to the point along the horizontal distance opposite which an airborne object reaches its highest point, simply use the equation

$$D = \frac{\mathrm{Tan}\ \theta \cdot V_0{}^2 \cdot \mathrm{Cos}^2\ \theta}{g}$$

And if you want to find the time necessary for an airborne object to reach any horizontal distance, use the equation

$$T = \frac{D}{V_0 \cdot \mathrm{Cos}\ \theta}$$

Below are three seldom-used equations that deal with propelling an object straight up into the air:

Velocity necessary to raise an object to a specific height

$$V = \sqrt{2gh}$$

Maximum height to which an object of known velocity will rise

$$h = \frac{V_0{}^2}{2g}$$

Time necessary for an object to rise to its maximum height

$$T = \frac{V_0}{g}$$

The first of the three preceding formulas will help you determine the vertical velocity necessary for a bump in the road to throw an object upward a known height. The second will give you the maximum height the bump can throw the object with any known vertical velocity. And the third will give you the time it takes to do it.

Well, that should cover about every approach that can be taken to a vaulting vehicle. Next we will turn to a vehicle moving up an incline.

Some parts of the country have roads with runaway truck ramps located part way down very long hills. When a vehicle makes use of a ramp, you can find the speed of the vehicle as it started up the ramp if you know the distance it traveled up the ramp, the drag factor of the surface of the ramp, and the angle of the ramp. The formula is

$$V_0^2 - V_f^2 = 2 \cdot \left[(g \cdot \text{Cos}\ \theta \cdot f) + (g \cdot \text{Sin}\ \theta) \right] \cdot D$$

This formula gives you the speed of a vehicle when the vehicle first begins moving up a slope. It incorporates the drag factor and the effect of gravity. In this equation you should use the drag factor for a level surface, since the other terms within the brackets account for the grade. (V_f = Final velocity)

The last formula will deal with hydroplaning. Hydroplaning occurs

when the front wheels ride up on a wedge of water and no longer make contact with the road surface. When this happens the vehicle can no longer be steered, and an accident may very well be the result.

For hydroplaning to occur, the depth of the water on the surface of the road must approximately equal the tread depth of the tire. The tread of a tire provides an escape for the water, and a wedge of water is slow to build if the water is forced up into the grooves of the tread design on the face of the tire. It must be noted that some tread designs are quite tight so that the water does not get readily forced up. A tire with such a tread will behave as though it were bald and will hydroplane at a lower speed. No matter what kind of tread a tire has, it will hydroplane if the speed gets high enough. But there has to be at least two-tenths of an inch of water on a road surface for any tire to hydroplane, no matter what its tread design or condition.

Here is the formula that gives you the speed necessary for a tire to hydroplane.

$$S = 10.35 \sqrt{\text{Air pressure in the tire in PSI}}$$

All you do is measure the pounds per square inch tire pressure and multiply the square root of this number by the constant 10.35.

This formula assumes an average tread depth, such as is found in a tire that is neither brand new nor bald. Since a bald tire will hydroplane at a slightly lower speed, you should adjust this formula for a bald tire by reducing the constant to about 9.

Well, this wraps it up. I hope you will find use for these miscellaneous formulas. Even if you have no use for them, they have helped make this manual a little thicker. The prestige of any book or manual increases by the square of the number of pages it has over 100.

REFRESHER MATHEMATICS

Appendix A of this manual is devoted to Refresher Mathematics. It is set up to be self-explanatory. It is necessary for you to go through Appendix A carefully before starting this course even though you have probably had this material at some point in your formal education. It is also important for you to work the problems, as this level of math will be used extensively.

The material in Appendix A was written by T. L. Aycock, formerly a Georgia State Trooper and presently a staff member of Northwestern University Traffic Institute. It is reproduced here with his explicit permission.

APPEXDIX A

REFRESHER MATHEMATICS

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REFRESHER MATHEMATICS

(TEXT)

I. Decimals

A. Division of Decimals

1. When a decimal appears in the divisor we eliminate the decimal by moving it to the right of the last number in the divisor. You must, however, move the decimal in the dividend the same number of places to the right.

NOTE: $\overset{\text{Quotient}}{\text{Divisor}/\overline{\text{Dividend}}}$

a. Examples

(1) $48.36 \div 4.26 = 4.26/\overline{48.36} =$

```
          11.352
426/4836.000
   -426
     576
    -426
     1500
    -1278
      2220
     -2130
       900
      -852
        48 (Remainder, or continued
              division)
```

I. A. 1. a. (2) $487.6 \div .58 = .58/\overline{487.6} = 58/\overline{48760.} =$ 840.69

(3) $.034 \div 1.49 = 1.49/\overline{.034} = 149/\overline{3.4} = .0228$

(4) $48 \div 1.4 = 1.4/\overline{48.0} = 14/\overline{480} = 34.29$

b. Alternate ways of indicating division:

$4 \div 2 = 2$

$\frac{4}{2} = 2$

$4/2 = 2$

$\overset{2}{2/\overline{4}}$

$\frac{2/4}{2}$

c. Number of digits in quotient. Some "rules of thumb" to follow are:

(1) Division should be carried out to at least the same number of decimal places contained in the original number with the greatest number of decimal places.

I. A. 1. c. (2) Answer should have at least three or four digits to have meaning unless it is exactly a whole number with less digits.

(3) Some problems may require greater accuracy depending on the desired use--this is a decision on your part.

2. Work Problems (Use extra sheet of paper)

a. 223 ÷ 1.7 =

b. 196 ÷ .98 =

c. 496.4 ÷ 186.54 =

d. .89 ÷ 1.6 =

e. 5.2 ÷ .9 =

f. 92.1 ÷ 15.46 =

g. .119 ÷ .03 =

h. 13.1 ÷ .9 =

i. 96.3 ÷ .003 =

j. .0004 ÷ .002 =

B. Multiplication of Decimals

1. When a decimal appears in the numbers to be multiplied you must count the total number of numerals to the right of the decimal points in

I. B. 1. both numbers and then, starting from your right, count off the same number of numerals to your left in the answer or product and there place your decimal point.

NOTE:

$$\begin{array}{rl} 4 & \text{Multiplicand} \\ \underline{x2} & \text{Multiplier} \\ 8 & \text{Product} \end{array}$$

a. Alternate ways of indicating multiplication:

4 x 2 = 8

4 · 2 = 8

(4) (2) = 8

$$\begin{array}{r} 4 \\ \underline{x2} \\ 8 \end{array}$$

b. Examples

(1) 36.4 x 2.4 =

$$\begin{array}{rl} 36.4 & \text{(1 decimal)} \\ \underline{x\ 2.4} & \text{(1 decimal)} \\ 1456 & \\ \underline{728\ } & \\ 87.36 & \text{(2 decimals)} \end{array}$$

(2) 2.63 x 0.45 =

$$\begin{array}{rl} 2.63 & \text{(2 decimals)} \\ \underline{x\ 0.45} & \text{(2 decimals)} \\ 1315 & \\ \underline{1052\ } & \\ 1.1835 & \text{(4 decimals)} \end{array}$$

I. B. 1. b. (3) 684 x 0.033 = 684 (0 decimals)

x 0.033 (3 decimals)

2052

2052

22.572 (3 decimals)

(4) 0.482 x 6 = 0.482 (3 decimals)

x 6 (0 decimals)

2.892 (3 decimals)

c. Work Problems (Use extra sheet of paper)

(1) 411.3 x 0.04 =

(2) 0.0468 x 6.2 =

(3) 420 x 0.21 =

(4) 15 x 0.030 =

(5) 5333.2 x 0.96 =

(6) 0.233 x 1.3 =

(7) 15.28 x 5.5 =

(8) 49.1 x 0.003 =

(9) 24 x 1.4 =

(10) 0.8743 x .033 =

II. Fractions

A. Understanding Fractions

Fractions are really nothing more than a way to show one number divided by another. They can always be divided just as the line indicates to yield a decimal number.

II. (cnt'd) A. NOTE: $\frac{\text{Numerator}}{\text{Denominator}}$

1. <u>Proper Fraction</u>: A proper fraction is a fraction whose value is less than one. In a proper fraction the numerator is less than the denominator.

a. Examples

(1) $\frac{3}{4}$ Numerator Denominator Less than one or 0.75

(2) $\frac{1}{9} =$ Less than one or 0.11

(3) $\frac{9}{10} =$ Less than one or 0.90

(4) $\frac{2}{4} =$ Less than one or 0.50

(5) $\frac{999}{1000} =$ Less than one or 0.999

2. <u>Improper Fraction</u>: An improper fraction is a fraction whose value is equal to or greater than one. In an improper fraction the numerator is equal to or greater than the denominator.

a. Examples

(1) $\frac{10}{2} =$ More than one or 5

II. A. 2. a. (2) $\frac{9}{3}$ = More than one or 3

(3) $\frac{8}{8}$ = Equal to one or 1

(4) $\frac{100}{25}$ = More than one or 4

(5) $\frac{100}{100}$ = Equal to one or 1

3. Equivalent Fractions: Equivalent fractions are fractions having the same value. They are made by multiplying both the numerator and the denominator by the same number, or dividing both the numerator and denominator by the same number.

a. Examples

(1) $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32} = \frac{32}{64}$

$$\frac{1 \times 32}{2 \times 32} = \frac{32}{64}$$

(2) $\frac{1}{3} = \frac{3}{9} = \frac{9}{27} = \frac{27}{71} = \frac{71}{213}$

$$\frac{71}{213} \div \frac{71}{71} = \frac{1}{3}$$

II. B. Changing Forms of Mixed Numbers and Improper Fractions

1. Mixed Numbers: A mixed number equals a whole number plus a fraction.

a. Examples

(1) $1\frac{1}{3}$

(2) $2\frac{5}{8}$

(3) $4\frac{7}{8}$

(4) $100\frac{1}{2}$

(5) $15\frac{1}{9}$

2. Changing a mixed number to an improper fraction

a. Procedure:

(1) Multiply the whole number by the denominator.

(2) Add the numerator to the product.

(3) Form an improper fraction by placing the results over the denominator.

(a) Examples

$$17\frac{2}{5} = 17 + \frac{2}{5}$$

Solution: $17 \times 5 = \begin{array}{r} 85 \\ +2 \\ \hline 87 \end{array}$ Answer $\frac{87}{5}$

II. B. 2. a. (3) (a) $9\frac{3}{10} = 9 + \frac{3}{10}$
(cnt'd)

Solution: $9 \times 10 = 90$ Answer $\frac{93}{10}$
$+3$
93

b. Work Examples (Use extra sheet of paper)

(1) $6\frac{2}{3} =$

(2) $5\frac{4}{5} =$

(3) $1\frac{1}{4} =$

(4) $9\frac{8}{10} =$

(5) $100\frac{4}{7} =$

II. C. Multiplying and Dividing Mixed Numbers and Fractions

1. Multiplying fractions

a. Examples (Proper Fractions)

(1) $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

II. C. 1. a. (2) $\frac{6}{9} \times \frac{4}{5} = \frac{24}{45}$

(3) $\frac{3}{16} \times \frac{1}{5} = \frac{3}{80}$

(4) $\frac{3}{100} \times \frac{4}{9} = \frac{12}{900}$

(5) $\frac{1}{6} \times \frac{19}{20} = \frac{19}{120}$

b. Examples (Improper Fractions)

(1) $\frac{8}{3} \times \frac{4}{2} = \frac{32}{6}$

(2) $\frac{5}{2} \times \frac{3}{2} = \frac{15}{4}$

(3) $\frac{8}{5} \times \frac{16}{5} = \frac{128}{25}$

(4) $\frac{2}{1} \times \frac{5}{3} = \frac{10}{3}$

(5) $\frac{9}{2} \times \frac{7}{3} = \frac{63}{6}$

c. Examples (Mixed Numbers)

First change mixed numbers to improper fractions and then handle as in (b) above.

II. C. 1. c. (1) $1\frac{1}{8} \times 2\frac{3}{8} = \frac{9}{8} \times \frac{19}{8} = \frac{171}{64} = 2\frac{43}{64}$

(2) $2\frac{3}{4} \times 3\frac{5}{8} = \frac{11}{4} \times \frac{29}{8} = \frac{319}{32} = 9\frac{31}{32}$

(3) $4\frac{1}{4} \times 5\frac{5}{6} = \frac{17}{4} \times \frac{35}{6} = \frac{595}{24} = 24\frac{19}{24}$

(4) $6\frac{1}{3} \times 5\frac{1}{4} = \frac{19}{3} \times \frac{21}{4} = \frac{399}{12} = 33\frac{3}{12} = 33\frac{1}{4}$

(5) $9\frac{1}{2} \times \frac{1}{4} = \frac{19}{2} \times \frac{1}{4} = \frac{19}{8} = 2\frac{3}{8}$

d. Work Examples (Use extra sheet of paper)

(1) $\frac{3}{4} \times \frac{5}{8} =$

(2) $\frac{5}{7} \times \frac{3}{8} =$

(3) $\frac{2}{16} \times \frac{4}{10} =$

(4) $\frac{9}{4} \times \frac{6}{4} =$

(5) $\frac{16}{15} \times \frac{9}{7} =$

(6) $\frac{8}{3} \times \frac{4}{1} =$

(7) $2\frac{6}{8} \times 3\frac{1}{3} =$

II. C. 1. d. (8) $31\frac{1}{4} \times 1\frac{3}{8} =$

(9) $1\frac{2}{3} \times 2\frac{1}{4} =$

(10) $2\frac{5}{7} \times 3\frac{2}{10} =$

2. Dividing Fractions

a. Note that $\frac{1}{3} \div \frac{2}{3}$ means $\dfrac{\frac{1}{3}\ \text{numerator}}{\frac{2}{3}\ \text{denominator}}$

Take the denominator fraction and invert (or interchange or "turn upside down") its numerator and denominator, and then multiply the fractions as shown before.

(1) $\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} = \frac{3}{6}$

b. Examples (Proper Fractions)

(1) $\frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} = \frac{3}{6} = \frac{1}{2}$

(2) $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$

(3) $\frac{6}{8} \div \frac{1}{3} = \frac{6}{8} \times \frac{3}{1} = \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}$

II. C. 2. b. (4) $\frac{1}{5} \div \frac{1}{8} = \frac{1}{5} x \frac{8}{1} = \frac{8}{5} = 1\frac{3}{5}$

(5) $\frac{2}{10} \div \frac{4}{5} = \frac{2}{10} x \frac{5}{4} = \frac{10}{40} = \frac{1}{4}$

c. Examples (Mixed Numbers)

(1) $2\frac{7}{8} \div 1\frac{2}{5} = \frac{23}{8} \div \frac{7}{5} = \frac{23}{8} x \frac{5}{7} = \frac{115}{56} = 2\frac{3}{56}$

(2) $5\frac{1}{3} \div 2\frac{1}{4} = \frac{16}{3} \div \frac{9}{4} = \frac{16}{3} x \frac{4}{9} = \frac{64}{27} = 2\frac{10}{27}$

(3) $2\frac{1}{6} \div 1\frac{1}{8} = \frac{13}{6} \div \frac{9}{8} = \frac{13}{6} x \frac{8}{9} = \frac{104}{54} = 1\frac{25}{27}$

(4) $3\frac{2}{9} \div 2\frac{3}{4} = \frac{29}{9} \div \frac{11}{4} = \frac{29}{9} x \frac{4}{11} = \frac{116}{99} = 1\frac{17}{99}$

(5) $8\frac{1}{2} \div \frac{1}{3} = \frac{17}{2} \div \frac{1}{3} = \frac{17}{2} x \frac{3}{1} = \frac{51}{2} = 25\frac{1}{2}$

d. Work Examples (Use extra sheet of paper)

(1) $\frac{1}{2} \div \frac{1}{3} =$

(2) $\frac{3}{4} \div \frac{5}{8} =$

(3) $\frac{5}{7} \div \frac{1}{3} =$

(4) $\frac{2}{5} \div \frac{1}{2} =$

II. C. 2. d. (5) $\frac{6}{7} \div \frac{3}{8} =$

(6) $1\frac{2}{3} \div 3\frac{1}{5} =$

(7) $2\frac{3}{5} \div 1\frac{4}{6} =$

(8) $3\frac{1}{5} \div 6\frac{1}{3} =$

(9) $4\frac{1}{9} \div 2\frac{1}{3} =$

(10) $5\frac{1}{2} \div 1\frac{1}{6} =$

D. Adding and Subtracting Fractions

1. Adding Fractions

a. Examples (Common Denominator)

(1) $\frac{3}{5} + \frac{2}{5} + \frac{1}{5} = \frac{3 + 2 + 1}{5} = \frac{6}{5} = 1\frac{1}{5}$

(2) $\frac{4}{7} + \frac{3}{7} + \frac{4}{7} = \frac{4 + 3 + 4}{7} = \frac{11}{7} = 1\frac{4}{7}$

(3) $\frac{2}{10} + \frac{3}{10} + \frac{4}{10} = \frac{2 + 3 + 4}{10} = \frac{9}{10}$

(4) $\frac{3}{16} + \frac{4}{16} + \frac{5}{16} = \frac{3 + 4 + 5}{16} = \frac{12}{16} = \frac{3}{4}$

(5) $\frac{8}{2} + \frac{3}{2} + \frac{1}{2} = \frac{8 + 3 + 1}{2} = \frac{12}{2} = 6$

II. D. 1. b. Examples (Different Denominators)

You must first determine what is the least common denominator (LCD). The least common denominator is the lowest number which, when divided by the denominator of each fraction, yields a whole number such as 1, 2, 3, etc. Sometimes the (LCD) is just the product of all the denominators.

You now divide the denominator of each fraction that is to be added into the least common denominator.

You now multiply the numerator of each fraction by the respective answer of the previous step.

(1) $\frac{2}{6} + \frac{3}{12} + \frac{5}{36} = \frac{2 \text{ x } 6}{36} + \frac{3 \text{ x } 3}{36} + \frac{5 \text{ x } 1}{36} =$

$\frac{12 + 9 + 5}{36} = \frac{26}{36} = \frac{13}{18}$

(2) $\frac{1}{3} + \frac{2}{5} + \frac{3}{9} = \frac{1 \text{ x } 15}{45} + \frac{2 \text{ x } 9}{45} + \frac{3 \text{ x } 5}{45}$

$\frac{15 + 18 + 15}{45} = \frac{48}{45} = 1 \frac{3}{45} = 1 \frac{1}{15}$

II. D. 1. b. (3) $\frac{4}{7} + \frac{3}{8} + \frac{2}{9} = \frac{4 \text{ x } (8 \text{ x } 9)}{7 \text{ x } 8 \text{ x } 9} + \frac{3 \text{ x } (7 \text{ x } 9)}{7 \text{ x } 8 \text{ x } 9} +$

$\frac{2 \text{ x } (7 \text{ x } 8)}{7 \text{ x } 8 \text{ x } 9} = \frac{4 \text{ x } 72}{7 \text{ x } 8 \text{ x } 9} + \frac{3 \text{ x } 63}{7 \text{ x } 8 \text{ x } 9} +$

$\frac{2 \text{ x } 56}{7 \text{ x } 8 \text{ x } 9} = \frac{288 + 189 + 112}{7 \text{ x } 8 \text{ x } 9} = \frac{589}{504} = 1 \frac{85}{504}$

(4) $\frac{3}{4} + \frac{2}{6} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} + \frac{4}{12} = \frac{9 + 4 + 4}{12} =$

$\frac{17}{12} = 1 \frac{5}{12}$

(5) $\frac{1}{9} + \frac{3}{5} + \frac{9}{45} = \frac{5}{45} + \frac{27}{45} + \frac{9}{45} = \frac{5 + 27 + 9}{45} =$

$\frac{41}{45}$

2. Subtracting Fractions

a. Examples (Common Denominators)

(1) $\frac{3}{4} - \frac{1}{4} = \frac{3 - 1}{4} = \frac{2}{4} = \frac{1}{2}$

(2) $\frac{5}{8} - \frac{3}{8} = \frac{5 - 3}{8} = \frac{2}{8} = \frac{1}{4}$

(3) $\frac{3}{5} - \frac{1}{5} = \frac{3 - 1}{5} = \frac{2}{5}$

II. **D.** 2. a. (4) $\frac{9}{16} - \frac{3}{16} = \frac{9 - 3}{16} = \frac{6}{16} = \frac{3}{8}$

(5) $\frac{5}{9} - \frac{3}{9} = \frac{5 - 3}{9} = \frac{2}{9}$

b. Examples (Different Denominators)

(1) $\frac{2}{9} - \frac{1}{10} = \frac{2 x 10}{90} - \frac{1 x 9}{90} = \frac{20 - 9}{90} = \frac{11}{90}$

(2) $\frac{6}{7} - \frac{3}{4} = \frac{6 x 4}{28} - \frac{3 x 7}{28} = \frac{24 - 21}{28} = \frac{3}{28}$

(3) $\frac{8}{9} - \frac{3}{4} = \frac{8 x 4}{36} - \frac{3 x 9}{36} = \frac{32 - 27}{36} = \frac{5}{36}$

(4) $\frac{9}{15} - \frac{1}{5} = \frac{9}{15} - \frac{3}{15} = \frac{9 - 3}{15} = \frac{6}{15} = \frac{2}{5}$

(5) $\frac{9}{23} - \frac{1}{21} = \frac{9 x 21}{23 x 21} - \frac{1 x 23}{23 x 21} = \frac{189 - 23}{483} =$

$\frac{166}{483}$

3. Work Examples (Adding and Subtracting Fractions)

a. Adding Fractions (Use extra sheet of paper)

(1) $\frac{1}{3} + \frac{2}{3} + \frac{1}{3} =$

(2) $\frac{2}{9} + \frac{3}{9} + \frac{4}{9} =$

II. **D.** 3. a. (3) $\frac{3}{8} + \frac{2}{4} + \frac{1}{2} =$

(4) $\frac{4}{6} + \frac{1}{4} + \frac{1}{9} =$

(5) $\frac{2}{5} + \frac{3}{10} + \frac{4}{15} =$

b. Subtracting Fractions (Use extra sheet of paper)

(1) $\frac{3}{6} - \frac{2}{6} =$

(2) $\frac{5}{8} - \frac{2}{8} =$

(3) $\frac{3}{9} - \frac{1}{7} =$

(4) $\frac{5}{8} - \frac{2}{7} =$

(5) $\frac{8}{9} - \frac{2}{3} =$

III. Repeated Multiplying of a Factor

A. Base, Exponent and Power

1. An EXPONENT or POWER is a number which indicates how many times another number, the BASE, is being used as a repeated FACTOR.

a. Example: $3^4 = 81$ or $3 \cdot 3 \cdot 3 \cdot 3 = 81$

III. (cnt'd) A. 1. a. Thus, since $3 \cdot 3 \cdot 3 \cdot 3$ or $3^4 = 81$, 3 is the BASE, 4 is the EXPONENT and 3^4 is read "3 to the fourth power."

b. Examples

(1) $6^2 = 36$ or $6 \cdot 6 = 36$

(2) $5^3 = 125$ or $5 \cdot 5 \cdot 5 = 125$

(3) $6^4 = 1296$ or $6 \cdot 6 \cdot 6 \cdot 6 = 1296$

(4) $2^5 = 32$ or $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

(5) $3^3 = 27$ or $3 \cdot 3 \cdot 3 = 27$

c. Work Examples (Use extra sheet of paper)

(1) $5^4 =$

(2) $6^3 =$

(3) $7^2 =$

(4) $8^3 =$

(5) $2^4 =$

IV. The Mathematics of Negative Numbers

A. Larger numbers can be subtracted from smaller numbers--the answer is a <u>negative</u> number.

1. A negative number is a number preceded by a minus sign (-). This can also be interpreted as a positive number multiplied by a minus one.

