The author(s) shown below used Federal funding provided by the U.S. Department of Justice to prepare the following resource:

Document Title:	**Graphical User Interface for Multi-Factorial Age-At-Death Estimation Method Using Fuzzy Integrals**
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Document Number:	**250466**
Date Received:	**December 2016**
Award Number:	**2011-DN-BX-K838**

Graphical User Interface for Multi-Factorial Age-At-Death Estimation Method Using Fuzzy Integrals

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ABSTRACT

Accurate and precise estimation of chronological age-at-death for a single skeleton is critical in forensic anthropology when developing a biological profile. Best practice standards indicate that age-at-death should be produced from the evaluation of multiple indicators of biological age from different regions of the skeleton when possible. However, there are few methods for conducting multi-factorial age-at-death estimations for a single skeleton. Because of this, most forensic anthropologist average information from multiple indicators using past experience and the skeletal remains present for a specific case. Still, each of the methods have different error rates and are most effective for different stages of life. Therefore, a standardized method for combining multiple indicators of age from a single skeleton into a single, accurate, and repeatable age-at-death estimation is needed in forensic anthropology. The purpose of this research project was to develop a graphical user interface (GUI) that uses algorithms based on fuzzy integrals and provides forensic scientists with a multifactorial age-at-death estimation, confidence in the estimation, informative graphs, and a standardized reproducible method to generate linguistic descriptions of the age-at-death estimation in medicolegal death investigations involving skeletal remains.

Fuzzy set theory is a powerful mathematical framework in which to model different types of uncertainty (e.g., probabilities, evidences, possibilities, etc.), perform computation (e.g., fuzzy logic), and it can be used to fuse (e.g., fuzzy integral) different information to provide a *confidence* in some hypothesis. In the case of age-at-death, it is used to provide a set of confidences for each age tested (ranging from 1 to 110 years). While the fuzzy integral came from the field of fuzzy set

theory, it belongs more to the field of aggregation operators. The fuzzy integral is a function generator, meaning it is a generic framework that can be used to produce a wealth of different aggregation operators based on a thing called the fuzzy measure. In this project, the fuzzy integral is used to provide a measure of strength of the hypothesis acquired by fusing (aggregating) distinct sources of information. The algorithm produces a decision regarding age-at-death using multiple interval-valued aging methods and does not require a population. Most prior multifactorial approaches are statistical and are generally based on the use of population information. As part of this grant we focused on the extension of the fuzzy integral to uncertain information and different application domains and skeletal age-at-death estimation. The most important extensions are the subnormal fuzzy integral, the generalized fuzzy integral, the non-direct fuzzy integral, and the application of the non-direct fuzzy integral to skeletal age-at-death estimating which are described in publications resulting from this grant.

While we published a number of manuscripts during the grant, the primary product of this project was to develop a user-friendly GUI for providing multifactorial age-at-death estimations. The GUI developed is freely available to forensic investigators and it allows forensic scientists to estimate age-at-death for a single skeleton using the age-at-death methods that they are most comfortable with and that are available based on the bones present, the condition of the bones, and the equipment available. The investigator can enter the score or stage for multiple age-at-death methods and receive a multifactorial age-at-death estimation, confidence in the estimation, and a reproducible, grounded linguistic way of interpreting the results. The graphs and linguistic terms can then be used in case reports and when testifying in court.

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INTRODUCTION

Statement of Problem

In forensic anthropological investigations, the estimation of chronological age-at-death based on skeletal morphology is a critical component to the biological profile. Accurate and precise estimations help narrow the search of potential missing persons and to aid in the identification of the skeleton. Age-at-death estimations that include too narrow of an error range can inadvertently exclude the missing individual from consideration, while estimations that include too broad of an error range will include an excess of irrelevant individuals.

Studies suggest that combining multiple indicators of biological age (multifactorial method) from different regions of the skeleton provides a more accurate estimation of chronological age than using any single age indicator (Bedford et al. 1993, Brooks 1955, Lovejoy et al. 1985a, SWGANTH 2010, Uhl and Nawrocki 2010). However, there are very few easy-to-use methods for combining multiple indicators for a single skeleton because the methods have different correlations with chronological age, different error rates, and are more effective for different stages of life (Nawrocki 2010, Uhl and Nawrocki 2010). As a result, most forensic anthropologist develop their own guidelines for combining multiple indicators based on their past experience and the skeletal remains present for a specific case (Christensen and Crowder 2009, Garvin and Passalacqua 2012), which is likely very subjective and unlikely reliable and reproducible. With increased demand by the courts for reliable and reproducible methods in forensic science (NAS 2009, NSTCCS 2014), a standardized protocol for combining multiple indicators of age that provides information about the confidence of the estimation is needed in forensic anthropology.

The purpose of this project was to facilitate accurate and reproducible multi-factorial age-at-death estimations based on human skeletal remains in the field of forensic anthropology by providing a standardized, easy-to-use, and interpretable method of combining multiple indicators of age. The goal of the project was to develop a graphical user interface (GUI) that uses algorithms based on fuzzy integrals and provides a multi-factorial age-at-death estimation for a single skeleton, confidence in the estimation, informative graphs, and a standardized reproducible approach to generate linguistic descriptions of age-at-death estimations. The GUI developed during this project will facilitate accurate multi-factorial age-at-death estimations and ensure consistency and reliability in the application of age-at-death estimations in forensic anthropology.

Project Goals and Objectives

The goal of this project was to develop a user-friendly GUI that forensic scientists conducting medicolegal death investigations could use to develop a multi-factorial age-at-death estimation. The GUI, which uses algorithms based on fuzzy integrals to produce the multi-factorial age-at-death estimation, also provides numeric, graphical, and linguistic information about the *quality* of the confidence of the estimation. Fuzzy set theory (Zadeh 1965) and linguistic summarization (Anderson et al. 2009, Kacprzyk and Wilbik 2010, Wu et al. 2010, Zadeh 1978) are used to provide a reproducible method of linguistically translating and interpreting the numeric results and measure the *specificity* of the age-at-death estimation (Anderson et al. 2011).

The objectives of the project were to 1) determine the age-at-death methods that need to be included in the GUI, 2) collect data to compile a database about the accuracy, error rate, and reproducibility of the methods on a wide variety of populations, 3) develop the necessary core libraries needed by the age-at-death algorithm, 4) extend the fuzzy measure procedure to discover the worth of combinations of different methods, 5) design, develop, and test the GUI, 6) develop a user manual, and 7) distribute the GUI.

METHODS

Age-At-Death Methods

There are a number of well-established and tested univariate methods for estimating age-at-death for adults from various regions of the skeleton. The majority of these are macroscopic methods based on degenerative changes in the skeleton. In order to determine the methods to utilize in the beta-version of the GUI, we conducted an extensive review of the forensic anthropological literature and a survey of members of the anthropology section of the American Academy of Forensic Sciences. The results suggest that the pubic symphysis, auricular surface, rib end morphology, and cranial sutures are frequently used by professional anthropologists to estimate age-at-death. Furthermore, many of the older methods (e.g., Gilbert and McKern 1973, Iscan et al. 1984, Katz and Suchey 1986, Lovejoy et al. 1985, McKern and Stewart 1957, Todd 1920) were more widely used than revised methods (e.g., Buckberry and Chamberlain 2002, DeGangi et al. 2009, Harnett 2010a, 2010b, Kimmerle et al. 2008, Osborn et al. 2004), regardless of the experience level of the investigator. Similar to the results of Garvin and Passalacqua (2012), our survey results indicate that the Suchey-Brooks method based on the pubic symphysis is the most commonly used method by practicing forensic anthropologists, but most participants stated that they try to find population specific methods. Dental methods, such as macroscopic evaluation of attrition, tooth transparency, and dental cementum annulation (Drusini et al. 1989, Lamendin et al. 1992, Prince and Ubelaker 2002, Robbins Schug et al. 2012), and histological methods (Ericksen 1991, Kerley 1965, Kerley and Ubelaker 1978, Robling and Stout 2000, Stout and Paine 1992 and others) were not commonly used by professional anthropologists even though the literature suggest they may provide more accurate and precise estimations of ag-at-death. As expected, most forensic anthropologists combine these techniques to establish a final age-at-death estimation by using experience, rather than statistical or mathematical methods.

