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Automatic Fingerprint Matching Using Extended Feature Set

Final Report

Award Number: 2007-RG-CX-K183

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Abstract

Fingerprint friction ridge features are generally described in a hierarchical order at three different levels, namely, Level 1 (ridge flow), Level 2 (minutiae points) and Level 3 (pores and ridge shape, etc.). Current Automated Fingerprint Identification Systems (AFIS) generally rely only on a subset of Level 1 and Level 2 features (minutiae and core/delta) for matching. On the other hand, latent print examiners frequently take advantage of a much richer set of features naturally occurring in fingerprints. It is believed that this difference may be one of the reasons for the superior performance of fingerprint examiners over AFIS, particularly in case of difficult latent matches. Fingerprint features, other than minutiae and core/delta, are also referred to as the extended feature set (EFS). The goal of this study is to i) develop algorithms for encoding and matching extended features, ii) develop fusion algorithms to combine extended features with minutiae information to improve fingerprint matching accuracy, and iii) understand the contributions of various extended features in latent fingerprint matching.

We study a number of extended features at all three levels, including ridge flow map, ridge wavelength map, ridge quality map, ridge skeleton, pores, dots, incipient ridges, and ridge edge protrusions. Feature extraction and matching algorithms are developed for each type of feature. Relative contribution of each feature towards the overall matching accuracy is evaluated by incrementally adding features to baseline features (minutiae and core/delta). The order of adding features is determined based on the amount of manual labour in feature marking and the estimated importance of features. Latent fingerprint databases, NIST SD27 and ELFT-EFS-PC, and several NIST rolled/plain fingerprint databases are used in our experiments.

Based on extensive experiments, we report the following findings: i) almost all the extended features lead to some improvement in latent matching accuracy, ii) extended features at higher level are more effective in improving latent matching accuracy than those at lower level, iii) high image resolution (at least 1000 ppi) is necessary but not sufficient for reliably capturing Level 3 features.

Based on our study, we would like to offer the following recommendations: i) extended features at Level 1 and Level 2 are strongly recommended to be incorporated into AFIS, ii) convenient GUI tools should be developed to help fingerprint examiners manually mark extended features (especially ridge skeleton) at Level 1 and Level 2 in latents, iii) it is crucial to improve the quality of enrolled fingerprints (so that a sufficient number of Level 3 features can be extracted) before Level 3 features can play an important role in AFIS.

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Executive Summary

Introduction

Over the past 40 years, Automated Fingerprint Identification Systems (AFIS) have played a major role in forensics and criminal investigations. However, these systems have not yet eliminated the need for manual examination and matching of fingerprints by experienced human experts, particularly for latent prints. This is mainly because “AFIS technology, since its onset, has utilized a very limited amount of fingerprint detail. Latent print experts rely on far more information in effecting individualizations/exclusions than just ending ridges and bifurcations” [50].

Fingerprint features are generally categorized into three levels [6]. Level 1 features are the macro details of the fingerprint such as ridge flow and pattern type. Level 2 features refer to the Galton characteristics or minutiae, such as ridge bifurcations and endings. Level 3 features include all dimensional attributes of the ridge such as ridge path deviation, width, shape, pores, edge contour, incipient ridges, breaks, creases, scars, and other permanent details. The current FBI standard of fingerprint resolution for AFIS is 500 ppi (50 microns pitch), which is inadequate to automatically and reliably extract Level 3 features, such as pores (60 microns in radius). As a result, state of the art AFIS technology is primarily based on Level 1 and Level 2 features.

With the advances in fingerprint sensing technology, many high resolution (i.e., 1000 ppi) sensors are now available that makes the extraction of extended features more feasible. The extended features that are well understood and often used by latent experts include minutiae shape, dots and pores [6]. They are utilized in latent matching when they are present in the input image and discernible to reach accurate conclusions. Unfortunately, there have very few systematic studies on automatic fingerprint identification using these extended features. Such studies are sorely needed with the planned Next Generation Identification (NGI) project launched by the FBI that will use fingerprints at 1000 ppi resolution.

At the 2005 ANSI/NIST fingerprint standard update workshop, SWGFAST [29] proposed a minimum scanning resolution of 1000 ppi for latent, ten-print, and palm print images and the inclusion of extended features in the FBI standard. This proposal and its support by the forensic community calls for an urgent need for systematic research in the use of extended feature set in automatic fingerprint identification. We propose to develop an automated system that would robustly extract and match some of the prominent extended features. This would enable us to quantify the discriminating power of the extended features by evaluating their statistical significance.

Methods

We study a number of extended features at all three levels, including ridge flow map, ridge wavelength map, ridge quality map, ridge skeleton, pores, dots, incipient ridges, and ridge edge protrusions. Feature extraction and matching algorithms are developed for each type of feature. Relative contribution of each feature to matching accuracy is evaluated by incrementally adding features to baseline features (minutiae and core/delta). The order of adding features is determined based on the amount of man-

ual effort needed in feature marking and the estimated importance of features. Latent fingerprint databases, NIST SD27 and ELFT-EFS-PC, and several NIST rolled/plain fingerprint databases are used in our experiments.

Based on a conventional fingerprint feature extraction algorithm, which outputs ridge skeleton image and minutiae, and a baseline minutiae matcher, which outputs minutiae correspondences and minutiae match score, encoding and matching algorithms are developed for extended features at three levels:

1. At Level 1, three types of extend features are considered, including ridge flow map, ridge wavelength map, and ridge quality map. These features are used in local matching stage to facilitate minutiae pairing as well as in global matching stage to help separate genuine matches and impostor matches.
2. At Level 2, a ridge skeleton matching algorithm is developed. Starting with the most similar minutiae pairs, the skeleton matching algorithm establishes skeleton correspondence through a skeleton propagation procedure. After skeleton matching is finished, a skeleton match score is computed.
3. At Level 3, four types of extended features are considered, including pores, dots, incipient ridges, and ridge edge protrusions. A topological level 3 feature matching algorithm is developed for latent to full fingerprint matching. Unlike most existing level 3 feature matching algorithms that only consider the feature location, the proposed algorithm enforces the topological relationship between level 3 features, minutiae, and ridge skeletons.

Because the public domain 1000 ppi fingerprint databases are few and small in size, we used different databases for extended features at different levels.

1. For experiments related to Level 1 and Level 2 features, 258 latent fingerprints and mated rolled fingerprints in NIST SD27 are used. A total of 29K rolled fingerprints in NIST SD4 and SD14 are additionally used as background data to increase the gallery size. All these fingerprint images are captured at 500 ppi.
2. For experiments related to Level 3 features, 242 latent fingerprints and mated rolled finger-prints in ELFT-EFS-PC are used. The gallery consists of 4,180 rolled fingerprints, where the additional rolled fingerprints are from NIST SD29 and SD30. All these fingerprint images are captured at 1000 ppi.
3. Because the poor quality of many latent and/or rolled fingerprints in ELFT-EFS-PC restricts the efficacy of Level 3 features, a simulated 1000 ppi partial fingerprint database is constructed from rolled fingerprints in NIST SD30 to study the potential values of Level 3 features. In all these databases, features in the latent prints are manually marked, but features in the rolled fingerprints are automatically extracted.

The first set of experiments is conducted to evaluate the proposed matching algorithms for Level 1 and Level 2 extended features and to study the utility of various Level 1 and Level 2 extended features.

1. The performance, in terms of the Cumulative Match Characteristic (CMC) curve, of the baseline minutiae matcher is first established by matching all latents against the 29K gallery.
2. Extended features are incrementally added (in the following order: ridge quality map, ridge flow map, ridge wavelength map, and finally ridge skeleton) and the test is repeated.
3. Performance improvement of each extended feature is analyzed for the whole database as well as separately for three quality levels of latent fingerprints (specified by fingerprint examiners).

The second set of experiments is conducted to evaluate the proposed encoding and matching algorithms for Level 3 extended features and to study the utility of various Level 3 extended features.

1. The joint distributions of the number of Level 3 features and the number of minutiae in latents of ELFT-EFS-PC database and the West Virginia University (WVU) latent database are analyzed.
2. The performance, in terms of the Cumulative Match Characteristic (CMC) curve, of the baseline minutiae matcher is first established by matching all latents in ELFT-EFS-PC against the 4180 rolled fingerprints.
3. Extraction accuracy of Level 3 features is evaluated on the simulated partial fingerprint database by comparing them to ground truth.
4. Matching experiments using Level 3 features are conducted on both ELFT-EFS-PC and the simulated partial fingerprint database.

Results

Experimental results are summarized as follows:

1. Extended features at Level 1 lead to the largest matching accuracy improvement than extended features at the other two levels. Among the Level 1 extended features, ridge quality map and ridge flow map are the most important.
2. Extended features at Level 2, namely ridge skeleton, improve the rank-1 recognition rate by 5%.
3. The “ugly” latents in NIST SD27 benefit the most from using extended features at Level 1 and Level 2. The rank-1 recognition rate for “ugly” latents using all extended features at Level 1 is already comparable to the rank-1 recognition rate for “good” latents using the current FBI standard features (minutiae + core/delta).
4. The contribution of Level 3 features on ELFT-EFS-PC is minor. But, on a simulated partial fingerprint database, they do show obvious improvement in the matching accuracy.

Conclusions

Based on the extensive experiments, the following conclusions are made:

1. All extended features at Level 1 and Level 2 should be incorporated into AFIS as soon as possible, since (i) these features are insensitive to image quality, (ii) do not rely on high resolution images, and (iii) do lead to significant improvement in the matching accuracy.
2. Algorithms and GUI tools should be developed to help fingerprint examiners manually mark extended features in latents, since it is a time-consuming task.
3. Utility of Level 3 features heavily rely on the image quality of both latents and enrolled fingerprints. Hence it is crucial to improve the quality of enrolled fingerprints (so that a sufficient number of Level 3 features can be extracted) before Level 3 features can play an important role in design and development of next generation AFIS.

1 Introduction

Over the past 40 years, Automated Fingerprint Identification Systems (AFIS) have played a major role in forensics and criminal investigations. However, these systems have not yet eliminated the need for manual examination of fingerprints by experienced human experts, particularly for latent prints. This is mainly because “AFIS technology, since its onset, has utilized a very limited amount of fingerprint detail. Latent print experts rely on far more information in effecting individualizations/exclusions than just ending ridges and bifurcations” [50].

Three types of fingerprint images are commonly used in law enforcement applications: ink, live-scan, and latent (see Fig. 1). Inking method is the earliest method for capturing and recording fingerprints. To capture the ink fingerprint images of a subject, the finger is coated with ink and pressed or rolled against a paper card. The print left on the card is then scanned to obtain a digital fingerprint image. Live-scan fingerprint images are obtained by using optical, capacitive or other types of sensors to directly image the finger. Latent fingerprint images are inadvertently left by persons on surfaces of objects and are lifted or photographed by using various techniques, e.g. chemical processing [3]. Compared to ink and live-scan fingerprint images, latent fingerprint images are characterized by small area, poor quality, and large non-linear distortion [56].

Fingerprint features are generally categorized into three levels [6]. Level 1 features are the macro details of the fingerprint such as ridge flow and pattern type. Level 2 features refer to the Galton characteristics or minutiae, such as ridge bifurcations and endings. Level 3 features include all dimensional attributes of the ridge such as ridge path deviation, width, shape, pores, edge contour, incipient ridges, breaks, creases, scars, and other permanent details. An example is given in Fig. 2 to show some of the three level features. The current FBI standard of fingerprint resolution for AFIS is 500 ppi (50 microns pitch), which is inadequate to automatically extract Level 3 features, such as pores (60 microns in radius). As a result, AFIS technology is primarily based on Level 1 and Level 2 features.

With the advances in fingerprint sensing technology, many high resolution (i.e., 1000 ppi) sensors are now available that makes the extraction of extended features more feasible. The extended features that are well understood and often used by latent experts include minutiae shape, dots and pores [6]. They have been relied upon in latent matching when they are present and discernible to reach accurate conclusions. Unfortunately, there have very few systematic studies on automatic fingerprint identification using these extended features.

At the 2005 ANSI/NIST fingerprint standard update workshop, SWGFAST [29] proposed a minimum scanning resolution of 1000 ppi for latent, ten-print, and palm print images and the inclusion of extended features in the FBI standard. This proposal and its strong endorsement by the forensic community calls for systematic research in the use of extended feature set in automatic fingerprint identification. We propose to develop an automated system that would robustly extract and match some of the prominent extended features. This would enable us to quantify the discriminating power of the extended features by evaluating their statistical significance.

1.1 Statement of the problem

We propose to develop an automated system that would robustly extract and match some of the prominent extended features for both latent-to-full and full-to-full fingerprint matching. Using this system, we will quantify the discriminating power of the extended features by evaluating their statistical significance.

1.2 Literature review

It is a common practice to improve the capability of a minutiae matcher by using Level 1 and Level 2 features. These include singular points and pattern type [24], ridge flow map (or orientation field) [24, 31–35], ridge wavelength map (or frequency map) [31, 36], skeleton [24, 25, 31, 37, 38], and crease [39]. However, these studies primarily address full fingerprint matching and, to our knowledge, there is no published algorithm on using extended features for latent matching. NIST has conducted an evaluation of latent fingerprint technology using extended feature set (ELFT-EFS) [55]. Extended feature set (EFS) was manually marked in the latent fingerprints, and their contribution to latent search was assessed by using matchers from the participants. The NIST evaluation showed that EFS did improve the latent search accuracy. However, because the ELFT-EFS test did not evaluate each extended feature separately, the contribution of individual features is not known from this evaluation.

There is a growing interest in using Level 3 features, such as pores [35, 40, 41, 53, 54], ridge contours [35, 41], dots and incipient ridges [42], for fingerprint matching. It is claimed that Level 3 features contain discriminating information and can improve the performance of matching rolled/plain to rolled/plain fingerprints. However, these conclusions are not easy to extend to latent fingerprint matching, because

- Latent fingerprints are generally of poor quality.
- Since latent images need to be matched against rolled/plain fingerprints, the repeatability or consistency of Level 3 features is critical. Repeatability of Level 3 features in images acquired with different techniques is much lower than that in [35, 41, 42] where the same sensor was used to capture both template and query fingerprints. The survey performed by Anthonioz et al. [43] among seventy latent examiners shows that there is no clear consensus on the repeatability of Level 3 features.
- Level 3 features such as pores and ridge edges are correlated with skeleton and ridge flow map. Therefore, it is not evident if the performance improvement reported in [35, 41, 42] is due to Level 3 features or Level 2 features that have been implicitly used.

1.3 Rationale for the research

The rationale for the proposed research and methods are as follows:

1. Latent examiners routinely use a rich set of fingerprint features in manual latent fingerprint matching. However, current AFIS rely mainly on minutiae and

core/delta. Thus it is reasonable to hypothesize that the large performance gap between AFIS and latent examiners is due to features utilized in matching.

2. Previous studies have shown that some of the extended features are able to improve the matching accuracy of minutiae matcher in full fingerprint matching. Thus it is reasonable to hypothesize that with proper algorithm design, extended features will also help improve latent matching accuracy.
3. Extended features are not independent from each other. In fact, higher level features can be determined from lower level features. For example, orientation field can be determined from ridge skeletons and ridge skeletons can be determined from ridge contours. Thus it is reasonable to incrementally incorporate each type of extended feature into AFIS and to study the additive value of each extended feature.
4. Higher level extended features are more insensitive to image quality and resolution than lower level extended features. Thus it is reasonable to use higher level features ahead of lower level features.

2 Methods

2.1 Features

The proposed system utilizes the following features: Level 1 features (reference points (core/delta), ridge quality map, ridge flow map, and ridge wavelength map), Level 2 features (minutiae and skeleton), and Level 3 features (dots, incipient ridges, pores, and ridge edge protrusions). Since all these features are defined in the CDEFFS document [30], we use terms that are consistent with these definitions.

- Reference points have location, direction and type (see [30]).
- Ridge flow map, ridge wavelength map, and ridge quality map are obtained by dividing the image into non-overlapping blocks of size 16×16 and assigning a single orientation, wavelength, and quality value to each block. We define three quality levels for a block: level 0 (background), level 1 (clear ridge flow and unreliable minutiae), and level 2 (clear minutiae).
- A minutia consists of five attributes, namely x and y coordinates, minutiae direction, type, and quality. The quality of minutia is defined to have two levels: 0 (unreliable) and 1 (reliable).
- A skeleton is one-pixel-wide ridge, which is traced in the thinned image and represented as a list of points.
- Level 3 features (dots, incipient ridges, pores, and ridge edge protrusions) are represented as a set of points.

2.2 Feature Extraction

2.2.1 Level 1 and Level 2

While the features at Level 1 and Level 2 have been manually marked for 258 latents in SD27 (see Fig. 3 for a latent in SD27 with manually marked features and Fig. 4 for three latents in ELFT-EFS-PC with manually marked features), the rolled fingerprints are automatically processed to obtain all the features. The feature extraction algorithm consists of two modules: preprocessing and postprocessing. In this work, Neurotechnology Verifinger 4.2 SDK [44] was used as a preprocessor. Due to the presence of background noise (characters and strokes on many fingerprints scanned from paper, such as the rolled prints in NIST SD4, SD14, and SD27), Verifinger produces many false minutiae. Therefore, a ridge validation algorithm is used to classify each ridge or ridge segment as true or false, and a minutiae validation algorithm is used to classify each minutia as false, reliable, or unreliable.

A minutia is deemed as false if it is close to the background region. A minutia is deemed as unreliable if it forms an opposite pair with another minutia. An opposite pair is a pair of minutiae which are spatially close but have opposite directions. Remaining minutiae are deemed as reliable.

Ridge validation consists of the following two steps: 1) each ridge is broken into multiple segments at unreliable positions; 2) the segments are grouped and those belonging to large groups are deemed as reliable.

A block containing true ridges is labelled as foreground; otherwise as background. For each sample point (at intervals of 6 pixels) on each ridge, the tangent direction is computed and the distance from the adjacent ridges on both sides is computed. The ridge flow and ridge wavelength in each foreground block are estimated by majority voting. Singular points are extracted by the Poincaré index method [45]. An example is given in Fig. 5 to show the results of these processing steps.

2.2.2 Level 3

Level 3 fingerprint features considered here include pores, dots, incipient ridges and ridge edge protrusions. Pores appear as bright blobs on ridges and the other three features appear between ridges (see Fig. 2(c)). We first discuss pore extraction, followed by extraction algorithm for the other three features.

Pores Pores, also known as sweat pores, are located on finger ridges. They are formed in the sixth month of gestation due to the sweat-gland ducts reaching the surface of the epidermis. Once the pores are formed, they are fixed on the ridges; typically, there are between 9 and 18 pores along a centimeter of a ridge [45]. A pore can be visualized as open on one print, but as closed on another print of the same finger depending on the finger pressure and whether it is exuding perspiration. As shown in Fig. 2(c), a closed pore appears as an isolated dot on the ridge, while an open pore is connected to one or both of the two valleys surrounding it. As a result, the shape and size of pores can vary from one impression to another, and therefore only the pore position is used in matching.