IV. (cnt'd) A. 1. Example: 10 is a positive number. It can be written as +10 or (+1) x 10 which means the number 10 multiplied by a plus one. A number which is not preceded by a minus sign is always considered a positive number. Positive numbers are the types of numbers you normally use.

-10 is a negative number. It is called a minus 10. It can also be written as (-1) x 10 = -10.

2. Subtraction can be considered as the addition of one number and the negative of another number.

Example: Subtract the number 2 from the number 4. The customary way of indicating this is

$$\begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array} \quad \text{This is also} \quad \begin{array}{r} +4 \\ -2 \\ \hline +2 \end{array}$$

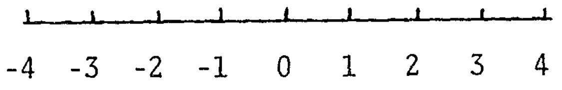
Using the idea of negative and positive numbers, we can write this as

$$4 + (-2) = 4 - 2 = 2$$

This will now be explained.

IV. A. 3. This latter example can be visualized by a "number line." Consider a horizontal line where the number zero is in the center. Mark off divisions to the right and to the left of zero. Number these divisions in increasing order to both the left and the right. Distinguish those numbers to the left by placing a minus sign in front of them.

Example:



Number Line

Those numbers to the left of zero are now the negative numbers. Those numbers to the right of zero are the positive numbers--but it is unnecessary to place plus signs in front of them--plus signs are understood by our convention.

4. Consider the addition of two positive numbers on the number line.

IV. (cnt'd) A. 4. Example: 4 + 2 = 6

This is done by moving two places to the right of number 4. This brings you to a plus 6.

Subtracting two from four, 4 - 2 = 2, is done by moving two places to the left of plus 4.

Now add a negative four to a plus two, -4 + 2 = -2

This is done by moving two places to the right from the number minus 4.

5. Note that in the previous discussion, "positive" means move to the right on the number line and "negative" means to move to the left or in the opposite direction from "positive." So, the positive and negative signs can be thought of as having a directional character.

6. The number line gives a physical interpretation of the addition or subtraction of positive and negative numbers. The mathematical laws for multiplying and dividing minus and plus signs

IV. A. 6. will provide a formal method of dealing with negative and positive numbers.

B. The Laws of Signs

1. Multiplying a positive number by another positive number results in a positive product. Dividing a positive number by a positive number results in a positive quotient.

Examples

Multiplication:

$(+) \times (+) = (+)$

$(+1) \times (+1) = (+1) = 1$

$2 \times 2 = 4$

Division:

$\frac{(+)}{(+)} = (+)$ $\frac{4}{2} = 2$

2. Multiplying a negative number by a negative number, or dividing a negative number by another negative number, results in a positive number.

Examples

Multiplication:

$(-) \times (-) = (+)$

$(-1) \times (-1) = (+1)$

$(-2) \times (-2) = 4$

IV. (cnt'd) B. 2. Division: $\frac{(-)}{(-)} = (+)$ $\frac{-4}{-2} = 2$

3. Multiplication or division by a positive number and a negative number (considering only two numbers) will always result in a negative number.

Examples

Multiplication:

$(+) \times (-) = (-)$

$(-) \times (+) = (-)$

$(-1) \times (+1) = (-1)$

$(-2) \times 2 = -4$

Division:

$\frac{(-)}{(+)} = (-)$

$\frac{(+)}{(-)} = (-)$

$\frac{-4}{2} = -2$

$\frac{4}{-2} = -2$

4. When more than two numbers are multiplied, or when more than two numbers form a fraction, the sign of the answer is negative only if the number of negative numbers is odd. Otherwise, the answer is a positive number.

IV. B. 4. Examples

Multiplication: (-1) x 2 x 3 = -6

4 x (-2) x (-3) = 24

(-1) x (-2) x (-3) = -6

Division:

$$\frac{4 \times (-2)}{4 \times 2} = -1$$

$$\frac{(-4) \times 2}{4 \times (-2)} = 1$$

$$\frac{(-4) \times (-2)}{(-2) \times 4} = -1$$

$$\frac{(-4) \times (-2)}{(-4) \times (-2)} = 1$$

$$\frac{8 \times (-3)}{2} = -12$$

$$\frac{8 \times 2 \times 3}{-6} = -8$$

5. When a sequence of numbers of different signs are added, the sign of the sum depends upon the magnitude of the negative and positive numbers.

Examples

-1 + 2 - 3 = -2

-1 + 3 + 4 - 6 = 0

-2 - 3 - 4 - 6 = -15

2 + 3 + 4 - 6 = 3

IV. B. 6. A sequence of numbers or groups can have subtractions indicated for negative numbers.

Examples

5 - (-4) =

(4 - 5) - (2 - 6) =

Recall that subtraction can be considered as the addition of one number and the negative of the other number, so the first example above can be written:

5 - (-4) = 5 + (-(-4))

Also recall that a negative number can always be written as the number times (-1), for example, -20 = (-1) x 20. Thus, the above can be written:

5 + (-(-4)) = 5 + ((-1)(-4))

The last term is a multiplication of two negative numbers which results in a positive number, so

5 + ((-1)(-4)) = 5 + (+4) = 9

Therefore, you can say that the negative of a negative number is positive. Similarly the

IV. (cnt'd) B. 6. negative of a positive number is negative.

Examples

$-(-4) = +4$

$-(+4) = -4$

$+(-4) = -4$

$(4 - 5) - (2 - 6) = (-1) - (-4)$

$= (-1) + (-1)(-4)$

$= -1 + 4 = 3$

V. Square Root

A. Terms Relating to Square Root: Definitions

1. The square root of a particular number is a number which when multiplied by itself, or squared, will equal that particular number.

2. The symbol $\sqrt{\ }$ indicates the principal square root of a number. This symbol is a modified form of the letter "r," the initial of the Latin word RADIX, meaning root.

V. A. 3. A RADICAL is an indicated root of a number or expression.

Examples:

$\sqrt{9}$, $\sqrt[3]{27}$, $\sqrt[5]{32}$

4. Radical Sign

Examples:

$\sqrt{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$

5. The RADICAND is the number or expression under the radical sign.

Examples: (Thus, 80 is the radicand of ...)

$\sqrt{80}$, $\sqrt[3]{80}$, $\sqrt[4]{80}$

6. The INDEX of a root is the small number written above and to the left of the radical sign. The index indicates which root is to be taken. In square roots, the index 2 is usually not indicated but understood.

a. Examples:

4 is the INDEX of the following:

$\sqrt[4]{16}$, $\sqrt[4]{26}$, $\sqrt[4]{96}$

b. Examples:

V. (cnt'd) A. 6. b. 2 is the INDEX of the following:

$\sqrt{4}$, $\sqrt{16}$, $^{2}\sqrt{186}$

B. Computing the Square Root of a Number

1. Square root: The square root of a number is that number, which if multiplied by itself, will give you the same number for which you seek a square root. If you wish to know the square root of 16 ($\sqrt{16}$), you would merely ask yourself, "What number, if multiplied by itself, would give me 16?" That number would be 4.

a. Examples

(1) $\sqrt{4} = 2$

(2) $\sqrt{25} = 5$

(3) $\sqrt{49} = 7$

(4) $\sqrt{36} = 6$

(5) $\sqrt{100} = 10$

2. Method for Finding Square Root: The question of finding square root becomes more difficult when you increase the digits of a number. The following is a procedure that can be used. Let us find

V. A. 2. the square root of 1398.76 -- $\sqrt{1398.76}$

a. To find the square root of 1398.76 you place the number 1398.76 inside a radical sign, $\sqrt{\quad} = \sqrt{1398.76}$

b. Your next step is to start from the right of the decimal and mark the digits off in pairs. Repeat to the left of the decimal point also.

$\sqrt{1398.76}$

c. Your next step is to place a decimal point directly above the decimal in the dividend.

$\sqrt{1398\overset{.}{.}76}$

d. Your next step is to find the nearest number to being the square root of the first digit or pair of digits from your left. You now place this number on top of the radical sign, directly above the second digit from your left.

V. (cnt'd) A. 2. d.

$$\begin{array}{l} \quad 3\ \ . \\ \sqrt{1398.76} \end{array}$$

e. Your next step is to square three--(3 · 3) and place that number below the first two digits from your left.

$$\begin{array}{l} \quad 3\ \ . \\ \sqrt{1398.76} \\ \quad 9 \end{array}$$

f. Your next step is to extend a line straight down from your radical sign.

$$\begin{array}{|l} \quad 3\ \ . \\ \sqrt{1398.76} \\ \quad 9 \\ \\ \\ \end{array}$$

g. Your next step is to subtract nine from thirteen.

$$\begin{array}{|l} \quad 3\ \ . \\ \sqrt{1398.76} \\ \ -9 \\ \ \ \overline{\ 4} \\ \\ \end{array}$$

h. Your next step is to bring down the next pair of digits alongside your remainder, 4.

V. A. 2. h.

$$\begin{array}{|l} \quad 3\ \ . \\ \sqrt{1398.76} \\ \ -9 \\ \ \ \overline{498} \\ \\ \end{array}$$

i. Your next step is to double your square root (3) and add a zero to the right of this number. You now place this number to the left of 498.

$$\begin{array}{r|l} & \quad 3\ \ . \\ & \sqrt{1398.76} \\ & \ -9 \\ 60 & \ \ \overline{498} \\ & \end{array}$$

j. Your next step is to divide 60, your first trial divisor, into your new dividend, 498, and place your answer to the right of 3 in your square root above the next pair of digits in the radicand.

$$\begin{array}{r|l} & \quad 3\ 7. \\ & \sqrt{1398.76} \\ & \ -9 \\ 60 & \ \ \overline{498} \\ & \end{array}$$

k. Your next step is to add the 7 in your square root to the 60, your first trial divisor, and then multiply this new trial divisor, 67, by that same number from the

V. A. 2. k. (cnt'd)

k. square root, 7, placing your answer under 498.

```
          3 7.
        √1398.76
          9
   60     498
   67     469
```

l. Your next step is to subtract 469 from 498.

```
          3 7.
        √1398.76
          9
   60     498
   67     469
           29
```

m. Your next step is to drop down the next pair of digits.

```
          3 7.
        √1398.76
          9
   60     498
   67     469
           29 76
```

n. Your next step is to double your square root, add a zero to the right of this number and then place this total to the left of your new dividend, 2976

V. A. 2. n. (cnt'd)

```
          3 7.
        √1398.76
          9
   60     498
   67     469
  740      29 76
```

o. Your next step is to divide 740 into 2976 and place this number directly to the right of the decimal in your square root, above the next pair of digits.

```
          3 7. 4
        √1398.76
          9
   60     498
   67     469
  740      29 76
```

p. Your next step is to add the last digit, 4, in your square root, to your new divisor, 740, yielding 744 and then multiply that number by the last number you placed in the square root, 4. You place the product of your multiplication under your new dividend, 2976.

```
          3 7. 4
        √1398.76
          9
   60     498
   67     469
  740      29 76
  744     -29 76
               0
```

V. A. 2. q. Since the difference between this product and the new dividend is zero, you now have the exact square root of 1398.76. If this difference was not zero, you would add a pair of zeros to the number you are taking the square root of and continue the process to a zero difference or the desired number of decimal places in the square root. Now square your answer to see if it is correct.

```
    37.4
    37.4
   149 6
  2618
 1122
 1398.76     -- Correct
```

3. Second Example of Working Square Root: This example will be worked the same way as the preceding example except this one will have a remainder.

Since we have described how to do square roots we will not do so in this example.

Let us now find the square root of 218. We will carry this square root out to hundredths.

V. A. 3. a.

```
           1 4. 7 6
         √ 218.0000
          | 1
  20      | 118
  24      |  96
  280     |  22 00
  287     |  20 09
  2940    |   1 9100
  2946    |   1 7676
          |     1424
```

b. In this example you will notice you have a remainder. To check your answer you should square your answer and add your remainder to the square of your answer.

c. 14.76 x 14.76 = 217.8576

You now add your remainder to 217.8576:

(1)
```
217.8576
  +.1424
218.0000
```

(2) As you can see you have worked the problem correctly.

4. Another example: find the square root of 220.

V. A. 4. a.

```
         1 6.
       √220.00
       | 1
    20 | 120
    26 | 156
       | -36        NO!
```

b.

```
         1 5.
       √220.00
       | 1
    20 | 120
    25 | 125
       |  -5       NO!
```

c.

```
          1 4. 8 3 2 3
        √220.00000000
        | 1
     20 | 120
     24 |  96
    280 |  24 00
    288 |  23 04
   2960 |     9600
   2963 |     8889
  29660 |      71100
  29662 |      59324
 296640 |      1177600
 296643 |       889929
        |          etc.
```

d. Actually, $\sqrt{220} = 14.832396$

5. Another Example of Working Square Root:

Sometimes you will find that your remainder will be larger than your divisor. In normal division this would indicate you had made a mistake, yet when using the above method to determine square

V. A. (cnt'd) 5. root, a remainder that is larger than your divisor may not mean you have made a mistake.

a. Let us now find the square root of 224 and we will also carry this problem out to hundredths. After working the problem we will check our answer.

b.

```
          1 4. 9 6
        √224.0000
        | 1
     20 | 124
     24 |  96
    280 |  28 00     As you can see, 28 is
    289 |  26 01     larger than 24 but if
   2980 |   1 9900   you use 25 as a divi-
   2986 |   1 7916   sor it would be too
        |     1984   large.
```

c. Now square your answer and add your remainder.

(1) 14.96 x 14.96 = 223.8016

(2)
```
223.8016
  +.1984
224.0000
```

(3) As you can see you have worked the problem correctly.

V. A. 6. Work Examples (Use extra sheet of paper). Work your problems to the nearest tenth.

a. $\sqrt{29}$

b. $\sqrt{329}$

c. $\sqrt{1569}$

d. $\sqrt{869}$

e. $\sqrt{6789}$

VI. Basic Algebra

A. Representing Numbers By Letters

1. In algebra, letters are used to represent numbers. By using letters and mathematical symbols, short algebraic statements replace lengthy verbal statements.

a. Example

Verbal: Seven times a number reduced by the same number equals six times the number.

Algebraic Statement: Let n = number

$7n - n = 6n$

(1) In the above example "7n" is used instead of "seven times a number." When multiplying a number by a letter,

VI. (cnt'd) A. 1. a. (1) the multiplication sign may be omitted. Multiplication may also be indicated by using a multiplication sign, a raised dot or parentheses around each quantity being multiplied.

(2) Thus, seven times a number may be shown by

$7 \times n,\ 7 \cdot n,\ 7(n)$ or $7n$.

(3) Omitting the multiplication sign, as in 7n, is the preferred method of indicating multiplication. However, the multiplication indication may not be omitted when multiplying two numbers.

b. Examples: Stating Products Without Multiplication Signs

(1) $7 \times y = 7y$

(2) $3 \times 5 \times a = 3 \times 5a = (3 \times 5)a = 15a$

(3) $1 \times w = 1w$

(4) $10 \times r \times s = 10rs$

(5) $.07 \times p \times q \times t = .07pqt$

VI. A. 1. c. Work Examples

(1) 9 x y x c =

(2) 15 x 4 x 2xy =

(3) 6 x c x d x e =

(4) 5 x 1 x 2 x r =

(5) 5 x 6 x a =

B. Interchanging Numbers in Addition

1. RULE: Interchanging addends (addends are numbers being added) does not change the sum.

a. Examples

(1) 5 + 3 = 8 or 3 + 5 = 8

(2) 9 + 6 = 15 or 6 + 9 = 15

(3) 1 + 2 = 3 or 2 + 1 = 3

(4) 6 + 5 + 6 = 17 or 5 + 6 + 6 = 17

(5) 10 + 2 + 3 = 15 or 3 + 10 + 2 = 15

b. Examples (Using Letters)

(1) a + b + c = c + b + a

(2) x + y + z = y + z + x

(3) c + a + d = a + c + d

(4) e + d + f = d + e + f

(5) k + g + h = g + h + k

VI. B. 2. RULE: Rearranging addends to a preferred order. Numerical addends are numbers used as addends. Literal addends are letters used to represent numbers being added.

a. Examples (Numerical Addends)

(1) 48 + 90 + 10 = 10 + 90 + 48 = 148

(2) 96 + 48 + 4 = 4 + 96 + 48 = 148

(3) 53 + 48 + 47 = 47 + 53 + 48 = 148

(4) 23 + 48 + 77 = 23 + 77 + 48 = 148

(5) 70 + 48 + 30 = 30 + 70 + 48 = 148

b. Examples (Literal Addends)

(1) d + c + b = b + c + d

(2) a + d + c = a + c + d

(3) z + y + x = x + y + z

(4) e + d + f = d + e + f

(5) h + j + i = h + i + j

c. Examples (Literal Addends preceded by Numerical Addends)

(1) b + a + 3 = a + b + 3

(2) 9 + f + e = e + f + 9

(3) e + 4 + c = c + e + 4

(4) x + b + 4 + c = b + c + x + 4

(5) 9 + 6 + c + b = b + c + 6 + 9

VI. B. 2. d. Work Examples

(1) $123 + 90 + 10 = 223$ or

(2) $w + a + b =$

(3) $3 + 9 + 1 = 13$ or

(4) $3 + d + a =$

(5) $14 + a + 20 + 80 + c =$

C. Interchanging Numbers in Multiplication

1. RULE: Interchanging factors does not change the product.

a. FACTORS are numbers being multiplied. Thus, in $5 \times 4 = 20$, 5 and 4 are the factors and 20 is the product.

b. NUMERICAL FACTORS: Numerical factors are numbers used as factors. Thus, $8 \times 2 = 16$, 8 and 2 are numerical factors.

c. LITERAL FACTORS: Literal factors are letters used to represent numbers being multiplied. Thus, in $ab = 30$, a and b are literal factors.

d. Examples (Numerical and Literal Factors)

VI. C. 1. d.

(1) $4 \times 2 = 8$ or $2 \times 4 = 8$

(2) $8 \times 4 = 32$ or $4 \times 8 = 32$

(3) $ba = 40$ or $ab = 40$

(4) $xy = 26$ or $yx = 26$

(5) $cba = abc$ or $bac = cab$

D. Order In Which Fundamental Operations Are Performed

1. RULE: In evaluating or finding the value of an expression containing numbers, the operations involved must be performed in a certain order. Do all multiplication first, then all division and finally addition and subtraction.

a. Examples (The Correct Order)

(1) $3 + 4 \times 2 =$

$4 \times 2 + 3 =$

$8 + 3 = 11$

(2) $9 + 4 \times 2 + 3 =$

$4 \times 2 + 9 + 3 =$

$8 + 12 =$

(3) $16 + 5 + 3 \times 4 =$

$4 \times 3 + 16 + 5 =$

$12 + 21 = 33$

VI. D. 1. a. (4) $15 - 3 + 4 \times 4 + 9 =$

$4 \times 4 + 15 + 9 - 3 =$

$16 + 24 - 3 =$

$40 - 3 = 37$

(5) $50 - 2 + 4 \times 3 \times 2 =$

$4 \times 3 \times 2 + 50 - 2 =$

$24 + 50 - 2 =$

$74 - 2 = 72$

b. Examples (The Incorrect Order). Note that the symbol ≠ means "not equal to."

