Document Title: Neural Network for the Fast Gaussian Distribution Test

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NEURAL NETWORK FOR THE FAST GAUSSIAN DISTRIBUTION TEST

There are several problems where it is very important to know whether the tested data are distributed according to the Gaussian law.
At the detection of the hidden information within the digitized pictures (steganography), one of the key factors is the analysis of the noise contained in the picture. The incorporated noise should show the typically Gaussian distribution. The departure from the Gaussian distribution might be the first hint that the picture has been changed – possibly new information has been inserted. In such cases the fast Gaussian distribution test is a very valuable tool.
The article describes the phase of the noise (in the picture) extraction and the distribution formation. The second phase of the noise analysis is performed by the neural network. The neural network is trained to recognize the Gaussian distribution of the noise.
The trained neural network successfully performs the fast Gaussian distribution test.

INTRODUCTION

The cryptographic method of steganography secures an important message by encrypting it to an unrecognized form of the data. Steganographic methods hide the encrypted message in cover carriers (pictures) so that it cannot be seen while it is transmitted on public communication channels such as computer network. Many steganographic methods embed a large amount of secret information in the first k LSBs (Lest Signifficant Bit) of the pixels of the cover images (Lou, Liu, 2002). By other words, the construction of the LSB insertion, the noise of the carrier picture is changed. Normally the noise of the original picture is the Gaussian noise. The change in the "noise" content also changes the shape of the noise distribution function, which is no longer normally distributed. Discovery that the image noise is not normally distributed can be the first hint that the original image has been changed.

The steganography is the cryptographic method that is very simple to implement and on the other hand, it is extremely hard to detect and even harder to decipher. One of the possible indices that the original image has been changed, is the noise analysis. Even if the image appers to have the Gaussian noise, this is not the proof that the image is "clean". The sophisticated methods of steganography also reshape the embeded information in such a way that the noise of the changed image again takes the Gaussian distribution profile (Lou, Liu, 2002).

In order to detect the distribution of the image noise, the special neural network method has been developed and tested. Two goals have been followed. The first is the robustness of the test and the other is the simplicity of its use. Since the determination of the image noise is not the 100% proof of the existance of the embeded information, there is no neccessity to build a very reliable distribution test. The coarse test is often good enough. Sometimes it would be enogh to implement the least square fitting to the sampled image noise distribution, with the function of Gaussian type and observe the least square error of the sampled points from the approximated function. Such test is good enough when the number of the sampled points is high enough.

METHOD

The method of the Gaussian distribution detection by the neural networks uses two approximation techniques to detect the shape of the noise distribution. First the image noise is amplitude – frequency analysed. The analysis results in the amplitude – frequency distribution. The image noise distribution is typically discrete distribution, which produces the relatively small value of amplitude samples (for 3 LSB there are only 9 amplitude stages). Therefore the noise amplitude distribution is far from the continuous i.e. normal distribution. This is a typical example of the discrete distribution – Poisson distribution (Cherkassky, Mulier, 1998).

THE DISTRIBUTIONS

Let's first review the relations between the two distributions – one discrete and another continuous (Spiegel, 1975). Both distributions and their relations are stated here as the point of reference.

THE NORMAL DISTRIBUTION

One of the most important examples of a continuous probability density distribution is the normal or Gaussian distribution. The density function for this distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^{2}/2\sigma^{2}} \qquad -\infty < x < \infty \qquad (1)$$

Where μ and σ are the mean and standard deviation respectively. The corresponding distribution function is given by

$$F(x) = P(x \le x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(v-\mu)^{2}/2\sigma^{2}} dv \qquad (2)$$

In such case we say that the random variable X is normally distributed with mean μ and variance σ^2.

If we let Z be the standardized variable corresponding to X, i.e. if we let

$$Z = \frac{X-\mu}{\sigma} \qquad (3)$$

then the mean or expected value of Z is 0 and the variance is 1. In such case the density function for Z can be obtained from 1 by formally placing $\mu = 0$ and $\sigma = 1$, yielding

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^{2}/2} \qquad (4)$$

This is often referred to as the standard normal density function or distribution. The corresponding distribution function is given by

$$F(z) = P(Z \le z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u/2}\, du = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^{2}/2}\, du \qquad (5)$$

We sometimes call the value z of the standardized variable Z the standard score. The function $F(z)$ is related to the extensively tabulated error function, depicted by $erf(z)$. We have

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{z} e^{-u^{2}} du \quad \text{and} \quad F(z) = \frac{1}{2}\left[1 + erf\left(\frac{2}{\sqrt{2}}\right)\right] \qquad (6)$$

THE POISSON DISTRIBUTION

Let X be a descrete random variable which can take on the values 0, 1, 2, ... such that the probability function of X is given by

$$f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x} \qquad x = 0, 1, 2, \ldots \qquad 7$$

where λ is a given positive constant. This distribution is called the Poisson distribution and a random variable having this distribution is called to be Poisson distributed.