Based on this collected information, we decided to include in the beta-version of our GUI methods associated with the auricular surface, pubic symphysis, sternal rib ends, cranial sutures, acetabulum, and sacrum. We combined a number of older and relatively new methods. The methods selected are presented in Table 1. In future versions of the GUI, we will select methods based on the methods commonly used and literature on how well they perform. For example, in our collection of data from a documented skeletal collection, we found that age estimations based on the sacrum poorly defined age-at-death and added very little information about the confidence. As a result, this method will not be provided as an option in the GUI. Other methods, such as cranial suture closure, have relatively low correlations with chronological age but still contributed to the estimation of age-at-death. Future versions of the GUI will also include dental and histological methods.

Table 1. Age-at-Death Methods Chosen for Beta-Version of Graphical User Interface

Method	Reference
Pubic Symphysis	Todd 1920, 1921, Katz and Suchey 1986, McKern and Stewart 1957, Gilbert and McKern 1973, Harnett 2010a
Auricular Surface	Lovejoy et al 1985, Buckberry and Chamberlin 2002
Sternal Rib End	Iscan et al. 1984, Hartnett 2010b
Cranial Sutures	Meindl and Lovejoy 1985
Acetabulum	Calce 2012
Sacrum	Passalacqua 2009

Fuzzy Integral, Fuzzy Measures and Extensions of the Fuzzy Integral

Overview

We developed a novel, multi-factorial approach to account for inaccuracy in aging methods by using the fuzzy integral to produce a confidence in skeletal age-at-death estimations. This method, which will be describe in detail below, has several advantages over others in that it is a multi-factorial method that allows investigators to use nearly any well-established and tested age-at-death indicator methods and fuse the information about the accuracy of the methods with other types of information that can be quantified, such as the quality of the bone (Anderson 2008, Anderson et al. 2010). No other method allows for the fusion of information about the quality of the bone with the accuracy of the methods. Other advantages of the fuzzy integral age-at-death method are that it can be easily used for a single skeleton, it can be used for both adult and immature skeletons, it can be customized to meet the investigator's needs on specific cases, and it provides informative graphs and a standardized way of linguistically translate and interpret results of the age-at-death estimations (Anderson et al. 2011).

The fusion of information using the fuzzy integral (Sugeno or Choquet) has a rich history. Much of the theory and several applications can be found in Grabisch et al. (1995, 2000). In this report, we just provide a background on the real (aka number) valued Sugeno and Choquet integral and we summarize our extensions and provide appropriate references--due to the fact that those extensions, applications and proofs span twelve publications.

With respect to skeletal age-at-death estimation, we consider a finite set of information sources, $X = \{x_1, \ldots, x_E\}$, and a function (h) that maps X into some domain (initially [0,1]) that represents the partial support of a hypothesis from the standpoint of each information source. Depending on the problem domain, X can be a set of experts, sensors, features, or pattern recognition algorithms. Herein, X is different age-at-death methods. The hypothesis is usually thought of as an alternative in a decision process or a class label in pattern recognition. Herein, the hypothesis is that the individual died at a specific age, e.g., 25. The fuzzy integral is used multiple times, once for each age under question. Both Sugeno and Choquet integrals take partial support for the hypothesis from the standpoint of each source of information and they fuse it with the (perhaps subjective) worth of each subset of X in a non-linear fashion. This worth is encoded in a fuzzy measure [1]. Initially, the function $h: X \rightarrow [0,1]$, and the measure $g: 2^X \rightarrow [0,1]$ took real number values in the interval [0,1]. Certainly, the output range for both function and measure can be (and have been) defined more generally, but it is convenient to think of them in the unit interval for *confidence* fusion.

Fuzzy Measure

The concept of the measure is one of the most significant concepts in mathematics. All of us have used measures in one form or another (e.g., determining the length of an interval $[a, b]$ according to $b - a$, measuring area, volume, etc). Classical measures are associated with the so-called additive property. While additivity is appropriate in many situations, it can be inadequate in many real-world scenarios. The fuzzy measure does not have the property of additivity. Instead, it has a weaker property of monotonicity related to the inclusion of sets. In context, it is noteworthy to state that the fuzzy measure is also frequently referred to simply as a monotone and normal measure, in particular in cases where it is not defined on fuzzy sets. Both the Sugeno and Choquet FIs are defined with respect to a fuzzy measure.

A measurable space is the tuple (X, Ω), where *X* is a set (e.g., the set of real numbers) and Ω is a σ-algebra or set of subsets of *X* such that:

1. $X \in \Omega$.
2. Let A be a subset of X. If $A \in \Omega$ then $A^c \in \Omega$.
3. If $A_n \in \Omega$ then $\bigcup_{n=1}^{\infty} A_n \in \Omega$.

For example, when X is the set of real numbers and Ω is the σ-algebra that contains the open subsets of X, then Ω is the Borel σ-algebra. The Sugeno fuzzy measure g is a set valued function $g: \Omega \to [0,1]$ that satisfies the following properties,

1. Boundary conditions:

   $g(\emptyset) = 0$ and $g(X) = 1$

2. Monotonicity:

   If A, B $\in \Omega$ and $A \subseteq$ B, then $g(A) \leq g(B)$

3. Continuity:

If $\{A_i\}$ is an increasing subsequence of subsets of Ω, then

$$g\left(\bigcup_{i=1}^{\infty} A_i\right) = lim_{i\to\infty} g(A_i)$$

Property 1 asserts that the "worth" of the empty set is 0, while the "worth" of all sets is 1. Property 2 is the monotonicity property of the FM (i.e. if A is a subset of B, then the "worth" of A is smaller or equal to that of B). Property 3 is not applicable when X is a finite set, as it is for this report. A benefit of the fuzzy measure is that it is monotone, which is a weaker property than additivity. Well-known measures that have the additive property is the familiar probability measure and the Lebesgue measure. Figure 1 shows the fuzzy measure lattice for three information sources.

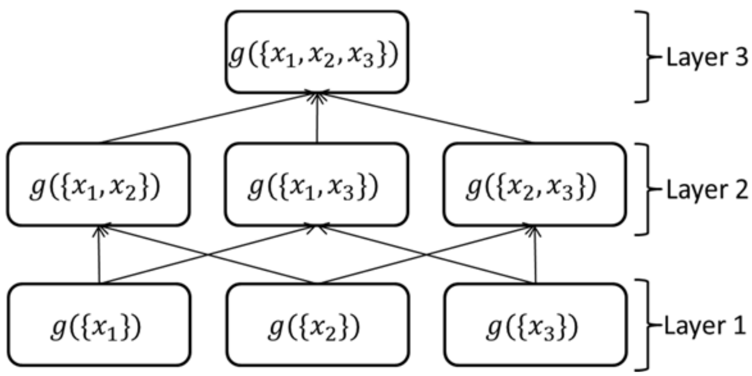

Fig. 1. Fuzzy measure lattice for three information sources.