(1) $3 + 4 \times 2 \neq$

$7 \times 2 = 14$ (Correct answer: 11)

(2) $9 + 4 \times 2 + 3 \neq$

$13 \times 2 + 3 =$

$26 + 3 = 29$ (Correct answer: 20)

(3) $16 + 5 + 3 \times 4 \neq$

$21 + 3 \times 4 =$

$24 \times 4 = 96$ (Correct answer: 33)

VI. D. 1. b. (4) $15 - 3 + 4 \times 4 + 9 =$

$12 + 4 \times 4 + 9 \neq$

$16 \times 4 + 9 =$

$64 + 9 = 73$ (Correct answer: 37)

(5) $50 - 2 + 4 \times 3 \times 2 =$

$48 + 4 \times 3 \times 2 \neq$

$52 \times 3 \times 2 =$

$156 \times 2 = 312$ (Correct answer : 72)

NOTE: Possible confusion of operations is best avoided by using parentheses.

E. The Uses of Parentheses

1. To treat an expression as a single number.

a. Examples: $2(x + y)$ represents twice the sum of x and y.

(1) $2(x - y)$ = Twice the difference between x and y.

(2) $2(x \cdot y)$ = Twice the product of x multiplied by y.

VI. E. 1. a. (3) $2(x \div y)$ = Twice the quotient of x divided by y.

2. To replace the multiplication sign.

   a. Examples: 4(5) represents the product of $4 \cdot 5$ or 4×5.

      (1) 4(6) represents the product of $4 \cdot 6$, or 4×6 or 6 times 4.

      (2) 8(6) represents the product of $8 \cdot 6$, or 8×6 or 8 multiplied by 6.

      (3) 9(4) represents the product of $9 \cdot 6$, or 9×6 or 9 multiplied by 6

3. To change or clarify the order of operations for evaluating by grouping the respective numbers with their operations.

   a. Examples

      (1) $2(4 + 3) = 2 \cdot 7 = 14$ as compared to $2 \cdot 4 + 3 = 8 + 3 = 11$

      (2) $8(5+2) = 8 \cdot 7 = 56$ as compared to $8 \cdot 5 + 2 = 40 + 2 = 42$

VI. E. 3. a. (3) $9(2 + 4) = 9 \cdot 6 = 54$ as compared to $9 \cdot 2 + 4 = 18 + 4 = 22$

4. "Nested" parentheses are often used in complex equations. Always work "from the inside out" by performing the operations within each basic group first.

   a. Examples

      (1) $4(10 + 2) - 3[2(4 - 3) + (6 - 1)] =$
      $4(12) - 3[2(1) + (5)] =$
      $48 - 3(2 + 5) = 48 - 3(7) = 48 - 21 = 27$

      (2) $2[6 + 4(3 + 2(5 - 3)\ (4 - 2))] =$
      $2[6 + 4(3 + 2(2)(2))] =$
      $2[6 + 4(3 + 8)] = 2\ [6 + 4\ (11)] =$
      $2(6 + 44) = 2(50) = 100$

F. Solving Simple Equations Using Inverse Operation

1. Addition and subtraction are inverse operations.

   a. Examples: addition of unknown solutions requiring subtraction.

      (1) $x + 3 = 8$ =

$$\begin{array}{rcr} x + 3 & = & 8 \\ - 3 & = & -3 \\ \hline x & = & 5 \end{array}$$

VI. F. 1. a. (2) $5 + y = 13 \;=\; \begin{array}{r} 5 + y = 13 \\ \underline{-5 \quad\; = -5} \\ y = \;\;8 \end{array}$

(3) $6 + a = 18 \;=\; \begin{array}{r} 6 + a = 18 \\ \underline{-6 \quad\; = -6} \\ a = 12 \end{array}$

b. Examples: subtraction of unknown solutions requiring addition.

(1) $x - 10 = 2 \;=\; \begin{array}{r} x - 10 = \;\;\,2 \\ \underline{+ 10 = +10} \\ x \qquad\; = \;12 \end{array}$

(2) $y - 3 = 14 \;=\; \begin{array}{r} y - 3 = 14 \\ \underline{+ 3 = +3} \\ y \qquad = 17 \end{array}$

(3) $x - 14 = 18 \;=\; \begin{array}{r} x - 14 = \;\;18 \\ \underline{+ 14 = +14} \\ x \qquad\; = \;\;32 \end{array}$

c. Work Examples (Use extra sheet of paper)

(1) $b + 16 = 32$

(2) $a + 5 = 10$

(3) $c + 2 = 14$

(4) $g - 3 = 18$

(5) $h - 6 = 4$

VI. F. 2. RULE: Multiplication and division are inverse operations.

a. Examples: Equations involving multiplication of unknown.

(1) $(3x = 12) = (\frac{3x}{3} = \frac{12}{3})$ or $x = 4$

(2) $(4y = 16) = (\frac{4y}{4} = \frac{16}{4})$ or $y = 4$

(3) $(16b = 32) = (\frac{16b}{16} = \frac{32}{16})$ or $b = 2$

b. Examples: Equations involving division of unknown.

(1) $(\frac{x}{3} = 12) = (\frac{x}{3} \cdot 3 = 12 \cdot 3)$ or $x = 36$

(2) $(\frac{y}{4} = 10) = (\frac{y}{4} \cdot 4 = 10 \cdot 4)$ or $y = 40$

(3) $(\frac{e}{2} = 4) = (\frac{e}{2} \cdot 2 = 4 \cdot 2)$ or $e = 8$

c. Work Examples (Use extra sheet of paper)

(1) $5q = 42$

(2) $9y = 28$

(3) $5z = 8$

(4) $6j = 9$

(5) $7x = 21$

VI. F. 2. c. (6) $\frac{y}{9} = 3$

(7) $\frac{q}{3} = 3$

(8) $\frac{z}{8} = 4$

(9) $\frac{b}{4} = 3$

(10) $\frac{c}{8} = 5$

G. Rules of Equality for Solving Equations

1. Equation: a statement indicating that two quantities are equal. Those quantities may be simple or complex combinations of numerical and literal factors where often the literal factor(s) are unknown(s).

   a. Solution of an equation is usually accomplished by separating the numerical and literal factors by placing the literal factors on the left-hand side of the equation and the numerical factors on the right-hand side. All operations indicated and possible are usually performed on the numerical factors.

2. Division Rule of Equality: To keep an equality, both sides may be divided by equal numbers, except zero.

VI. G. 2. a. Examples

(1) $7\ x = 35$

Answer: $\frac{7x}{7} = \frac{35}{7}$ or $x = 5$

(2) $35y = 7$

Answer: $\frac{35y}{35} = \frac{7}{35}$ or $y = \frac{1}{5}$

(3) $0.3a = 9$

Answer: $\frac{0.3a}{0.3} = \frac{9}{0.3}$ or $z = 30$

(4) $7x - 2x = 55$

Answer: $\frac{5x}{5} = \frac{55}{5}$ or $x = 11$

(5) $9z = 18$

Answer: $\frac{9z}{9} = \frac{18}{9}$ or $z = 2$

3. Multiplication Rule of Equality: To keep an equality, both sides may be multiplied by equal numbers.

   a. Examples

   (1) $\frac{x}{8} = 4$

   Answer: $\frac{x}{8} \cdot 8 = 4 \cdot 8$ or $x = 32$

   (2) $\frac{a}{0.5} = 4$

   Answer: $\frac{a}{0.5} \cdot 0.5 = 4 \cdot 0.5$ or $a = 2$

VI. G. 3. a. (3) $\frac{c}{6} = 5$

Answer: $\frac{c}{6} \cdot 6 = 5 \cdot 6$ or $c = 30$

(4) $\frac{b}{8} = 6$

Answer: $\frac{b}{8} \cdot 8 = 6 \cdot 8$ or $b = 48$

(5) $\frac{z}{10} = 7$

Answer: $\frac{z}{10} \cdot 10 = 7 \cdot 10$ or $z = 70$

4. Subtraction Rule of Equality: To keep an equality, equal numbers may be subtracted from both sides.

a. Examples

(1) $r + 8 = 13$

Answer:
$$\begin{array}{rcr} r + 8 & = & 13 \\ - 8 & = & -8 \\ \hline r \quad\; & = & 5 \end{array}$$

(2) $15 + x = 60$

Answer:
$$\begin{array}{rcr} 15 + x & = & 60 \\ -15 \quad\; & = & -15 \\ \hline x & = & 45 \end{array}$$

(3) $d + 6.9 = 20.8$

Answer:
$$\begin{array}{rcr} d + 6.9 & = & 20.8 \\ - 6.9 & = & -6.9 \\ \hline d \quad\;\; & = & 13.9 \end{array}$$

VI. G. 4. a. (4) $c + 16 = 34$

Answer:
$$\begin{array}{rcr} c + 16 & = & 34 \\ - 16 & = & -16 \\ \hline c \quad\;\; & = & 18 \end{array}$$

(5) $z + 17 = 69$

Answer:
$$\begin{array}{rcr} z + 17 & = & 69 \\ - 17 & = & -17 \\ \hline z \quad\;\; & = & 52 \end{array}$$

5. Addition Rule of Equality: To keep an equality, equal numbers may be added to both sides.

a. Examples

(1) $w - 10 = 19$

Answer:
$$\begin{array}{rcr} w - 10 & = & 19 \\ + 10 & = & +10 \\ \hline w \quad\;\; & = & 29 \end{array}$$

(2) $h - 14 = 6$

Answer:
$$\begin{array}{rcr} h - 14 & = & 6 \\ + 14 & = & +14 \\ \hline h \quad\;\; & = & 20 \end{array}$$

(3) $c - 3 = 5$

Answer:
$$\begin{array}{rcr} c - 3 & = & 5 \\ + 3 & = & +3 \\ \hline c \quad\; & = & 8 \end{array}$$

(4) $d - 8 = 6$

Answer:
$$\begin{array}{rcr} d - 8 & = & 6 \\ + 8 & = & +8 \\ \hline d \quad\; & = & 14 \end{array}$$

VI. G. 5. a. (5) $e - 16 = 2$

Answer:
$$\begin{array}{rcr} e - 16 & = & 2 \\ \underline{+ 16} & \underline{=} & \underline{+16} \\ e & = & 18 \end{array}$$

6. The preceding rules can be expressed with a general statement: what you do to one side of the equation, you must also do to the other.

7. Work Examples (Dealing with Rules of Equality)
   a. Work Examples (Division Rule Applies)
      (1) $3x = 23$
      (2) $4y = 24$
      (3) $5z = 35$
      (4) $6q = 48$
      (5) $7c = 49$

   b. Work Examples (Multiplication Rule Applies)
      (1) $\frac{x}{3} = 9$
      (2) $\frac{y}{4} = 16$
      (3) $\frac{b}{9} = 2$
      (4) $\frac{q}{3} = 20$
      (5) $\frac{b}{10} = 2$

VI. G. 7. c. Work Examples (Addition Rule Applies)
      (1) $c - 5 = 15$
      (2) $b - 6 = 24$
      (3) $e - 2 = 10$
      (4) $h - 13 = 2$
      (5) $j - 14 = 24$

   d. Work Examples (Subtraction Rule Applies)
      (1) $x + 4 = 16$
      (2) $y + 5 = 23$
      (3) $z + 7 = 10$
      (4) $q + 10 = 24$
      (5) $k + 3 = 5$

H. Using Two or More Operations to Solve Equations

1. In equations more than one operation can be performed to separate unknowns as needed to solve the equations.
   a. Thus, in $2x + 7 = 19$, two operations are required for solution--division and subtraction, subtraction performed first.
   b. Also, in $\frac{x}{5} - 3 = 7$, two operations needed for solution are multiplication and addition, addition performed first.

VI. H. 1. c. Examples (Using two operations)

(1)
$$\begin{array}{rcr} 2x + 7 &=& 19 \\ -\ 7 &=& -7 \\ \hline 2x \quad &=& 12 \end{array}$$

$$\frac{2x}{2} = \frac{12}{2} \qquad x = 6$$

(2)
$$\begin{array}{rcr} 3x - 5 &=& 7 \\ +\ 5 &=& +5 \\ \hline 3x \quad &=& 12 \end{array}$$

$$\frac{3x}{3} = \frac{12}{3} \qquad x = 4$$

(3)
$$\begin{array}{rcr} \frac{x}{5} - 3 &=& 7 \\ +\ 3 &=& +3 \\ \hline \frac{x}{5} \quad &=& 10 \end{array}$$

$$\frac{x}{5} = 10 \qquad \frac{x}{5} \cdot 5 = 10 \cdot 5$$

$$x = 50$$

(4)
$$\begin{array}{rcr} \frac{x}{3} + 5 &=& 7 \\ -\ 5 &=& -5 \\ \hline \frac{x}{3} \quad &=& 2 \end{array}$$

$$\frac{x}{3} \cdot 3 = 2 \cdot 3 \qquad x = 6$$

VI. H. 1. c. (5) $8x + 4x - 3 = 9$

$$\begin{array}{rcr} 12x - 3 &=& 9 \\ +\ 3 &=& +3 \\ \hline 12x \quad &=& 12 \end{array}$$

$$\frac{12x}{12} = \frac{12}{12}$$

$$x = 1$$

(6) $13x + 4 + x = 39$

$$\begin{array}{rcr} 14x + 4 &=& 39 \\ -\ 4 &=& -4 \\ \hline 14x \quad &=& 35 \end{array}$$

$$\frac{14x}{14} = \frac{35}{14}$$

$$x = 2\ 1/2$$

(7)
$$\begin{array}{rcr} 10 &=& 7 + x - \frac{x}{2} \\ -7 &=& -7 \\ \hline 3 &=& x - \frac{x}{2} \end{array}$$

$$3 = \frac{x}{2} \qquad 2 \cdot 3 = \frac{x}{2} \cdot 2$$

$$6 = x \qquad \text{or} \qquad x = 6$$

VI. H. 1. c. (8)

$$\begin{array}{rcl} 5n & = & 40 - 3n \\ +3n & = & \quad + 3n \\ \hline 8n & = & 40 \end{array}$$

$$\frac{8n}{8} = \frac{40}{8} \qquad n = 5$$

(9)

$$\begin{array}{rcl} 9n & = & 40 - 3n \\ +3n & = & \quad + 3n \\ \hline 12n & = & 40 \end{array}$$

$$\frac{12n}{12} = \frac{40}{12} \qquad n = 3\ 1/3$$

(10)

$$\begin{array}{rcl} 4u + 5 & = & 5u - 30 \\ -5 & = & \quad - 5 \\ \hline 4u & = & 5u - 35 \\ -5u & = & -5u \\ \hline -u & = & \quad - 35 \end{array}$$

$$\frac{-u}{-1} = \frac{-35}{-1} \qquad u = 35$$

(11) $\frac{8}{x} = 2$

$x(\frac{8}{x}) = 2x$

$\frac{8}{2} = \frac{2x}{2}$

$4 = x$ or $x = 4$

VI. H. 1. c. (12) $12 = \frac{3}{y}$

$12y = (\frac{3}{y})y$

$\frac{12y}{12} = \frac{3}{12}$

$y = 1/4$

(13)

$$\begin{array}{rcl} 10 - w & = & 3 \\ +w & = & \quad + w \\ \hline 10 & = & 3 + w \\ -3 & = & -3 \\ \hline 7 & = & w \end{array}$$

$w = 7$

(14)

$$\begin{array}{rcl} 25 - 3n & = & 13 \\ +3n & = & \quad + 3n \\ \hline 25 & = & 13 + 3n \\ -13 & = & -13 \\ \hline 12 & = & 3n \end{array}$$

$\frac{12}{3} = \frac{3n}{3}$

$4 = n$ or $n = 4$

VI. H. 1. c. (15) $25 = \frac{5}{4}x$

$$4 \cdot 25 = 4 \cdot \frac{5}{4}x$$

$$\frac{100}{5} = \frac{5x}{5}$$

$20 = x$ or $x = 20$

(16) $48 - \frac{5}{3}w = 23$

$$\begin{array}{rcl} +\frac{5}{3}w & = & +\frac{5}{3}w \\ \hline 48 & = & 23 + \frac{5}{3}w \\ -23 & = & -23 \\ \hline 25 & = & \frac{5}{3}w \end{array}$$

$$3 \cdot 25 = 3 \cdot \frac{5}{3}w$$

$$\frac{75}{5} = \frac{5w}{5}$$

$15 = w$ or $w = 15$

VI. H. 1. d. Check the final result by substituting the result of the unknown into the original statement of the equation.

e. Examples (Numbers in parentheses refer to the previous examples in "c")

(1) $2x + 7 = 19$, $x = 6$

$$\begin{aligned} 2(6) + 7 &= 19 \\ 12 + 7 &= 19 \\ 19 &= 19 \end{aligned}$$

(2) $3x - 5 = 7$, $x = 4$

$$\begin{aligned} 3(4) - 5 &= 7 \\ 12 - 5 &= 7 \\ 7 &= 7 \end{aligned}$$

(3) $\frac{x}{5} - 3 = 7$, $x = 50$

$$\begin{aligned} \frac{50}{5} - 3 &= 7 \\ 10 - 3 &= 7 \\ 7 &= 7 \end{aligned}$$

(4) $\frac{x}{3} + 5 = 7$, $x = 6$

$$\begin{aligned} \frac{6}{3} + 5 &= 7 \\ 2 + 5 &= 7 \\ 7 &= 7 \end{aligned}$$

VII. Speed and Skid Formula

A. Basis for the Formula

1. The formula is worked out from basic principles of physics.

a. Every object or body in motion has energy because of its motion.

VII. A. 1. b. This energy can do damage in an accident or do work in sliding the vehicle or other objects on a roadway or other surface.

c. Work needed to slide on a surface depends on what the surface skid resistance is and the distance the vehicle slides.

d. A vehicle will keep on sliding until the work done in sliding uses up all of the vehicle's energy of motion. Here, of course, we assume that no other outside forces interfere with the movement of the vehicle.

e. Therefore, to do the work of sliding to a stop in a certain distance, on a certain roadway surface, the vehicle must have had a certain amount of energy.

(1) By knowing a vehicle had to have had a certain amount of energy to slide a certain distance on a certain roadway surface we can use calculations

VII. (cnt'd) A. 1. e. (1) to estimate this energy in miles per hour.

2. The basic formula for calculating speed from skidmarks is based on the fact that the energy of motion of a vehicle can be used up as work done by friction in sliding to a stop.

B. Formula for Determining Speed

1. Abbreviations: To reduce writing required, abbreviations are used to represent commonly used quantities.

a. D = Distance in feet (ft)

b. S = Speed in miles per hour (MPH)

c. m = Grade, slope, measured up or down per foot of horizontal distance (ft/ft). Upgrade = + m, downgrade = - m.

d. f = Coefficient of friction in pounds of force to slide per pound of weight (lb/lb). Coefficient of friction is slipperiness ($f = F/W$).

e. 5.47 = The square root of the constant 29.94. (This means that 5.47 times

VII. B. 1. e. 5.47 will most nearly equal 29.94.)
(cnt'd)

2. We could say the speed of an automobile is equal to five point four seven times the square root of the average distance the vehicle skidded, this expressed in feet, times the coefficient of friction of the surface on which the automobile skidded. Instead of using this verbal description we simply use the following equation.

   a. $S = 5.5\sqrt{Df}$ This equation is much shorter and simpler than the above verbal description.