RELATION BETWEEN THE POISSON AND NORMAL DISTRIBUTION

It can be shown that if X is the Poisson random variable of (7) and $(X-\lambda)/\sqrt{\lambda}$ is the corresponding standardized random variable, then

$$\lim_{\lambda\to\infty} P\left(a \le \frac{X-\lambda}{\sqrt{\lambda}} \le b\right) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-u^2/2}\, du \qquad 8$$

i.e. the Poisson distribution approaches the normal distribution as $\lambda \to \infty$ or $(X-\lambda)/\sqrt{\lambda}$ is asymptotically normal.

SOME PROPERTIES OF THE NORMAL AND THE POISSON DISTRIBUTIONS

In the following table some important properties of the general normal distribution and the Poisson distribution are listed.

Table1: important properties of the normal and Poisson distributions (Spiegel, 1975)

	Normal distribution	Poisson distribution
Mean	μ	$\mu = \lambda$
Variance	σ^2	$\sigma^2 = \lambda$
Standard deviation	σ	$\sigma = \sqrt{\lambda}$
Coefficient of skewness	$\alpha_3 = 0$	$\alpha_3 = 1/\sqrt{\lambda}$
Coefficient of kurtosis	$\alpha_4 = 3$	$\alpha_4 = 3 + (1/\lambda)$
Moment generating function	$M(t) = e^{\mu t + (\sigma^2 t^2/2)}$	$M(t) = e^{\lambda(e^t - 1)}$
Characteristic function	$\phi(\omega) = e^{i\mu\varpi - (\sigma^2\omega^2/2)}$	$\phi(\omega) = e^{\lambda(e^{i\varpi} - 1)}$

THE NEURAL NETWORK AND LEAST SQUARE APPROXIMATORS

In order to detect whether the given noise is at least asymptotically normally distributed we perform two approximations of the amplitude-frequency noise distribution. Figure 1 shows the general idea of two approximations comparrison. Aproximations are needed to enlarge the resolution of sampled distribution and thus to perform a better analysis.

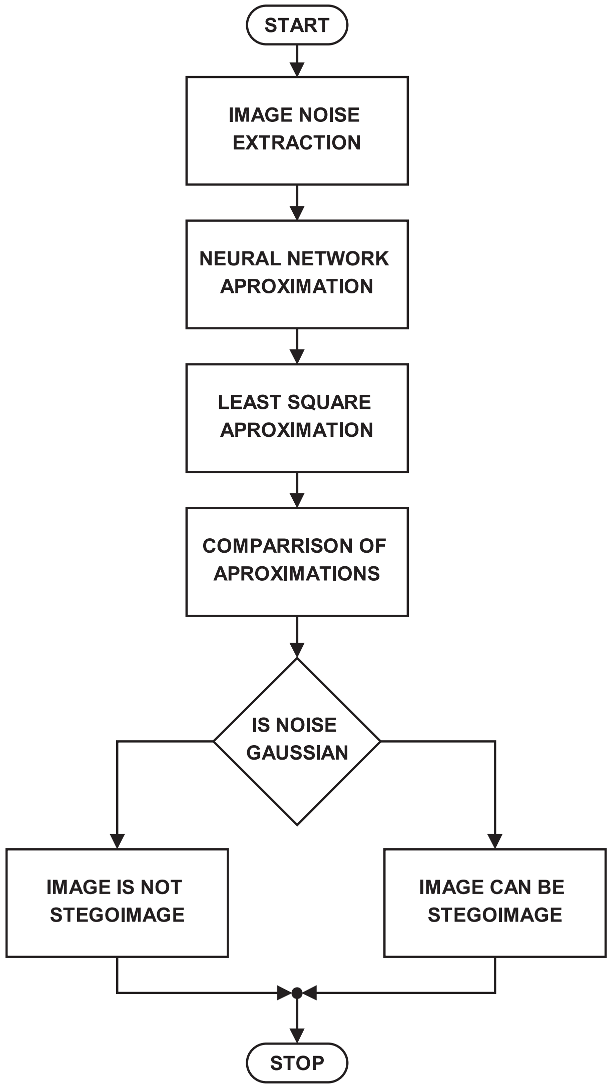

Figure 1: The noise shape detection by two approximations – one model-les and another with the pre-given model

EXPERIMENTAL RESULTS

The experimetal work was dedicated to design the adequate feedforward neural network (Haykin, 1999) to perform the approximation of the sampled noise distribution function. For the preliminary testing the Cheshire Neuraliyst 1.4 was used. The least square approximation of the sampled noise data was obtained by the MathWorks Matlab 5.2. Figures 2 and 3 show the result of two noise data samples. On the figure 2 there are two approximations of the image noise data. The dot marked curve is the result of the neural networ k approximation while the other curve represents the least square approximation of the same noise data distribution. On the figure 3, the image noise is distorted. The result of the least square approximation always produces the Gaussian function, while the neural natwork approximation fits the given data without the limitation of the curve shape – this is why it is called model-les aproximation. From both examples (figure 2 and 3) it is obvious that such a test can produce the criteria function whether the tested noise distribution is Gaussian or not. Here it is the matter of a distance assesment of the two approximations and setting of the criteria how much should both approximated functions depart from one another that we can still say that the analysed noise profile is still of a Gaussian shape.