The basic idea of the proposed pore extraction method is to model the spatial appearance of pores in fingerprint images and detect them via filtering the images with suitable matched filters. In [53], it was shown that along the ridge tangential orientation, the intensity profile across the pore has a Gaussian shape irrespective of whether it is open or closed (see Figs. 6(a) and 6(b)). Based on this observation, an anisotropic pore model was established and an adaptive pore extraction algorithm was proposed [53]. One drawback of the method is that it sets the scale parameter in the pore model as a constant multiple of local ridge period. Such a constant ratio parameter is however difficult to specify for all fingerprints, especially when large distortion exists across the fingerprint images such as latents. As an improvement to the method in [53], we propose a new pore matched filter based on the automatic scale selection technique [58]. Let X and Y be the horizontal (column) and vertical (row) axes of the global image coordinate system (x and y are the corresponding coordinates), and V and U denote the local ridge tangential and normal orientation, respectively. Let θ be the local ridge (tangential) orientation with respect to the X axis. The proposed pore matched filter is defined as

$$P_{POR}^{\gamma}(v, u; t_V, t_U, \theta) = -t_V^{3/4} g_{VV}(v; t_V) g(u; t_U), \quad (1)$$

where

- $(v, u) = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$,
- $g(u; t_U) = 1/(\sqrt{2\pi t_U})e^{-u^2/(2t_U)}$ is Gaussian along the ridge normal orientation and constant along the ridge tangential orientation,
- $g_{VV}(v; t_V) = (v^2 + t_V)/(\sqrt{2\pi t_V^3})e^{-v^2/(2t_V)}$ is Laplacian along the ridge tangential orientation and constant along the ridge normal orientation,
- and t_V and t_U are, respectively, the variances along the ridge tangential orientation and the ridge normal orientation.

Note that unlike [53], we describe the intensity appearance of a pore along the ridge tangential orientation by using a Laplacian kernel because it is more robust to noise. The Gaussian kernel along the other orientation is used merely for smoothing the noise along the ridge normal orientation.

In order to apply the above pore matched filters, we first divide the fingerprint image into blocks and estimate the local ridge orientation θ . We then instantiate a pore matched filter for each block that has dominant ridge orientation (called a well-defined block) according to eq. 1. The parameter t_U in the pore matched filter is set to a constant because it is used merely for noise smoothing. As for the parameter t_V , a multiscale setting is adopted so that pores of varying sizes can be detected. More specifically, a set of pore matched filters are constructed for each well-defined block and convolved with the block. The maximum response among the sets of pore matched filters is binarized, resulting in the pore map; candidate pore pixels have value 1 and the non-pore pixels have value 0.

The pore map contains some falsely detected pores. To remove them, the following post-processing steps are conducted: (i) candidate pores which are not on ridges are removed; (ii) connected components on the pore map whose area is either too small or too large are discarded; (iii) connected components on the pore map are removed if the intensity of their pixels is too low. After these post-processing operations, many spurious pores are excluded, and each connected component in the post-processed pore map corresponds to a pore. The centroids of these detected pores are recorded. Fig. 7(a) shows a portion of a rolled ink fingerprint image, and Fig. 7(b) shows the pores detected in it by the proposed method. The pore extraction results of the latent fingerprint image in Fig. 1(c) are shown in Fig. 7(c). Due to the poor quality of latent fingerprint images, more false pores are detected in the latent than in the rolled image. Yet, most of the true pores are correctly extracted in latents. Therefore, the automatic pore extraction algorithm proposed here may provide useful information to latent examiners and cut down their workload in manually marking pores.

Dots, Incipient Ridges, and Ridge Edge Protrusions (DIP) While typical ridges stretch over a large area of fingerprints and their width varies from $100\mu m$ to $300\mu m$ [45], there are occasionally some ridges which are quite short or substantially thin (see Fig. 2(c)). These are actually dots and incipient ridges, two additional types of level 3 features in fingerprints [6]. Unlike pores, which are present in almost every finger, dots and incipient ridges can be found in fingerprints of only about 45% of the population

and 13.5% of the fingers [35]. They reside in fingerprint valleys and, if observed in small areas, have been claimed to be distinctive for differentiating fingerprints.

Along a ridge, variations in ridge width can be observed. This gives rise to ridge edge features, including protrusions, indentations, and discontinuities (see Fig. 2(c)), among which protrusions are the most notable ridge edge feature. A ridge edge protrusion refers to an abrupt increase in ridge width that is not long enough to be called a bifurcation. Although ridge edge protrusions, dots, and incipient ridges are defined as different features, their appearance in fingerprint images can be greatly affected by finger pressure and imaging conditions [35], and consequently, they can be confused with each other in different impressions of the same finger. As shown in Fig. 8, a dot in one impression can appear as a ridge edge protrusion in the other impression, and an incipient ridge can appear as a series of separated dots. Therefore, we do not distinguish among these three types of level 3 features for extraction and matching, but collectively label them as a single feature type (denoted as DIP).

In order to extract the DIP features, a procedure similar to that for pore extraction is applied, but with matched filters designed for DIP. Fig. 6 shows that the intensity profiles along the ridge normal orientation are shaped as a full or half negative Gaussian. Therefore, we define the following matched filters for the DIP features,

$$P_{DIP}^\gamma(v, u; t_V, t_U, \theta^\perp) = t_U^{3/4} g(v; t_V) g_{UU}(u; t_U), \quad (2)$$

where θ^\perp is the local ridge normal orientation at the DIP feature (perpendicular to θ). The DIP matched filters are applied for each block that has dominant ridge orientation with t_V set to a constant, and t_U to a multiscale setting. The resulting DIP map then goes through the following post-processing steps. First, the candidate DIP pixels which are not in the valleys are removed, because the DIP features should reside in valleys only. Second, the connected components in the DIP map of either too small or too large area are discarded. Third, those components in the DIP map whose intensity is too high are removed. After these post-processing operations, many spurious DIP are excluded. The remaining connected components in the DIP map are then thinned to single-pixel curves. If a curve bends too much, i.e. the maximum distance from its pixels to the chord (straight line connecting its two ends) is too large, it is divided into two curves at the pixel which is farthest from the chord. Finally, the centroids of these curves are recorded to represent the extracted DIP features in the fingerprint. The DIP extraction results of the proposed method for an example rolled ink fingerprint fragment are shown in Fig. 9(a) and Fig. 9(b) (note that if the length of a detected DIP is larger than the local ridge period, it is displayed as an incipient ridge. See the blue lines in Fig. 9(b)). The DIP extraction results in the latent fingerprint image in Fig. 4(b) are shown in Fig. 9(c). Despite the poor quality of the latent, most of the true DIP features have been correctly extracted by the proposed method, but there are many false detections (most of which are due to ridge edge features).

2.3 Matching

2.3.1 Level 1 and Level 2

To understand the relative importance of various extended features, they are incrementally used for matching and the performance gains are examined. Starting with the baseline matching algorithm, which uses only minutiae, additional features (reference points, overall image characteristics and skeleton) are incrementally used. This order is roughly based on the required time in manual feature marking. The baseline matching algorithm is not only a matcher for minutiae-only templates, but also serves as a framework to match and fuse various extended features. We provide a detailed description of the baseline matcher and then describe the approaches to using various extended features.

Baseline Matching Algorithm The baseline matching algorithm takes only minutiae as input and consists of the following steps:

1. Local minutiae matching: Similarity between each minutia of latent fingerprint and each minutia of rolled fingerprint is computed.
2. Global minutiae matching: Using each of the five most similar minutia pairs found in Step 1 as an initial minutia pair, a greedy matching algorithm is used to find a set of matching minutia pairs.
3. Matching score computation: A matching score is computed for each set of matching minutia pairs and the maximum score is used as the matching score between the latent and rolled prints.

Local minutiae matching In local minutiae matching, the similarity between each minutia of latent fingerprint and each minutia of rolled fingerprint is computed. Since the basic properties of a minutia, like location, direction and type, are not very distinctive features, additional features, which are collectively referred to as a descriptor, are computed for each minutia. Figure 10 show five types of features that have been used as minutiae descriptors in the literature [31, 46, 47]. In the baseline algorithm, a neighboring minutiae-based descriptor is used, since only minutiae information is available.

The neighborhood of a minutia is defined to be a circular region with an 80-pixel radius. All minutiae lying in this neighborhood are called the neighboring minutiae. Let p and q be the two minutiae whose similarity needs to be computed. For each neighboring minutia p_i of p , we examine if there is a neighboring minutia of q whose location and direction are similar to those of p_i . If such a minutia exists, p_i is deemed as a matching minutia; otherwise p_i is checked against the following two criteria: 1) the minutia is unreliable, 2) it does not fall in the foreground region (the convex hull of minutiae) when mapped to the other fingerprint based on the alignment parameters between p and q . If p_i satisfies either one of these two criteria, it will not be penalized; otherwise, it will be penalized. The above process is also applied to the neighboring

minutiae of q . The similarity between two neighboring minutiae-based descriptors is computed as:

$$s_m = \frac{m_p + 1}{m_p + u_p + 3} \cdot \frac{m_q + 1}{m_q + u_q + 3}, \quad (3)$$

where m_p and m_q denote the number of neighboring minutiae of p and q that match, u_p and u_q denote the number of penalized unmatched neighboring minutiae of p and q , the value 1 in the numerator is used to deal with the case where no neighboring minutiae are available, and the value 3 in the denominator is empirically chosen to favor the case where there are more neighboring minutiae that match. Note that m_p may be different from m_q since we do not establish a one-to-one correspondence between minutiae.

Global minutiae matching Given the similarity among all minutia pairs, the one-to-one correspondence between minutiae is established in the global minutiae matching stage. Greedy strategy is used to find matching minutia pairs in the decreasing order of similarity. In order to give priority to those minutia pairs that are not only similar to each other but also dissimilar with other minutiae, a normalized similarity measure s_n is defined based on similarity s as:

$$s_n(i, j) = \frac{(N_m^L + N_m^R - 1) \cdot s(i, j)}{\sum_{k=1}^{N_m^R} s(i, k) + \sum_{k=1}^{N_m^L} s(k, j) - s(i, j)} \quad (4)$$

where $s(i, j)$ denotes the similarity between minutia i and minutia j , and N_m^L and N_m^R denote the number of minutiae in the latent and rolled, respectively. All minutia pairs are sorted in the decreasing order of normalized similarity, and each of the top 5 minutia pairs is used to align the two sets of minutiae. Minutiae are examined according to the decreasing order of their similarity; minutiae that are close in both location and direction, and have not been matched to other minutiae are deemed as matching minutiae. After all the minutia pairs have been examined, a set of matching minutiae is returned.

Matching score computation The matching score between two fingerprints is a measure that reflects the likelihood that they are from the same finger. A desired property of matching scores is that the score for fingerprints that have many matched minutiae and few unmatched minutiae in the common area should be very high, the score between fingerprints that appear obviously different should be very low, and the score between fingerprints that share small common area or whose common areas are of poor quality should be in the middle.

Computing matching scores or simply scoring is typically approached in two ways: formula-based and classifier-based. In formula-based approach [32,48], an empirically chosen formula is used to compute matching scores. In classifier-based approach [31, 49], scoring is regarded as a two-category classification problem. A pair of fingerprints is classified by a traditional classifier, such as Artificial Neural Network (ANN) or Support Vector Machine (SVM), as a genuine match or an impostor match based on a feature vector extracted from matching these two fingerprints. A major problem with classifier-based approach is that the training targets of all genuine matches are the

same, say 1, no matter how many minutiae are matched. Similarly, the training targets of all impostor matches are also the same, say 0, no matter how many minutiae in the common area are unmatched. This violates the desired property for matching scores. It is also not practical to use a classifier-based scoring approach in latent matching since obtaining manually marked latents is very difficult. For the above two reasons, we adopted a formula-based scoring approach in this paper.

Our scoring method is described as follows. When fewer than three minutiae are matched, the matching score S_M is set to 0; otherwise S_M is the product of a quantitative score S_{mn} and a qualitative score S_{mq} :

$$S_M = S_{mn} \cdot S_{mq}. \quad (5)$$

The quantitative score S_{mn} is computed as $M_m/(M_m + 8)$, where M_m denotes the number of matched minutiae and the value 8 is an estimate of the average number of matching minutiae for low quality latents. The qualitative score is computed as:

$$S_{mq} = S_d \cdot \frac{M_m}{M_m + U_m^L} \cdot \frac{M_m}{M_m + U_m^R} \quad (6)$$

where S_d is the average similarity of descriptors for all matching minutiae, and U_m^L and U_m^R denote the number of penalized unmatched minutiae in latent and rolled prints, respectively.

Additional Features

Reference Points Using the spatial transformation between the two images, which is estimated based on the matched minutiae, the reference points (if present) of the latent are transformed into the coordinate system of the rolled print. The distance and angle difference between reference points of the same type are computed and compared to predefined thresholds (30 for distance and $\pi/4$ for angle). If both values are less than their respective thresholds, the reference points are deemed as matched. The accumulated matching score is computed as:

$$S_R = S_M + C_r \cdot S_r, \quad (7)$$

where S_r denotes the matching score based on reference points, namely, the number of matched reference points and C_r is a constant value empirically set as 0.03.

Ridge Quality Map Ridge quality map is used in local minutiae matching and matching score computation stages to ignore the unmatched minutiae of one fingerprint that are mapped to the low quality region (quality level 0 or 1) of the other fingerprint. As will be shown in the experimental results section, this modification significantly improves the matching accuracy. The accumulated matching score S_Q is computed by Eqs. (5) and (7).

Ridge Flow Map Ridge flow map is used in two stages: local minutiae matching and matching score computation.

For every minutia, a local coordinate system is defined with the minutia as the origin and its direction as the positive x axis. A set of fixed sample points is defined [32] and the local ridge flow at these sample points form the flow descriptor. The similarity of two descriptors is computed as the mean value of the similarity of all valid sample points (a sample point falling in background region is deemed as invalid). The similarity between the flow at two sample points is computed as $s_f = \exp(-|\Delta\theta|/(\pi/16))$, where $\Delta\theta$ denotes the angle between the two flows. If the number of common valid sample points is less than 25% of the total number of sample points, the similarity of two minutiae is set to 0. The similarity between two minutiae is computed as the weighted sum of the neighboring minutiae-based similarity and flow-based similarity:

$$s = w_m \cdot s_m + (1 - w_m) \cdot s_f, \quad (8)$$

where the weight w_m for the neighboring minutiae-based descriptor is empirically set as 0.6, due to its superior performance compared to flow-based descriptor.

The ridge flow maps of latent and rolled prints are aligned using the spatial transformation estimated based on the matched minutia pairs. The matching score S_f based on ridge flow is the product of a quantitative score S_{fn} and a qualitative score S_{fq} . The quantitative score S_{fn} is computed as $N_b/(N_b + 100)$, where N_b is the number of blocks where the difference in flow is less than $\pi/8$ and the value 100 is an estimate of the average number of 16×16 blocks in low quality latents. The qualitative score S_{fq} is computed as $(1 - 2 \cdot D_f/\pi)$, where D_f is the mean of the difference of flow values in all overlapping blocks.

The accumulated matching score S_F between two fingerprints is computed as:

$$S_F = S_M + C_r \cdot S_r + C_f \cdot S_f, \quad (9)$$

where the constant C_f is empirically set as 0.2.

Ridge Wavelength Map Ridge wavelength map is used in two stages: local minutiae matching and matching score computation.

Wavelength-based minutia descriptor is composed of the ridge wavelength at the same set of sample points as ridge flow-based descriptor. The similarity between the wavelength of two sample points is computed as $s_w = \exp(-|\Delta w|/3)$, where Δw denotes the wavelength difference at the two sample points. The similarity between two minutiae is computed as the weighted sum of the neighboring minutiae-based similarity, flow-based similarity, and wavelength-based similarity:

$$s = w_m \cdot s_m + w_f \cdot s_f + (1 - w_m - w_f) \cdot s_w, \quad (10)$$

where the weights w_m and w_f for the neighboring minutiae-based and flow-based descriptors are empirically set as 0.6 and 0.2, respectively.

The ridge wavelength maps of latent and rolled prints are aligned using the spatial transformation estimated based on the matched minutia pairs. The matching score S_w based on wavelength is the product of a quantitative score S_{wn} and a qualitative score

S_{wq} . The quantitative score S_{wn} is computed as $N_b/(N_b + 100)$, where N_b is the number of blocks where the difference in wavelength is less than 3 pixels and the value 100 is an estimate of the average number of 16×16 blocks in low quality latents. The qualitative score S_{wq} is computed as the average similarity of wavelength in all overlapping blocks.

The accumulated matching score S_W between two fingerprints is computed as:

$$S_W = S_M + C_r \cdot S_r + C_f \cdot S_f + C_w \cdot S_w, \quad (11)$$

where the constant C_w is empirically set as 0.2.

Skeleton Minutiae can be deemed as an abstract representation of ridge skeleton. However, the skeleton image contains more information than minutiae. The skeleton matching algorithm is similar in spirit to the “ridges in sequence” idea recommended by SWGFAST [50]. Hara and Toyama [25] describe an interesting skeleton matching algorithm, which consists of the following steps: 1) select the most reliable minutiae pair from all the matched minutiae pairs as the base paired minutiae (BPM); 2) remove minutiae pairs that are inconsistent with BPM; 3) modify the two skeleton images to make them more similar; and 4) incrementally match skeleton points guided by the matched minutiae or skeleton points. While their approach needs at least three pairs of correctly matched minutiae to guide the skeleton matching process, our approach needs only a pair of correctly matched minutiae as starting point, which is useful in matching latent prints with very small area.

The proposed skeleton matching algorithm is an improved version of the algorithm in [37]. Its main steps are briefly described as follows.

1. Similarity between minutiae of two fingerprints is computed.
2. For each of the five most similar minutiae pairs, steps 3 to 5 are performed to establish correspondence between skeletons of two fingerprints and compute a matching score. The maximum value of these scores is used as the skeleton matching score.
3. The associated skeletons of the initial minutiae pair are assumed to be matched and used as a reference.
4. Skeletons adjacent to reference skeleton pair are aligned according to reference skeleton pair and then matched. Newly matched skeletons are used a new reference. This step is iteratively performed until no more skeletons can be matched.
5. A skeleton matching score is computed.