Fuzzy Integral

Both the Sugeno and Choquet integrals take partial support for a hypothesis from the standpoint of each source of information, $h: X \to [0,1]$, and fuse it with the (perhaps subjective) "worth" of each subset of X (encoded in a fuzzy measure) in a non-linear fashion. The Sugeno and Choquet fuzzy integrals are defined as:

$$\int_S h \circ g = S_g(h) = \bigvee_{i=1}^{N} \left(h(x_{(i)}) \wedge g(\{x_{(1)}, \dots, x_{(i)}\})\right)$$

and

$$\int_C h \circ g = C_g(h) = \sum_{i=1}^{N} h(x_{(i)}) \left(g\left(A_{(i)}\right) - g\left(A_{(i-1)}\right)\right)$$

where $X = \{x_1, \dots, x_N\}$ has been sorted such that $h\left(x_{(1)}\right) \geq h\left(x_{(2)}\right) \geq \cdots \geq h(x_{(N)})$, $A_{(i)} = \left\{x_{(1)}, \dots, x_{(i)}\right\}$ and $g\left(A_{(0)}\right) = 0$. Note, other formulations such as the Mobius fuzzy integral exist in which sorting is not required. This finite realization of the actual definition highlights the fact that the Sugeno integral represents the *best pessimistic agreement* between the objective evidence in support of a hypothesis (the h function) and the subjective worth of the supporting evidence (the fuzzy measure g). That is, we compute an intersection (∧, a t-norm in fuzzy set theory) on each of the inner equations terms (a pessimistic operation). We can think of this as not believing more in what a particular input tells us than what we believe in that subset of sources. We then union (∨, a t-conorm in fuzzy set theory) of those results (an optimistic operation). Thus, we take the "best" (optimistic) of the "smallest" (a pessimistic operation) of these computed values. On the other hand, the Choquet integral is a direct extension of the Lebesque integral--and the Lebesque integral is "recovered" for an additive measure. There is not a similar story for the Choquet integral. It is more grounded in classical integral theory (from Calculus). It is important to note that these two integrals do not necessarily provide the same answer. The min and max (intersection and union) operations of the Sugeno restrict the result to be one of the inputs or one of the fuzzy measure values. However, the Choquet integral can, and does, return any value between the min and max of the inputs (depending on the selection of the fuzzy measure).

Extensions of the Fuzzy Integral

In our work directly leading up to this grant and subsequent works supported both fully and partially from this grant, a total of twelve publications appeared from our research team (Anderson et al. 2012, 2014, 2015, Havens et al. 2013, 2014, 2015, Hu et al. 2013, 2014, Price et al., 2013, Wagner and Anderson 2013, Wagner et al 2013). The majority of these works are focused on the formal extension of the fuzzy integral to uncertain information, different application domains and skeletal age-at-death estimation. The extensions most relevant to this particular grant are summarized in Algorithms 1-4.

Algorithm 1 is the subnormal fuzzy integral (SuFI). SuFI is the way to calculate the fuzzy integral when the integrands (our "h" information) are fuzzy set valued and do not have a height of one--note a fuzzy set is called 'normal' (height of 1) if there exists at least one element for which the fuzzy set is equal to 1. For example, in skeletal age-at-death estimation, a fuzzy set captures the uncertainty in the degree to which a particular age (the "domain") belongs to a target concept (e.g. aging method Y says that the individual died at this age). As we outline in Algorithm 4, our inputs are not always normal fuzzy sets. That is, their height can, and often is, less than 1. However, before we can address such a complicated scenario of extending the integral we must start with a simple scenario such as all inputs are normal fuzzy sets.

Algorithm 1 Computation of the SuFI algorithm

Input the $\Re$-valued FM g ▷ *e.g., use the Sugeno λ-FM, learn g from data, manually specify g, etc.*
Input partial support function $\hat{\hat{H}}$ ▷ *sub-normal FN*
Calculate $\beta = \bigwedge_{i=1}^{N} Height(\hat{\hat{H}}_i))$ ▷ *minimum height of partial support FSs*
for each $\alpha \in (0, \beta]$ **do** ▷ *note, this step is discretized in practice*
    $[\int \hat{\hat{H}} \circ g]_\alpha = [\int [^\alpha \hat{\hat{H}}]^- \circ g, \int [^\alpha \hat{\hat{H}}]^+ \circ g]$ ▷ *the $\Re$-based (integrand and measure) FI*
end for

Grabisch (Grabish et al. 1995, 2000) and Dubois and Prade (1987) were responsible for proposing and proving SuFI. The next algorithm our group extended was the generalized fuzzy integral (gFI), Algorithm 2. The gFI is capable of fusing sets that are not normal (i.e., do not have a height of 1). The gFI can even handle non-convex fuzzy sets, whereas SuFI could only deal with normal and convex sets (which are referred to in the fuzzy set community as a fuzzy number).

Algorithm 2 Algorithm to calculate the *generalized FI* (gFI)

Input the $\Re$-valued FM g ▷ *e.g., use the Sugeno λ-FM, learn g from data or manually specify g, etc.*
Input FS-valued partial support function H ▷ *i.e., H_i for $i = \{1, ..., N\}$*
Calculate $\beta = \bigwedge_{i=1}^{N} Height(H_i)$ ▷ *minimum height of partial support FSs*
for each $\alpha \in (0, \beta]$ **do** ▷ *note, this step is discretized in practice*
  $[\int H \circ g]_\alpha = (\int [{}^\alpha H]_1 \circ g)$ ▷ *first gFI calculation, I-based integrand and $\Re$-based FM FI*
  for k = 2 to $\mathcal{M}_\alpha$ **do** ▷ *all discontinuous interval combinations at α*
    $[\int H \circ g]_\alpha = [\int H \circ g]_\alpha \bigcup (\int [{}^\alpha H]_k \circ g)$ ▷ *the I-based integrand and $\Re$-based FM FI*
  end for
end for

The third algorithm we extended is the non-direct fuzzy integral (NDFI). It is called non-direct because we did not use Zadeh's Extension Principle (which helps us extend functions from real-valued inputs to set-valued inputs) to extend the integral. It is still a legit extension, however its extension is not done in a "conventional" manner.

Algorithm 3 Algorithm to calculate the *non-direct FI* (NDFI)

Input the $\Re$-valued FM g ▷ *e.g., use the Sugeno λ-FM, learn g from data, manually specify g, etc.*
Input the FS-valued partial support function H ▷ *i.e., H_i for $i = \{1, ..., N\}$*
Discretize the output domain, $D = \{d_1, ..., d_{|D|}\}$ ▷ *e.g., $D = \{0, 0.01, ..., 1\}$*
Initialize the (FS) result to $R[d_k] = 0$
for each $d_k \in D$ **do** ▷ *for each output domain location*
  for each $i \in \{1, ..., N\}$ **do** ▷ *for each input*
    Let $z_i = H(d_k)$ ▷ *i.e., $\mathbf{z}$ is the vector of memberships at d_k*
    $R[d_k] = \int \mathbf{z} \circ g$ ▷ *calculated using the $\Re$-valued (integrand and measure) FI of $\mathbf{z}$ with g*
  end for
end for

However, SuFI, gFI and NDFI do not directly solve skeletal age-at-death estimation. Last, we extended NDFI to skeletal age-at-death estimation in Algorithm 4.

Algorithm 4 NDFI algorithm for skeletal age-at-death estimation for forensic anthropology

Input fuzzy measure g ▷ *e.g., use the Sugeno λ-FM, learn g from data, manually specify g, etc.*
Input bone quality weathering values, $\{q_1, ..., q_N\}$ ▷ *Where $q_i \in [0,1]$*
Input age-at-death intervals for each aging method, $\{\bar{v}_1, ..., \bar{v}_N\}$ ▷ *Where $\bar{v}_i$ is an age interval, e.g., $\bar{v}_i = [5, 20]$ years*
Discretized the output domain, $D = \{d_1, ..., d_{|D|}\}$ ▷ *e.g., $D = \{1, 2, ..., 110\}$*
Initialize the (FS-valued) result to $R[d_k] = 0$
for each $d_k \in D$ **do** ▷ *i.e., each discrete age*
  for $i = 1$ to N **do** ▷ *Calculate the partial support function at d_k*
    if $d_k \geq v_i^-$ and $d_k \leq v_i^+$ **then** ▷ *Where $-$ and $+$ are the left and right endpoints, e.g., $[v_i^-, v_i^+]$*
      $z_i = q_i$ ▷ *Age method i indicates possible age-at-death, use bone quality q_i*
    else
      $z_i = 0$ ▷ *Age method i indicates not a possible age-at-death, so no support in the hypothesis*
    end if
  end for
  $R[d_k] = \int \mathbf{z} \circ g$ ▷ *Fuzzy membership at d_k is the $\Re$-valued (integrand and measure) FI of $\mathbf{z}$ with g*
end for

Note, the reader can refer to the list of disseminations from this project for full mathematical details, proofs, etc. We would like to note that while gFI is a "classical extension", we showed that NDFI actually produces results that are more intuitive and useful for age-at-death estimation.