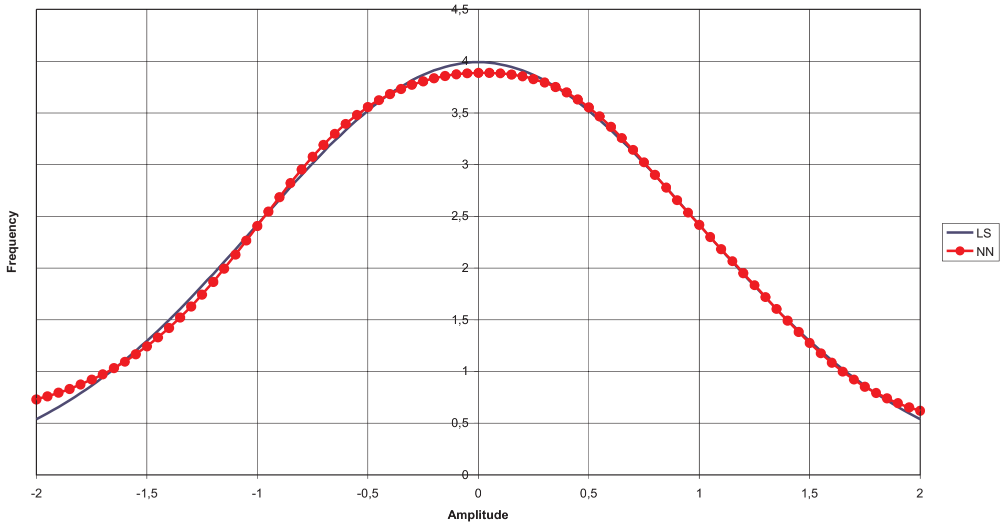

Figure 2: The result of two approximations. The dot marked curve is the neural network approximation (NN), the curve without the markings is the least square approximation to the normal distribution function. While the sampled noise distribution curve was Gaussian, both approximations were quite similar

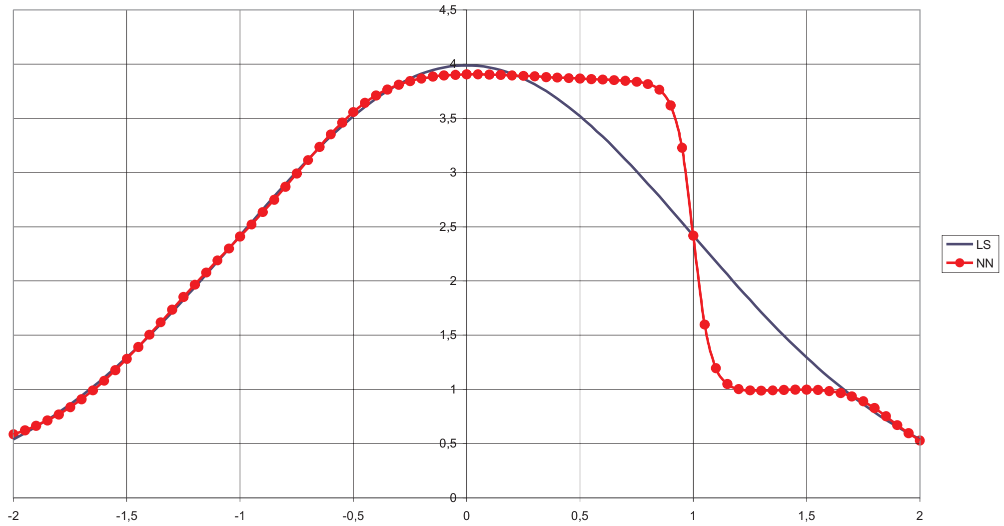

Figure 3: The result of two approximations and with distorted noise distributions. The dot marked curve is the neural network approximation (NN), the curve without the

markings is the least square approximation to the normal distribution function. While the sampled noise distribution curve was not Gaussian, both approximations are significantly different.

DISCUSSION

The shape of the noise that is present in every digital picture is assumed to be normally (Gaussian) distributed. The steganographic methods use the noise level of the picture to embed the new information without notable change in the picture quality. However, the shape of the noise amplitude – frequency distribution is change. The noise is no longer normally distributed.

The new method of noise distribution shape detection was proposed and tested. The new method uses two types of approximation of the sampled noise amplitude distribution. The first method is called "model-les" and uses the feedforward neural network to perform the approximation. The second method is the classical least square approximation that uses the given gaussian function as the template (model) for approximation. At this approximation method, the best fit of the Gaussian function is found to the sampled noise amplitude distribution.

The comparison of the two approximations gives the answer whether the noise in the picture is normally distributed. In the case of normal noise distribution, both approximations produce almost the same results.

The experimental work proved that the feedforward neural network with two hidden layers, can approximate the sampled noise amplitude – frequency distribution with a smooth approximation function.

The answer to the question, how much can both approximated distributions differ from one another that it is still possible to confirm the gaussian nature of the noise, remains the subject of the further research.

ABOUT THE AUTHORS

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