The differences from the algorithm in [37] lie in the computation of minutiae similarity and skeleton matching score. The similarity between minutiae is now computed using the composite minutiae descriptor based on neighboring minutiae, ridge flow, and wavelength features. The similarity computation is described in previous subsections. This composite descriptor is more robust to noise than the ridge structure based

descriptor used in [37]. The skeleton matching score is computed as the product of a quantitative score S_{sn} and a qualitative score S_{sq} :

$$S_s = S_{sn} \cdot S_{sq}. \quad (12)$$

The quantitative score S_{sn} is computed as:

$$S_{sn} = \frac{M_s}{M_s + 400}, \quad (13)$$

where M_s denotes the number of matched skeleton points and the value 400 is an estimate of the average number of skeleton sample points in low quality latents. The qualitative score is computed as:

$$S_{sq} = \frac{M_s}{M_s + U_s^L} \cdot \frac{M_s}{M + U_s^R}, \quad (14)$$

where U_s^L and U_s^R denote the number of unmatched skeleton sample points of latent and rolled prints in their common region, respectively.

The accumulated matching score S_S is obtained by combining S_s and S_W computed in Eq. (11):

$$S_S = S_W + C_s \cdot S_s, \quad (15)$$

where the constant C_s is empirically set as 1. For efficiency, skeleton matching is performed only for the top 100 candidates found by the minutiae matcher.

2.3.2 Level 3

Algorithm Overview Given a latent fingerprint, it is first matched with the exemplars in the background database by using a minutiae matcher (VeriFinger [44] was used in our experiments). The rank 1 minutiae match score is then examined to determine if it is necessary to invoke the level 3 feature matching module. Specifically, if the rank 1 minutiae match score is already above a prespecified threshold, the matcher will directly output the identification results (e.g. the list of top N candidates); otherwise, the level 3 features will be further compared, and the final identification results will be based on the fusion of the matching results of minutiae and level 3 features. Fig. 11 illustrates this algorithm.

We first match the minutiae because i) minutiae have already been shown to be stable and discriminative, and ii) minutiae form the basis of all the available AFIS. In the rest of this section, we will describe the three modules of the proposed level 3 feature matching method, i.e. ridge correspondence establishment, pore matching, and DIP matching.

Ridge Correspondence Establishment The level 3 feature matching method proposed here differs from existing methods in that it matches level 3 features along the ridges and incorporates the topological relationship between level 3 features, minutiae, and ridges. Given a query latent F_q and an exemplar full fingerprint F_t , the proposed matcher first establishes the ridge correspondences between the two fingerprints. To

facilitate ridge matching process, the ridges in each fingerprint are traced and sampled at a constant interval (in our experiments, the interval is set to 10 pixels, which is the allowed tolerance of location displacement between two matched level 3 features). During the ridge tracing and sampling, the associated minutiae (if any) are recorded for each of the ridges, and the neighboring ridges and the neighboring sampling points on the left-hand and right-hand sides at each sampling point of the ridge are also recorded.

Algorithm 1 Ridge Correspondence Establishment

Require: MM : Mated minutiae pairs between F_q and F_t ; R_q, R_t : Ridges in F_q and F_t

Ensure: s_r : Similarity between ridges in F_q and F_t ; MR : Mated ridge pairs between F_q and F_t , and corresponding sampling points on them

```

1:  $s_r \leftarrow 0, MR \leftarrow \text{NULL}$ 
2: for each pair of mated minutiae,  $\{M_1, M_2\}$ , in  $MM$  do
3:   for each pair of ridges,  $\{R_1, R_2\}$ , associated with  $M_1$  and  $M_2$  do
4:     Generate candidate aligned ridge pairs  $CR = \{RSP_1, 0, RSP_2, 0\}$  from
        $\{R_1, R_2\}$ 
5:      $MRSP \leftarrow \text{IntraRidgeMatch}(CR)$ 
6:     if  $|MRSP| > 4$  then
7:        $(mr, s) \leftarrow$ 
          $\text{InterRidgePropagation}(R_q, R_t, MRSP)$ 
8:       if  $s_r < s$  then
9:          $s_r \leftarrow s, MR \leftarrow mr$ 
10:      end if
11:    end if
12:  end for
13: end for

```

Algorithm 1 describes the ridge correspondence establishment. Suppose a set of mated minutiae are found between F_q and F_t by the minutiae matcher. Fig. 12 shows an example latent and its mated rolled fingerprint in ELFT-EFS-PC. There are three pairs of mated minutiae in them. From each pair of mated minutiae, several pairs of candidate aligned ridges can be obtained from the ridges associated with the two minutiae in the pair. A candidate aligned ridge pair is defined as $CR = \{RSP_1, PR_1; RSP_2, PR_2\}$, where RSP_1 and RSP_2 are the two candidate ridges (or ridge segments) represented by their sampling points and the first sampling points on them are assumed to be matched (here, the sampling points corresponding to the mated minutiae) and PR_1 and PR_2 are the parent ridges from which this candidate aligned ridge pair is generated. For the candidate aligned ridge pairs generated from mated minutiae, the parent ridges are set to 0, which means they have no parent ridges. The parent ridges will be used in Inter-Ridge propagation to ensure that only the sampling points neighboring to the parent ridges are matched during the propagation. For example, for the mated minutiae pair $\{F_q.M_1, F_t.M_1\}$ in Fig. 12(a), each of the three ridges associated with $F_q.M_1$ is paired with the corresponding ridge associated with $F_t.M_1$, resulting in three pairs of candidate aligned ridges. Note that if a ridge ending is mated with a ridge bifurcation, we will have two pairs of aligned ridges, while if two ridge

endings are mated, we will get only one pair of aligned ridges.

From each of these aligned ridge pairs, the two ridges in the pair are first compared by the Intra-Ridge matching procedure. If the two ridges can be matched (i.e. more than four sampling points are matched between them), the Inter-Ridge propagation procedure is invoked to match the remaining ridges in the two fingerprints based on the mated sampling points on the two ridges. After all the aligned ridge pairs have been considered, the ridge correspondences obtained from the one which gives the highest similarity between the ridges in the two fingerprints are taken as the final result. Next, we introduce the two main procedures, Intra-Ridge matching and Inter-Ridge propagation, involved in ridge correspondence establishment.

Given a candidate aligned ridge pair $CR = \{RSP_1, PR_1, RSP_2, PR_2\}$, Intra-Ridge matching is used to find the corresponding sampling points on the two aligned ridges (or ridge segments). This is essentially a string matching problem given that the first sampling points in RSP_1 and RSP_2 are matched. We employ the dynamic programming technique [37] to find the longest sequence of mated sampling points on the two ridges, $MRSP = \{RSP_1^m, RSP_2^m\}$, such that (i) the indices of mated sampling points monotonously increase in both RSP_1^m and RSP_2^m , (ii) changes between indices of adjacent mated sampling points are less than 3 (i.e. no more than 3 sampling points can be skipped during matching), and (iii) if $PR_i \neq 0$, all the mated sampling points in RSP_i^m should have PR_i as their neighboring ridges ($i = 1, 2$). In our implementation, two mated sampling points should satisfy i) the absolute difference between the distances from them to the first sampling points is below a given threshold (i.e. 10 pixels), and ii) the absolute difference between the ridge curvatures at them is also below a given threshold (i.e. 15 degrees). We measure the distance between two sampling points on a ridge by using the absolute difference between their indices, which is similar to geodesic distance. The ridge curvature at a sampling point is measured by the change in local ridge orientation at the point with respect to the ridge orientation at the first sampling point.

Given the mated points between two ridges, the similarity between the ridges is computed as follows. Because short ridges are mostly unreliable, if there are fewer than 4 mated points between two ridges, we discard them. Otherwise, we further examine the neighboring ridge structures of the mated sampling points. Let $n_{msp} = |MRSP|$ be the number of mated sampling points found on RSP_1 and RSP_2 . For all the mated sampling points in RSP_i^m ($i = 1, 2$), we examine on left-hand and right-hand sides, respectively, if the neighboring ridges of each two adjacent sampling points are different or not, resulting in two feature vectors, $NR_i^l \in \{0, 1\}^{n_{msp}-1}$ and $NR_i^r \in \{0, 1\}^{n_{msp}-1}$, in which ‘0’ means same ridge and ‘1’ means different ridges. NR_1^l and NR_1^r are then compared with NR_2^l and NR_2^r , respectively, and the number of the same entries between them is counted, denoted as n_{NR}^l and n_{NR}^r for the left-hand and right-hand sides, respectively. The similarity between the neighboring ridge structures of the mated sampling points on the two ridges is then calculated by

$$s_N = 0.5 \times \frac{n_{NR}^l}{n_{msp} - 1} + 0.5 \times \frac{n_{NR}^r}{n_{msp} - 1}. \quad (16)$$

If the mated sampling points on the two ridges have very low similarity between their neighboring ridge structures, they are also discarded. Fig. 12(b) shows the Intra-Ridge

matching results for the ridges $F_q.R_1$ and $F_t.R_1$ associated with the mated minutiae $F_q.M_1$ and $F_t.M_1$ in Fig. 12(a).

Given a set of mated sampling points on the two ridges found by the Intra-Ridge matching procedure, the Inter-Ridge propagation procedure, as sketched in Algorithm 2, matches all the remaining ridges. A queue (denoted as Q) is constructed to store the candidate aligned ridge pairs. The queue is initialized by generating candidate aligned ridge pairs from each pair of mated sampling points. The candidate aligned ridge pairs are the neighboring ridges on the corresponding sides of the mated sampling points.

After the initialization of Q , we start the main loop of the Inter-Ridge propagation procedure to compare the ridges in each of the candidate aligned ridge pairs in Q until Q is empty. The first candidate in Q is popped out and matched by the Intra-Ridge matching procedure. If more than four mated sampling points are established, new candidate aligned ridge pairs are generated and pushed into Q . When Q is empty, the Inter-Ridge propagation procedure terminates with a set of mated ridge pairs as well as the corresponding mated sampling points. Figs. 12(c) and 12(d) shows the mated ridge pairs found between the two example fingerprints as the procedure Inter-Ridge propagation proceeds from the mated ridge pairs shown in Fig. 12(b).

Let \bar{s}_N , \bar{d}_{loc} , and \bar{d}_{ori} be the average similarity between neighboring ridge structures of all the mated ridge pairs, the average location displacement and the average orientation difference between all the mated sampling points, respectively. Let n_{MRSP} and n_{RSP} denote the total number of mated sampling points on the ridges in the two fingerprints and the total number of sampling points on ridges in the query fingerprint, respectively. Then the similarity between the ridges in the two fingerprints is defined as

$$s_r = 0.3 \times \bar{s}_N + 0.2 \times \frac{10 - \bar{d}_{loc}}{10} + 0.2 \times \frac{15 - \bar{d}_{ori}}{15} + 0.3 \times \frac{n_{MRSP}}{n_{RSP}}. \quad (17)$$

Pore Matching Once the correspondences between ridges are obtained, the level 3 features can be matched along the mated ridges. To implement this, we need to first associate the level 3 features with the ridges. In this section we discuss the matching of pores; the matching of DIP features will be discussed in the next section. Recall that pores in a fingerprint are all located on ridges. Hence, for each pore, we find the closest ridge to it and its projection point on this ridge. The pores on the same ridge are then grouped together and ordered along the ridge tracing direction.

Given a pair of mated ridges, the correspondences between pores on these two ridges are found using the following method. For each pore on a ridge, POR_1 , we first find its closest sampling point SP_1 on the ridge (denote the ridge as R_1). If SP_1 does not have a mated sampling point, then POR_1 does not have any mated pores; otherwise, we find the nearest pore, POR_2 , to the mated sampling point, SP_2 , of SP_1 . The location displacement d_i between POR_i and SP_i ($i = 1, 2$) is calculated as the difference between the sampling indices of the projection point of POR_i and SP_i . The location displacement between the two pores, POR_1 and POR_2 , is then defined

Algorithm 2 Inter-Ridge Propagation

Require: $MRSP$: Mated sampling points on two ridges in F_q and F_t ; R_q, R_t : Ridges in F_q and F_t

Ensure: s_r : Similarity between ridges in F_q and F_t ; MR : Mated ridge pairs between F_q and F_t , and corresponding sampling points on them

- 1: $MR \leftarrow MRSP$
 - 2: Initialize the queue of candidate aligned ridge pairs, Q , based on $MRSP$
 - 3: **while** Q is not empty **do**
 - 4: Retrieve the first candidate aligned ridge pair in Q : CR
 - 5: $mrsp \leftarrow IntraRidgeMatch(CR)$
 - 6: **if** $|mrsp| > 4$ **then**
 - 7: Append $mrsp$ to MR
 - 8: Generate new candidate aligned ridge pairs based on $mrsp$
 - 9: Push the new candidate aligned ridge pairs into Q
 - 10: **end if**
 - 11: **end while**
 - 12: Calculate the similarity between the ridges in F_q and F_t : s_r
-

as $d_{loc} = |d_1 - d_2|$. If d_{loc} is smaller than a given threshold (i.e. 10 pixels for 1000ppi fingerprint images), POR_1 is mated with POR_2 . After all the pores in the latent are examined, we get the mated pores between the two fingerprints. Fig. 13(a) shows the mated pores obtained between a latent and its mated exemplar.

To calculate the pore match score, we compare the neighboring ridge structures and pore distribution of the mated pores on each pair of mated ridges (recall that the pores on a ridge are ordered, so are the mated pores). A comparison of neighboring ridge structures is the same as being described for mated sampling points on ridges. As for the neighboring pore distribution, if two mated pores both have a neighboring pore on its left-hand side or right-hand side neighboring ridge, the location displacement between the neighboring pores is calculated. Let $d_{NP}^{l,i}$ be the location displacement between the neighboring pores on the left-hand side of the i_{th} mated pores on the mated ridges, and $d_{NP}^{r,i}$ the location displacement between the neighboring pores on the right-hand side of them. The similarity between the neighboring ridge structures and pore distribution of the mated pores in the two ridges can be then calculated as

$$s_N = 0.4 \times \frac{n_{NR}^l}{n_{mp} - 1} + 0.4 \times \frac{n_{NR}^r}{n_{mp} - 1} + 0.1 \times \frac{\sum_{i=1}^{n_{NP}^l} (10 - d_{NP}^{l,i})}{10 \times n_{NP}^l} + 0.1 \times \frac{\sum_{i=1}^{n_{NP}^r} (10 - d_{NP}^{r,i})}{10 \times n_{NP}^r}, \quad (18)$$

where n_{mp} is the number of mated pores on the two ridges and n_{NP}^l and n_{NP}^r are the number of cooccurring neighboring pores on the left-hand and right-hand sides,

respectively. Finally, the pore match score between the two fingerprints is defined as

$$s_{POR} = 0.8 \times s_r + 0.2 \times \left(0.3 \times \bar{s}_N + 0.3 \times \frac{10 - \bar{d}_{loc}}{10} + 0.4 \times \frac{n_{MP}}{n_P} \right), \quad (19)$$

where \bar{s}_N is the average similarity between neighboring ridge structures and pore distribution of all the mated pores on the mated ridge pairs, \bar{d}_{loc} is the average location displacement between all the mated pores, and n_{MP} and n_P denote the total number of mated pores and the number of pores in the query latent fingerprint, respectively. It is worth mentioning that the above match score measures the similarity between fingerprints by considering not only the location displacement between mated pores and the number of mated pores, but also the consistency of the ridge structures and feature distribution surrounding the mated pores, whereas existing methods [35, 54] consider only the location displacement or the number of mated pores.

DIP Matching The matching of DIP features is also constrained along mated ridges. Unlike pores, DIP features reside on valleys rather than ridges. Therefore, we associate each DIP feature with two ridges that are on the left-hand and right-hand sides of the valley on which it resides. Given a DIP feature DIP_1 which is associated with two ridges R_{11} and R_{12} , the nearest sampling point to its projection on the ridge R_{11} is first found, denoted as SP_1 . If SP_1 does not have mated sampling points, then DIP_1 does not have mated DIP features; otherwise, the nearest DIP feature to the mated sampling point SP_2 on the mated ridge R_{21} of R_{11} is found, denoted as DIP_2 . Let d_i be the location displacement between DIP_i and SP_i ($i = 1, 2$), then the location displacement between DIP_1 and DIP_2 is $d_{loc} = |d_1 - d_2|$. Let R_{22} be the other ridge associated with DIP_2 . DIP_1 and DIP_2 are mated DIP features only if i) $d_{loc} \leq 10$ and ii) R_{12} and R_{22} are mated ridges. After enumerating all the DIP features in F_q , we obtain the mated DIP features. Fig. 13(b) shows the mated DIP features in a latent and its mated exemplar.

Let \bar{d}_{loc} be the average location displacement between all the mated DIP features and n_{MDIP} and n_{DIP} be the total number of mated DIP features and the number of DIP features in the query latent fingerprint, respectively. The DIP match score between the two fingerprints is then defined as

$$s_{DIP} = 0.8 \times s_r + 0.2 \times \left(0.3 \times \frac{10 - \bar{d}_{loc}}{10} + 0.7 \times \frac{n_{MDIP}}{n_{DIP}} \right). \quad (20)$$

3 Results

3.1 Statement of results

3.1.1 Level 1 and Level 2

Database To evaluate the latent fingerprint matching algorithm, 258 latent fingerprints in NIST SD27, which also contains the mated rolled prints, were matched against a large background database of rolled prints. This is the only public domain database available containing mated latent and rolled prints. Since there are only 257 (excluding one duplicate image) rolled fingerprints in SD27, to make the latent-to-rolled matching problem more realistic, we expand the background database by adding fingerprints from NIST SD4 and SD14 databases. There are 2,000 different fingers and 2 rolled impressions per finger in SD4, and 27,000 fingers and 2 rolled impressions per finger in SD14. These fingerprints were also scanned from paper and have similar characteristics to the rolled prints in SD27. The 29,000 file fingerprints in SD4 and SD14 are combined with the 257 rolled images in SD27 to form a background database containing 29,257 rolled prints. We search the 258 latents against this background database of 29,257 rolled prints. All these fingerprint images are scanned at 500 ppi.

Matching Accuracy The Cumulative Match Characteristic (CMC) curve of the proposed algorithm in searching all 258 latents against the background database of 29,257 rolled prints is shown in Fig. 14. A CMC curve plots the rank- k identification rate against k , for $k = 1, 2, \dots, 20$. The rank- k identification rate indicates the proportion of times the mated fingerprint occurs in the top k matches. A rank-1 identification rate of 74.0% and a rank-20 identification rate of 82.9% were achieved by our system. Note that no systematic procedure has been used to select the best parameters in matching score computation, due to lack of a large number of latents. The matching accuracy can be further improved by fusing the matching results of latent-to-rolled and latent-to-plain, as shown in [51].

Latent Quality Fingerprint quality has a significant impact on the matching accuracy of fingerprint matchers. The number of minutiae is the most important indicator of fingerprint quality [49, 52]. We conducted an experiment to examine the impact of subjective quality and the number of minutiae on matching accuracy, respectively.