3. Let us now make the assumption that we know the quantity of each of the aforementioned abbreviations.

   a. Let (D) have a quantity of 200 feet and (f) have the quantity of .80.

4. We can now substitute 200 for (D) and .80 for (f) and by working the equation we can determine the quantity of (S).

VII. B. 4. a. Example

$S = 5.47\sqrt{Df} =$

$S = 5.47\sqrt{200(.80)} =$

$S = 5.47\sqrt{160} =$

$S = 5.47(12.65) =$

$S = 69.20 =$

$S = 69$ MPH

5. If we make the assumption that the quantity of (D) is 250, would this change the estimated quantity of (S)? Let's check and see.

   a. Example

$S = 5.47\sqrt{Df} =$

$S = 5.47\sqrt{250(.80)} =$

$S = 5.47\sqrt{200} =$

$S = 5.47(14.14) =$

$S = 77.34 =$

$S = 77$ MPH

      (1) As you can see, there is a difference in the two estimated quantities of (S).

VII. B. 6. If we change the quantity of (f) from .80 to .75 and let the quantity of (D) remain at 200, would this change our estimated quantity of (S)? Let's check and see.

a. Example

$S = 5.47\sqrt{Df} =$

$S = 5.47\sqrt{200(.75)} =$

$S = 5.47\sqrt{150} =$

$S = 5.47(12.25) =$

$S = 67.00$

$S = 67$ MPH

(1) As you can see, there is a difference in the estimated value of (S).

7. The purpose of the preceding examples is twofold. First, the examples give you the opportunity of following the working of the speed equation three times. Second, by changing the quantities in each equation it becomes obvious that the accurate estimate of the quantity of either (D) or (f) has a great effect on the accurate estimate of the quantity of (S). The importance of placing accurate data into an equation is emphasized.

VII. (cnt'd) B.

8. There is one additional abbreviation that can be added to the speed equation. This additional abbreviation is (m), which stands for grade. As indicated before, you add or subtract grade from the coefficient of friction. If the test skids were made on a level surface and the accident skids were made going up a grade, you would add (+m) to the coefficient of friction. If the accident vehicle was going down a grade you would subtract (-m) from the coefficient of friction.

a. Example: Let us assume the accident vehicle was skidding up a 5 percent grade (+.05).

$S = 5.47\sqrt{D(f + m)} =$

$S = 5.47\sqrt{200(.80 + .05)} =$

$S = 5.47\sqrt{200(.85)} =$

$S = 5.47\sqrt{170} =$

$S = 5.47(13.04) =$

VII. B. 8. a. $S = 71.34$

$S = 71$ MPH

(1) As you can see grade does make a difference in estimating the quantity of (S).

9. We have discussed the equation that enables you to accurately estimate the speed of a skidding vehicle. We know that the equation loses value in proportion to the amount of inaccurate data placed in the equation. At this time we will not go into the methods used to accurately estimate the quantity of (D). We will, however, discuss the equation used to estimate the quantity of (f).

C. Formula for Determining Coefficient of Friction

1. Abbreviations

a. S = Speed in miles per hour

b. D = Distance in feet

c. f = Coefficient of friction--pounds of force needed to slide an object on a level surface, divided by the weight of the object.

VII. C. 1. d. 29.95 = Constant

2. Let us assume we know the quantity of (D) and (S). By knowing these two quantities we can calculate the quantity of (f) with the following formula.

a. $f = \frac{S^2}{29.95D}$

3. Let us assume that the quantity of (S) is 30 MPH and the quantity of (D) is 40 ft. We can now estimate the quantity of (f) using the coefficient of friction formula.

a. Example

$$f = \frac{S^2}{29.95D}$$

$$f = \frac{30^2}{29.95(40)}$$

$$f = \frac{900}{1198}$$

$$f = .7513$$

$$f = .75$$

VII. C. 3. b. Example

$$f = \frac{S^2}{29.95D}$$

$$f = \frac{32^2}{29.95(40)}$$

$$f = \frac{1024}{1198}$$

$$f = .8547$$

$$f = .85$$

(1) Notice the difference in our estimate of the quantity (f).

c. Example: We will change the quantity of (D) to 35 ft and once again assume the quantity of (S) is 30 MPH.

$$f = \frac{S^2}{29.95D}$$

$$f = \frac{30^2}{29.95(35)}$$

$$f = \frac{900}{1048.25}$$

$$f = .8585$$

$$f = .86$$

VII. C. 3. c. (1) Once again we see the difference in our estimates of the quantity of (f).

4. In the preceding examples we have a chance to see how the coefficient of friction formula works and once again we can see the need of accurate data, if our estimates are to be accurate.

D. Formula for Determining Skidding Distances

1. Abbreviations

   a. The abbreviations used in this formula are the same as those used in the coefficient of friction formula.

2. Let us assume we know the quantity of (f) and (S). We can now estimate the quantity of (D). We will use the following formula.

   a. $D = \frac{S^2}{29.95(f \pm m)}$

3. Let us assume that the quantity of (S) is 45 MPH and the quantity of (f) is .75.

VII. D. 3. a. Example

$$D = \frac{S^2}{29.95f}$$

$$D = \frac{45^2}{29.95(.75)}$$

$$D = \frac{2025}{22.46}$$

$$D = 90.16$$

$$D = 90 \text{ ft}$$

b. Example: Now change the quantity of (f) from .75 to .85.

$$D = \frac{S^2}{29.95f}$$

$$D = \frac{45^2}{29.95(.85)}$$

$$D = \frac{2025}{25.46}$$

$$D = 79.54$$

$$D = 80 \text{ ft}$$

c. Example: Now change the quantity of (S) to have a value of 40 MPH.

$$D = \frac{S^2}{29.95f}$$

VII. (cnt'd) D. 3. c.

$$D = \frac{40^2}{29.95(.75)}$$

$$D = \frac{1600}{22.46}$$

$$D = 71.24$$

$$D = 71 \text{ ft}$$

(1) Notice the difference in our estimate of the quantity of (D).

VIII Derivation of Speed-Skid Formula from Basic Laws of Energy and Motion

A. Basic Laws of Energy and Motion

1. Energy = Force, expressed in pounds, times distance, expressed in feet.
2. Energy (kinetic) = One-half of mass times velocity, expressed in feet per second, squared.
3. Mass = Weight, expressed in pounds, divided by acceleration of gravity. We assume the acceleration of gravity to be 32.2 ft/sec/sec.
4. Velocity (in feet per second) = 1.47 times speed, expressed in miles per hour.

VIII. A. 5. Coefficient of Friction = Horizontal force divided by vertical force. Both are expressed in pounds.

B. Abbreviations

1. $E = $ Energy (work done) $= F \times D$
2. $E = \frac{1}{2} MV^2$ (kinetic energy)
3. $M = \frac{W}{32.2}$
4. $V = 1.47S$
5. $f = \frac{F}{W}$
6. $W = $ Weight of vehicle and vertical force
7. $F = $ Force (horizontal force)
8. $D = $ Distance in feet

C. Derivation of the Speed-Skid Formula

1. The first step is to combine the two known values of energy.

   a. $E = \frac{1}{2} MV^2$

   b. $E = FD$

   $= FD = \frac{1}{2} MV^2$

2. The second step is to solve for (F) in the coefficient of friction formula.

   a. $f = \frac{F}{W}$ or $Wf = \frac{F}{W} \cdot W$ or

   $Wf = F$

VIII. C. 3. The third step is to replace (F) in the formula of step one with the value of (F), which is (Wf).

   a. $FD = \frac{1}{2} MV^2 = WfD = \frac{1}{2} MV^2$

4. We now convert mass to weight on the right side of the equation.

   a. $WfD = \frac{1}{2} MV^2$ or $WfD = \frac{1}{2} (\frac{W}{32.2}) V^2$

5. The next step is to change velocity to miles per hour. We do this by dividing the number of feet in a mile by the number of seconds in an hour. There are 5,280 feet in one mile and 3,600 seconds in one hour. (5280/3600) = 1.466. We already know that velocity is changed to MPH by 1.466 times speed (MPH = S). In our equation we have (V^2), so when we make our change from velocity to MPH we must square both 1.466 -- 1.466^2 and S--S^2.

$$WfD = \frac{1}{2} \left(\frac{W}{32.2}\right) V^2 \quad \text{or}$$

$$WfD = \frac{1}{2} \left(\frac{W}{32.2}\right) \left(\frac{5280}{3600}\right)^2 S^2 \quad \text{or}$$

$$WfD = \frac{1}{2} \left(\frac{W}{32.2}\right) (1.466)^2 S^2$$

VIII. C. 6. The next step is to cancel out weight (W) on both sides of the equation.

$$WfD = \frac{1}{2}\left(\frac{W}{32.2}\right)(1.466)^2\ S^2 \qquad \text{or}$$

$$fD = \frac{1}{2}\left(\frac{1}{32.2}\right)(1.466)^2\ S^2$$

7. The next step is to reduce the equation to a workable formula.

a. $fD = \frac{1}{2}\left(\frac{1}{32.2}\right)(1.466)^2\ S^2$

b. $fD = \frac{1}{64.4}(1.466)^2\ S^2$

(Here we multiplied $\frac{1}{2}$ by $\frac{1}{32.2}$ and got $\frac{1}{64.4}$.)

c. $fD = \frac{1}{64.4}(2.15)\ S^2$

(Here we squared 1.466 and got 2.15.)

d. $fD = \frac{2.15}{64.4}\ S^2$

(Here we multiplied $\frac{1}{64.4}$ by 2.15 and got $\frac{2.15}{64.4}$.)

VIII. C. 7. e. $fD = \frac{1}{29.95}\ S^2$

(Here we reduced the fraction $\frac{2.15}{64.4}$ by dividing both numbers by 2.15 and got $\frac{1}{29.95}$.)

f. $fD = \frac{S^2}{29.95}$

(Here we multiplied $\frac{1}{29.95}$ by S^2 and got $\frac{S^2}{29.95}$.)

8. We are now in a position to solve for (f) or (S) or (D). First let us solve for (f).

a. $fD = \frac{S^2}{29.95}$

b. $f = \frac{S^2}{29.95D}$

Here we divided both sides of the equation with (D).

$$\frac{fD}{D} = \frac{S^2}{29.95D}$$

By doing this we move (D) to the right side of the equation.

VIII. C. 8. c. I am sure, at this point, everyone would agree that we could use $f = \frac{S^2}{29.95D}$ to determine the quantity of (f) if we knew the quantities of (S) and (D).

9. Now let us solve for (S).

a. $fD = \frac{S^2}{29.95}$

b. $29.95fD = S^2$

Here we multiply both sides of the equation by 29.95. This method is used to isolate (S^2) on one side of the equation.

$fD = \frac{S^2}{29.95}$ or $29.95\ fD = \frac{S^2}{29.95}$.

$29.95 = 29.95\ fD = S^2$

c. $S^2 = 29.95\ fD$

Here we simply moved (S^2) to the left side of the equation. If $29.95\ fD = S^2$, then $S^2 = 29.95\ fD$.

d. $\sqrt{S^2} = \sqrt{29.95fD}$

VIII. (cnt'd) C. 9. d. Here we are taking the square root of each side of the equation. We do this because we want to reduce (S^2) to (S).

e. $S = 5.47\sqrt{fD}$

Here we have the square root of (S^2) and 29.95.

$\sqrt{S^2} = S \qquad \sqrt{29.95} = 5.47$

f. With the formula $S = 5.47\sqrt{fD}$ or $S = 5.47\sqrt{Df}$ we can estimate the quantity of (S) if we know the quantity of (f) and (D).

10. Now let us solve for (D).

a. $fD = \frac{S^2}{29.95}$

b. $D = \frac{S^2}{29.95f}$

Here we have divided both sides of the equation by (f).

$\frac{fD}{f} = \frac{S^2}{29.95f}$

We have now moved (f) to the right side of

VIII. (cnt'd) C. 10. b. the equation, leaving (D) isolated on the left side of the equation.

IX Special Equations Dealing with Speed, Distance, and Time

A. Critical Speed on Curves

1. You can only travel so fast on a given curve. The mere fact that the vehicle starts to slide while in a curve, without application of brakes, indicates that the forces pulling the vehicle off the curve and toward a straight line of travel, are greater than those forces that are trying to hold the vehicle on the curve.

2. When we are dealing with the estimation of a vehicle's speed while traveling on a curve, we are really dealing with centripetal force and its relationship to inertia.

    a. CENTRIPETAL FORCE: This force holds a vehicle on a curve by accelerating the vehicle toward the center of the curve.

    b. INERTIA: The property of matter which causes it to remain at rest, or if it is set in motion, causes it to continue in

IX. (cnt'd) A. 2. b. motion in a straight line unless it is acted upon by some external force.

3. At this time we will not discuss the derivation of the critical speed formula. We will simply say that a formula has been devised that will give us a "S" (speed in MPH) at the time centripetal force equals inertial force. At this speed the vehicle will start to sideslip.

4. In this critical speed formula we will be dealing with three unknowns. The unknowns will be speed, coefficient of friction, and radius.

5. Critical Speed Formula: For us to use this formula we must know two of the three unknowns that are used to calculate (S), critical speed. (R) is one of the unknowns and (R) stands for radius. Radius, in this formula, refers to the radius of the curve on which the center of mass was traveling. In another subject area we will be discussing how to determine the radius in which the center of mass is traveling. We must

also know the quantity of (f), the coefficient of friction.

In some cases the (f), the coefficient of friction, will include superelevation. The percent of superelevation will be the rise or fall of the bank in the curve. Superelevation is obtained by taking measurement across the roadway.

IX. A. 6. In the following example we will make the assumption that (f) has a quantity of .80 and (R) has a quantity of 300 feet.

a. Example

(1) $S = 3.87\sqrt{Rf}$

(2) $S = 3.87\sqrt{300(.80)}$

(3) $S = 3.87\sqrt{240}$

(4) $S = 3.87(15.49)$

(5) $S = 59.95$

(6) S = 60 MPH

b. Example: In this equation let (f) = .75.

(1) $S = 3.87\sqrt{Rf}$

(2) $S = 3.87\sqrt{300(.75)}$

(3) $S = 3.87\sqrt{225}$

(4) $S = 3.87(15.00)$

(5) $S = 58.05$

(6) S = 58 MPH

As you can see, by changing one of the unknowns, even just a little, you change the quantity of (S).

c. Example (In this equation let R = 250, and f = .80)

(1) $S = 3.87\sqrt{Rf}$

(2) $S = 3.87\sqrt{250(.80)}$

IX. (cnt'd) A. 6. c. (3) $S = 3.87\sqrt{200}$

(4) $S = 3.87(14.14)$

(5) $S = 54.72$

(6) $S = 55$ miles per hour

B. Combined Speed Formula

1. The error in combining speeds is that of adding the two calculated speeds. This is a gross error in that it overestimates the vehicle's combined speed.

2. To combine two speeds we must first square each speed and then add the two together. Now that we have added the squares, we find the square root of the sum of the squares, and this gives us the correct combined minimum speed of the vehicle.

3. Let us assume we have two speeds to combine. One speed is 40 MPH and the other speed is 60 MPH. Let us now combine these two speeds with this formula:

$$S_c = \sqrt{S_1{}^2 + S_2{}^2}$$

a. Example

IX. B. 3. a. (1) $S_c = \sqrt{S_1{}^2 + S_2{}^2}$

(2) $S_c = \sqrt{40^2 + 60^2}$

(3) $S_c = \sqrt{1600 + 3600}$

(4) $S_c = \sqrt{5200}$

(5) $S_c = 72.11$

(6) $S_c = 72$ MPH

b. Example: Let $S_1 = 30$ MPH and $S_2 = 70$ MPH.

(1) $Sc = \sqrt{S_1{}^2 + S_2{}^2}$

(2) $S_c = \sqrt{30^2 + 70^2}$

(3) $S_c = \sqrt{900 + 4900}$

(4) $S_c = \sqrt{5800}$

(5) $S_c = 76.16$

(6) $S_c = 76$ MPH

IX. B. 3. c. Example: Let S_1 = 50 MPH and S_2 = 50 MPH.

(1) $S_c = \sqrt{S_1{}^2 + S_2{}^2}$

(2) $S_c = \sqrt{50^2 + 50^2}$

(3) $S_c = \sqrt{2500 + 2500}$

(4) $S_c = \sqrt{5000}$

(5) S_c = 70.71

(6) S_c = 71 MPH

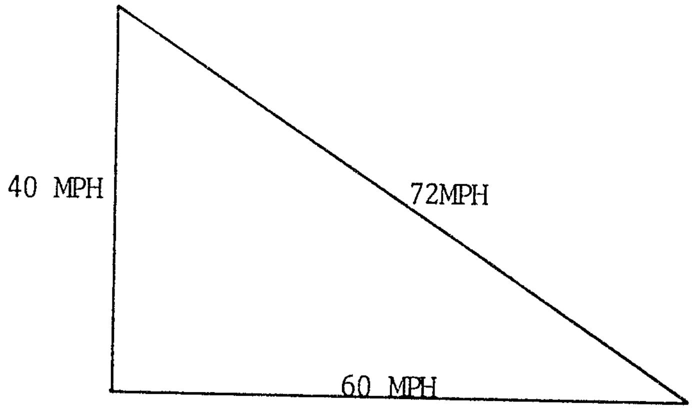
Notice: In the three examples we see that had we just added the two speeds, we would have gotten 100 MPH.

4. In the preceding examples we used the combined speed formula to calculate combined minimum speed. The same calculation can also be done by the use of a simple scale diagram.

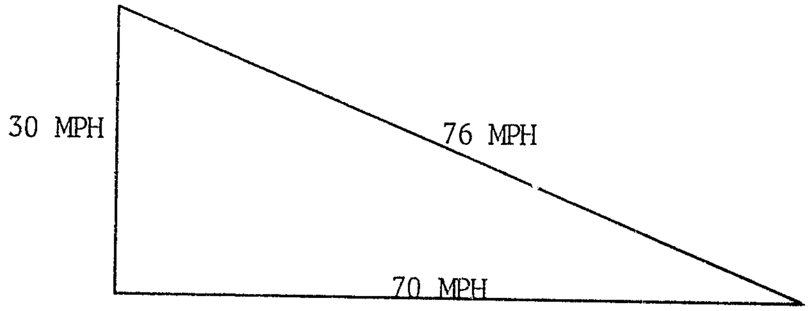
Measure (S_1) of the example 3(a) along one leg of a right angle and (S_2) along the second leg

IX. (cnt'd) B. 4. of the same right angle. Now measure the distance between the ends of each leg of the right angle. This distance is your combined speed.

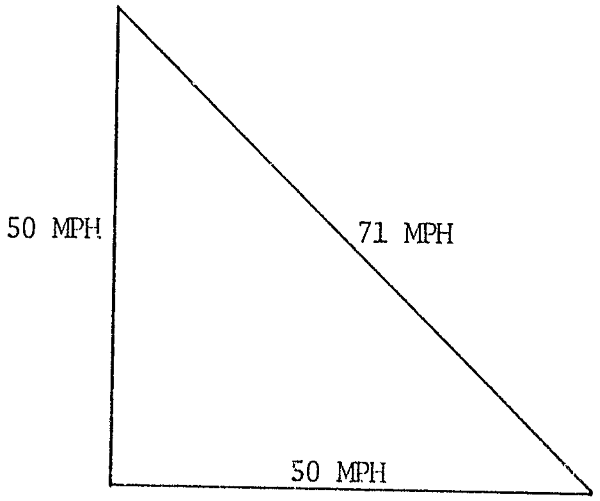
a. Example: S_1 = 40 MPH and S_2 = 60 MPH.



b. Example: Let S_1 = 30 MPH and S_2 = 70 MPH.



c. Example: Let S_1 = 50 MPH and S_2 = 50 MPH.



IX. B. 4. Notice: As you can see, you get the same answer by using this method as you do if you use the formula.
(cnt'd)

C. Equation for Fall Speed

1. Abbreviations
   - a. S = Speed in MPH
   - b. D = Horizontal distance in feet
   - c. h = Vertical distance of the fall in feet
   - d. m = Grade of hill at take-off in percent
   - e. 2.74 = Mathematical constant

2. When we deal with (h) or (m) in this equation, we must remember that (h) will be a (+ h) if the vehicle lands higher than its take-off point and it will be a (- h) if it lands lower than its take-off point; (m) will be (+ m) if the vehicle is going uphill at take-off and will be a (- m) if the vehicle is going downhill at take-off.