RESULTS: GRAPHICAL USER INTERFACE

One of the main goals of the project was to develop a user-friendly graphical user interface for combining multiple, commonly used indicators of age-at-death. The current web-based GUI and accompanying user manual will be freely available to forensic scientists. A stand-alone desktop-based interface will also become available in the future. The GUI allows forensic investigators to enter age-at-death stages and composite scores selected based on their method preference and the skeletal remains available. The output is a multi-factorial age-at-death estimation or interval, graphical representation, and linguistic interpretation. The procedure for using the interface is outlined below using the selection of the Suchey-Brooks pubic symphysis and Meindl ectocranial suture closure methods as an example. To use the interface the investigator performs the following:

1) Open the webpage (Figure 2).

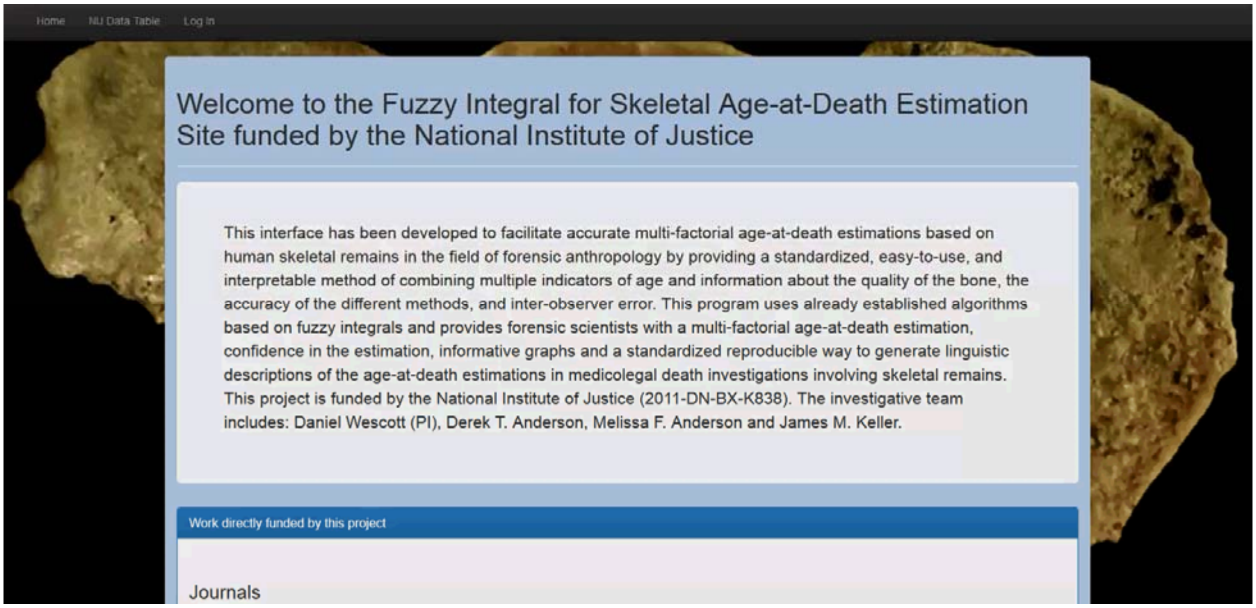

Figure 2. GUI homepage.

2) Click on the Data Table tab to view a table with all the methods available for selection.

3) Select the chosen methods and input the stage using the dropdown tab (Figures 3 and 4).

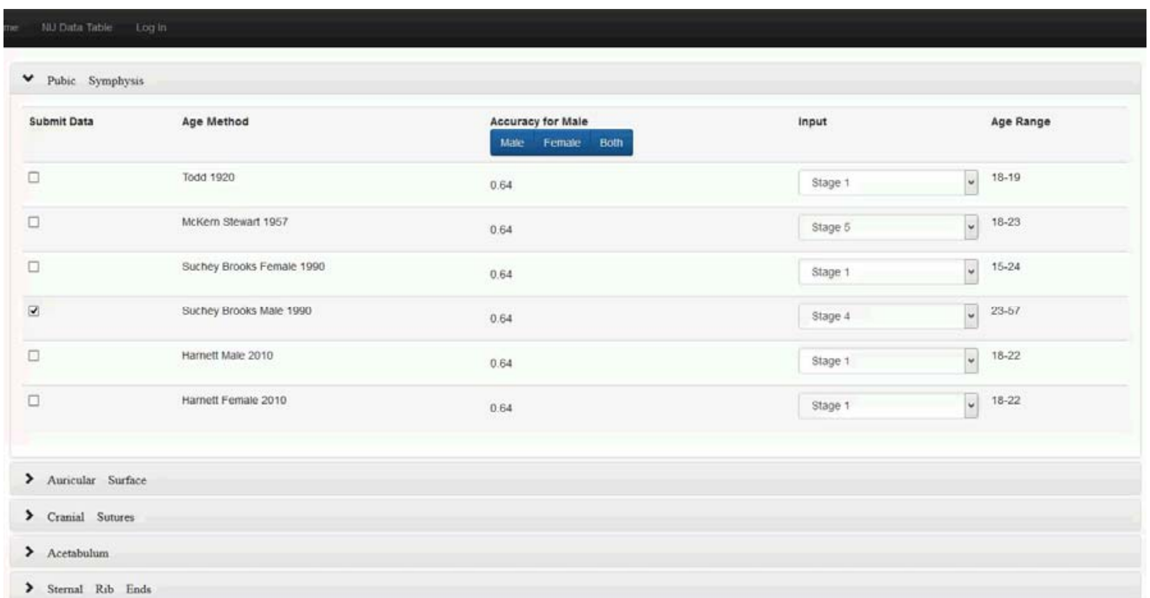

Figure 3. Data Table showing methods and the selection of the Suchey-Brooks method.

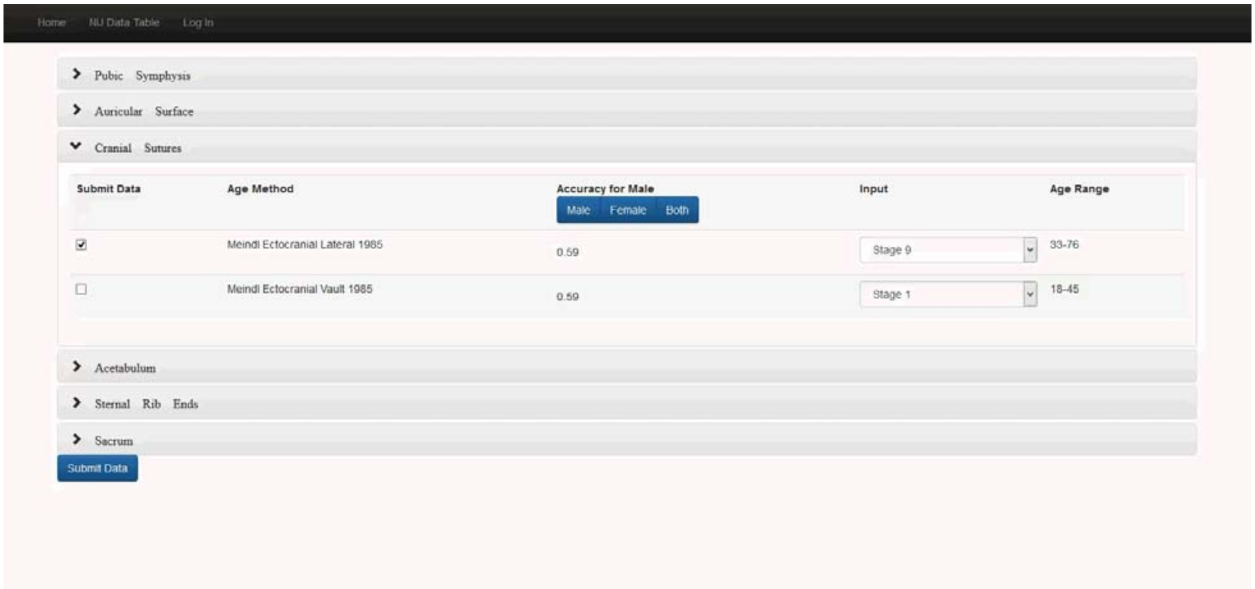

Figure 4. Data Table showing methods and the selection of the Meindl ectocranial suture closure method.