The 258 latent prints in SD27 were subjectively classified by latent examiners into three quality levels, namely: Good, Bad, and Ugly. There are 88 Good, 85 Bad, and 85 Ugly latent prints in SD27. Figure 15a shows the CMC curves of the proposed algorithm separately for Good, Bad, and Ugly quality latent prints. As expected, the matching performance for Good quality latents is significantly better than those for the latents belonging to the other two quality groups. Three examples of successful identification (one from each quality group) are shown in Fig. 16. In all these three cases, the mated rolled print was found at rank 1. It should be noted that although there are only 4 matching minutiae in the Ugly latent (Fig. 16), our algorithm still identified it correctly at rank-1.

Based on the distribution of the number n of minutiae in latents in SD27, these latents are classified into three types: Large ($n \geq 21$), Medium ($13 < n < 22$), and Small ($n \leq 13$). There are 86 Large, 85 Medium, and 87 Small latents in SD27. Figure 15b shows the CMC curves of the proposed algorithm separately for these three types of latent prints. The curves in Fig. 15b are quite consistent with those in Fig. 15a. This indicates that the number of minutiae has similar capability as subjective quality in predicting latent matching performance.

Although the quality of latent prints is a good indicator of the matching performance, the identification result of a given latent print depends on both the latent and its mated rolled print. If a large number of spurious minutiae are detected in the overlapping region of latent and rolled prints, the matching algorithm will fail as shown in Fig. 17.

Importance of Extended Features Figure 18a plots the rank-1 identification rates for all the 258 latents when extended features are incrementally used. The largest accuracy improvement is due to singularity feature; ridge quality map and ridge flow map also significantly improve the matching accuracy. Figure 18b shows the rank-1 identification rates separately for each quality level when extended features are incrementally used. It can be observed that Ugly quality latents benefit the most from the use of extended features. Figure 19 shows the matched minutiae and skeletons between a latent and its mated rolled print. In this example, with the incremental use of extended features, the rank of the mated rolled print is 206 (minutiae), 114 (singularity), 5 (quality), 2 (flow), 2 (wavelength), and 1 (skeleton), respectively.

Speed The experiments were conducted on a PC with Intel Core2 Duo CPU and Windows XP operating system. The automatic feature extraction takes 580 ms for a rolled print in NIST SD4 and 735 ms for a print in NIST SD27 and SD14. It takes around eight minutes to match a latent against all the 29,257 rolled prints.

3.1.2 Level 3

Databases Two latent databases were used in this study. One is the ELFT-EFS-PC database [55], which has 242 1000 ppi latent fingerprints (most of them are from the same source as the latents in NIST SD27) with 1000 ppi mated full prints. The level 3 features in these latents have already been manually marked by latent examiners. The background database consists of 4,180 1000 ppi fingerprint images, which were collected from the mated fingerprints of the latents and the “B” session fingerprint images in the NIST SD29 and the NIST SD30 datasets. The second latent database was collected at West Virginia University (WVU). It has 127 latents in which the level 3 features have been manually marked. While these latents are at 1000 ppi, the full fingerprints in the background database are only at 500 ppi. In an earlier study [57], we investigated the utility of pores in the context of varying fingerprint image quality and resolution by using the rolled ink fingerprint images in NIST SD30 and a commercial minutiae matcher (VeriFinger [44]). It was reported that automatic pore extraction and the resulting matching accuracy are significantly affected by fingerprint image quality.

Further, it is only at high resolution (1000 ppi) and for good quality fingerprint images that the pores can improve the fingerprint verification accuracy and even then only marginally. Hence, the WVU database is not suitable for studying the utility of level 3 features in latent matching (although many latents in it have large number of pores, the mated rolled images at 500 ppi do not have sufficient number of pores), and we simply used it for reporting the statistics of level 3 features in latents.

Statistics of Level 3 Features in Latents The statistics of level 3 features in the latents in the ELFT-EFS-PC and WVU databases have been collected based on the manual markup data. Fig. 20 shows the number of level 3 features (i.e. pores and DIP features) with respect to the number of minutiae. It can be seen that there is a large variance in the number of level 3 features across different latent fingerprints. In ELFT-EFS-PC, very few of the latents (only 6 of 242 latents) have any pores. Fig. 21 shows three example latents in ELFT-EFS-PC, which have 91, 17, and 0 pores marked by the latent examiners, respectively. Given such a small number of latents in ELFT-EFS-PC which have pores, pore matching is not likely to improve the latent matching accuracy on this database. Since no public domain latent database is suitable for studying the utility of pores, we constructed a simulated partial fingerprint database and investigated the effectiveness of pores on that dataset (see Section 3.1.2). On the other hand, many (79 out of 242) latents in ELFT-EFS-PC do have DIP features. So, we will use the ELFT-EFS-PC database for studying the effectiveness of DIP features.

Feature Detection Accuracy Sixty partial fingerprints (320×240 pixels) were cropped from the 1000 ppi rolled ink fingerprint images ($\sim 1500 \times 1500$ pixels) in NIST SD30. The pores, dots, incipient ridges, and ridge edge protrusions in these sixty partial fingerprint images were manually marked for the purpose of evaluating their automatic detection accuracy. In order to study the impact of image quality on the automatic detection accuracy, the 60 partial fingerprint images were divided into two quality groups (good and bad) according to their image quality evaluated by the method in [59]. Two pore detection methods were considered, i.e. the proposed method and the method in [35]. These two methods differ in that the proposed method conducts filtering in the spatial domain using an anisotropic model, whereas the method in [35] applies filtering in the frequency domain with an isotropic model. Table 1 gives the average pore detection accuracy along with the standard deviation of the two methods on the ground truth dataset; R_t , the true detection rate, is defined as the ratio of the number of true detected pores to the total number of ground truth pores, and R_f , the false detection rate, is defined as the ratio of the number of falsely detected pores to the total number of detected pores. These results show that the detection accuracy of both the methods degrades as the fingerprint image quality goes down. As the fingerprint image quality changes from good to bad, the true detection rate decreases by about 5%, and the false detection rate increases more by about 10%. According to the standard deviation of the detection accuracy, in general, the automatic pore detection methods are more robust on good quality fingerprint images than on bad quality fingerprint images. Table 2 presents the detection accuracy of the proposed DIP detection method ([35] did not present a DIP feature extractor), where N_m and N_s denote the numbers of missing features and spu-

rious features, respectively. Note that the average numbers of observable DIP features on good and bad quality fingerprint images in the ground truth dataset are 12 and 27, respectively. Again, poor quality fingerprint images cause more missing features as well as more spurious features. These results show that improving the quality of full fingerprint images in the background database is very important for the effectiveness of level 3 features in latent fingerprint matching. We will further demonstrate this in the next section.

Latent Fingerprint Matching We have evaluated the latent matching performance with pores and DIP features on the 242 latents in ELFT-EFS-PC. Features (including minutiae, ridge skeletons, pores, and DIP) were manually marked in latents, and automatically extracted from full fingerprints in the background database by using VeriFinger and the proposed level 3 feature extraction methods. As discussed in Section 2.3.2, level 3 features are matched only when the rank 1 minutiae match score of a latent is smaller than the given threshold (in our experiments, it was empirically set to 80 according to the raw match scores of VeriFinger on ELFT-EFS-PC which ranged from 0 to 248). For the sake of efficiency, we matched level 3 features only between the latent and the top 100 candidate exemplars retrieved by VeriFinger. The match scores of VeriFinger and level 3 features were then combined by using the weighted sum rule [60]. Before the fusion, the minutiae match scores of each query latent fingerprint were normalized by using the max-min normalization method [60] based on the maximum and minimum scores between it and the exemplars.

According to the experimental results, among the 6 latents in ELFT-EFS-PC which have any pores, three are already correctly identified at rank 1 by VeriFinger, two (L030G with 91 pores and L201U with 17 pores, see Figs. 21(a) and 21(b)) are correctly matched with their true mates after rank 1 but before rank 100, and the remaining one (L014G with 83 pores, see Fig. 4(b)) is correctly matched at rank 2874. By applying the proposed level 3 feature based latent matching method, however, the identification results of the latents L030G and L201U are not improved. This is because i) the number of pores is small or the pores are sparsely distributed in the latent (e.g. L201U), or ii) the corresponding region of the latent in its mated full fingerprint image is of poor quality and has few pores (e.g. L030G). Fig. 22 shows the latent L030G and its mated full fingerprint. As can be seen, although there are nine mated minutiae in them, no corresponding pores are found in the full fingerprint for any of the marked pores in the latent because of the poor quality of the corresponding region in the full fingerprint.

Fig. 23 presents the Cumulative Match Curves of VeriFinger and combination of VeriFinger and the proposed DIP matcher on the ELFT-EFS-PC database. At rank 1, VeriFinger correctly identified 65 latents; this number was improved to 75 after incorporating the proposed DIP matcher. In addition, many of the other latents had the ranks of their true mates improved. For example, for the latent shown in Fig. 13(b), its true mate was ranked at 82 by VeriFinger; after incorporating the proposed DIP matcher, the rank was improved to 4. From these results, we can see that level 3 features, when reliably present in both latents and their mated full fingerprints, are indeed useful in improving the latent matching accuracy.

Unfortunately, there are many difficulties in using level 3 features on the available latent databases. For one thing, there are very few latents in the latent databases with sufficient number of level 3 features. This is not only because of the generally low quality of latent fingerprints, but also because latent experts often are not able to mark some of the level 3 features. While we can clearly see a large number of pores in the latent shown in Fig. 1(c), according to the markup data in ELFT-EFS-PC, three of the four latent experts did not mark any pores in it. Another concern is that the quality of the exemplar full fingerprints in these databases is not good enough to automatically extract reliable level 3 features. As a consequence, it is not possible to utilize the level 3 features even though they are present in latents (as shown in Fig. 22). Such problem of reproducibility of level 3 features in exemplar full fingerprints has also been acknowledged in a recent survey on level 3 features among latent examiners [43].

Simulated Partial Fingerprint Recognition The experimental results presented above not only show the potential of level 3 features in improving latent matching accuracy, but also demonstrate the difficulty of using level 3 features in existing latent databases due to the small number of latents having level 3 features and the poor quality of mated full fingerprints. In order to better understand the utility of level 3 features, in particular pores, we constructed an additional set of 131 simulated partial fingerprint images of small area (320×240 pixels). They were cropped from the 1000ppi “B” session rolled ink fingerprint images ($\sim 1500 \times 1500$ pixels) in NIST SD30. The pores in these 131 partial fingerprints were manually marked, whereas the minutiae and ridge skeletons were extracted by VeriFinger [44]. The background database in the experiments was the same as in our experiments with ELFT-EFS-PC except that the exemplars from the “B” session rolled ink fingerprint images in NIST SD30 were substituted with the corresponding “A” session images in the database (since the simulated partial fingerprint images as the query fingerprints were cropped from the “B” session images). All features in the exemplars were automatically extracted by VeriFinger and the proposed level 3 feature extraction methods. Next, we report the performance of the proposed pore matching method, and compare it with VeriFinger and the ICP based pore matching method [35] to show the effectiveness of the proposed method and the utility of pores.

The identification accuracy on this dataset is presented in Fig. 24. Using VeriFinger, among the 131 query partial fingerprints, 80 fingerprints were correctly identified at rank 1. After incorporating the proposed pore matcher, 27 additional fingerprints were successfully identified at rank 1. Compared with the results on ELFT-EFS-PC, these results are much more promising, which show the effectiveness of level 3 features in matching partial fingerprints of small area which will otherwise pose challenge to minutiae based AFIS because they contain a limited number of minutiae. Moreover, the importance of collecting good quality full fingerprints can also be seen from these results. With the fast development of fingerprint imaging techniques and the widespread use of high resolution (1000 ppi) live-scan fingerprint scanners, we believe that it is becoming feasible to collect good quality fingerprints. This will facilitate extraction of reliable level 3 features and thereby further improve the latent matching accuracy of existing AFIS by incorporating level 3 features.

Fig. 25 compares the fingerprints which are correctly identified at rank 1 by both VeriFinger and the proposed method with those which are correctly identified after rank 1 by VeriFinger but at rank 1 after incorporating the proposed method. Interestingly, the improvement due to level 3 features is only in situations where fingerprints have small number of minutiae or low minutiae match scores. This indicates that i) minutiae matchers usually work very well when there is a sufficient number of minutiae, and ii) the contribution of level 3 features is more effective for fingerprints which have few minutiae or low minutiae match scores. In [43], about one-third of the participant latent examiners reported that they do not consider level 3 features when level 2 features are of sufficient quality and are sufficient in number, which confirms our findings here. Moreover, these observations also justify the proposed level 3 feature based latent fingerprint matching algorithm sketched in Section 2.3.2.

For comparison, the ICP based level 3 feature matching method in [35] has also been implemented. Given a pair of mated minutiae between two fingerprints, the method in [35] first uses the two mated minutiae to align the level 3 features (here, pores) in the two fingerprints, and then employs the ICP algorithm to further align and match the level 3 features. The match score for the two fingerprints was finally computed based on the average distance, \bar{d} , between the obtained mated pores: $s_{POR} = 1 - \bar{d}/d_{Max}$, where d_{Max} is the maximum distance between mated pores (in our experiments, the distance threshold between two mated pores was set to 10 pixels). The CMC of the ICP based method in Fig. 24(a) shows that its performance is much worse than the proposed method: Only 10 fingerprints are correctly identified at rank 1. By fusing its scores with the scores of VeriFinger, we did not get consistent improvement in the identification accuracy (see Fig. 24(b)).

Fig. 26 shows the pore matching results on an example partial fingerprint and its mated full fingerprint (cropped for display purposes) by using the proposed method and the ICP based method, respectively. Obvious distortion can be observed between the two fingerprints (note the polygons highlight the corresponding minutiae). As a consequence, in the ICP based method, most pores are falsely matched (see Fig. 26(b)). On the contrary, most pores are correctly matched by the proposed method despite the large distortion (see Fig. 26(a)). Based on the match scores, the rank of the mated fingerprint in this pair is 1 (the proposed method) and 16 (the ICP based method), respectively, while VeriFinger ranks it at 5. These results illustrate the advantage of the proposed method in considering ridge structures and inter-feature topology when matching level 3 features in fingerprints.

3.2 Tables

Table 1: Average pore detection accuracy and standard deviation

Method	Proposed		Method in [35]	
Quality	Good	Bad	Good	Bad
$R_t(\%)$	73 ± 9.5	67 ± 14.6	69 ± 16.0	63 ± 12.5
$R_f(\%)$	20 ± 12.2	30 ± 14.4	27 ± 11.7	40 ± 18.1

Table 2: Average DIP detection accuracy and standard deviation

Quality	Good	Bad
N_m	6 ± 5.6	12 ± 8.5
N_s	4 ± 3.6	10 ± 7.0

3.3 Figures

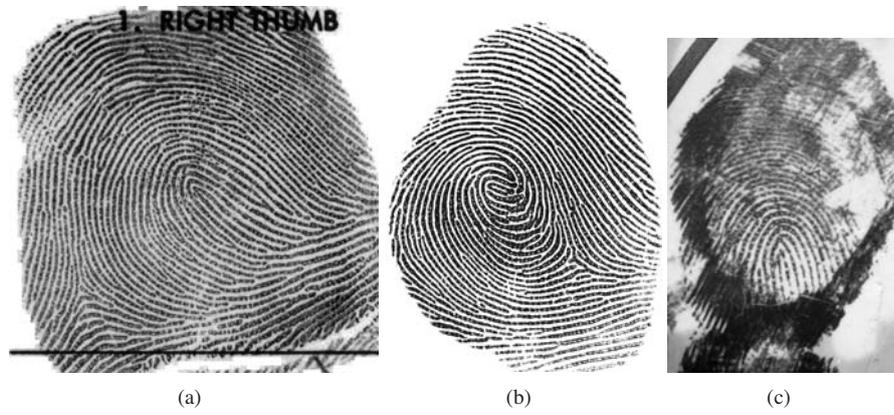


Figure 1: Three types of fingerprint images: (a) Ink, (b) live-scan, and (c) latent fingerprints.

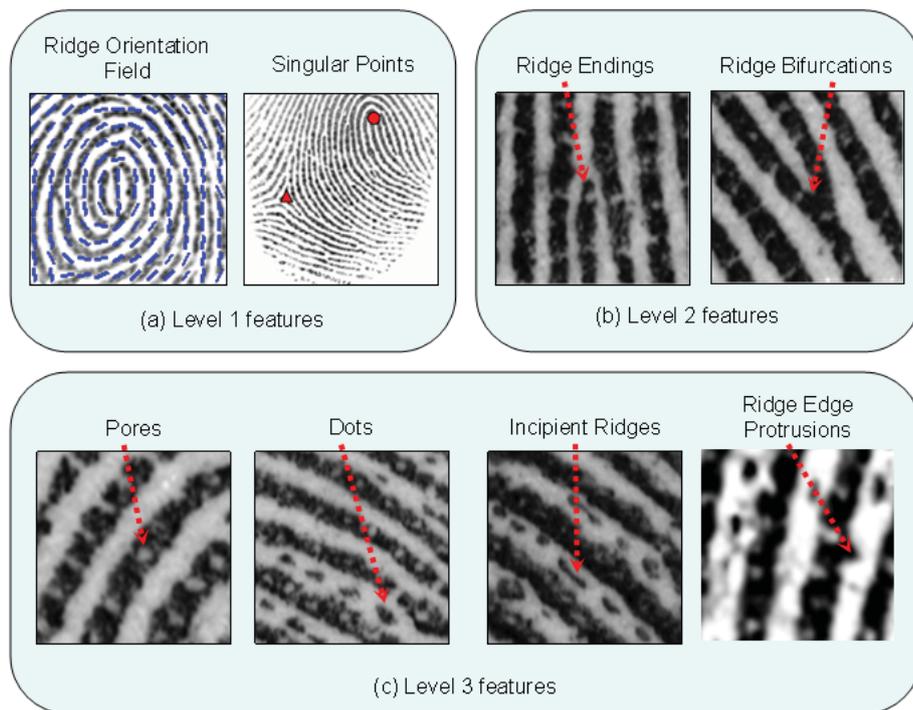


Figure 2: Fingerprint features: (a) Level 1 (ridge orientation field and singular points), (b) level 2 (minutiae, i.e. ridge endings and ridge bifurcations), (c) and level 3 (pores, dots, incipient ridges, and ridge edge protrusions).