3. In this formula, as in the previous formulas that we have discussed, accurate data is of the greatest importance.

IX. C. 4. Let us now make some assumption about some of the unknowns in the formula. Let m = +.05 and D = 60 ft and h = 14 ft. In this case let h = -h = -14.

a. Example

(1) $S = 2.74D/\sqrt{+mD - (-h)}$

(2) $S = 2.74(60)/\sqrt{(+.05)\ (60) - (-14)}$

(3) $S = 164.4/\sqrt{3 + 14}$

(4) $S = 164.4/\sqrt{17}$

(5) $S = 164.4/4.12$

(6) $S = 39.90$

(7) S = 40 MPH

b. Example: Let D = 50 ft, m = -.05 and h = -20 ft.

(1) $S = 2.74D/\sqrt{mD - h}$

(2) $S = 2.74(50)/\sqrt{(-.05)\ 50 - (-20)}$

IX. C. 4. b. (3) $S = 137/\sqrt{-2.5 + 20}$

(4) $S = 137/\sqrt{17.5}$

(5) $S = 137/4.18$

(6) $S = 32.78$

(7) $S = 33$ MPH

c. Example: Let $D = 80$ ft, $h = -10$ ft and $m = 0$.

(1) $S = 2.74D/\sqrt{mD - h}$

(2) $S = 2.74(80)/\sqrt{(0)(80) - (-10)}$

(3) $S = 2.74(80)/\sqrt{10}$

(4) $S = 219.2/\sqrt{10}$

(5) $S = 219.2/3.16$

(6) $S = 69.36$

(7) $S = 69$ MPH

IX. D. Equation for Flip or Vault

1. In the following formula the abbreviations have the same meaning as they did in the fall formula.
   a. In this equation the constant is 3.87

2. Sometimes it will be difficult to obtain all the data necessary to accurately estimate speed by using the flip or vault formula. Sometimes you may need to obtain an expert who has the tools and skill to determine the quantity of (D) and (h) and sometimes you can only guess. When you guess, you must remember this causes your calculations to be less than accurate.

   There is a way that you can obtain reasonably accurate estimates of the quantity of (h) and (D) but this will not be discussed at this time.

3. Let us now work about three examples of the flip and vault formula.
   a. Example: Let $h = -10$ ft and $d = 30$ ft.
      (1) $S = 3.87D/\sqrt{D - h}$

      (2) $S = 3.87(30)/\sqrt{30 - (-10)}$

IX. D. 3. a. (3) $S = 116.1/\sqrt{30 + 10}$

(4) $S = 116.1/\sqrt{40}$

(5) $S = 116.1/6.32$

(6) $S = 18.37$

(7) S = 18 MPH

b. Example: Let h = -5 ft and D = 20 ft.

(1) $S = 3.87D/\sqrt{D - h}$

(2) $S = 3.87(20)/\sqrt{20 - (-5)}$

(3) $S = 77.4/\sqrt{20 + 5}$

(4) $S = 77.4/\sqrt{25}$

(5) $S = 77.4/5$

(6) $S = 15.48$

(7) S = 15 MPH

IX. D. 3. c. Example: Let h = +5 and D = 25.

(1) $S = 3.87D/\sqrt{D - h}$

(2) $S = 3.87(25)/\sqrt{25 - (+5)}$

(3) $S = 96.75/\sqrt{25 - 5}$

(4) $S = 96.75/\sqrt{20}$

(5) $S = 96.75/4.47$

(6) $S = 21.64$

(7) S = 22 MPH

E. Equations Dealing with Distance, Velocity, and Time

1. Definitions

a. Motion: An object is in motion when it changes its place or position.

b. Velocity: The distance a body moves in a given direction in a unit of time is called velocity.

IX. E. 1. c. Uniform Motion: When a body moves through the same distance in each unit of time, its velocity is uniform.

d. Accelerated Motion: The rate of change of velocity is called acceleration.

2. Laws of Accelerated Motion

a. The change in velocity of a uniformly accelerated body is proportional to the time.

$$V = AT$$

b. The distance traveled by a uniformly accelerated body starting from rest is directly proportional to the square of the time it travels.

$$D = \frac{1}{2} AT^2$$

c. The distance traveled by a body uniformly accelerated from rest varies as the square of the velocity.

$$V^2 = 2 AD \quad \text{or} \quad V = \sqrt{2 AD}$$

IX. E. 3. Abbreviations

a. V = Velocity in ft/sec

b. V_0 = Initial velocity in ft/sec

c. V_f = Final velocity in ft/sec

d. D = Distance in feet

e. T = Time in seconds

f. A = Acceleration in ft/sec^2

4. Let us now work some problems dealing with acceleration, time, distance, and velocity.

Let us assume that we want to know how far a vehicle will travel and we know the vehicle will be accelerating at a uniform rate of 32.2 ft/sec^2 and will continue at this rate of acceleration for 3 sec. The vehicle starts from zero velocity.

a. Example

(1) $D = V_0 \cdot T + \frac{1}{2} AT^2$

(2) $D = 0 \cdot 3 + \frac{1}{2} (32.2)(3)^2$

(3) $D = \frac{1}{2} . (32.2)(3)^2$

(4) $D = 16.1 \ (9)$

IX. E. 4. a. (5) $D = 144.9$ ft

b. Examples: Let $T = 15$ sec, $V_0 = 0$, and $A = 4$ ft/sec^2.

(1) $D = V_0 \cdot T + \frac{1}{2} AT^2$

(2) $D = 0 \cdot 15 + \frac{1}{2} (4)(15)^2$

(3) $D = \frac{1}{2} (4)(15)^2$

(4) $D = 2 \cdot 225$

(5) $D = 450$ ft

c. Example: Let $T = 10$ sec, $V_0 = 0$, and $A = 20$ ft/sec^2.

(1) $D = V_0 \cdot T + \frac{1}{2} AT^2$

(2) $D = 0 \cdot 10 + \frac{1}{2} (20)(10)^2$

(3) $D = \frac{1}{2} (20)(10)^2$

(4) $D = 10 \cdot 100$

(5) $D = 1{,}000$ ft

IX. E. 5. Let us now assume we want to know the final velocity (V_f) of a vehicle when we know the acceleration rate and the distance traveled.

a. Example: Let $D = 144.9$, $V_0 = 0$, and $A = 32.2$ ft/sec^2.

(1) $V_f^2 = V_0^2 + 2 \cdot A \cdot D$

(2) $V_f^2 = 0 + 2 \cdot 32.2 \cdot 144.9$

(3) $V_f^2 = 64.4 \cdot 144.9$

(4) $V_f^2 = 9331.56$

(5) $\sqrt{V_f^2} = \sqrt{9331.56}$

(6) $V_f = 96.6$ ft/sec

b. Example: Let $D = 400$, $V_0 = 0$ and $A = 26$ ft/sec^2.

(1) $V_f^2 = V_0^2 + 2 \cdot A \cdot D$

(2) $V_f^2 = 0 + 2 \cdot 26 \cdot 400$

(3) $V_f^2 = 52 \cdot 400$

IX. E. 4. a. (5) $D = 144.9$ ft

b. Examples: Let $T = 15$ sec, $V_0 = 0$, and $A = 4$ ft/sec^2.

(1) $D = V_0 \cdot T + \frac{1}{2} AT^2$

(2) $D = 0 \cdot 15 + \frac{1}{2} (4)(15)^2$

(3) $D = \frac{1}{2} (4)(15)^2$

(4) $D = 2 \cdot 225$

(5) $D = 450$ ft

c. Example: Let $T = 10$ sec, $V_0 = 0$, and $A = 20$ ft/sec^2.

(1) $D = V_0 \cdot T + \frac{1}{2} AT^2$

(2) $D = 0 \cdot 10 + \frac{1}{2} (20)(10)^2$

(3) $D = \frac{1}{2} (20)(10)^2$

(4) $D = 10 \cdot 100$

(5) $D = 1{,}000$ ft

IX. E. 5. Let us now assume we want to know the final velocity (V_f) of a vehicle when we know the acceleration rate and the distance traveled.

a. Example: Let D = 144.9, $V_0 = 0$, and $A = 32.2$ ft/sec^2.

(1) $V_f^2 = V_0^2 + 2 \cdot A \cdot D$

(2) $V_f^2 = 0 + 2 \cdot 32.2 \cdot 144.9$

(3) $V_f^2 = 64.4 \cdot 144.9$

(4) $V_f^2 = 9331.56$

(5) $\sqrt{V_f^2} = \sqrt{9331.56}$

(6) $V_f = 96.6$ ft/sec

b. Example: Let D = 400, $V_0 = 0$ and $A = 26$ ft/sec^2.

(1) $V_f^2 = V_0^2 + 2 \cdot A \cdot D$

(2) $V_f^2 = 0 + 2 \cdot 26 \cdot 400$

(3) $V_f^2 = 52 \cdot 400$

IX. (cnt'd) E. 5. b. (4) $V_f^2 = 20,800$

(5) $\sqrt{V_f^2} = \sqrt{20,800}$

(6) $V_f = 144.2$ ft/sec

c. Example: Let D = 400, $V_0 = 20$, and A = 20 ft/sec^2.

(1) $V_f^2 = V_0^2 + 2 \cdot A \cdot D$

(2) $V_f^2 = 20^2 + 2 \cdot 20 \cdot 400$

(3) $V_f^2 = 400 + 40 \cdot 400$

(4) $V_f^2 = 400 + 16,000$

(5) $V_f^2 = 16,400$

(6) $\sqrt{V_f^2} = \sqrt{16,400}$

(7) $V_f = 128.1$ ft/sec

6. If we know the acceleration rate of a vehicle and the time it is accelerating, can we estimate final velocity at the end of a certain

IX. E. 6. time? The answer is yes.

a. Example: Let $V_0 = 0$, A = 32.2 ft/sec^2, and T = 3.

(1) $V_f = V_0 + A \cdot T$

(2) $V_f = 0 + 32.2 \cdot 3$

(3) $V_f = 32.2 \cdot 3$

(4) $V_f = 96.6$ ft/sec

b. Example: Let $V_0 = 0$, A = 26 ft/sec^2, and T = 6.

(1) $V_f = V_0 + A \cdot T$

(2) $V_f = 0 + 26 \cdot 6$

(3) $V_f = 26 \cdot 6$

(4) $V_f = 156$ ft/sec

c. Example: Let $V_0 = 20$ ft/sec, A = 15 ft/sec^2, and T = 4.

(1) $V_f = V_0 + A \cdot T$

IX. E. 6. c. (2) $V_f = 20 + 15 \cdot 4$

(3) $V_f = 20 + 60$

(4) $V_f = 80$ ft/sec

7. Can we determine time if we know the initial velocity, final velocity and rate of acceleration of a vehicle? The answer is yes.

a. Example: Let $V_0 = 0$, $V_f = 180$ ft/sec, and $A = 32.2$.

(1) $T = \frac{V_f - V_0}{A}$

(2) $T = \frac{180 - 0}{32.2}$

(3) $T = \frac{180}{32.2}$

(4) $T = 5.6$ sec

b. Example: Let $V_0 = 0$, $V_f = 200$, and $A = 20$ ft/sec^2.

(1) $T = \frac{V_f - V_0}{A}$

(2) $T = \frac{200 - 0}{20}$

IX. E. 7. b. (3) $T = \frac{200}{20}$

(4) $T = 10$ sec

c. Example: Let $V_0 = 20$ ft/sec, $V_f = 190$ ft/sec, and $A = 30$.

(1) $T = \frac{V_f - V_0}{A}$

(2) $T = \frac{190 - 20}{30}$

(3) $T = \frac{170}{30}$

(4) $T = 5.7$ sec

8. General Problems

a. If 5 seconds are required for an automobile starting from rest to attain a velocity of 35 MPH, then, A = MPH/sec.

(1) $A = \frac{V_f - V_0}{T}$

(2) $A = \frac{35 \text{ MPH} - 0 \text{ MPH}}{5 \text{ sec}}$

(3) $A = 7$ MPH/sec

IX. E. 8. b. If a vehicle takes 10 seconds to reach a velocity of 90 MPH, what will its acceleration be in miles per hour per second?

(1) $A = \frac{V_f - V_0}{T}$

(2) $A = \frac{90 - 0}{10}$

(3) A = 9 MPH/sec

c. What speed will an automobile starting from rest acquire in 15 seconds if it is uniformly accelerated at the rate of 4 MPH/sec?

(1) V = AT

(2) V = 4(15)

(3) V = 60 MPH

d. How much time is required for an automobile that is accelerating at 4 MPH/sec to reach 40 MPH?

(1) V = AT or $T = \frac{V}{A}$

(2) $T = \frac{V}{A}$

IX. E. 8. d. (3) $T = \frac{40 \text{ MPH}}{4 \text{ MPH/sec}}$

(4) T = 10 sec

e. It an automobile's brakes, when applied, stop the automobile in 6 seconds and the automobile is traveling 30 MPH, what is the average rate of deceleration or negative acceleration?

(1) V = AT or $A = \frac{V}{T}$

(2) $A = \frac{30 \text{ MPH}}{6 \text{ sec}}$

(3) A = 5 MPH/sec

f. You may have noticed that in the preceding problems we have been dealing with velocity as both ft/sec and MPH. You may also have noticed that we have dealt with acceleration and deceleration as ft/sec^2 and MPH/sec.

The following will explain how you can change MPH into ft/sec or ft/sec into MPH.

IX. E. 8. f. (1) If a vehicle moves one mile in one hour, we can say the vehicle moves 5,280 ft (one mile) in 3600 sec (one hour). To find out how many feet this vehicle travels in one second, we divide 3,600 sec into 5,280 ft.

$3600/\overline{5280.000}$ or 1.47 ft/sec

(2) If a vehicle is traveling 60 MPH, then we would multiply miles per hour by 1.47 and thereby convert miles per hour into ft/sec.

ft/sec = MPH x 1.47

ft/sec = 60 MPH x 1.47

ft/sec = 88

(3) If a vehicle is traveling 88 ft/sec and you want to know the speed in MPH, you would divide 88 ft/sec by 1.47.

MPH = ft/sec ÷ 1.47

MPH = 88 ft/sec ÷ 1.47

MPH = 59.86 or 60

APPENDIX B

(PROBLEMS)

GEOMETRIC CONSTRUCTION

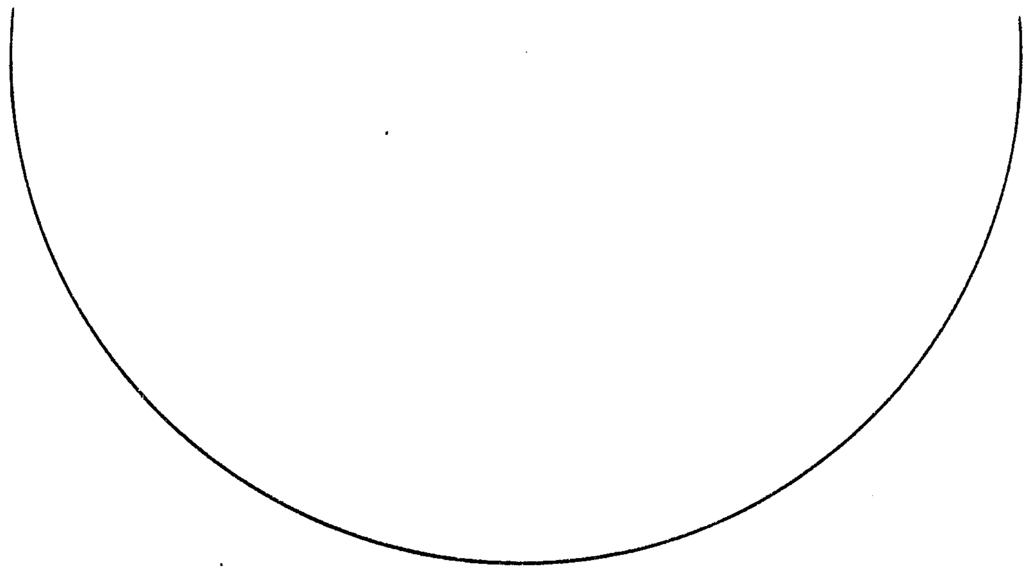
BISECT THIS LINE.

(SEE PAGE 2,
CHAPTER I)

GEOMETRIC CONSTRUCTION

BISECT THIS CURVE.

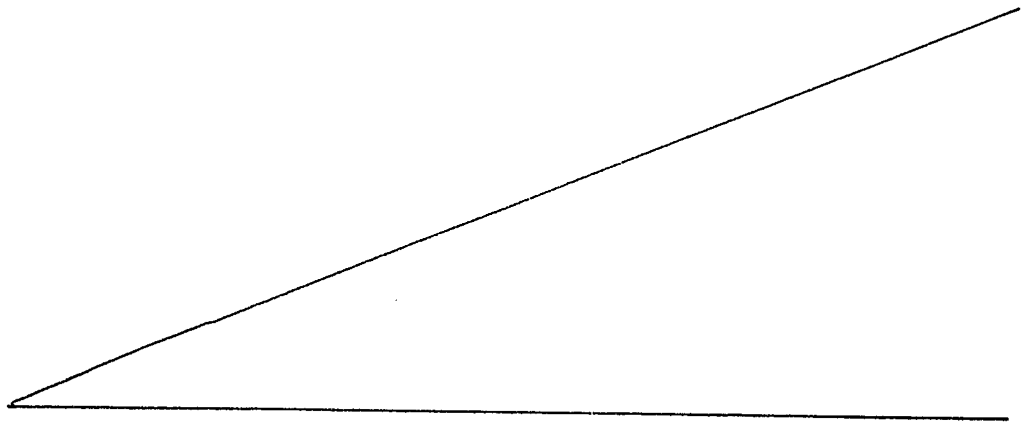
(SEE PAGE 2)



GEOMETRIC CONSTRUCTION

BISECT THIS ANGLE.

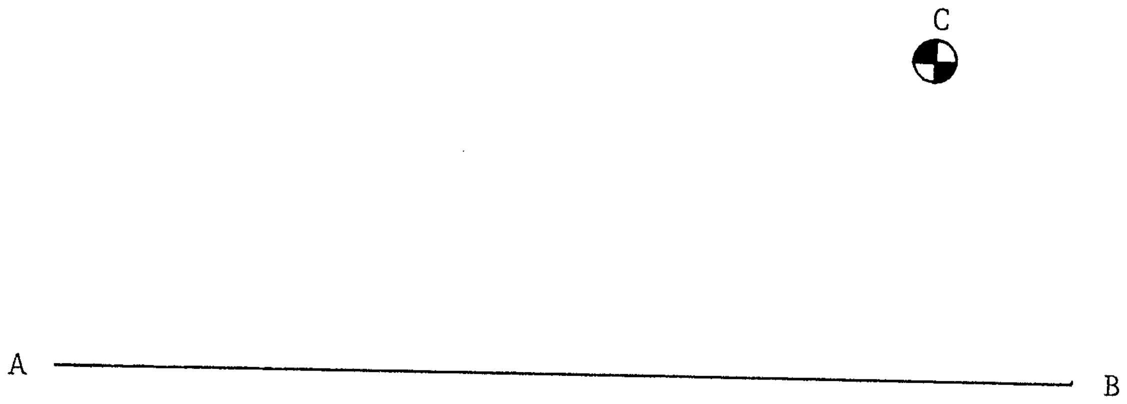
(SEE PAGE 3)



GEOMETRIC CONSTRUCTION

DRAW A STRAIGHT LINE THROUGH POINT "C"
PARALLEL TO LINE "AB".

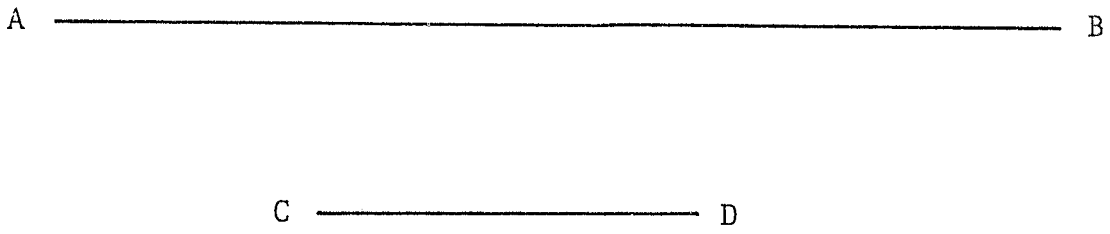
(SEE PAGE 4)



GEOMETRIC CONSTRUCTION

DRAW A LINE PARALLEL TO "AB" AT A DISTANCE
EQUAL TO THE LENGTH OF "CD".