4) Click “Submit Data.” The results will display a multi-factorial age range, graph, linguistic data, and Tuple output (Figure 5).

The output provides reproducible vocabulary (fuzzy sets) about 1) the support of membership of the age-at-death fuzzy set into one of four classes (i.e., specific, interval, inconclusive, and reconsideration), 2) the specificity or information about how specific the fuzzy set is with respect to being able to narrow the age-at-death estimation to a single value, and 3) confidence in the age-at-death estimation. The graph *class* is used to simplify interpretation of the age-at-death estimation (Anderson et al. 2010). A fuzzy set that has high membership into a “specific” class graph indicates that the age-at-death methods have a high degree of confidence for a specific age. A more common type of graph, the “interval” graph suggests that there is similar confidence for a consecutive range of ages but the length of the plateau is less than 30 years. If the length of the plateau in an interval graph is greater than 30 years then the fuzzy set is considered inconclusive (Anderson et al. 2010). Finally, if there are two or more peaks with similar confidence the graph is placed in the "reconsideration" class. Reconsideration graphs suggest that the results of the methods selected are too variable to make a conclusion about age-at-death. That is, the age-at-death methods are in extreme disagreement. The *specificity* provides information how specific the fuzzy set is with regards to being able to narrow down the age range to a single value (Anderson et al. 2011). The specificity can be exact, high, moderate, or low. As the maximum fuzzy integral value decreases, so does the confidence in the amount of attainable specificity for the fuzzy set. Finally, the confidence in the age-at-death estimation can be classified as high, moderate, low, or no support (Anderson et al. 2011).

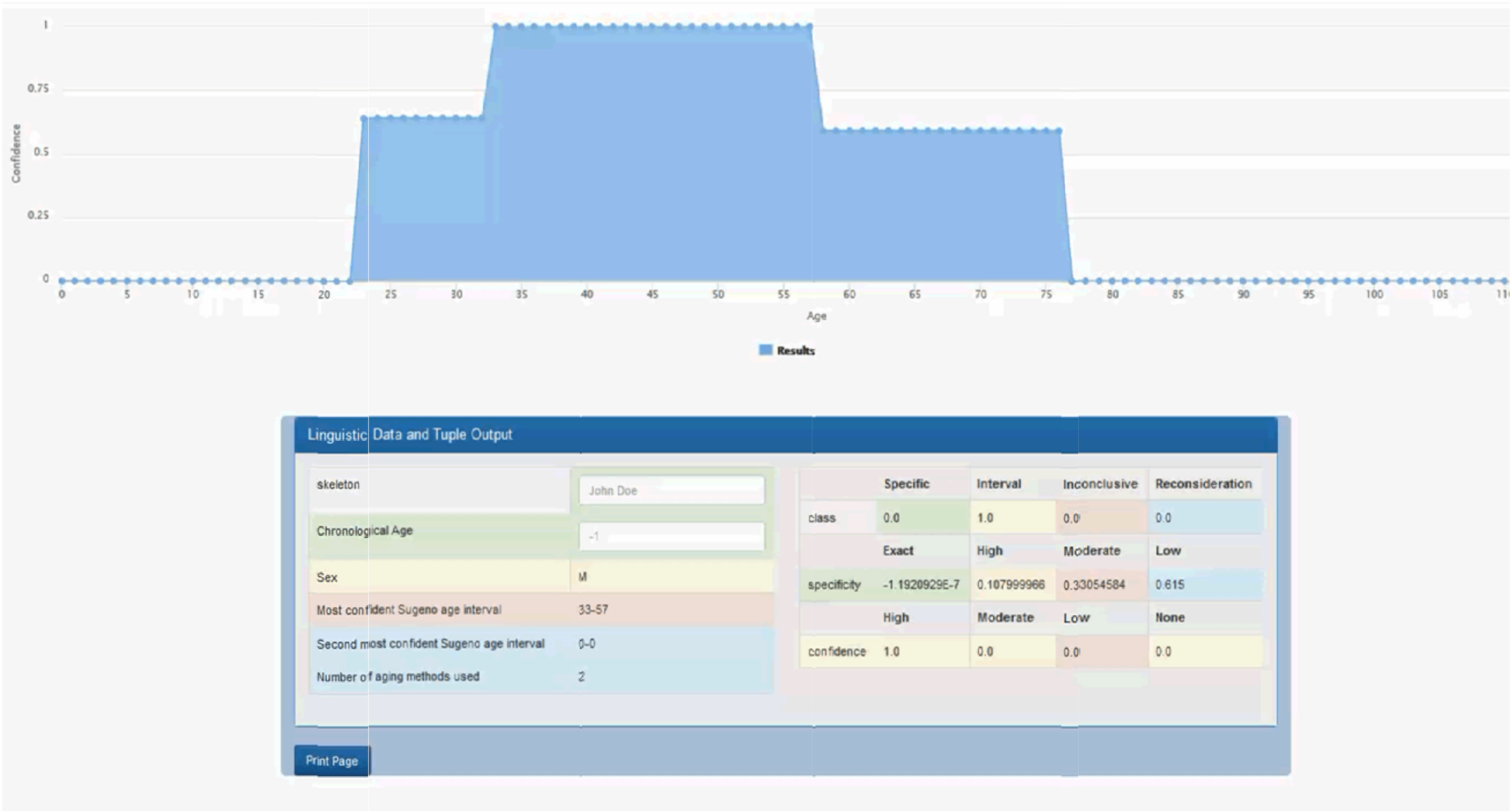

Figure 5. Results based on data input for the pubic symphysis and cranial suture.

CONCLUSIONS

Implications for Policy and Practice

Age estimation is a crucial component of the biological profile in forensic anthropological case work. Best practices in anthropology recommend utilizing multiple indicators of age when making an estimation based on skeletal remains (SWGANTH 2010). However, no specific guidelines are provided for the best way to combine the various methods used. Forensic anthropologists are recommended to synthesize the available information keeping in mind the reference sample for the method, the forensic anthropologist's skill at the method, and the condition of the remains. However, with the call for strengthening the forensic sciences and using reliable and reproducible methods, there is a strong need for an standardized method for conducting multi-factorial age-at-death estimation that utilize methods commonly employed by forensic anthropologists. The graphical user interface developed in this project is freely available to forensic scientists and it will allow them to input skeletal data into the GUI and receive results that provide an age-at-death estimation, a measure of the confidence in the estimation, and graphs that can be presented in a report and during testimony.

MAIN BODY OF TECHNICAL REPORT

INTRODUCTION

Statement of Problem

Accurate and precise estimations of chronological age-at-death based on skeletal remains are critical in forensic anthropological analyses to help narrow the search of potential missing persons and to aid in the identification of the skeleton. Age-at-death estimations that include too narrow of an error range can inadvertently exclude the missing individual from consideration, while estimations that include too broad of an error range will include an excess of irrelevant individuals. Studies suggest that combining multiple indicators of biological age (multi-factorial method) from different regions of the skeleton provides a more accurate estimation of chronological age than using any single age indicator (Bedford et al. 1993, Brooks 1955, Lovejoy et al. 1985a, SWGANTH 2010, Uhl and Nawrocki 2010). However, there are relatively few standardized multi-factorial approach for the estimation of age-at-death for a *single* skeleton (Boldsen et al. 2002, Uhl and Nawrocki 2010). As a result, forensic anthropologists develop their own guidelines for combining multiple indicators of age, often based on their past experience and the skeletal remains present for a specific case (Christensen and Crowder 2009, Garvin and Passalacqua 2012). Unfortunately, the most recent "best practices" for estimation of age-at-death developed by the Scientific Working Group in Forensic Anthropology (SWGANTH 2010) provide no clear guidelines for combining multiple indicators of age. Therefore, standards or best practices for the acceptance and interpretation of age-at-death estimations based on multiple skeletal indicators of age are needed in forensic anthropology (Christensen and Crowder 2009). A standardized protocol for combining multiple indicators of age will ensure consistency and reliability in the application of age-at-death estimations in forensic anthropology.