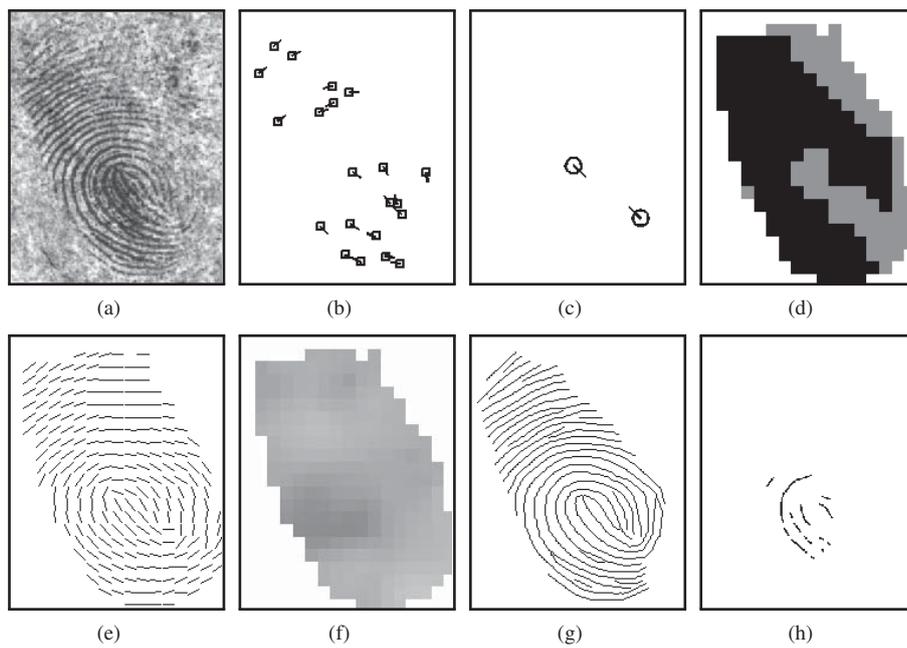


Figure 3: Features in a latent fingerprint. (a) Grayscale image, (b) minutiae, (c) singular points (cores), (d) ridge quality map (darkness indicates high quality level), (e) ridge flow map, (f) ridge wavelength map, (g) skeletonized image, (h) dots and incipient ridges.

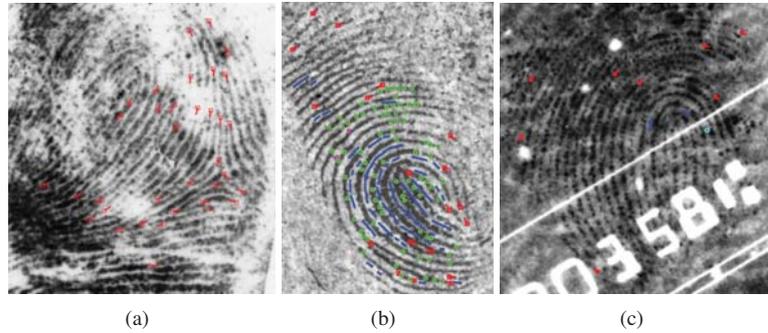


Figure 4: Some example latent fingerprints from the ELFT-EFS-PC database. Minutiae (marked by red rectangles), pores (green circles), dots (cyan circles), incipient ridges (blue lines), and ridge edge protrusions (magenta circles) in latents were manually marked by latent experts. Some latents may have small number of minutiae together with some extended features (c). In the ELFT-EFS-PC database (255 latents), there are, on average, about 18 minutiae in a latent, compared to 133 minutiae in a rolled ink fingerprint in the background database (4180 full fingerprints). Some latents in this database have only 4 minutiae.

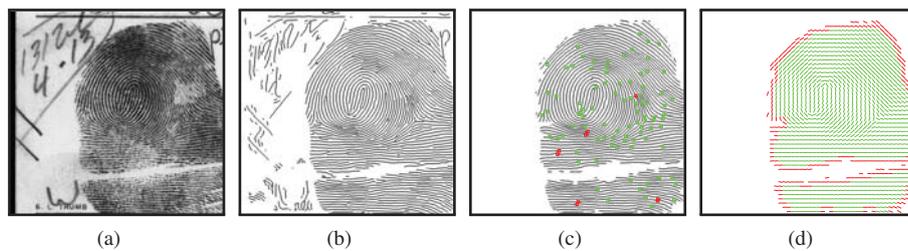


Figure 5: Feature extraction in a rolled fingerprint. (a) Gray image, (b) thinned image, (c) ridges and minutiae (green: reliable minutiae, red: unreliable minutiae), (d) ridge flow map and ridge quality map (green: reliable blocks, red: unreliable blocks).

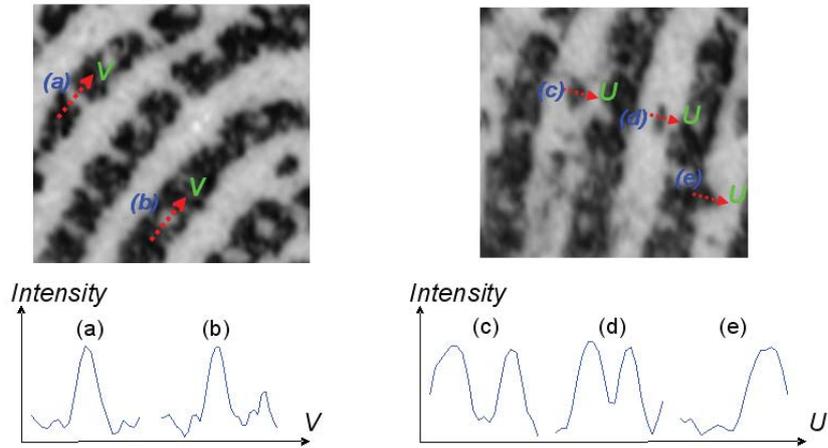


Figure 6: Properties of level 3 features. (a) and (b): Intensity profile across a pore along the ridge tangential orientation has a Gaussian shape. (c), (d), and (e): Intensity profile across a dot, an incipient ridge, or a ridge edge protrusion along the ridge normal orientation has the shape of a full or half negative Gaussian.

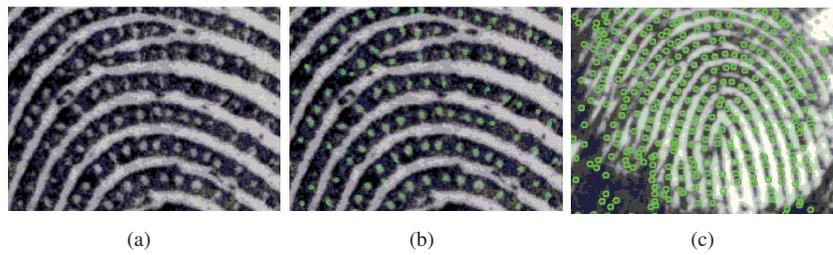


Figure 7: Example pore extraction results. (a) Part of a rolled ink fingerprint image in NIST SD30. (b) Pores detected in (a). (c) Pores detected in the latent fingerprint image in Fig. 1(c).

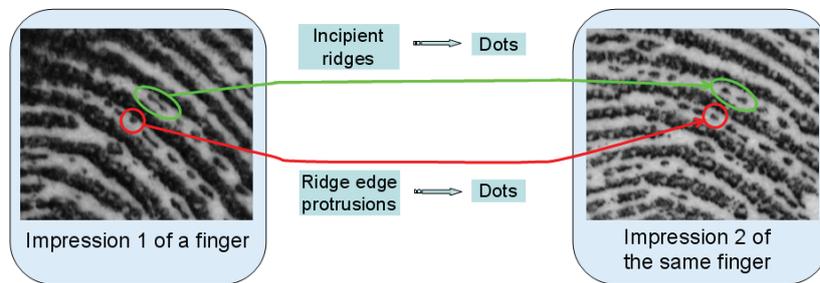


Figure 8: Dots, incipient ridges, and ridge edge protrusions are easily confused with each other in different impressions of the same finger. We thus unify them into a single feature type.



Figure 9: Example DIP extraction results. (a) Part of a rolled ink fingerprint image in NIST SD30. (b) DIP features detected in (a). (c) DIP features detected in the latent fingerprint image in Fig. 4(b).

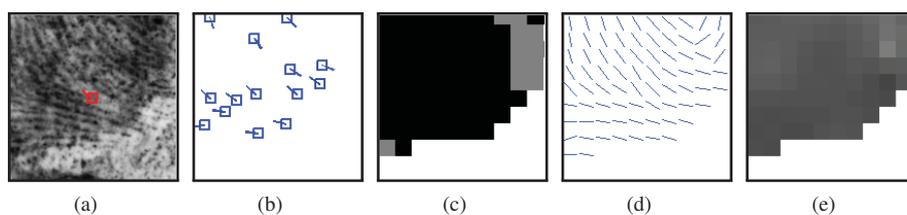


Figure 10: Minutia descriptors. (a) Local grayscale image, (b) neighboring minutiae, (c) local ridge quality map, (d) local ridge flow map, and (e) local ridge wavelength map.

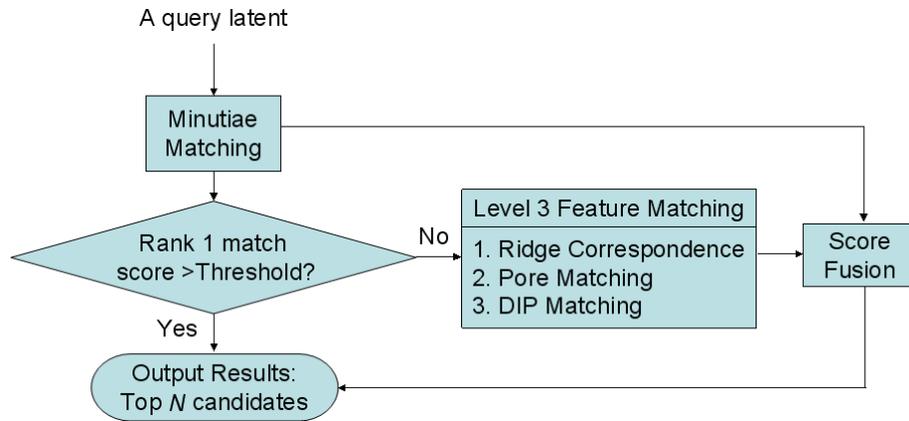


Figure 11: An overview of the proposed latent fingerprint matching algorithm which utilizes level 3 features.

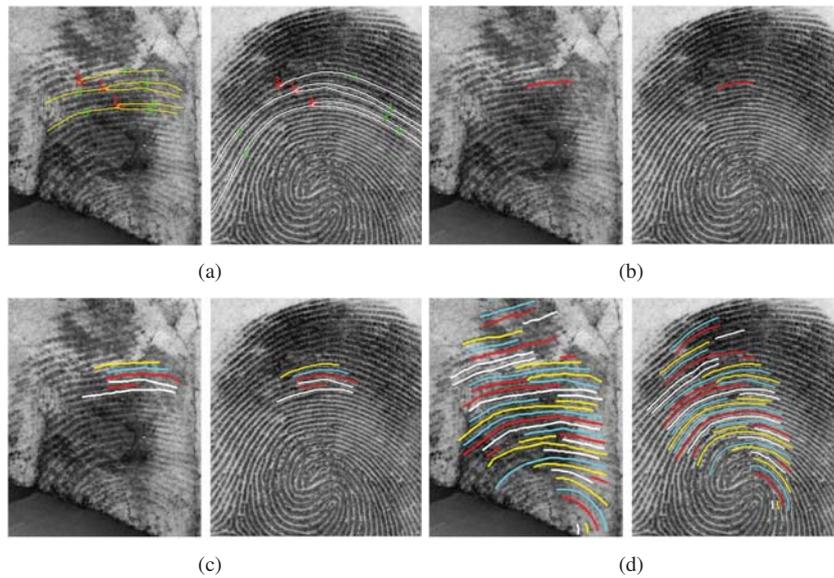
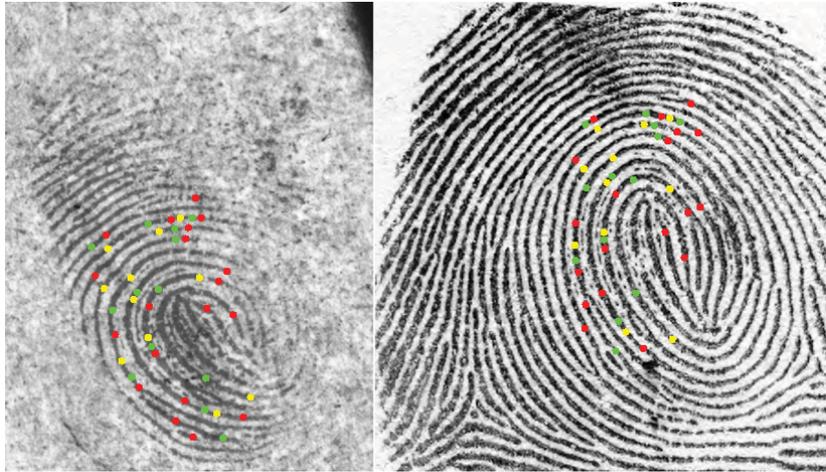
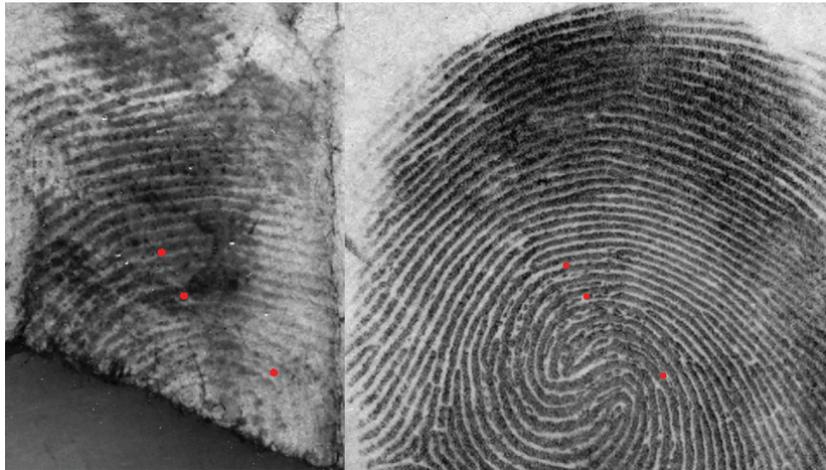


Figure 12: Ridge correspondence establishment results for the latent L177B and its mated exemplar in ELFT-EFS-PC. (a) Three pairs of mated minutiae are found between the latent and its exemplar. (b) The ridges $F_q.R_1$ and $F_t.R_1$ associated with the mated minutiae pair $\{F_q.M_1, F_t.M_1\}$ are matched by Intra-Ridge matching. (c) The mated ridges found at an intermediate step as the procedure Inter-Ridge propagation proceeds from the mated ridges in (b). (d) The final mated ridges between the two fingerprints obtained by the proposed method. Corresponding ridges are marked by the same color.



(a)



(b)

Figure 13: Example level 3 feature matching results. (a) Mated pores in the latent shown in Fig. 4(b) and its mated exemplar. Corresponding pores are marked by the same color. (b) Mated DIP features in the latent shown in Fig. 12(a) and its mated exemplar.

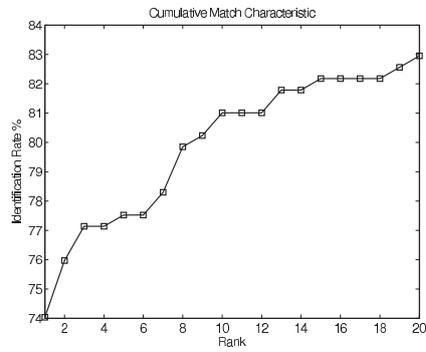


Figure 14: CMC curve of the proposed algorithm in matching 258 latents against a background database of 29,257 rolled prints.

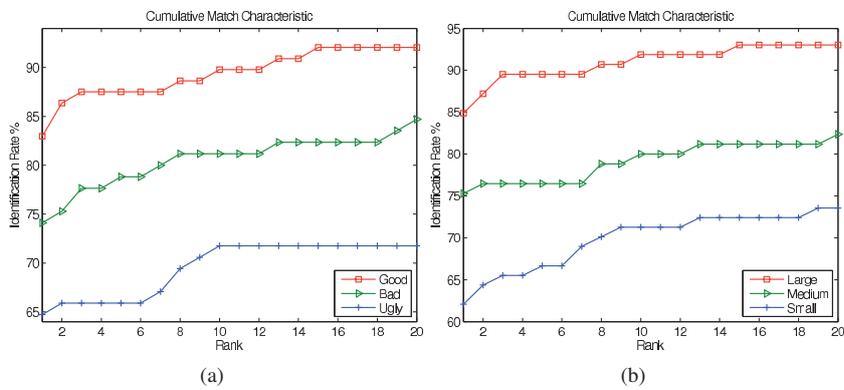


Figure 15: CMC curves for different types of latents. (a) Three types of latents according to subjective quality: Good (88), Bad (85), and Ugly (85). (b) Three types of latents according to the number of minutiae: Large (86), Medium (85), and Small (87).

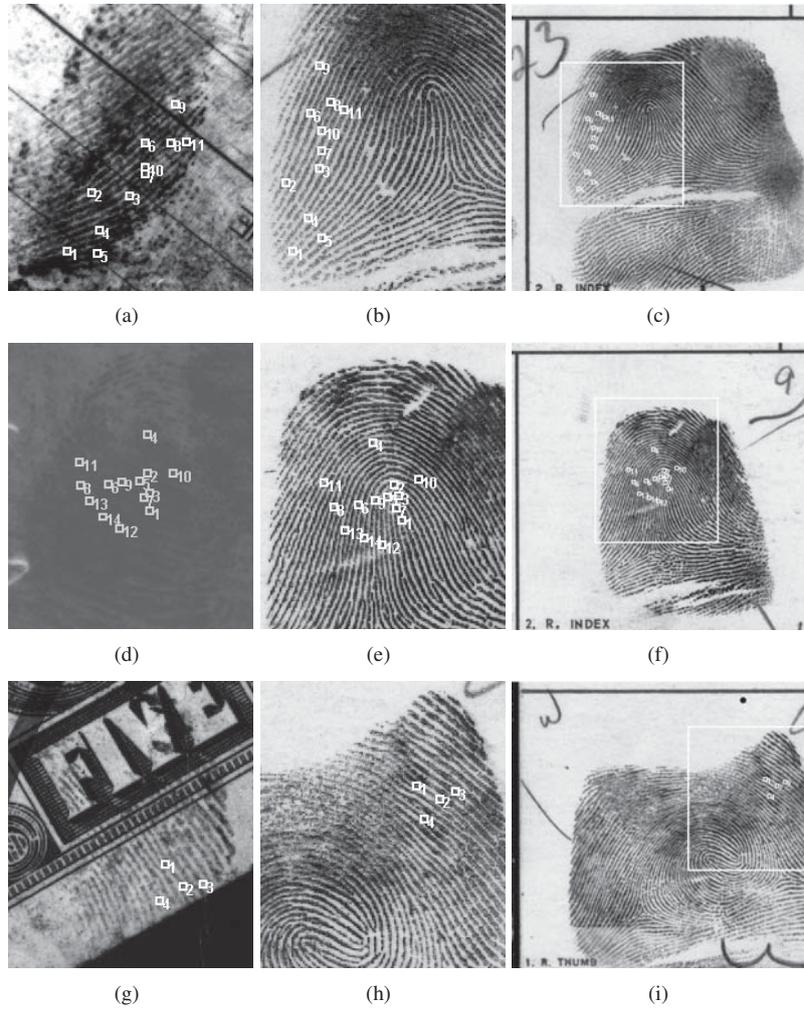


Figure 16: Examples of successful matchings. Three latents (classified as (a) good, (d) bad and (g) ugly by latent examiners), the corresponding regions in the mated rolled prints ((b), (e), (h)), and the mated rolled prints ((c), (f), (i)). In all these three cases, our algorithm found the true mate at rank one.