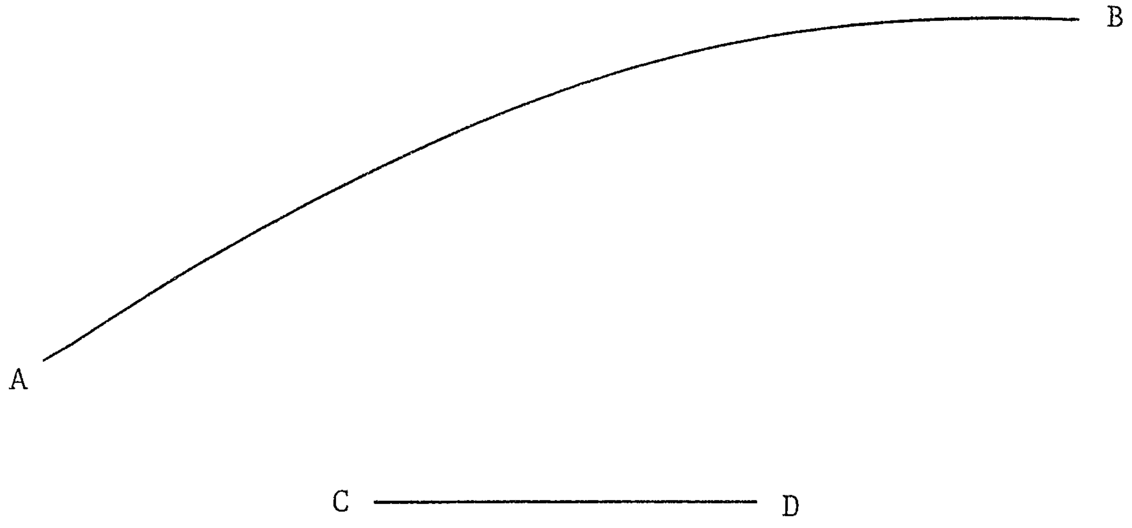
(SEE PAGE 5)



GEOMETRIC CONSTRUCTION

DRAW A CURVED LINE PARALLEL TO "AB" AT A DISTANCE EQUAL TO THE LENGTH OF "CD".

(SEE PAGE 6)



GEOMETRIC CONSTRUCTION

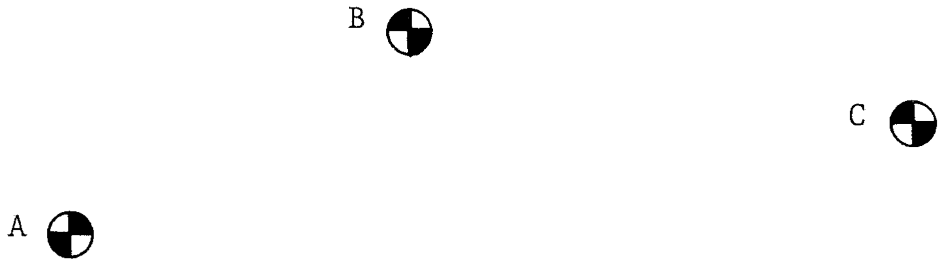
DIVIDE THIS LINE INTO 5 EQUAL PARTS.

(SEE PAGE 7)

GEOMETRIC CONSTRUCTION

DRAW AN ARC THROUGH POINTS "A B C".

(SEE PAGE 8)



MINIMUM SPEED FROM SKIDMARKS

ACCIDENT SKIDS

DISTANCE = 250 FEET

TEST SKIDS RUN IN SAME PLACE AND SAME DIRECTION

#1 SPEED = 30 MPH DISTANCE = 47 FEET

#2 SPEED = 30 MPH DISTANCE = 54 FEET

#3 SPEED = 30 MPH DISTANCE = 50 FEET

1. WHAT DRAG FACTOR DO YOU USE? ________
2. WHAT IS THE MINIMUM SPEED FOR THE 250 FEET ACCIDENT SKIDS? ________

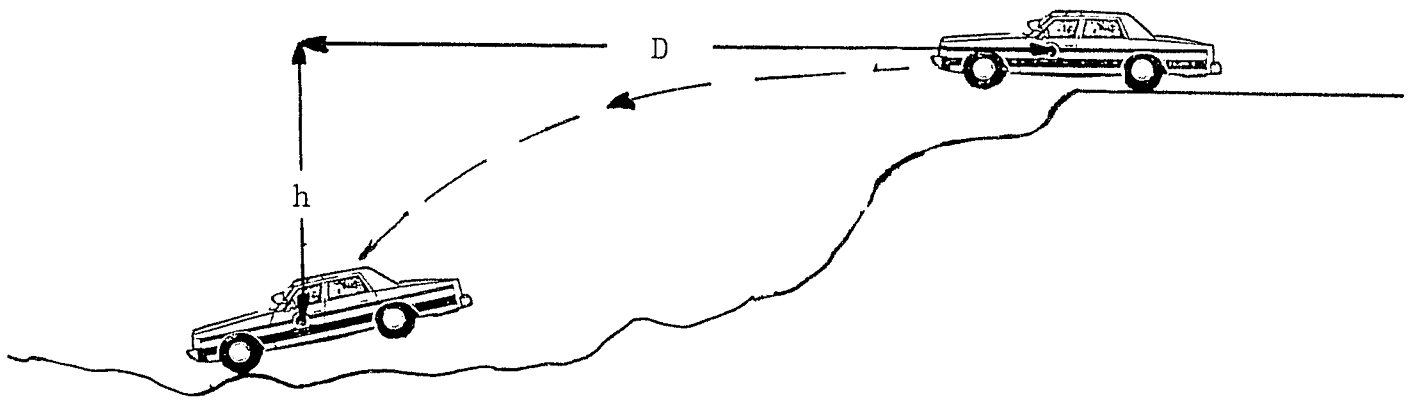
FALL SPEED

CALCULATE THE SPEED.

D = 80 FEET

h = 8.2 FEET

S = ________



FALL SPEED

CALCULATE THE SPEED.

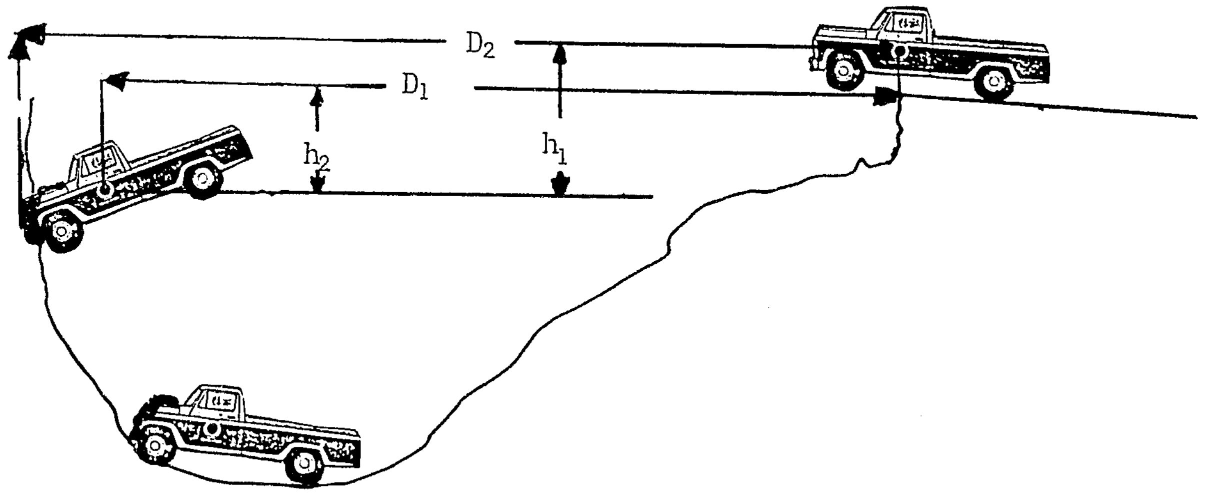
D_1 = 30 FEET

D_2 = 38 FEET

h_1 = 3 FEET

h_2 = 1.3

S = ______



WHICH MEASUREMENTS ARE CORRECT?

FALL SPEED

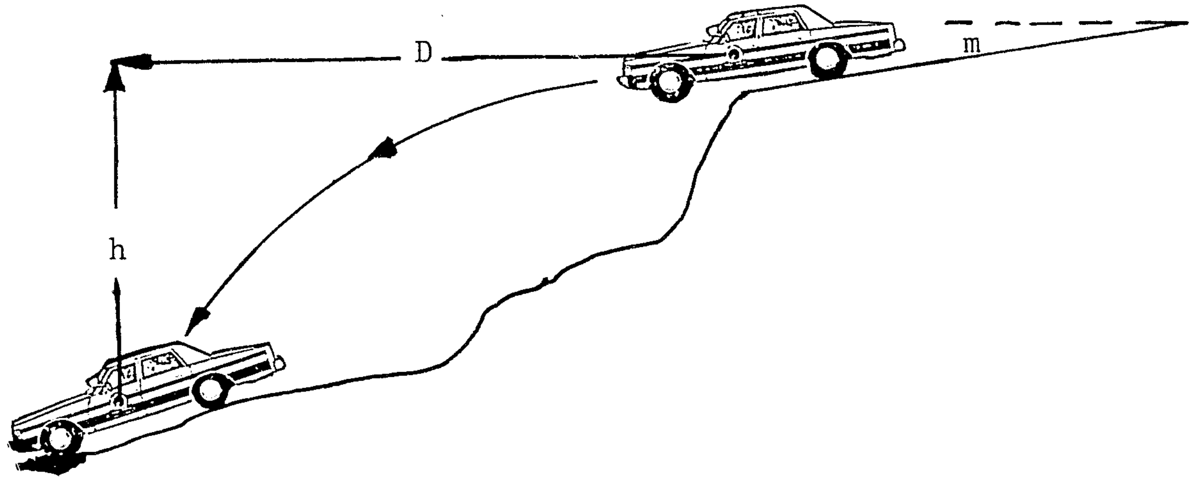
CALCULATE THE SPEED.

D = 20 FEET

h = 6 FEET

m = -3%

S = ________



FALL SPEED

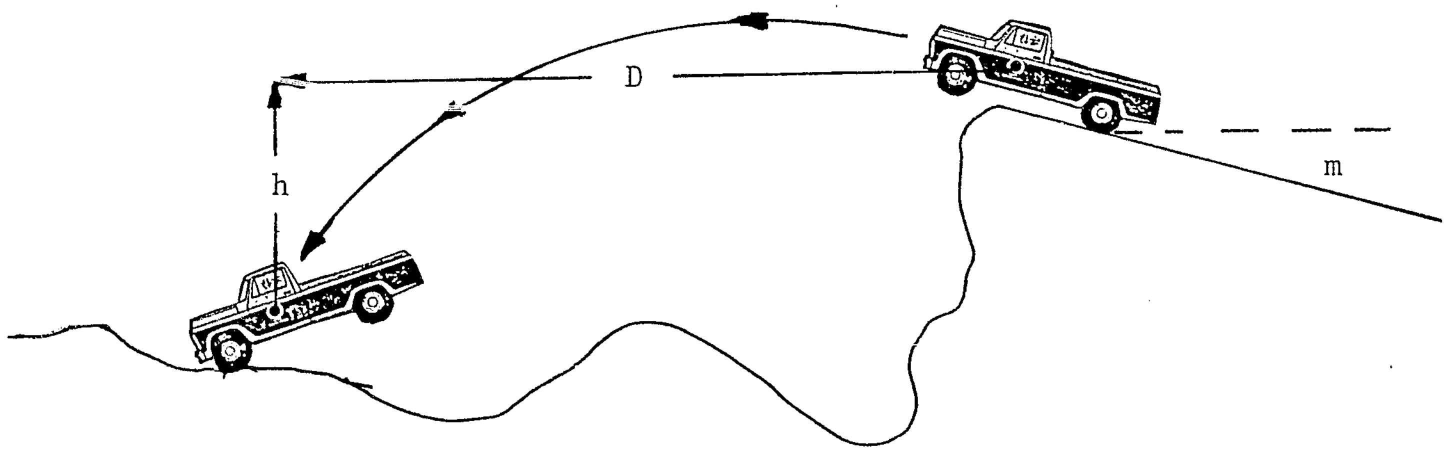
CALCULATE THE SPEED.

D = 50 FEET

h = 5 FEET

m = +6%

S = ________



VAULT SPEED

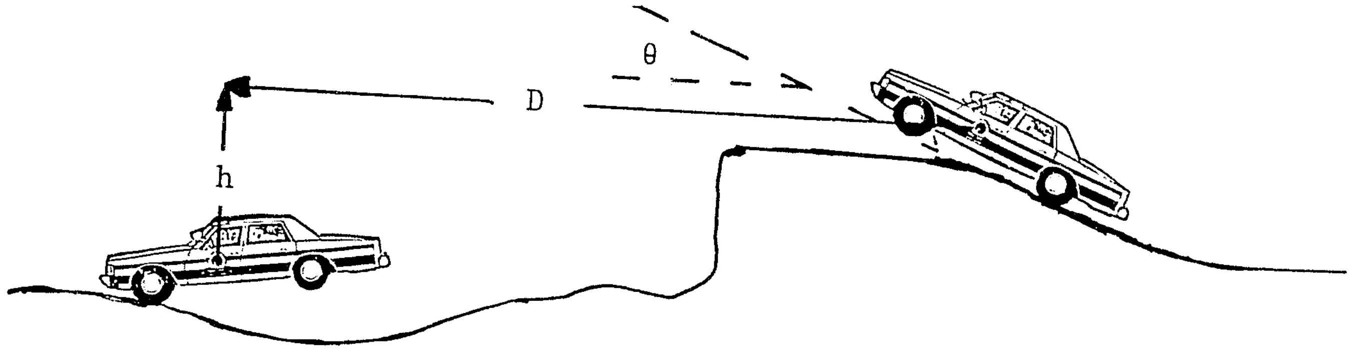
CALCULATE THE SPEED.

D = 100 FEET

h = 2 FEET

θ = 20^0

S = ________



VAULT SPEED

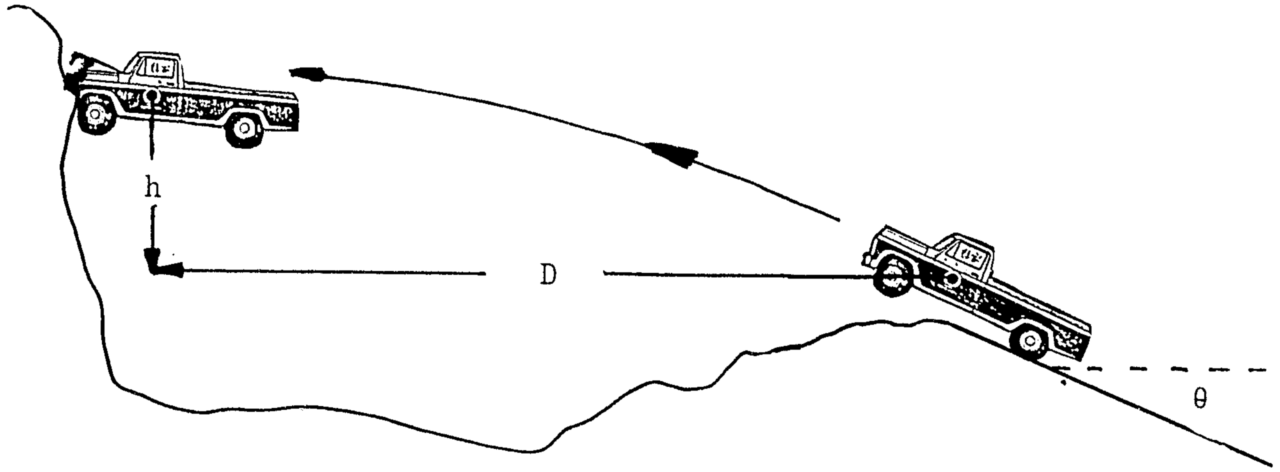
CALCULATE THE SPEED.

D = 100 FEET

h = 3 FEET

θ = 20^0

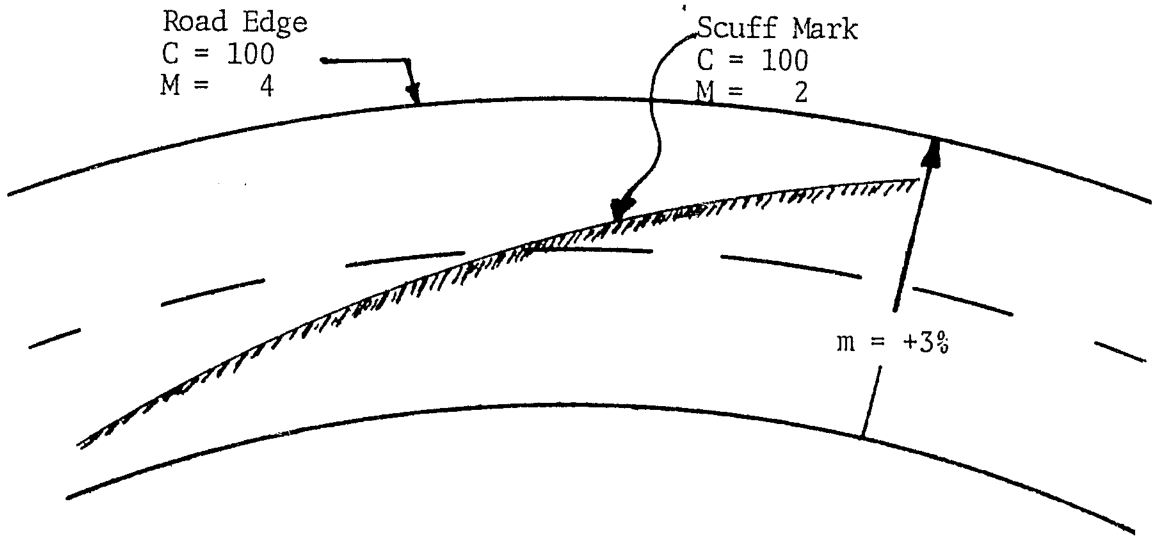
S = ________



CRITICAL SPEED

(f) FOR LEVEL SURFACE = .80

1. WHAT IS THE CRITICAL SPEED OF THE CURVE? _______
2. WHAT IS THE CRITICAL SPEED OF THE SCUFF MARK? _______



KINETIC ENERGY / COMBINED SPEED

1. What is the kinetic energy of each of the following vehicles?

A. S = 50 MPH W = 4000 lbs. Ke = __________

B. S = 25 MPH W = 8000 lbs. Ke = __________

2. What is the momentum of each of the above vehicles and how do the momentums compare?

Note: Momentum = Mass x Velocity

- - - - - - - - - - -

3. If a vehicle skids with all four wheels locked for 100 feet over four different surfaces and comes to a stop without hitting anything, how fast was the vehicle going at the start of the skid?

Surface # 1	D = 25 ft	f = .70
Surface # 2	D = 25 ft	f = .90
Surface # 3	D = 25 ft	f = .32
Surface # 4	D = 25 ft	f = .08

S = _________

4. A vehicle slides to a stop in 150 feet. Two wheels were on the slab with (f) = .70 and two wheels were on the gravel shoulder with (f) = .35. What was its speed at the start of the skid?

S = _________

TANGENT OFFSET EQUATION

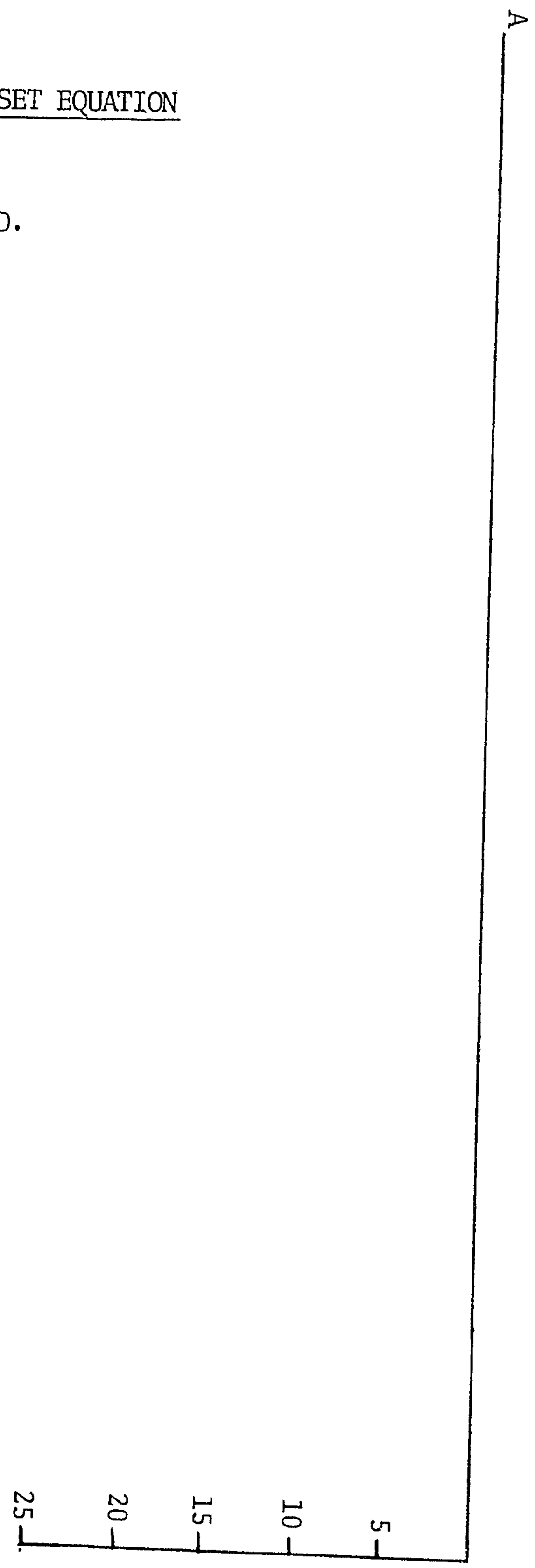
FROM POINT "A" CONSTRUCT A ROAD.