The purpose of this research project was to develop a graphical user interface (GUI) that uses algorithms based on fuzzy integrals and provides a multi-factorial age-at-death estimation for a single skeleton, confidence in the estimation, informative graphs, and a standardized reproducible approach to generate linguistic descriptions of age-at-death estimations. The GUI uses fuzzy integrals to produce an age-at-death estimation and a measure of confidence or trust in the estimation. Fuzzy integral acquired fuzzy sets are then used to provide results about the age-at-death estimation that are reproducible. Unlike other multi-factorial methods, the current approach allows investigators to estimate age-at-death for a single skeleton by applying the well-established age methods they are comfortable using and that are available to them based on the bones present, the condition of the bones, and the equipment they have accessible. Furthermore, unlike other methods, the GUI allows the investigator to incorporate additional information about the quality of the bone and in the future include inter-observer error and other quantifiable variable about the uncertainty of the method.

Literature Citations and Review

Age-at-Death Estimation from Skeletal Remains

While accurate estimation of age is critical for the development of a biological profile in medicolegal cases involving skeletal remains, the estimation of age is one of the most difficult tasks facing the forensic anthropologist. There are a number of well-established and tested univariate methods for estimating age-at-death for various regions of the skeleton. The most commonly used methods for adults include age-related morphological changes in the pubic symphysis (e.g., Todd 1920, Katz and Suchey 1986, McKern and Stewart 1957, Gilbert and McKern 1973), auricular surface of the ilium (Buckberry and Chamberlain 2002, Lovejoy et al. 1985b, Osborn et al. 2004), sternal end of the ribs (Iscan et al. 1984), and cranial suture closure (Meindl et al. 1985), tooth root transparency (Lamendin et al. 1992, Prince and Ubelaker 2002) and cortical bone histology (Ericksen 1991, Kerley 1965, Kerley and Ubelaker 1978, Robling and Stout 2000, Stout and Paine 1992 and others) for adult remains. However, all of these univariate estimation methods, especially for estimating age in adults, have a high degree of inaccuracy and bias because of differences between chronological and biological (development or senescence) age due to genetic variation and environmental or lifestyle differences (e.g., diet, disease, and activity levels), atypical variation, repeatability and reproducibility of methods, and taphonomic alterations of the skeletal morphology (Boldsen et al. 2002, Hoppa 2000, Hoppa and Vaupel 2002, Meindl et al. 1985, Schmitt et al. 2002). Most univariate methods are also influenced by the age-at-death distribution of the test or reference sample causing age mimicry or bias towards the composition of the test or reference sample (Boldsen et al. 2002, Konigsberg and Frankenberg 2002, Meindl et al. 1985).

Forensic anthropologists overcome some of these problems by examining multiple indicators of age. Each indicator of age tracks different aspects of the aging process and therefore provides a different portion of the overall variance in aging. Therefore, multiple indicators provide a more accurate estimation of age. Even poor indicators of age (those with low correlations with chronological age such as cranial suture closure) can contribute to the estimation of age-at-death. Furthermore, combining multiple age indicators developed on different reference samples avoids some of the problems associated with age mimicry (Lovejoy et al. 1985a), although not all (Algee-Hewitt 2013). The problem is that there is no standardized method to combine multiple indicators of age in medicolegal cases involving a single skeleton because the methods have different correlations with chronological age, different error rates, and are most effective for different stages of life (Nawrocki 2010, Uhl and Nawrocki 2010).

Multi-factorial Methods for Estimating Age-at-Death

Most anthropologists agree that the use of multiple indicators of age provide more accurate estimations of death than univariate methods. The SWGANTH (2010) document on age states that "Most research suggest that for adults, consulting multiple age indicators provides more accurate results than using single indicators" (p.3). However, the document provides no guidelines for combining the multiple indicators of age into a single age-at-death estimation.

Ascadi and Neimesckeri (1970) proposed one of the earliest multi-factorial methods they called the "complex" method. The complex method uses observations of the pubic symphysis, radiographic translucency of the proximal femur and humerus, and endocranial suture closure. However, the complex method is rarely used in forensic anthropology because it is frequently limited by incomplete or damaged remains and radiographs may not be available to the forensic anthropologist. Furthermore, the weighted average of the method causes an attraction to the middle and therefore tends to over age young individuals and underage older individuals.

Later, Lovejoy et al. (1985a) presented the Multifactorial Summary Age (MSA) method that combined pubic symphyseal, iliac auricular surface, cranial suture closure, trabecular involution of the proximal femur, and dental wear. The Lovejoy et al. (1985a) method, however, requires an assemblage of skeletons to be seriated, and therefore is not applicable to most forensic anthropological cases. Furthermore, Saunders et al. (1992) examined the accuracy of single indicators of age and the MSA method on an historic cemetery sample and found that the MSA method did not provide more accurate estimations of age than many of the univariate methods.

Uhl and Nawrocki (2010) examined four methods of combining multiple age indicators. In their study, they used the pubic symphysis (Brooks and Suchey 1990), auricular surface (Osborne et al. 2004), sternal rib end (Iscan et al. 1984), and cranial suture closure (Nawrocki, 1998). The methods they tested included: 1) averaging the point estimates for the four methods, 2) using the total minimum range of the 95% prediction intervals for each method, 3) using the total maximum range of the 95% prediction intervals for each method, and 4) a multiple linear regression equation utilizing all four variable weighted by accuracy. Averaging of the four indicators was found to be easy to use and improved the accuracy of the age estimation. However, a limitation to this method is that it does not provide a valid approach of producing an error range or predictive interval (Uhl and Nawrocki 2010). The two methods using the range of the 95% prediction intervals were also found to provide a more accurate estimation of age than single indicators. While these methods do provide age range estimations, neither provides a good point estimate other than using the average of the ranges (Uhl and Nawrocki 2010). Furthermore, the total maximum range method provided such large intervals that the age estimation is impractical for forensic anthropology. Uhl and Nawrocki (2010) argued that the multiple regression method was the most accurate and provided a valid point estimate and error range. However, to use the multiple regression method all four age indicators must be present and scored using the same method as the original study. As a result, this multiple regression method may be limited in many forensic anthropological cases. While the multiple regression method can provide a confidence interval, it does not provide a confidence in the point estimate.

In 2002, Boldsen and colleagues proposed transition analysis as a multi-factorial method that allows forensic anthropologists to combine different indicators of age in a single skeleton. This method for estimating age-at-death uses information about the age at which a skeletal feature transition from one stage or phase to the next higher stage to calculate a likelihood of death for a specific age and confidence intervals about the point estimate (Boldsen et al. 2002, Konigsberg et al. 2008). Multiple indicators can be combined to produce a single probabilistic estimate (Hurst 2010, Wright and Yoder 2003). To avoid age mimicry, data from more representative or appropriate age-at-death distribution can be used as a prior informative when conducting transition analyses (Boldsen et al. 2002, Konigsberg et al. 2008). Other advantages of the transition analysis method are that it provides better coverage than traditional methods and can be used for single skeletons (Konigsberg et al. 2008). While this multi-factorial method has considerable promise, transition analysis is not commonly used in forensic anthropology because of its statistical complexity and the lack of an easy to use tool for forensic anthropologist to derive an age-at-death estimation. A computer software program called ADBOU was developed by Boldsen and Milner and uses transition analyses to calculate the maximum likelihood point age estimates and confidence intervals (Hurst 2010). However, the program is limited to adult skeletons where the cranial sutures, iliac auricular surface, and pubic symphysis morphology can be observed and scored using the protocol outlined by Milner and Boldsen (2012). In addition, the "forensic" prior distribution used in ADBOU software was calculated from 1996 homicide data collected by the

Center for Disease Control, which may not be an acceptable analogue for forensic anthropological cases (Hurst 2010).