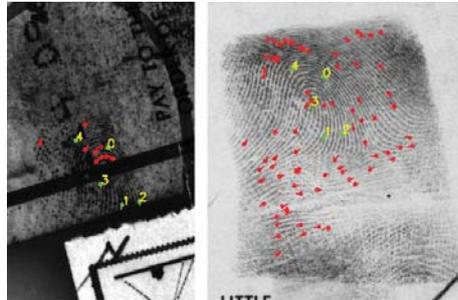


Figure 17: Example of an incorrect match. The mated rolled print (right) of the latent (left) was ranked 200 by our algorithm. Many spurious minutiae are detected in the rolled print.

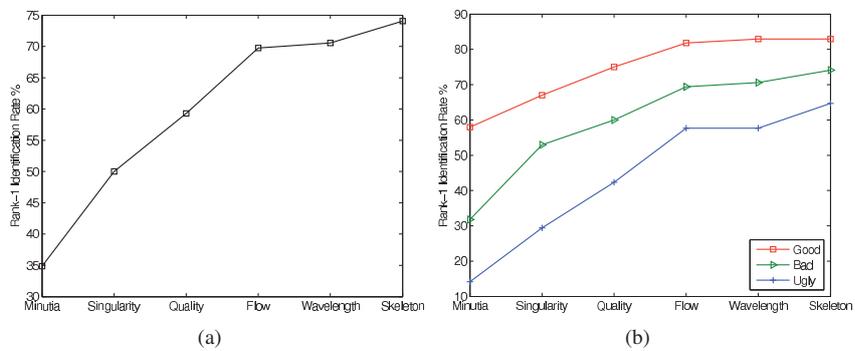
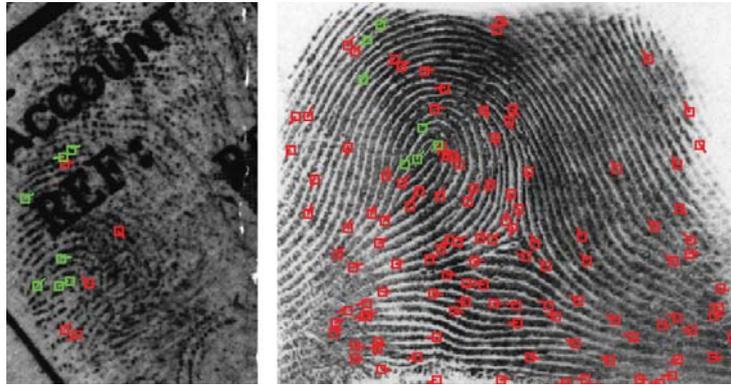
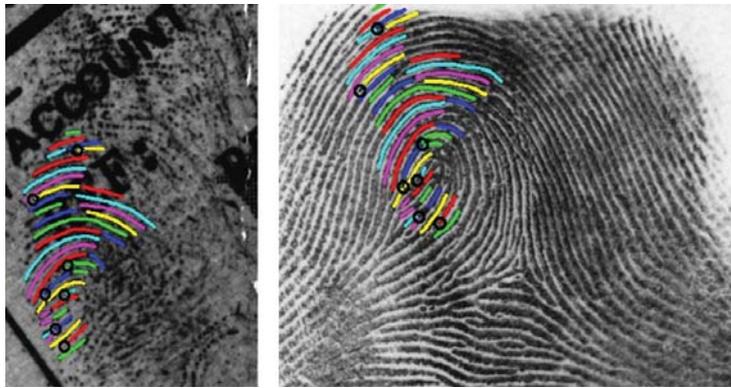


Figure 18: Plot of rank-1 identification rates vs. features. (a) All 258 latents, (b) Good, Bad and Ugly quality latent prints.

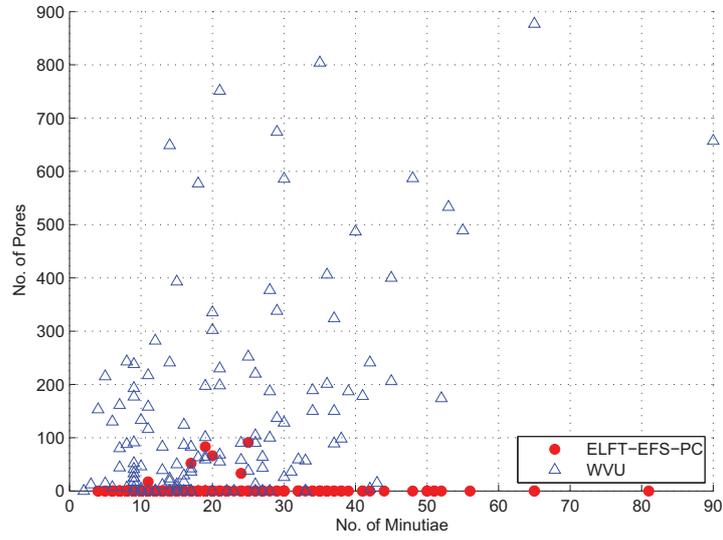


(a)

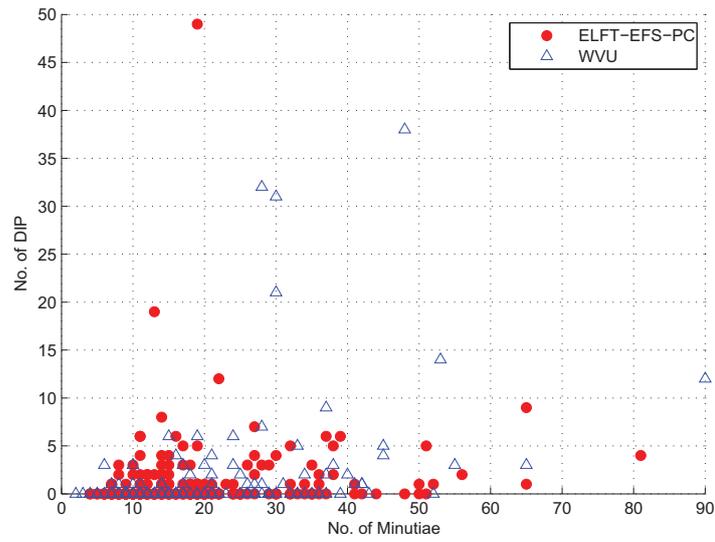


(b)

Figure 19: The matching result of a pair of mated fingerprints. (a) Minutiae matching, (b) skeleton matching.



(a)



(b)

Figure 20: The number of features in the latents in the ELFT-EFS-PC and WWU databases. (a) Number of pores vs. Number of minutiae. (b) Number of DIP features vs. Number of minutiae.

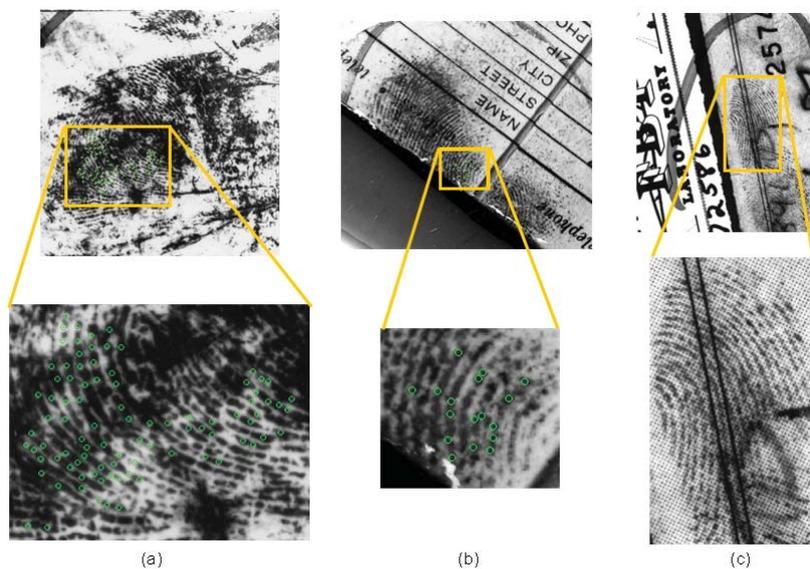


Figure 21: Example latents in ELFT-EFS-PC with markup pores. (a) 91 pores, (b) 17 pores, and (c) 0 pores.

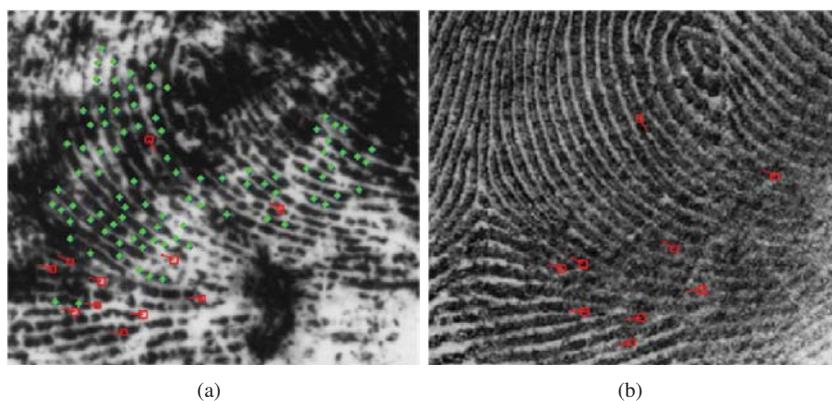


Figure 22: Poor quality of exemplar fingerprints significantly degrades the utility of level 3 features. (a) Latent L030G in ELFT-EFS-PC, and (b) its mated full fingerprint image. Nine mated minutiae are found between them. But none of the 91 pores marked in the latent (as shown in (a)) has any corresponding pore in the mated full fingerprint.

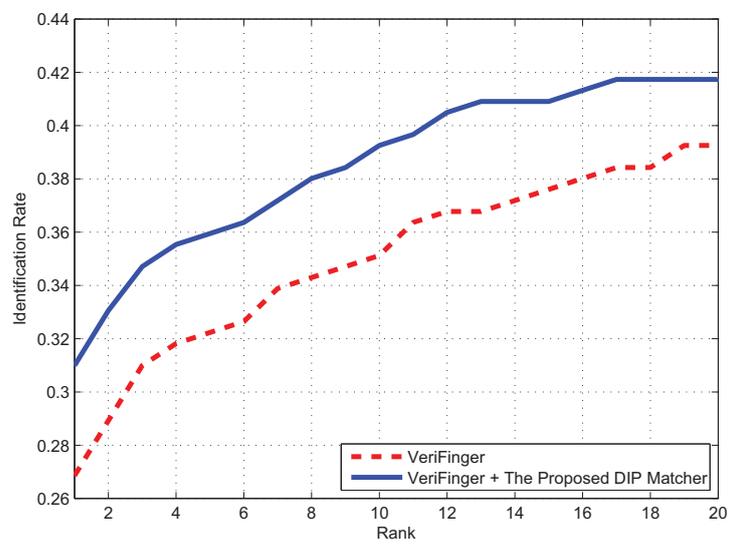
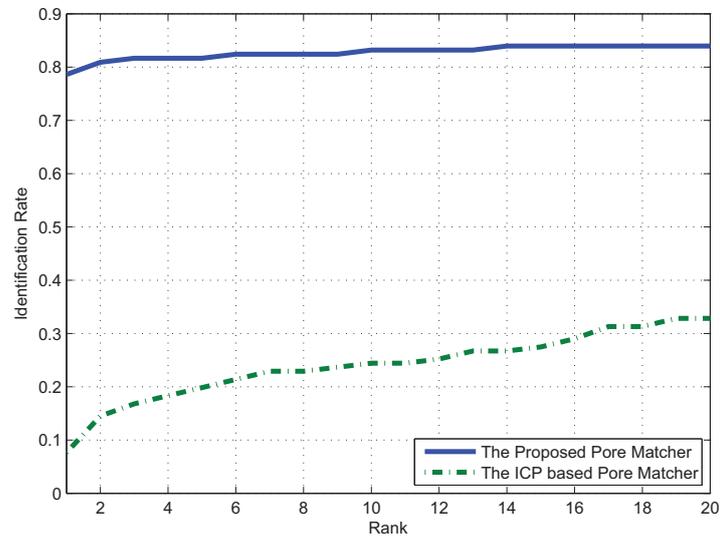
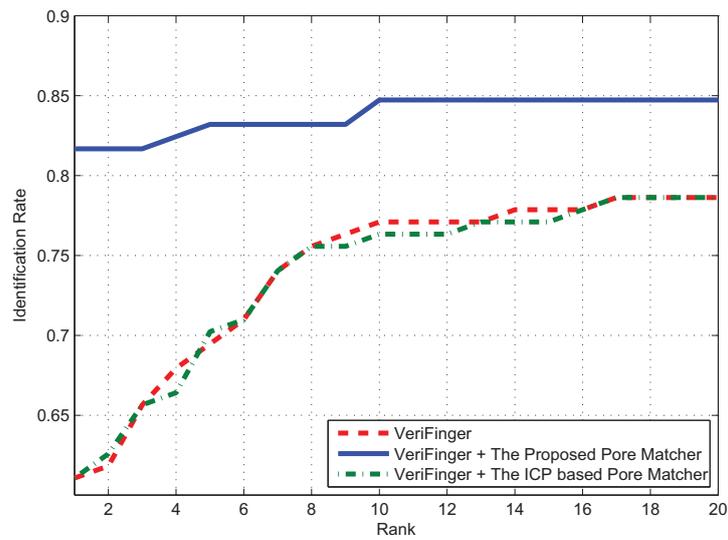


Figure 23: Identification accuracies of VeriFinger and combination of VeriFinger and the proposed DIP matcher on the ELFT-EFS-PC database.



(a)



(b)

Figure 24: Identification accuracy of the 131 simulated partial fingerprints and 4180 background exemplars. (a) Performance of the proposed pore matcher and the ICP based pore matcher. (b) Performance of combining VeriFinger and pore matchers.

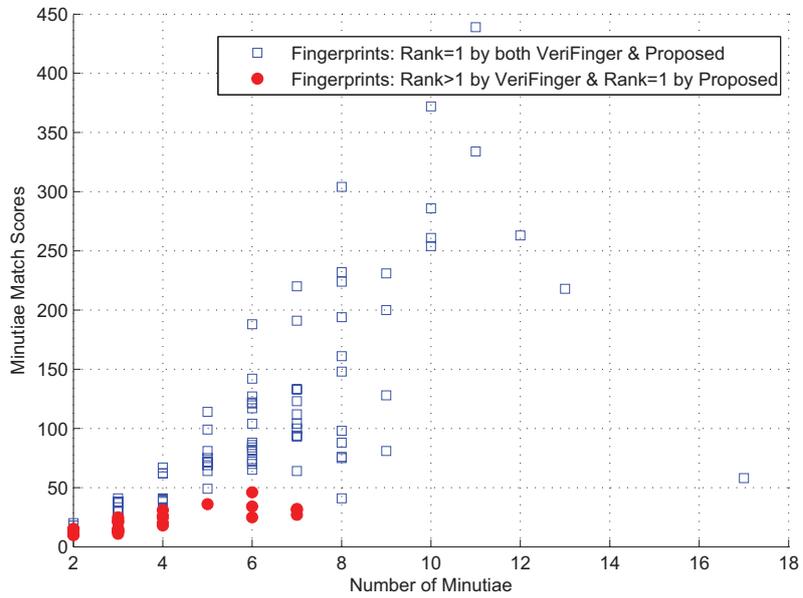


Figure 25: Number of minutiae vs. the minutiae match scores in fingerprints with different identification results.

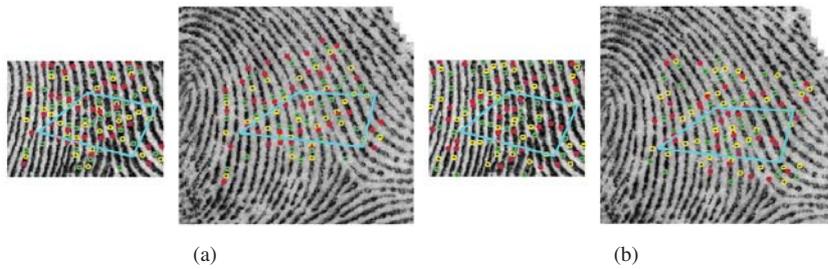


Figure 26: Pore matching results on an example partial fingerprint and its mated full fingerprint (cropped for display purposes) by using (a) the proposed method and (b) the ICP based method. The polygons highlight the corresponding reference minutiae between the fingerprints. Due to the large distortion in the fingerprints, the ICP based method mis-matches many pores; on the contrary, the proposed method is more robust to distortion, and can still correctly match most of the pores.

4 Conclusions

4.1 Discussion of findings

Experimental results are summarized as follows:

1. Extended features at Level 1 lead to the largest matching accuracy improvement than extended features at the other two levels. Among the Level 1 extended features, ridge quality map and ridge flow map are the most important.
2. Extended features at Level 2, namely ridge skeleton, improve the rank-1 recognition rate by 5%.
3. The “ugly” latents in NIST SD27 benefit the most from using extended features at Level 1 and Level 2. The rank-1 recognition rate for “ugly” latents using all extended features at Level 1 is already comparable to the rank-1 recognition rate for “good” latents using the current FBI standard features (minutiae + core/delta).
4. The contribution of Level 3 features on ELFT-EFS-PC is minor. But, on a simulated partial fingerprint database, they do show obvious improvement in the matching accuracy.

4.2 Implications for policy and practice

Based on the extensive experiments, we would like to offer the following recommendations:

1. All extended features at Level 1 and Level 2 should be incorporated into AFIS as soon as possible, since (i) these features are insensitive to image quality, (ii) do not rely on high resolution images, and (iii) do lead to significant improvement in the matching accuracy.
2. Algorithms and GUI tools should be developed to help fingerprint examiners manually mark extended features (especially ridge skeleton) at Level 1 and Level 2 in latents, since it is a time-consuming task and general-purpose image editing software (such as Photoshop) is not convenient for this task.
3. Utility of Level 3 features heavily rely on the image quality of both latents and enrolled fingerprints. Since it is generally difficult or not possible to improve the quality of latent prints, it is crucial to improve the quality of enrolled fingerprints so that a sufficient number of Level 3 features can be extracted. This is necessary for Level 3 features to play an important role in design and development of next generation AFIS.