OUTSIDE MEASUREMENT

C = 100 M = 2

ROAD WIDTH = 24 FEET

SCALE 1:120



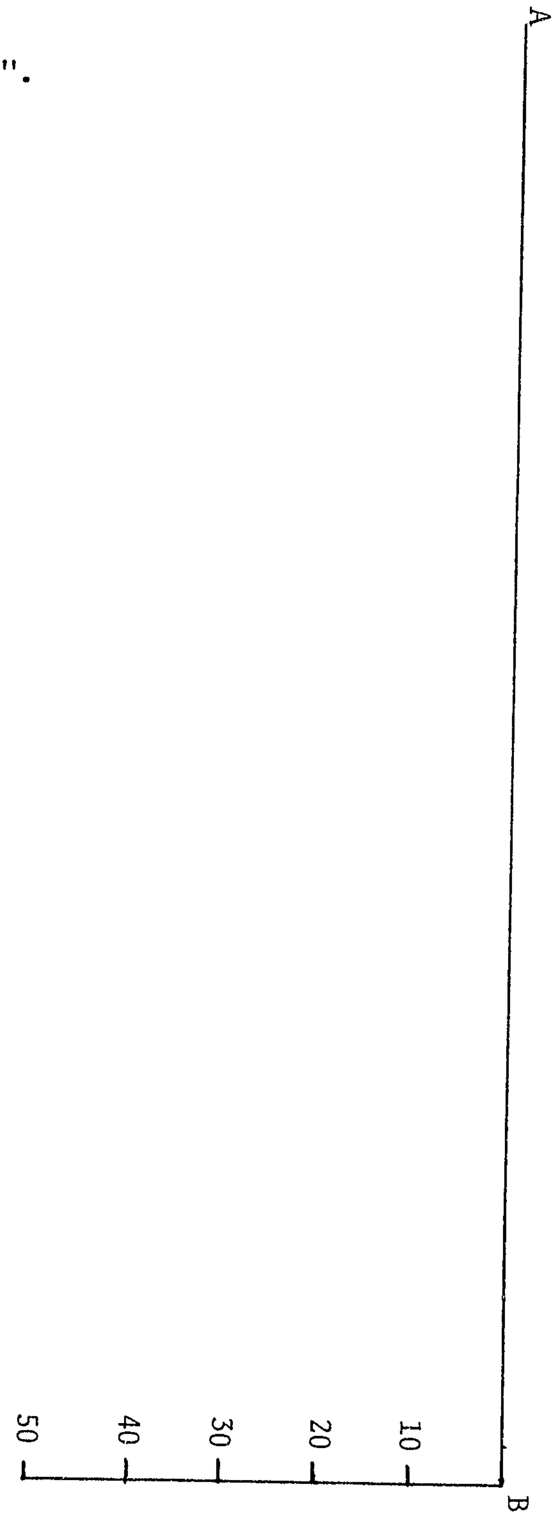
TANGENT OFFSET EQUATION

CONSTRUCT A ROAD FROM "A" TO "B".

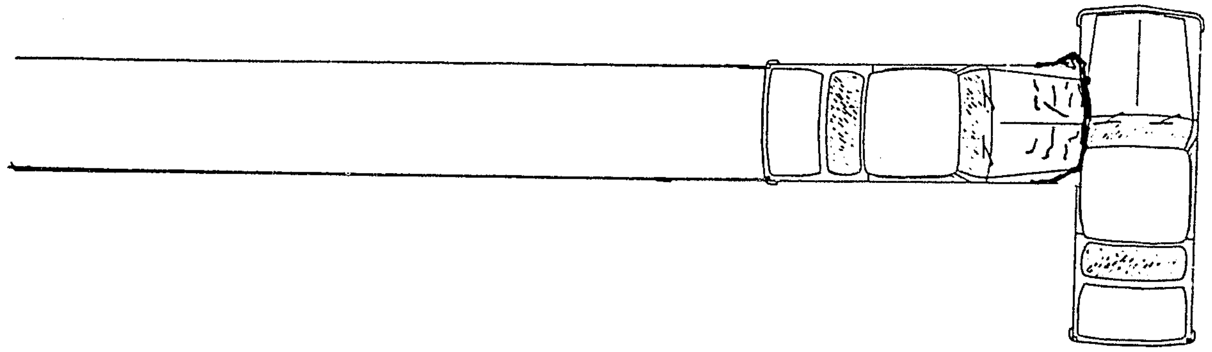
OUTSIDE RADIUS = 400 FEET

WIDTH = 24 FEET

SCALE 1:240



QUADRATIC EQUATIONS



WHAT IS THE TIME IN SECONDS FOR THE ABOVE SKID? (USE A QUADRATIC EQUATION.) SEE PAGES 61 - 67

T = UNKNOWN

V_0 = 88 FPS

f = .80

D = 100 FEET

T = ________SECONDS

TIME-DISTANCE EQUATIONS

1. In a start from stop a vehicle accelerates to 40 MPH in 400 feet. What is its acceleration factor?

2. In a start from stop a vehicle accelerates to 37 MPH in 12 seconds. What is its acceleration factor?

3. A vehicle accelerates with an acceleration factor of .12 for 10 seconds. How much speed will it gain?

4. A vehicle skids on a drag factor of .80 for 3 seconds. How much speed will it lose?

5. A driver says he stopped at a stop sign. He says he had accelerated to 20 MPH and had traveled 40 feet when he was hit. His car has an acceleration factor of .15. Is he telling the truth?

6. A vehicle has an acceleration factor of .12. How long in seconds will it take to go 100 feet from a stop?

TIME-DISTANCE EQUATIONS

7. A vehicle skids 150 feet to a stop on a surface that has a drag factor of .80. How long in seconds will it take?

8. If a car is going 65 MPH when the brakes are applied, how far will it skid in 2.3 seconds if (f) = .75 ?

9. A driver slams on his brakes doing 90 MPH. What will his speed be after 4 seconds of skidding? Assume (f) = .80.

10. A car starts to skid from 80 MPH. What will its speed be after it has slid 150 feet? Assume (f) = .72.

11. How long in seconds will this speed reduction take?

CONSERVATION OF MOMENTUM

PROBLEM #1

$\theta = 50^0$

$\Phi = 10^0$

$D_1 = 50$ FEET

$D_2 = 70$ FEET

$W_1 = 4800$ LBS.

$W_2 = 3900$ LBS.

TEST SKID

$S = 30$ MPH

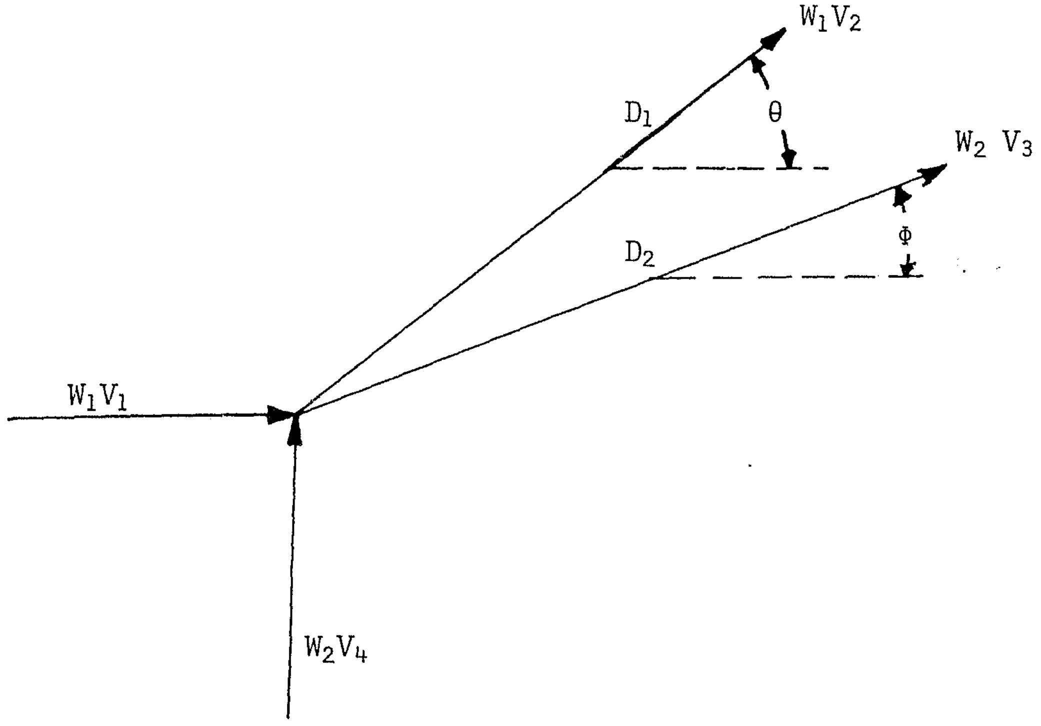
$D = 42$ FEET

4 WHEEL SKID BOTH VEHICLES

CALCULATE V_1 AND V_4.

$V_1 =$ __________

$V_4 =$ __________



CONSERVATION OF MOMENTUM

PROBLEM #2

W_1 = 4560 LBS.

W_2 = 3980 LBS.

θ = 40^0

Φ = 20^0

Ψ = 60^0

f = .80

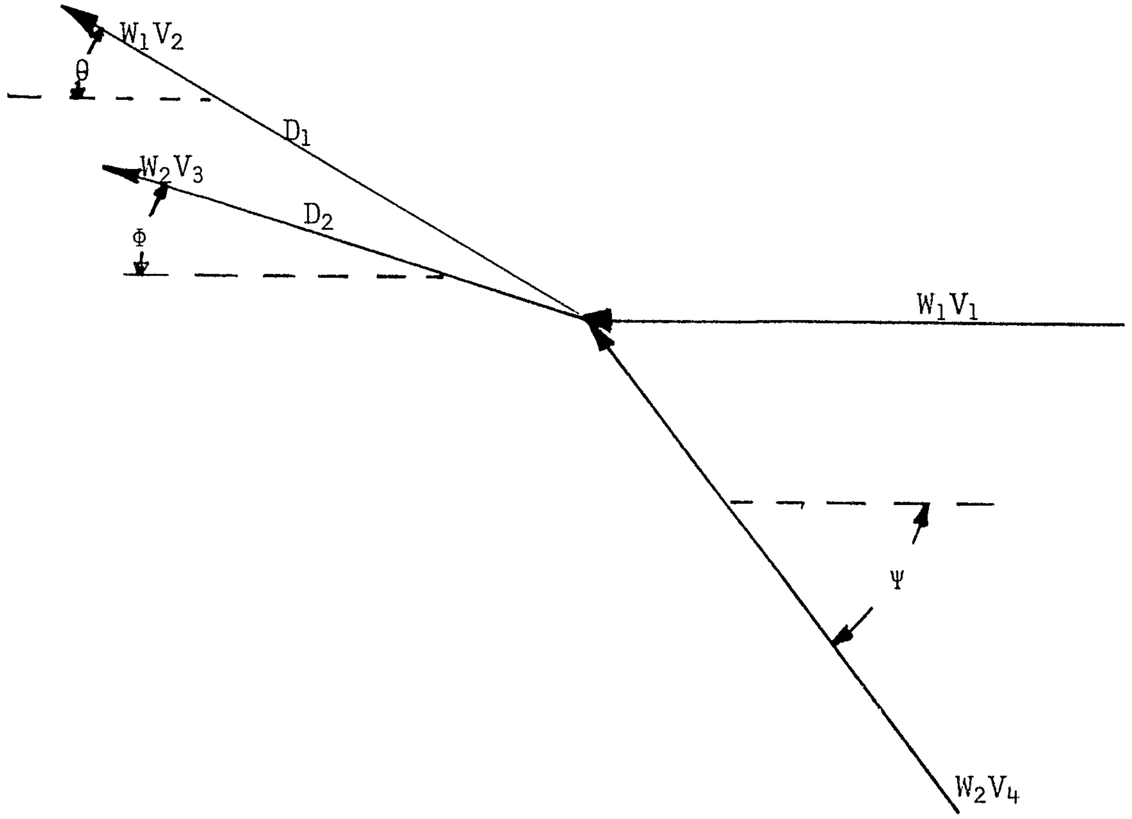
D_1 = 100 FEET, 2 WHEEL SKID

D_2 = 80 FEET, 4 WHEEL SKID

CALCULATE V_1 AND V_4.

V_1 = ________

V_4 = ________



CONSERVATION OF MOMENTUM

PROBLEM #3

W_1 = MUSTANG
ONE OCCUPANT = 2881 LBS.

W_2 = BUICK
NINE OCCUPANTS = 6400 LBS.

f = .70

BUICK APPROACH $\theta = 29^0$

BUICK DEPARTURE $\Phi = 4^0$

MUSTANG DEPARTURE $\Psi = 19^0$

D_1 = 31 FEET, 2 WHEEL SKID

D_2 = 14 FEET, 2 WHEEL SKID

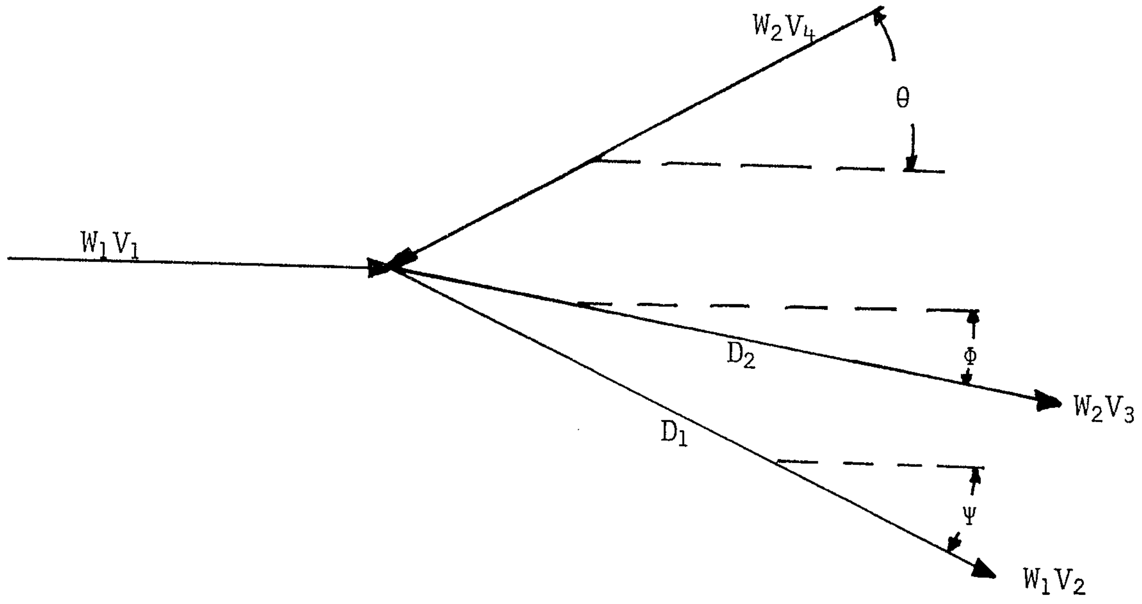
D_2 = 10 FEET, 4 WHEEL SKID

D_2 = 27 FEET, 2 WHEEL SKID

CALCULATE V_1 AND V_4.

V_1 = ______

V_4 = ______



CONSERVATION OF MOMENTUM

PROBLEM #4

W_1 = 1975 PONTIAC BONNEVILLE
4 DR. H.T.

W_2 = 1973 MERCEDES
4 DR. SEDAN 280 SE/8

(LOOK UP THE WEIGHTS OF THE VEHICLES IN A N.A.D.A BOOK.)

PONTIAC APPROACH $\theta = 24^0$

PONTIAC DEPARTURE $\Phi = 21^0$

MERCEDES DEPARTURE $\Psi = 59^0$

f = .80 TAR

f = .40 GRAVEL

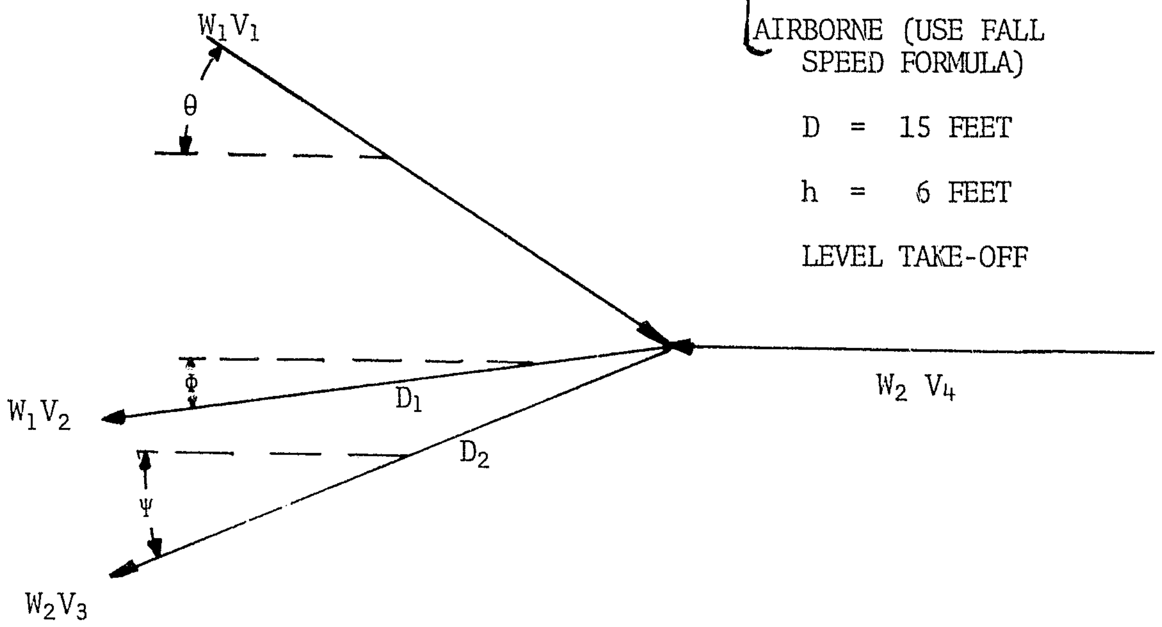
D_1 = { 40 FEET TAR, 2 WHEEL SKID; AND
20 FEET GRAVEL, 2 WHEEL SKID }

D_2 = { 20 FEET TAR 1 WHEEL SKID; AND
AIRBORNE (USE FALL SPEED FORMULA) }

D = 15 FEET

h = 6 FEET

LEVEL TAKE-OFF



CALCULATE V_1 AND V_4.

V_1 = ______

V_4 = ______

CONSERVATION OF MOMENTUM

PROBLEM #5

W_1 = 1972 CADILLAC
4 DR. DEVILLE
5 OCCUPANTS, 160 LBS. EA.

W_2 = 1970 OLDS CUTLASS F-85
4 OCCUPANTS, 180 LBS. EA.