Anderson and colleagues (Anderson 2008, Anderson et al. 2010) developed a novel multi-factorial approach to account for inaccuracy in aging methods by using the Sugeno fuzzy integral to produce a confidence in skeletal age-at-death estimations. This method, which will be describe in detail below, has several advantages over others in that it is a multi-factorial method that allows investigators to use nearly any well established and tested age-at-death indicator methods and fuse the information about the accuracy of the methods with other types of information that can be quantified, such as the quality of the bone (Anderson 2008, Anderson et al. 2010). No other method allows for the fusion of information about the quality of the bone with the accuracy of the methods. Other advantages of the method are that it can be easily used for a single skeleton, it can be used for both adult and immature skeletons, it can be customized to meet the investigator's needs on specific cases, and it provides informative graphs and a standardized way of linguistically translate and interpret results of the age-at-death estimations. Unlike transition analysis, the fuzzy integral does not model a population or produce likelihood estimates, but rather produces results for assessing the age of a skeleton without knowledge of the population from which any method was developed.

Rationale for Research

The goal of this project was to develop a user-friendly GUI that produces a multi-factorial age-at-death estimation as well as information (numeric, graphical, and linguistic) about the *quality* of the confidence of the estimation. Fuzzy set theory (Zadeh 1965) and linguistic summarization (Anderson et al. 2009, Kacprzyk and Wilbik 2010, Wu et al. 2010, Zadeh 1973) were used to provide a reproducible method of linguistically translating and interpreting the numeric results and measure the *specificity* of the age-at-death estimation.

METHODS

Age-At-Death Methods

There are a number of well-established and tested univariate methods for estimating age-at-death for adults from various regions of the skeleton. The majority of these are macroscopic methods based on degenerative changes in the skeleton. In order to determine the methods to utilize in the beta-version of the GUI, we conducted an extensive review of the forensic anthropological literature and a survey of members of the anthropology section of the American Academy of Forensic Sciences. The results suggest that the pubic symphysis, auricular surface, rib end morphology, and cranial sutures are frequently used by professional anthropologists to estimate age-at-death. In our survey of professionals, the pubic symphysis was the preferred skeletal region for estimating age-at-death followed by the auricular surface and rib ends. Examination of cranial sutures and dental attrition were also noted as preferred methods. Many of the older methods (e.g., Gilbert and McKern 1973, Iscan et al. 1984, Katz and Suchey 1986, Lovejoy et al. 1985, McKern and Stewart 1957, Todd 1920) were more widely used than revised methods (e.g., Buckberry and Chamberlain 2002, DeGangi et al. 2009, Hartnett 2010a,b, Kimmerle et al. 2008, Osborn et al. 2004), regardless of the experience level of the investigator. However, several individuals commented that the preferred method changed depending on their initial impression of age. Similar to the results of Garvin and Passalacqua (2012), our survey results indicate that the Suchey-Brooks method based on the pubic symphysis is the most commonly used method by practicing forensic anthropologists, but most participants stated that they try to find population specific methods. Dental methods, such as macroscopic evaluation of attrition, tooth transparency, and dental cementum annulation (Drusini et al. 1989, Lamendin et al. 1992, Prince and Ubelaker 2002, Robbins Schug et al. 2012), and histological methods (Ericksen 1991, Kerley 1965, Kerley and Ubelaker 1978, Robling and Stout 2000, Stout and Paine 1992 and others) were not commonly used by professional anthropologists even though the literature suggest they may provide more accurate and precise estimations of ag-at-death.

As expected, most forensic anthropologists combine these techniques to establish a final age-at-death estimation by using experience rather than statistical or mathematical methods. For example, many experts narrow the age range by weighting the known accuracy and precision of the methods and focus on areas of overlap of the multiple methods. Other experts tend to average the ages for multiple indicators using their intuition rather than any statistical method. However, some professionals rely most heavily on one method, such as the pubic symphysis, and adjust their age estimation based on whether other indicators are outside the range provided by the primary method.

Based on this information collected, we decided to include in the beta-version of our GUI methods associated with the auricular surface, pubic symphysis, sternal rib ends, cranial sutures. For these regions we combined a number of older and relatively new methods. In addition, we included some newer methods based on the acetabulum and sacrum because they were similar to the preferred methods reported. The methods selected are described below. In future versions of the GUI, we will select methods based on the methods commonly used and literature on how well they perform. For example, in our collection of data from a documented skeletal collection, we found that age estimations based on the sacrum poorly defined age-at-death and added very little information about the confidence. As a result, this method will not be provided as an option in later versions of the GUI.

Pubic Symphysis

1. Todd Method (Reference: Todd TW. Age changes in the pubic bone: I. the white male pubis. *American Journal of Physical Anthropology* 1920;3:285-334 and Todd TW. Age changes in the pubic bone II-IV: the pubis of the male negro-white hybrid, the pubis of the white female, the pubis of the female negro-white hybrid. *American Journal of Physical Anthropology* 1921;4:1-70). This method divides macroscopic changes in the pubic bone into ten phases with associate age ranges.
2. Suchey-Brooks Method (Reference: Katz D, Suchey JM. Age determination of the male os pubis. *American Journal of Physical Anthropology* 1986;69:427-435. This method is a revision of the Todd method. Cast of the stages and separate standards for males and females are available.
3. Hartnett Method (Reference: Hartnett KM. Analysis of age-at-death estimation using data from a new, modern autopsy sample—part I: pubic bone. *Journal of Forensic Sciences* 2010;55:1145-1151). This method is a revision of the Suchey-Brooks method with the addition of a new phase (phase seven) for older individuals.
4. McKern and Stewart Method (Reference: McKern TW, Stewart TD. Skeletal age changes in young American males analyzed from the standpoint of age identification. Technical Report EP-45, Quartermaster Research and Development Command, Natick, MA. 1957).

This method is a three-component system where the dorsal plateau, ventral rampart, and symphyseal rim development are scored on a scale of 0 to 5 and added to give a total score.

5. Gilbert and McKern Method (Reference: Gilbert BM, McKern TW. A method for aging the female os pubis. American Journal of Physical Anthropology 1973;38:31-38). This method follows the component score system of McKern and Stewart (1957) but provides descriptions for females.

Auricular Surface

6. Lovejoy Method (Reference: Lovejoy CO, Meindl RS, Pryzbeck TR, and Mensforth RP. Chronological metamorphosis of the auricular surface of the ilium: a new method for the estimation of adult skeletal age at death. *American Journal of Physical Anthropology* 1985;68:15-28). This method examines seven different morphological traits of the auricular surface (i.e., grain and density, macroporosity, microporosity, billowing, striations, transverse organization, attributes of the apex, and changes in the retroauricular areas) to create eight distinct phases that correspond to age.
7. Buckberry-Chamberlin Method (Reference: Buckberry JL, Chamberlin AT. Age estimation from the auricular surface of the ilium: a revised method. *American Journal of Physical Anthropology* 2002;119:231-239). This is a revision of the original Lovejoy et al (1985) method. The method evaluates features of the auricular surface to develop a composite score associated with seven stages.

Sternal Rib End

8. Iscan Method (Reference: Işcan MY, Loth SR, Wright RK. Metamorphosis of the sternal rib end: a new method to estimate age at death in white males. *American Journal of Physical Anthropology* 1984;65:147-156 and Iscan MY, Loth SR. Determination of age from the sternal rib in white females: a test of the phase method. *Journal of Forensic Sciences* 1986;31:900-999). Rib end pit depth, pit shape, and rim and wall configuration are scored to develop a total component score. Casts are available.
9. Hartnett Method (Reference: Harnett KM. Analysis of age-at-death estimation using data from a new, modern autopsy sample – part II: sternal end of the fourth rib. *Journal of Forensic Sciences* 2010;55:1152-1156). This method revises the phases developed by Iscan et al. (1984) creating a seven phase method.