4.3 Implications for further research

The proposed matching algorithm is still inferior to the performance of experienced latent examiners, which may be caused by three major differences between the methodologies used by latent examiners and automatic matchers.

- Approaches used in matching ridge skeleton and minutiae (or Level 2 features) are different. Latent examiners employ a “ridges in sequence” method [50] in the matching process, which is robust to noise and distortion. While the proposed skeleton matching algorithm tries to mimic such a method, it is not robust in the presence of large amounts of noise and distortion. The minutiae matching algorithm is also prone to spurious minutiae and distortion.
- The approach used to match the detailed ridge features (or Level 3 features) is different. When latent examiners compare the detailed ridge features in fingerprints, there is no explicit separation between feature extraction and matching stages. The separation of feature extraction and matching in automatic systems leads to some information loss. In addition, the automatic feature extractor may not be able to extract Level 3 features from rolled prints that are always compatible with the features marked by latent examiners.
- The approach to utilizing negative evidence is different. Latent examiners can determine a pair of fingerprints as unmatched based on a single unmatched minutia which is located in the good quality region of the two fingerprints. This is a risky proposition for fingerprint algorithms.

We plan to improve the latent matching accuracy by reducing these differences.

Our study shows that extended features at Level 3 lead to much smaller contribution to matching accuracy compared to extend features at Level 1 and Level 2. Since the image quality of enrolled fingerprints in the databases used in this study is not sufficiently good for extracting Level 3 features, the experiments on Level 3 features should be repeated when other fingerprint databases with better image quality become available. The experiments reported here utilized a 500 ppi database and a 1000 ppi database, but not a database of mixed resolution images. As pointed out by a reviewer of this report, “the real world state of AFIS databases will be of mixed resolution.” This calls for additional experiments involving extended features on mixed resolution databases.

Manual feature markings for poor quality latent fingerprints is a time consuming and tedious task. Considering that latent examiners often have to process many latents within a limited time period, significant attention should be paid to the automatic latent feature extraction problem. Given the performance gap between automatic and semi-automatic latent matching systems, human intervention is likely to be necessary for some time. One way to reduce manual processing is to define a latent fingerprint quality measure, which is continuously updated when latent examiners are marking features. Once the quality measure reaches a predefined threshold, the latent examiners are notified that the image quality is already good enough to perform a latent search.

5 Bibliography

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6 Dissemination

6.1 Publications

6.1.1 Journal Papers

1. A. K. Jain and J. Feng, "Latent Fingerprint Matching," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 33, no. 1, pp. 88-100, 2011.
2. J. Feng and A. K. Jain, "Fingerprint Reconstruction: From Minutiae to Phase," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 33, no. 2, pp. 209-223, 2011.

6.1.2 Conference Papers

1. Q. Zhao and A. K. Jain, "On the Utility of Extended Fingerprint Features: A Study on Pores," IEEE Computer Society Workshop on Biometrics, pp. 9-16, San Francisco, June 2010.
2. J. Bohn and V. Despiegel, "Fingerprint skeleton matching based on local descriptor," IEEE 3rd International Conference on Biometrics: Theory, Applications, and Systems (BTAS), pp. 1-6, Washington, Sept. 2009.
3. J. Feng, S. Yoon, and A. K. Jain, "Latent Fingerprint Matching: Fusion of Rolled and Plain Fingerprints," Proc. International Conference on Biometrics (ICB), Italy, June 2009.
4. J. Feng, and A. K. Jain, "FM Model Based Fingerprint Reconstruction from Minutiae Template," Proc. International Conference on Biometrics (ICB), pp. 544-553, Italy, June 2009.
5. Y. Chen and A. K. Jain, "Beyond Minutiae: A Fingerprint Individuality Model with Pattern, Ridge and Pore Features," Proc. International Conference on Biometrics (ICB), pp. 523-533, Italy, June 2009.
6. J. Feng, and A. K. Jain "Filtering Large Fingerprint Database for Latent Matching," Proc. International Conference on Pattern Recognition (ICPR), pp. 1-4, Tampa, Dec. 2008.
7. A. K. Jain, J. Feng, A. Nagar and K. Nandakumar, "On Matching Latent Fingerprints," IEEE Computer Society Workshop on Biometrics, pp. 1-8, Alaska, June 2008.

6.1.3 Technical Reports

1. Q. Zhao, J. Feng and A. K. Jain, "Latent Fingerprint Matching: Utility of Level 3 Features," MSU Technical Report, MSU-CSE-10-14, August, 2010 (Submitted to the IEEE Transactions on Information Forensics and Security).

6.2 Presentations

6.2.1 Conference Presentations

This research has been presented at the following conferences:

1. IEEE Computer Society Workshop on Biometrics, San Francisco, California, June 2010.
2. International Conference on Biometrics: Theory, Applications, and Systems (BTAS), Washington D.C., September 2009.
3. International Conference on Biometrics (ICB), Italy, June 2009.
4. International Conference on Pattern Recognition (ICPR), Tampa, Florida, December 2008.
5. IEEE Computer Society Workshop on Biometrics, Anchorage, Alaska, June 2008.

6.2.2 Invited Talks

This research has also been presented in invited talks at the following institutions:

1. Chosun University, Dec 2009;
2. Yonsei University, Nov 2009;
3. University of Maryland, Oct 2009;
4. Korea University, Oct 2009;
5. Pusan National University, Oct 2009;
6. Tsinghua University, June 2009;
7. University of North Texas, April 2009;
8. University of Florida, Oct 2008;
9. Chulalongkorn University, Nov 2008;
10. TCS, Hyderabad, India, Nov 2008;
11. Istanbul Technical University, Sept. 2008;
12. Cogent, Aug. 2008.

Appendix

The research conducted by MorphoTrak team under this NIJ project is described in the following paper attached here.

J. Bohn and V. Despiegel, "Fingerprint skeleton matching based on local descriptor," IEEE 3rd International Conference on Biometrics: Theory, Applications, and Systems (BTAS), pp. 1-6, Washington, Sept. 2009.

Fingerprint Skeleton Matching based on Local Descriptor

Julien Bohné and Vincent Despiégl

Abstract— In this paper, we present a new fingerprint matching algorithm based on a local skeleton descriptor. This descriptor uses ridge count information to encode minutiae locations in a small neighborhood. Taking advantage of ridge count properties, our descriptor is robust to distortions. We developed an efficient algorithm to match our descriptor and a strategy to combine matchings of many local descriptors. Our algorithm obtains interesting results on both tenprint-to-tenprint and latent-to-tenprint matchings.

I. INTRODUCTION

FINGERPRINT matching is extensively used in biometric identification. As database size is still growing, we need increasingly accurate algorithms to make the distinction between very similar non matching fingerprints.

Most fingerprint matchers mainly use positions and orientations of minutiae (bifurcations and endpoints of the ridges) to compute a similarity score between two fingerprints. Several authors have proposed to use additional information to build more accurate algorithms. Jain *et al.* [1] propose to use skin pores as discriminative features. Skeleton is another common feature used to enhance the matching. In [2] Feng *et al.* enforce the minutiae matching by following the ridges to avoid wrong matchings. Ridge count has been used as complementary information to minutiae as in [3] where Sha *et al.* use the ridge count between pairs of minutiae. Octant [4] is a standard method based on the ridge count, the ridge counts of the closest minutiae in eight directions are used to improve the minutiae matching. In the EFS format [5], the NIST proposes a standardized extended feature set including skeleton, dots and pores.

Distortion is one of the major issues in fingerprint matching. Most authors do not take the distortion explicitly into account and deals with it by making the matching tolerant to small displacements of features. In [6], Bazen and Gerez propose to cancel the distortion using a non-rigid registration.

In this paper we propose a method based on a local skeleton descriptor which encodes the position of minutiae of a region in a specific coordinate system. The descriptor is detailed in Section 2. In Section 3 we describe an algorithm to match two skeleton descriptors and a method to merge matchings of many descriptors to compute a global similarity score between two fingerprints. Experiments which demonstrate the accuracy of our algorithm on tenprint-to-tenprint (TPTP) and latent-to-tenprint (LTP)

matchings are shown in Section 4.

II. LOCAL SKELETON DESCRIPTOR

When minutiae positions are defined by their location in the image, the distortion affects both 2D coordinates. In this section we present a local coordinate system in which only one coordinate is sensitive to distortions by using the ridge count. The ridge count between two points is defined as the number of ridges crossed by the line segment linking these two points. When the line linking the two points is close to the orthogonal to the ridge flow it is very unlikely that any distortion could affect the ridge count. Our local skeleton descriptor presented in the next paragraphs takes advantage of this property.

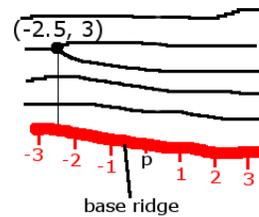


Fig. 1. Local coordinate system

The neighborhood of a point p is described by the relative minutiae positions as follows: we draw a curve segment centered in p called “base ridge” which follows the direction of the ridge flow. An orientation is arbitrarily chosen on this segment to define a 2D coordinate system. Minutiae are then projected on this base ridge following the orthogonal to the ridge flow direction. The first coordinate of a minutia is the algebraic curvilinear distance between the projection of the minutia on the base ridge and p . The second coordinate is the signed ridge count between the minutia and the projection of the minutia on the base ridge (Fig. 1).

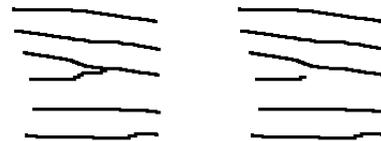


Fig. 2. Endpoint / bifurcation switch.

Minutiae can be divided in two types: endpoints and bifurcations (Fig. 2). Unfortunately this distinction is not always stable because an endpoint often becomes a bifurcation when the finger is pressed harder on the sensor. In addition it is sometimes very hard to classify a minutia into one category especially in poor quality images. Instead of

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using minutiae valence, we split minutiae into two categories: "positive minutiae" have the same orientation as the base ridge and "negative minutiae" have an opposite orientation relative to the base ridge (Fig. 3). As opposite to the endpoint/bifurcation distinction, these classes are very stable. Each minutia is represented by a triplet $M = (x, y, type)$ in the descriptor. Fig. 5 shows the description of a skeleton.

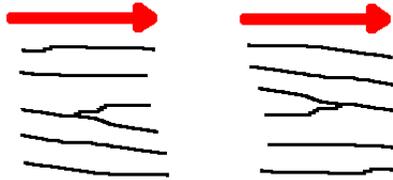


Fig. 3. Positive minutia / Negative minutia.

We empirically tuned the size of the region described by one descriptor to 200 pixels long (500dpi) along the ridge flow and 20 ridges high.

Valence errors (transformations of endpoints into bifurcations) can change the ridge count coordinate of minutiae. To be less sensible to valence errors, we compute the ridge count coordinate so that the type switch only induces a difference of 0.5 in the ridge count. When a minutia is a bifurcation, the ridge count value is equal to the number of ridges between the minutia and its projection on the base ridge. When it is an endpoint, the absolute value of the number of crossed ridges decreased by 0.5 (Fig. 4). By doing so there is only a difference of 0.5 in the ridge count coordinate with the case where a valence error occurs.

Projecting of minutiae is equivalent to unfolding the fingerprint before registering the minutiae into our coordinate system. The resulting descriptor does not take into account neither the directions of the ridge flow nor the distortions affecting it. The descriptor is sensitive only to distortions collinear to the ridge flow direction. In [7] Bazen and Gerez present a coordinate system also computed by unfolding

regions of fingerprints. The fingerprint is cut into regions so that the unfolding process does not tear the fingerprint. We do not address this problem explicitly so the description is quite unstable close to singular points. Since we use many descriptors to describe one fingerprint, many of them are not affected by the tearing.

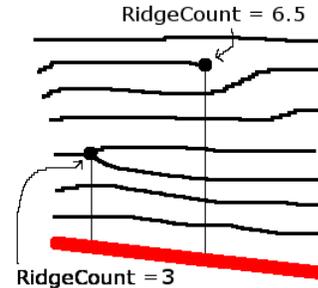


Fig. 4. Ridge count for bifurcation and endpoint minutiae

As the orientation of the base ridge is chosen arbitrarily, the same skeleton region can be described by two different descriptors. These two descriptors are linked by a rotation of 180 degrees and the switch of minutiae types (positive or negative).

Our local skeleton descriptor can be compared to the RCS presented in [9]. RCS also uses the projection into a distance-along-the-ridge / ridge count coordinate system. As opposed to our descriptor, the ridge flow direction is not used for the projection. In addition, whereas all sampled ridge points are recorded to build a descriptor, our representation is much more compact as only minutiae projections are computed. These differences lead to two very different matching strategies.

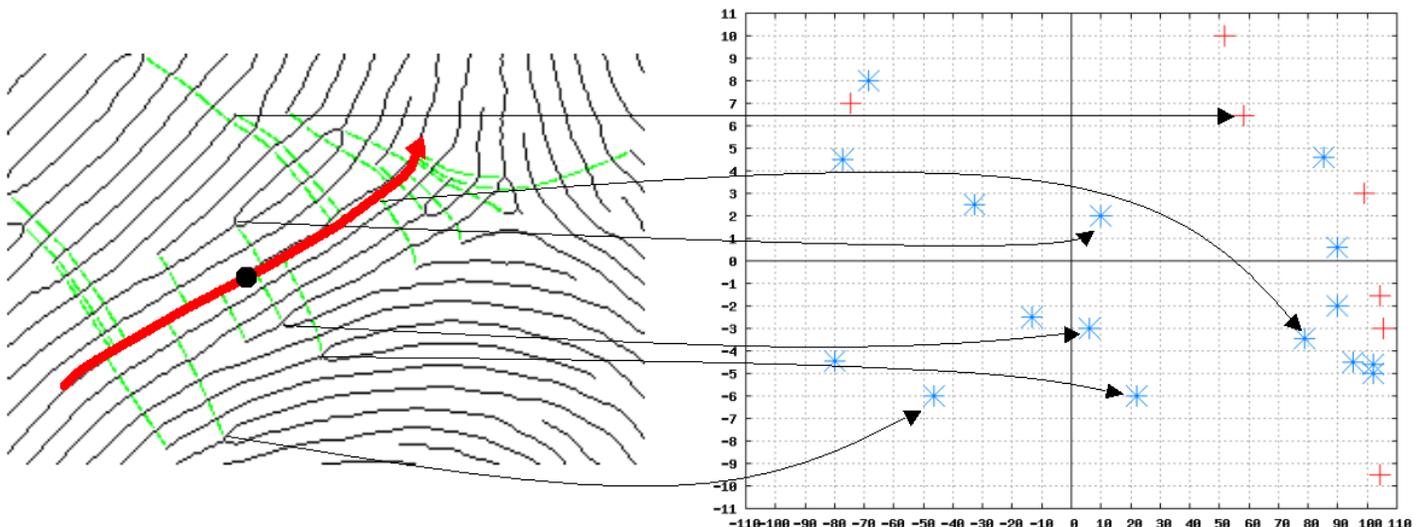


Fig. 5. Local skeleton descriptor corresponding to a skeleton. Positive minutiae are represented by blue stars and negative minutiae by red crosses

III. FINGERPRINT MATCHING ALGORITHM USING LOCAL SKELETON DESCRIPTORS

A. Outline

Our matching algorithm can be summarized as follows:

1. Fingerprints are registered based on matched minutiae using our standard minutiae matcher;
2. N pairs of corresponding points are selected in the two fingerprints and a descriptor is computed for each point;
3. Corresponding descriptors are matched: they are first aligned then the similarity between the two descriptors is computed;
4. The average of the N scores is outputted.

B. Matching of two local skeleton descriptors

The descriptor presented in the previous section is sensitive to some perturbations. The matching algorithm is designed to address the following issues:

1. The distortion along the ridge flow direction;
2. A small relative displacement of the origins of the descriptors;
3. The arbitrary choice of the orientation of base ridges;
4. The appearance / disappearance of minutiae between the search print and the reference print;
5. The noise in the location of the minutiae in the direction of the ridge flow. The location of a minutia can shift up to fifteen pixels at 500dpi when an endpoint becomes a bifurcation.

The matching algorithm is composed of two stages. The first stage performs the alignment of the two descriptors and addresses issues of translations, distortions and the arbitrary choice of the base ridges orientations. Once the two descriptors are registered, the second stage finds pairs of minutiae between the search print and the reference print using dynamic programming. These pairs are used to compute a similarity score between the two descriptors.

1) Registering stage

The objective of this stage is to register the descriptor from the reference fingerprint on the descriptor from the search fingerprint to best superpose matching minutiae. As our local descriptor is around 200 pixels long along the ridge flow direction, we consider that the distortion along the ridge flow direction can be approximated by a simple linear stretch. The transformation is defined by s the linear stretch along the ridge flow, tx the translation along the ridge flow and ty the translation orthogonal to the ridge flow. The main issue in estimation of the best transformation is that matching minutiae pairs are unknown. We therefore think in a probabilistic way and seek for the transformation which best superposes the biggest minutiae set.

To find the optimal parameters (s, tx, ty) we use a generalized Hough transform. Each couple of pairs of potentially matched minutiae votes for each (s, tx, ty) in the discretized parameter space proportionally to the probability that (s, tx, ty) superposes the two pairs of minutiae. We

consider that two minutiae can potentially match if they are of same type (positive or negative). As the multiplication is the natural operator in the scale space, the algorithm works in log scale. Votes are accumulated in the parameter space considering every couple of pairs of potentially matched minutiae:

$$Vote(\log(s), tx, ty) = \sum_{\substack{M_{Src}^i, M_{Src}^k \in \text{minutiae from search print} \\ M_{Ref}^j, M_{Ref}^l \in \text{minutiae from reference print} \\ \text{with } type_{Src}^i = type_{Ref}^j \text{ and } type_{Src}^k = type_{Ref}^l}} P(\log(s), tx, ty | M_{Src}^i, M_{Ref}^j, M_{Src}^k, M_{Ref}^l)$$

To be robust to the arbitrary choice for the base ridge orientation, two parameter spaces are used: one for each possible orientation of the reference base ridge. To register the two descriptors, we choose the (s, tx, ty) which receives the most votes in the parameter space.

As the two variables TX and TY are independent we have:

$$P(S = s, TX = tx, TY = ty | M_{Src}^i, M_{Ref}^j, M_{Src}^k, M_{Ref}^l) \\ = P(S = s, TX = tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l) \times P(TY = ty | y_{Src}^i, y_{Ref}^j, y_{Src}^k, y_{Ref}^l)$$

In the remaining part of this section we describe how to compute these expressions.