(LOOK UP VEHICLE WEIGHTS IN A N.A.D.A. BOOK.)

f = .45 - SHOULDER

f = .73 - SLAB

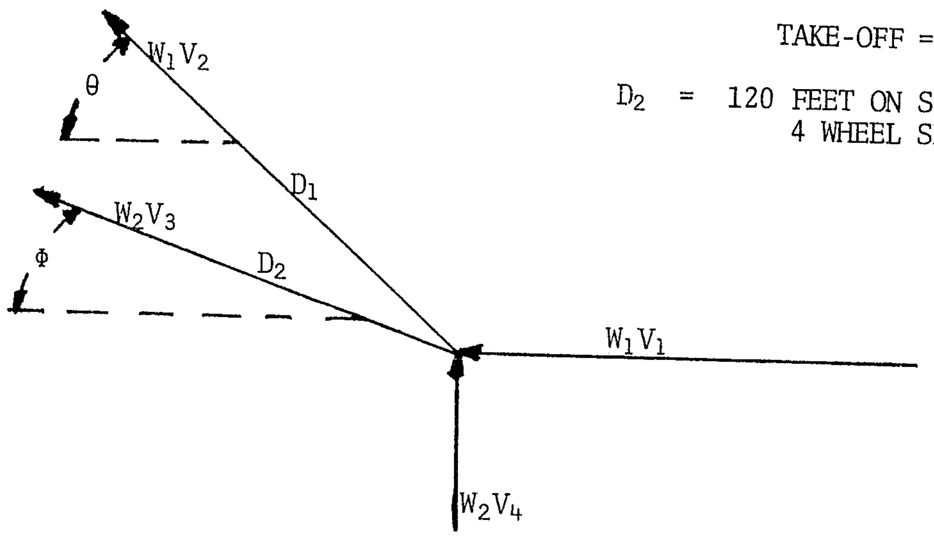
D_1 = { 30 FEET ON SHOULDER 2 WHEEL SKID; AND
AIRBORNE (USE VAULT FORMULA) }

D = 100 FEET

h = 8 FEET BELOW TAKE-OFF

TAKE-OFF = $+30^0$

D_2 = 120 FEET ON SLAB, 4 WHEEL SKID



CADILLAC DEPARTURE $\theta = 43^0$

OLDS DEPARTURE $\Phi = 21^0$

CALCULATE V_1 AND V_4.

V_1 = ______

V_4 = ______

CONSERVATION OF MOMENTUM

PROBLEM #6

W_1 = 1972 FORD TRUCK
C 8000

W_2 = 1973 CHRYSLER NEW YORKER
4 DR. SEDAN

(FIND VEHICLE WEIGHTS IN
A N.A.D.A. BOOK.)

FORD DEPARTURE $\theta = 8^0$

CHRYSLER DEPARTURE $\Phi = 3^0$

CHRYSLER APPROACH $\Psi = 78^0$

$f = .70$

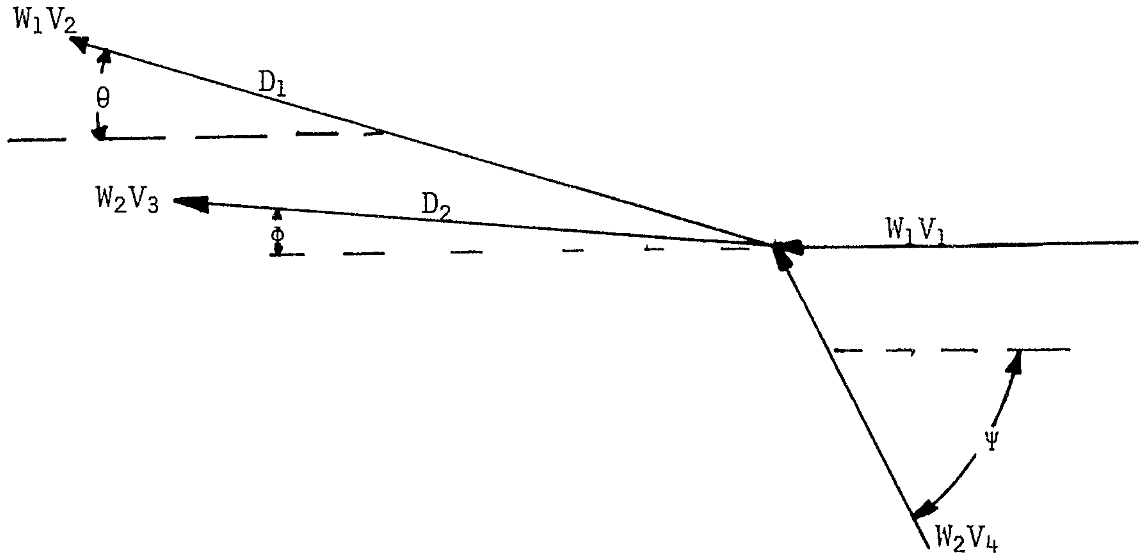
D_1 = 50 FEET, 4 WHEEL SKID

D_2 = 100 FEET, 2 WHEEL SKID

CALCULATE V_1 AND V_4.

V_1 = ______

V_4 = ______



CONSERVATION OF MOMENTUM

PROBLEM #6 CNT'D

A. ASSUME THE CHRYSLER STARTED UP FROM A STOP SIGN. ASSUME AN ACCELERATION FACTOR OF .12. HOW LONG HAD THE CHRYSLER BEEN MOVING?

B. HOW FAR IN FEET HAD THE CHRYSLER MOVED?

C. HOW FAR AWAY IN FEET WAS THE FORD TRUCK WHEN THE CHRYSLER STARTED UP?

This next section of Appendix B consists of a group of eight (8) questions involving weight shift in determining speed from skid-marks. The equations to be used are found in Chapter XII, Weight Shift. Questions 1 - 8 deal with automobiles, straight trucks, and truck tractors <u>not</u> hooked to semi-trailers (bob-tailed tractors). Remember that a 3-axle rig is treated as if it had 2 axles, with the rear axle located between the rear tandems or bogeys (See text).

Use the following data in answering the next 8 questions:

h = 24 inches
L = 120 inches
μ_F = .80
μ_R = .80

Total weight = 4000 lbs.
Left front, W_1 = 1000 lbs.
Right front, W_2 = 1000 lbs.
Left rear, W_3 = 1000 lbs.
Right rear, W_4 = 1000 lbs.

Skidding distances:

For vehicle = 200 ft.
Left front, d_1 = 200 ft.
Right front, d_2 = 200 ft.
Left rear, d_3 = 200 ft.
Right rear, d_4 = 200 ft.

W_F = 2000 lbs.
W_R = 2000 lbs.
g = 32.2
V = Velocity in FPS
S = Speed in MPH

1. Calculate the weight shift and speed assuming the vehicle skids the 200 feet with all wheels skidding.

A. ΔW_1 = __________lbs.

B. S = __________MPH

Use the minimum speed formula to answer C.

C. S = __________MPH

D. How do B and C compare?

2. Calculate the weight shift and speed assuming the vehicle has brakes on the front wheels only. $\mu_R = 0$; d_3 & $d_4 = 0$

A. ΔW = __________lbs.

B. S = __________MPH

Use the minimum speed formula and take 50% of .80 as the drag factor to answer C.

C. S = __________MPH

D. How do B and C compare?

3. Calculate the weight shift and speed assuming the vehicle has brakes on the rear wheels only. $\mu_F = 0$; d_1 & $d_2 = 0$

A. ΔW = __________lbs.

B. S = __________MPH

Use the minimum speed formula and take 50% of .80 as the drag factor to answer C.

C. S = __________MPH

D. How do B and C compare?

4. Calculate the weight shift and speed assuming the vehicle has brakes on the left front wheel only. $\mu_R = 0$; $W_1\ \mu_F = 1000 \times .80$; $W_2\ \mu_F = 0$; $d_1 = 200$ feet; d_2, d_3 & $d_4 = 0$

A. ΔW = __________lbs.

B. S = __________MPH

Use the minimum speed formula and take 25% of .80 as the drag factor to answer C.

C. S = __________MPH

D. How do B and C compare?

5. Calculate the weight shift and speed assuming the vehicle has brakes on the right rear wheel only. $\mu_F = 0$; $W_3\mu_R = 0$; $W_4\mu_R = 1000 \times .80$; d_1, d_2 & $d_3 = 0$; $d_4 = 200$ feet

A. ΔW = __________lbs.

B. S = __________MPH

Use the minimum speed formula and take 25% of .80 as the drag factor to answer C.

C. S = __________MPH

D. How do B and C compare?

6. Calculate the weight shift and speed assuming the vehicle has brakes on the right front, left front, and right rear wheels only. $W_3\mu_R = 0$; $d_3 = 0$

A. ΔW = __________lbs.

B. S = __________MPH

Use the minimum speed formula and take 75% of .80 as the drag factor to answer C.

C. S = __________MPH

D. How do B and C compare?

7. Calculate the weight shift and speed assuming the vehicle has brakes on the left front, right rear, and left rear wheels only. $W_2\mu_F = 0$; $d_2 = 0$

A. $\Delta W =$ ________lbs.

B. S = ________MPH

Use the minimum speed formula and take 75% of .80 as the drag factor to answer C.

C. S = ________MPH

D. How do B and C compare?

8. Calculate the weight shift and speed assuming the vehicle has brakes on its left front and right rear wheels only. $W_2\mu_F = 0$; $W_3\mu_R = 0$; d_2 & $d_3 = 0$

A. $\Delta W =$ ________lbs.

B. S = ________MPH

Use the minimum speed formula and take 50% of .80 as the drag factor to answer C.

C. S = ________MPH

D. How do B and C compare?

9. Calculate the speed in MPH for a truck-tractor semi-trailer combination assuming the tractor has no brakes on the front or steering axle. Use the data given below.

The long speed equation using weight shift is needed for this one. Refer to the truck diagram on page 131 to identify what the various symbols relate to.

Please note that the numbers listed below are just numbers to plug into the equation. They are not intended to be realistic, and it may not be possible for a truck-tractor semi-trailer combination to have these weights and produce these distances.

Plug in your calculator and throw your spouse, children, pets, clergy, or whoever is with you, out of the house and give it a try. Remember the brackets and remember that you multiply and divide, then add and subtract.

Good Luck!

W_T = 15,000 lbs.
W_L = 50,000 lbs.
D = 200 feet = length of skidmarks
μ_1 = 0 (no brakes)
μ_2 = .80
μ_3 = .80
L_1 = 5.6 feet
L_2 = 5.6 feet
L_3 = 20 feet
L_4 = 20 feet
L_5 = 2.8 feet
h_T = 5.5 feet
h_H = 4 feet
h_L = 7 feet
$L_1 + L_2$ = 11.2
$L_1 + L_2 + h_T\mu_2$ = 15.6
$L_3 + L_4 + \mu_3 h_H$ = 43.2
$L_3 + L_4 + \mu_2 h_H$ = 43.2
$L_5 - L_2$ = -2.8
$L_1 + L_5$ = 8.4
$h_T - h_H$ = 1.5
$L_2 - L_3 - L_4 - L_5$ = -37.2

S = __________MPH

MISCELLANEOUS FORMULAS

This next section contains 12 problems requiring the use of the miscellaneous formulas presented in Chapter XIII.

1. A truck-tractor semi-trailer combination attempts to go around a curve whose radius is 200 feet. The curve is flat. As the semi enters the curve, the trailer tips over. The height of the center of mass of the trailer is 7 feet. The width of the trailer is 8 feet. What is the speed of the semi?

2. Using the information given for Problem 1, what will the speed of the semi be if the curve has a superelevation of +5%?

3. The driver of a car traveling down the road comes upon a truck broken down on the asphalt. If the driver of the car is to successfully change lanes to go around the truck, how much distance in feet will be needed? Assume the speed of the car is 55 MPH, the lateral distance the car must move is 10 feet, the lateral acceleration is .30, and the driver's reaction time is .75 seconds.

MISCELLANEOUS FORMULAS

4. Assume the same information as for Problem 3. What distance will the driver of the car need to simply swerve to miss the truck?

5. Assume an object is launched into the air. Its horizontal distance of travel is 50 feet and it lands 20 feet lower than take-off.
   A. What angle of take-off will give you the minimum speed?
   B. What will the minimum speed be?

6. A car is launched into the air at 55 MPH. Its take-off angle is 25 degrees. It lands 6 feet lower than take-off. What will its horizontal distance of travel be?

7. Assume a car is launched into the air. Its take-off angle is 30 degrees. It travels a horizontal distance of 100 feet and lands 5 feet below take-off.
   A. How high above the level of take-off is the car at the at the 50 foot mark?
   B. How high above the level of take-off will the car be at the high point in its arc?
   C. At what distance from the point of take-off will the car reach the high point in its arc?

MISCELLANEOUS FORMULAS

   D. How long in seconds will the car be in the air?

8. An object is to be propelled straight up into the air to a height of 10 feet. What velocity is required?

9. What is the maximum height an object will reach if it is propelled upward at 44 FPS?

10. How long in seconds will it take the object in Problem 9 to reach its maximum height?

11. Assume a large truck loses its brakes coming down a mountain. The driver uses a runaway truck ramp to stop his truck. The ramp has an upward angle of 35 degrees. The gravel surface of the ramp will develop a retardant action equal to a drag factor of 2.0. The truck travels 100 feet over the loose gravel before stopping. How fast was the truck going when it entered the ramp?

12. The driver of a semi claims he had an accident because his front wheels started to hydroplane. You find that his front tires have 80 lbs. of air pressure and an average tread depth. How fast was he going if he did hydroplane?

ANSWERS TO THE PROBLEMS IN APPENDIX B

Note: Final answers are underlined. Appearing above some of the final answers, and not underlined, are answers to intermediate steps in the calculations. Appearing below some of the final answers are formulas used in the calculations.

PAGES 271 - 278 (Geometric Construction Exercises)

MINIMUM SPEED FROM SKIDMARKS

PAGE 279...... 1. .60
2. 67 MPH

FALL SPEED

PAGE 280...... 76 MPH

PAGE 281...... 47 MPH
Measurements D_1 and h_1 are correct because they both measure distance traveled by center of mass.

PAGE 282...... 23 MPH

PAGE 283...... 48 MPH

VAULT SPEED

PAGE 284...... 46 MPH

PAGE 285...... 50 MPH

CRITICAL SPEED

PAGE 286...... Adjusted f = .83
Radius of road = 314 feet
Radius of scuff = 626 feet
1. 62 MPH
2. 87 MPH

KINETIC ENERGY/COMBINED SPEED

PAGE 287 ...... 1. A. 333719.86
B. 166859.93
2. 9105.5898
The momentum is the same for both vehicles.
3. 23 MPH = Minimum speed on Surface #1
26 MPH = Minimum speed on Surface #2
15 MPH = Minimum speed on Surface #3
7 MPH = Minimum speed on Surface #4
38 MPH
4. 47 MPH

TANGENT OFFSET EQUATION

PAGE 288...... The outside of the curve should cross the 5.7 mark.
Radius = 626 feet

PAGE 289. ..... The outside of the curve should cross the 30.8 mark.

QUADRATIC EQUATIONS

PAGE 290 ...... 1.4 seconds

TIME-DISTANCE EQUATIONS

			SEE PAGE
PAGE 291	1.	f = .13	72 (4)
	2.	f = .14	71 (3)
	3.	38.64 FPS or 26.3 MPH	73 (6)
	4.	77.28 FPS or 52.7 MPH	73 (6)
	5.	no	74 (8)
	6.	7.2 seconds	74 (9)
PAGE 292......	7.	3.4 seconds	74 (9)
	8.	155.2 feet	76 (11)
	9.	28.9 FPS or 19.7 MPH	77 (12)
	10.	82.45 FPS or 56.2 MPH	77 (13)
	11.	1.5 seconds	73 (7)

CONSERVATION OF MOMENTUM

PAGE 293....... (PROBLEM #1)

f = .71

V_2 = 32 MPH

V_3 = 38 MPH

θ 50^0: Sin .766 Cos .643

Φ 10^0: Sin .174 Cos .985

V_1 = 50.9 MPH

V_4 = 36.7 MPH

$$W_1V_1 = W_1V_2 \text{ Cos } \theta + W_2V_3 \text{ Cos } \Phi$$

$$W_2V_4 = W_1V_2 \text{ Sin } \theta + W_2V_3 \text{ Sin } \Phi$$

PAGE 294 ...... (PROBLEM #2)

V_2 = 34 MPH

V_3 = 44 MPH

θ 40^0: Sin .643 Cos .766

Φ 20^0: Sin .342 Cos .940

Ψ 60^0: Sin .866 Cos .500

V_1 = 42 MPH

V_4 = 46 MPH

$$-W_1V_1 - W_2V_4 \text{ Cos } \Psi = -W_1V_2 \text{ Cos } \theta - W_2V_3 \text{ Cos } \Phi$$

$$W_2V_4 \text{ Sin } \Psi = W_1V_2 \text{ Sin } \theta + W_2V_3 \text{ Sin } \Phi$$

PAGE 295 ...... (PROBLEM #3)

V_2 = 18 MPH

V_3 = 24 MPH

θ 29^0: Sin .485 Cos .875

Φ 4^0: Sin .070 Cos .998

Ψ 19^0: Sin .326 Cos .946

V_1 = 85 MPH

$V_4 = 8$ MPH

$W_1V_1 - W_2V_4 \text{ Cos } \theta = W_1V_2 \text{ Cos } \Psi + W_2V_3 \text{ Cos } \Phi$

$-W_2V_4 \text{ Sin } \theta = - W_1V_2 \text{ Sin } \Psi - W_2V_3 \text{ Sin } \Phi$

PAGE 296 . . . . . (PROBLEM #4)

$W_1 = 4503$ lbs.

$W_2 = 3386$ lbs.

$V_2 = 24$ MPH

$V_3 = 19$ MPH

θ 24^0: Sin .407 Cos .914

Φ 21^0: Sin .358 Cos .934

Ψ 59^0: Sin .857 Cos .515

$V_1 = 51$ MPH

$V_4 = 101$ MPH

$W_1V_1 \text{ Cos } \theta - W_2V_4 = - W_1V_2 \text{ Cos } \Phi - W_2V_3 \text{ Cos } \Psi$

$- W_1V_1 \text{ Sin } \theta = - W_1V_2 \text{ Sin } \Phi - W_2V_3 \text{ Sin } \Psi$

PAGE 297 . . . . . . (PROBLEM #5)

$W_1 = 5562$ lbs.

$W_2 = 4131$ lbs.

$V_2 = 40$ MPH

$V_3 = 51$ MPH

θ 43^0: Sin .682 Cos .731

Φ 21^0: Sin .358 Cos .934

$V_1 = 64$ MPH

$V_4 = 54$ MPH

$- W_1V_1 = - W_1V_2 \text{ Cos } \theta - W_2V_3 \text{ Cos } \Phi$

$W_2V_4 = W_1V_2 \text{ Sin } \theta + W_2V_3 \text{ Sin } \Phi$

PAGE 298 . . . . . . (PROBLEM #6)

$W_1 = 8215$ lbs.

$W_2 = 4355$ lbs.

$V_2 = 32$ MPH

$V_3 = 32$ MPH

θ 8^0: Sin .139 Cos .990

Φ 3^0: Sin .052 Cos .999

Ψ 78^0: Sin .978 Cos .208

$V_1 = 47$ MPH

$V_4 = 10$ MPH

$- W_1V_1 - W_2V_4 \text{ Cos } \Psi = - W_1V_2 \text{ Cos } \theta - W_2V_3 \text{ Cos } \Phi$

$W_2V_4 \text{ Sin } \Psi = W_1V_2 \text{ Sin } \theta + W_2V_3 \text{ Sin } \Phi$

PAGE 299 . . . . . . (PROBLEM #6 CNT'D)

A. 3.7 seconds

SEE PAGE
72 (5)

B. 27.7 feet 74 (8)
C. 254.9 feet 78 (16)

WEIGHT SHIFT

PAGE 301 ...... 1. A. 640 lbs.
B. 69 MPH
C. 69 MPH
D. Same
2. A. 380 lbs.
B. 53 MPH
C. 49 MPH
D. C is 4 MPH too low.
PAGE 302 ...... 3. A. 275 lbs.
B. 45 MPH
C. 49 MPH
D. C is 4 MPH too high.
4. A. 190 lbs.
B. 36 MPH
C. 34 MPH
D. C is 2 MPH too low.
PAGE 303 ...... 5. A. 137 lbs.
B. 33 MPH
C. 34 MPH
D. C is 1 MPH too high.

PAGE 303...... 6. A. 480 lbs.
B. 62 MPH
C. 60 MPH
D. C is 2 MPH too low.
PAGE 304 ...... 7. A. 480 lbs.
B. 57 MPH
C. 60 MPH
D. C is 3 MPH too high.
8. A. 320 lbs.
B. 48 MPH
C. 49 MPH
D. Actually B is 48.9 MPH and C is 49.1 MPH; so for all practical purposes they are the same.
PAGE 306 ...... 9. 56 MPH
Note: This speed indicates skidding on about 66% of the full drag factor of .80.

MISCELLANEOUS FORMULAS

PAGE 307 ...... 1. 60.66 FPS or 41.37 MPH
2. 63.26 FPS or 43.15 MPH
3. 269.9 feet
PAGE 308 ...... 4. 165.21 feet
5. A. 34 degrees

B. V = 33 FPS or S = 22.61 MPH

PAGE 308 . . . . 6. 166.68 feet

7. A. 13.16 feet

B. 13.28 feet

C. 45.98 feet

PAGE 309 . . . . D. 1.97 seconds

8. 25.37 FPS

9. 30.06 feet

10. 1.36 seconds

11. 81 MPH

12. 92.57 MPH. The semi probably did not hydroplane.

END