Let us first focus on the two first parameters: s and tx .

From 2 pairs of matched minutiae (M_{Src}^i, M_{Ref}^j) and (M_{Src}^k, M_{Ref}^l) the parameters s and tx superposing the abscissa of the 2 pairs can be computed,

$$\begin{cases} x_{Src}^i = s \times x_{Ref}^j - tx \\ x_{Src}^k = s \times x_{Ref}^l - tx \end{cases} \Rightarrow \begin{cases} s = \frac{x_{Src}^i - x_{Src}^k}{x_{Ref}^j - x_{Ref}^l} \\ tx = x_{Src}^i - s \times x_{Ref}^j \end{cases}$$

The matcher needs to be robust to noise in the minutiae locations. To take explicitly this noise into account we want to compute the probability distribution function $P(s, tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l)$ considering a gaussian noise with a known standard deviation. Unfortunately this distribution cannot be accurately approximated by a 2D gaussian centered on the best (s, tx) with a fixed covariance matrix because the distances $\|x_{Src}^i - x_{Src}^k\|$ and $\|x_{Ref}^j - x_{Ref}^l\|$ act as levers on the computation of s (Fig. 6). However it is possible to compute the exact value of $P(S = s, TX = tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l)$.

Using the Bayes theorem, $P(S = s, TX = tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l)$ can be split:

$$P(S = s, TX = tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l) = \\ P(TX = tx | s, x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l) P(S = s | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l)$$

Let X_{Src}^i be a gaussian random variable of law $\mathcal{N}(x_{Src}^i, \sigma_{Src}^i)$ and X_{Ref}^j be a gaussian random variable of law $\mathcal{N}(x_{Ref}^j, \sigma_{Ref}^j)$, let $D_{Src} = X_{Src}^i - X_{Src}^k$, D_{Src} is a gaussian random variable of law $\mathcal{N}(\mu_{D_{Src}} = x_{Src}^i - x_{Src}^k, \sigma_{D_{Src}} = \sqrt{\sigma_{Src}^i{}^2 + \sigma_{Src}^k{}^2})$ and D_{Ref} is a gaussian random variable of law

$\mathcal{N}(\mu_{D_{Ref}} = x_{Ref}^j - x_{Ref}^l, \sigma_{D_{Ref}} = \sqrt{\sigma_{Ref}^j{}^2 + \sigma_{Ref}^l{}^2})$. The scale variable

$S = D_{Src} / D_{Ref}$ is a ratio of two gaussian random variables. The distribution of this type of variable has a well known

expression [10]:

$$P(S = s | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l) = \frac{e^{-c/2}}{\pi \sigma_{D_{Src}} \sigma_{D_{Ref}}} + \frac{b(s)}{\sqrt{2\pi \sigma_{D_{Src}} \sigma_{D_{Ref}}} a(s)^{3/2}} \exp\left(-\frac{1}{2}\left(c - \frac{b(s)^2}{a(s)}\right)\right) \operatorname{erf}\left(\frac{b(s)}{\sqrt{2a(s)}}\right)$$

$$\text{with } a(s) = \frac{s^2}{\sigma_{D_{Src}}^2} + \frac{1}{\sigma_{D_{Ref}}^2}, b(s) = \frac{\mu_{d_{Src}} s}{\sigma_{D_{Src}}^2} + \frac{\mu_{d_{Ref}}}{\sigma_{D_{Ref}}^2}, c = \frac{\mu_{d_{Src}}^2}{\sigma_{D_{Src}}^2} + \frac{\mu_{d_{Ref}}^2}{\sigma_{D_{Ref}}^2}$$

Given a scale s , the pdf of the random variable TX representing the translation in x can easily be computed. As we consider a gaussian noise on minutiae positions, the probability of TX given S considering each of the two minutiae pairs is gaussian too.

$$P(TX = tx | s, x_{Src}^i, x_{Ref}^j) = \mathcal{N}\left(x_{Src}^i - sx_{Ref}^j, \sqrt{\sigma_{Src}^i{}^2 + s^2 \sigma_{Ref}^j{}^2}\right), \text{ likewise}$$

$$P(TX = tx | s, x_{Src}^k, x_{Ref}^l) = \mathcal{N}\left(x_{Src}^k - sx_{Ref}^l, \sqrt{\sigma_{Src}^k{}^2 + s^2 \sigma_{Ref}^l{}^2}\right), \text{ the}$$

product of two gaussian is also a gaussian,

$$P(tx | s, x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l) = \mathcal{N}(\mu_{TX}, \sigma_{TX}) \text{ with :}$$

$$\sigma_{TX} = \frac{1}{\sqrt{\frac{1}{\sigma_{Src}^i{}^2 + s^2 \sigma_{Ref}^j{}^2} + \frac{1}{\sigma_{Src}^k{}^2 + s^2 \sigma_{Ref}^l{}^2}}} \text{ and}$$

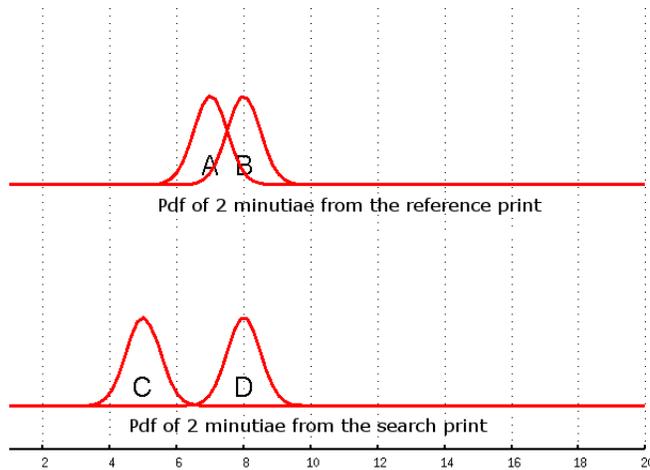
$$\mu_{TX} = \sigma_{TX}^2 \left(\frac{x_{Src}^i - sx_{Ref}^j}{\sigma_{Src}^i{}^2 + s^2 \sigma_{Ref}^j{}^2} + \frac{x_{Src}^k - sx_{Ref}^l}{\sigma_{Src}^k{}^2 + s^2 \sigma_{Ref}^l{}^2} \right)$$

The multiplication being the natural operator in the scale space, it is more convenient to work in log scale using a change of variables:

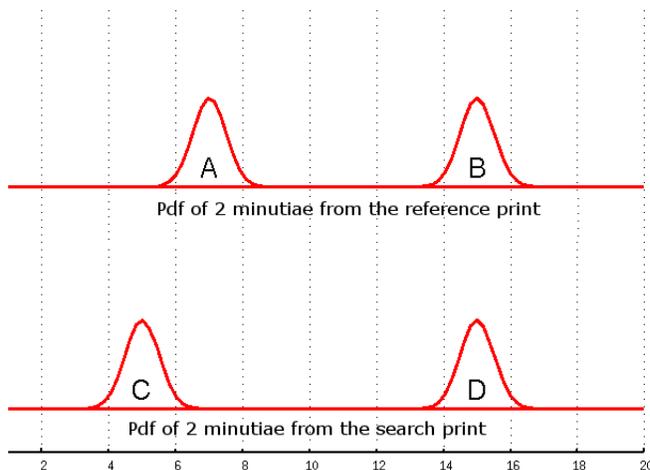
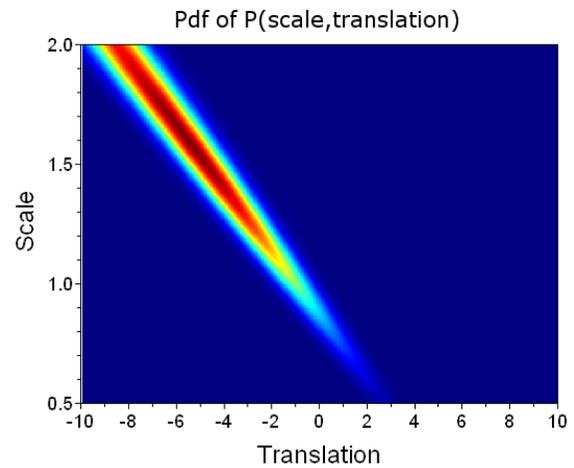
$$P(\log(s), tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l) = s \times P(s, tx | x_{Src}^i, x_{Ref}^j, x_{Src}^k, x_{Ref}^l)$$

We now focus on the variable TY , $P(TY = ty | y_{Src}^i, y_{Ref}^j)$ is easy to compute because there is no scale parameter involved. Considering a gaussian noise on minutiae positions, $P(TY = ty | y_{Src}^i, y_{Ref}^j)$ is also gaussian:

$$P(TY = ty | y_{Src}^i, y_{Ref}^j) = \frac{1}{\sqrt{2\pi \sigma_{ty}}} e^{-\frac{(y_{Src}^i - y_{Ref}^j + ty)^2}{2\sigma_{ty}^2}}$$



(a) Small distances between minutiae



(b) Large distances between minutiae

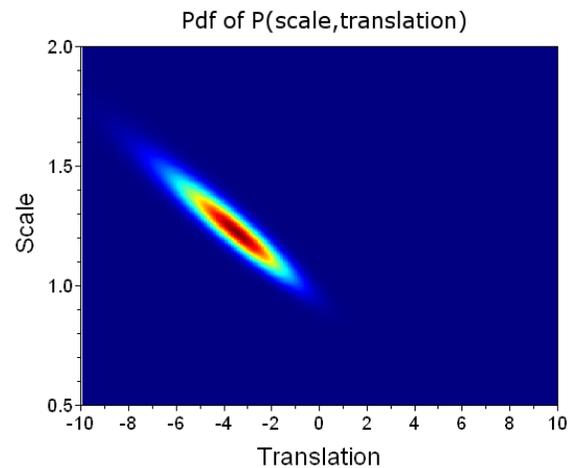


Fig. 6. Examples of pdf of (s, tx) function of minutiae positions considering A matches with C and B matches with D. We can observe the influence of distances between minutiae on the distribution entropy

2) Minutiae matching and score computation

When the descriptors are registered, the probability that two minutiae match is function of their type (positive or negative) and of the distance between them:

$$P(\text{match}(M_{Src}^i, M_{Ref}^j)) \propto \begin{cases} e^{-\frac{(y_{Src}^i - y_{Ref}^j)^2}{2\sigma_y^2} - \frac{(x_{Src}^i - x_{Ref}^j)^2}{2\sigma_x^2}} & \text{if } type_{Src}^i = type_{Ref}^j \\ 0 & \text{otherwise} \end{cases}$$

σ_x needs to be much bigger than σ_y because the ridge count is more stable than the location along the ridge flow direction. The matching needs to be less tolerant in areas of high minutiae density to avoid matching any of these areas with any other high density area. Therefore σ_x and σ_y are functions of the local minutiae density.

Finding the best matches is a bipartite graph matching problem. From a cost matrix $cost_{ij} = 1 - P(\text{match}(M_{Src}^i, M_{Ref}^j))$ we search for the set A of matches minimizing the sum of matching costs with the constraint that no minutia is used more than once. The value of the minimum can be used to compute a similarity score.

$$Score = -\min_A \left(\sum_{(i,j) \in A} cost_{ij} \right)$$

under constraints $(i,j) \in A \Rightarrow \neg \exists (i,k) \in A$ with $k \neq j$.

The Kuhn-Munkres algorithm [9] also called Hungarian algorithm solves this problem with polynomial complexity using dynamic programming. With the previously presented cost matrix, the score always increases when adding matches, even if those matches are highly improbable. This is not the sought behaviour because it automatically boosts the score when there are many minutiae.

The tuning of the tradeoff between the influence of true and wrong matches can be achieved by subtracting a constant $\alpha \in [0,1]$ to each value of the cost matrix. If α is close to 0, good matches increase the score but wrong ones do not have any effect. If α is close to 1, wrong matches decrease the score but good ones do not have any effect. In our implementation, we set $\alpha=0.6$ to balance the influence of good and wrong matches.

Some minutiae cannot be matched between the search and the reference. Instead of matching minutiae at any cost we would rather consider some minutiae as outliers. It can be done by simply adding lines and columns for outliers in the cost matrix:

$$cost = \begin{matrix} \begin{matrix} \# \text{ search minutiae} \\ \left[\begin{matrix} 1 - P(\text{match}(M_{Src}^i, M_{Ref}^j)) - \alpha & (\text{penalty} - \alpha) / 2 \\ (\text{penalty} - \alpha) / 2 & 0 \end{matrix} \right] \end{matrix} \\ \begin{matrix} \# \text{ reference minutiae} \\ \# \text{ search minutiae} \end{matrix} \end{matrix}$$

with $penalty - \alpha$ the threshold below which two minutiae are not matched.

Minutiae matched with lines and columns of penalties are considered as outliers.

We use the score computed by the Hungarian algorithm fed by this matrix as similarity score between two descriptors.

C. Similarity score of two fingerprints

To compute a global similarity score between two fingerprints, we merge scores obtained from many local

descriptors.

We first compute matched minutiae pairs with our standard minutiae matcher and use these matches to register the two fingerprints using a thin plate spline. This strategy is close to that of Bazen and Gerez in [6] but unlike them we do not use an iterative scheme. Descriptors with the same location on the two registered fingerprints are then compared. The global similarity score is the mean of the similarity scores of all descriptors pairs.

We want to describe locations which are more likely to give discriminative descriptors. The selection of the descriptors used to compute the global similarity score is based on the minutiae density on the search fingerprint. Areas with very high minutiae density are not discriminative because they often contain many false minutiae. Areas with very low minutiae density are not discriminative because of the lack of information.

We also want selected descriptors to be scattered over the fingerprint. The number of extracted descriptors usually varies from 3 to 10. A greedy selection which balances these two objectives is done as follows:

1. A descriptor is computed for every point of a regular grid to make a candidate list;
2. A potential is computed for each descriptor function of the local minutiae density in the descriptor;
3. The descriptor d with the highest potential is selected and removed from the candidate list;
4. If the potential $p(d)$ is below a threshold or there are more than N descriptors selected, exit;
5. The potential of every descriptor d' is updated with

$$\text{the rule: } p^{k+1}(d') = p^k(d') \times \left(1 - e^{-\frac{dst(d,d')^2}{2\sigma^2}} \right)$$

with $dst(d,d')$ being the Euclidian distance between d and d' locations;

6. Go to step 3.

IV. EXPERIMENTS

A. Tenprint-to-tenprint matching

We evaluate our local skeleton descriptor matching algorithm on a proprietary database composed of 30 000 rolled fingerprints on which skeletons are automatically extracted by our proprietary coder. We run 15 000 genuine tests and 900 000 impostor tests. We compare our skeleton algorithm with our standard minutiae matcher. We can see in Fig. 7 that the skeleton algorithm makes around twice as many false acceptance as the minutiae matcher. This difference of accuracy can be explained by the fact that the skeleton algorithm does not use orientation information and that it only performs many local matchings and does not use global coherence as the minutiae matcher does. In addition, the skeleton algorithm uses correspondences from the minutiae matcher to register the two fingerprints, therefore when the minutiae matcher fails completely the skeleton algorithm fails too. However the information used by the skeleton algorithm is complementary of the minutiae, so the

fusion of the two algorithms brings a substantial improvement in accuracy over the minutiae matcher alone. The fusion just consists of averaging the score of the two matchers.

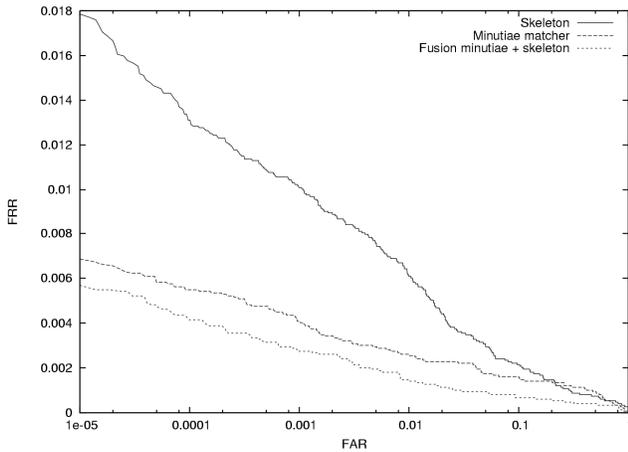


Fig. 7. ROC in TPTP matching

B. Latent-to-tenprint matching

Latent fingerprints are taken on crime scenes by investigators. This type of fingerprints is particularly hard to match because of many issues including strong noise, non uniform background and small surface. Skeletons and minutiae are hand marked by an expert on the latent fingerprints and are automatically extracted on reference fingerprints by our proprietary coder. We conduct experiments on the NIST27 latent database. This database is composed of 258 latent fingerprints and their corresponding TP. An additional 1000 rolled fingerprints database has been used to evaluate the accuracy of our skeleton matcher shown in Fig. 8.

The NIST27 database is divided into 3 parts of different difficulty. Our algorithm performs quite well on good quality fingerprints but the accuracy lowers when the quality decreases. Our algorithm is especially sensitive to the surface of the fingerprint. In LTTP the fusion with the minutiae matcher does not improve the accuracy.

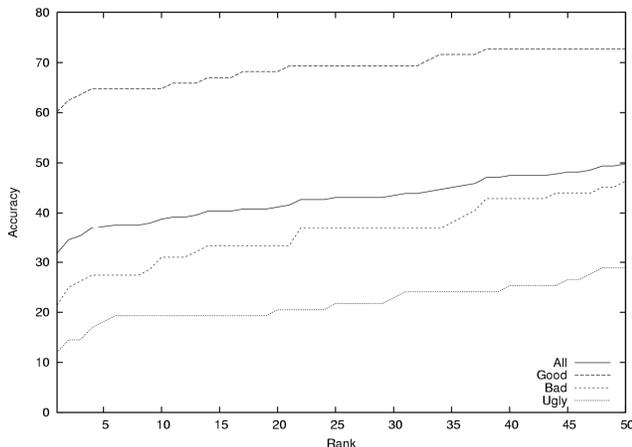


Fig. 8. Accuracy in LTTP matching

V. CONCLUSION

In this paper, a local skeleton descriptor and an associated matching algorithm have been presented. The descriptor takes advantage of the discriminative power of the ridge count information to build a compact and distortion robust representation of neighbouring minutiae. The two-step matching algorithm first registers couple of descriptors using a voting scheme and then matches them using dynamic programming.

Our algorithm achieves good accuracy especially in TPTP matching. Since our descriptor uses only the topological information from the skeleton, one of the most promising directions for further improvements is to combine our descriptor with another one based on the directional field like FingerCode [11] for example. Since the presented algorithm uses a minutiae matcher to perform the global matching, it would be interesting to develop another strategy to avoid it. This should lead to accuracy improvements especially for the fusion with the minutiae matcher.

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