Cranial Sutures

10. Meindl and Lovejoy Ectocranial Method (Reference: Meindl RS, Lovejoy CO. Ectocranial suture closure: a revised method for the determination of skeletal age at death based on the lateral-anterior sutures. *American Journal of Physical Anthropology* 1985;68:57-66). Small regions of the lateral-anterior sutures are scored and a composite score is develop to estimate age.

Acetabulum

11. Calce Method (Reference: Calce SE. A new method to estimate adult age-at-death using the acetabulum. American Journal of Physical Anthropology 2012;148:11-23). The method condenses the technique set forth by Rissech et al. (2006) and examines age-related

morphological changes in the acetabular groove, acetabular rim porosity, and apex activity. It is primarily useful for narrowing age to broad stages of young, middle, or older adult.

Sacrum

12. Passalacqua Method (Reference: Passalacqua NV. Forensic age-at-death estimation from the human sacrum. Journal of Forensic Sciences 2009;54:255-262). This method uses seven morphological traits of the sacroiliac joint to create a series of six age-related phases.

While these twelve methods were selected for the beta-version, in future versions of the GUI, we will modify the methods included based on how commonly they are used and how well they add to the age-at-death estimation. Methods that added little information regarding the confidence in the age-at-death estimation (e.g., Passalacqua) will be removed. Methods such as cranial suture closure, however, are still commonly used by forensic anthropologists and provide some information to the estimation of age-at-death by separating young, middle, and older adults and therefore may be retained. Future versions of the GUI will also include histological and dental methods as well as methods developed for estimating age in non-adult remains.

Fuzzy Integral, Fuzzy Measures, and Extensions of the Fuzzy Integral

Overview

We developed a novel, multi-factorial approach to account for inaccuracy in aging methods by using the fuzzy integral to produce a confidence in skeletal age-at-death estimations. This method, which will be describe in detail below, has several advantages over others in that it is a multi-factorial method that allows investigators to use nearly any well-established and tested age-at-death indicator methods and fuse the information about the accuracy of the methods with other types of information that can be quantified, such as the quality of the bone (Anderson 2008, Anderson et al. 2010). No other method allows for the fusion of information about the quality of the bone with the accuracy of the methods. Other advantages of the fuzzy integral age-at-death method are that it can be easily used for a single skeleton, it can be used for both adult and immature skeletons, it can be customized to meet the investigator's needs on specific cases, and it provides informative graphs and a standardized way of linguistically translate and interpret results of the age-at-death estimations.

The fusion of information using the fuzzy integral (Sugeno or Choquet) has a rich history. Much of the theory and several applications can be found in Grabisch et al. (1994, 2000). In this report, we just provide a background on the real (aka number) valued Sugeno and Choquet integral and we summarize our extensions and provide appropriate references--due to the fact that those extensions, applications and proofs span twelve publications.

With respect to skeletal age-at-death estimation, we consider a finite set of information sources, $X = \{x_1, \ldots, x_E\}$, and a function (h) that maps X into some domain (initially [0,1]) that represents the partial support of a hypothesis from the standpoint of each information source. Depending on the problem domain, X can be a set of experts, sensors, features, or pattern recognition algorithms. Herein, X is different age-at-death methods. The hypothesis is usually thought of as an alternative in a decision process or a class label in pattern recognition. Herein, the hypothesis is that the individual died at a specific age (e.g., 25 years). The fuzzy integral is used multiple times, once for each age under question. Both Sugeno and Choquet integrals take partial support for the hypothesis from the standpoint of each source of information and they fuse it with the (perhaps subjective) worth of each subset of X in a non-linear fashion. This worth is encoded in a fuzzy measure [1]. Initially, the function $h: X \rightarrow [0,1]$, and the measure $g: 2^X \rightarrow [0,1]$ took real number values in the interval [0,1]. Certainly, the output range for both function and measure can be (and have been) defined more generally, but it is convenient to think of them in the unit interval for *confidence* fusion.

Fuzzy Measure

The concept of the measure is one of the most significant concepts in mathematics. All of us have used measures in one form or another, e.g., determining the length of an interval $[a, b]$ according to $b - a$, measuring area, volume, etc. Classical measures are associated with the so-called additive property. While additivity is appropriate in many situations, it can be inadequate in many real-world scenarios. The fuzzy measure does not have the property of additivity. Instead, it has a weaker property of monotonicity related to the inclusion of sets. In context, it is noteworthy to state that the fuzzy measure is also frequently referred to simply as a monotone and normal measure, in particular in cases where it is not defined on fuzzy sets. Both the Sugeno and Choquet FIs are defined with respect to a fuzzy measure.

A measurable space is the tuple (X, Ω), where X is a set (e.g., the set of real numbers) and Ω is a σ-algebra or set of subsets of X such that:

4. $X \in \Omega$.
5. Let A be a subset of X. If $A \in \Omega$ then $A^c \in \Omega$.
6. If $A_n \in \Omega$ then $\bigcup_{n=1}^{\infty} A_n \in \Omega$.

For example, when X is the set of real numbers and Ω is the σ-algebra that contains the open subsets of X, then Ω is the Borel σ-algebra. The Sugeno fuzzy measure g is a set valued function $g: \Omega \rightarrow [0,1]$ that satisfies the following properties,

4. Boundary conditions:

   $g(\emptyset) = 0$ and $g(X) = 1$

5. Monotonicity:

   If A, B ∈ Ω and $A \subseteq$ B, then $g(A) \leq g(B)$

6. Continuity:

   If $\{A_i\}$ is an increasing subsequence of subsets of Ω, then

$$g\left(\bigcup_{i=1}^{\infty} A_i\right) = lim_{i \rightarrow \infty} g(A_i)$$

Property 1 asserts that the "worth" of the empty set is 0, while the "worth" of all sets is 1. Property 2 is the monotonicity property of the FM, i.e. if A is a subset of B, then the "worth" of A is smaller or equal to that of B. Property 3 is not applicable when X is a finite set, as it is for this article. A benefit of the fuzzy measure is that it is monotone, which is a weaker property than additivity. Well-known measures that have the additive property is the familiar probability measure and the Lebesgue measure. Figure 6 shows the fuzzy measure lattice for three information sources.

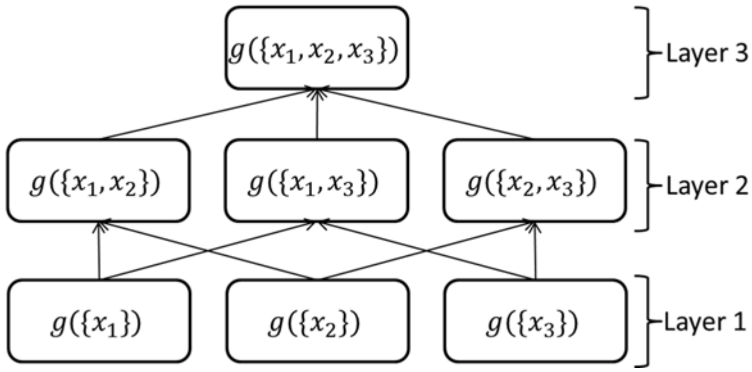

Fig. 6. Fuzzy measure lattice for three information sources.

Fuzzy Integral

Both the Sugeno and Choquet integrals take partial support for a hypothesis from the standpoint of each source of information, $h: X \rightarrow [0,1]$, and fuse it with the (perhaps subjective) "worth" of each subset of X (encoded in a fuzzy measure) in a non-linear fashion. The Sugeno and Choquet fuzzy integrals are defined as:

$$\int_S h \circ g = S_g(h) = \bigvee_{i=1}^{N} \left(h(x_{(i)}) \wedge g(\{x_{(1)}, \dots, x_{(i)}\})\right)$$

and

$$\int_C h \circ g = C_g(h) = \sum_{i=1}^{N} h(x_{(i)}) \left(g\left(A_{(i)}\right) - g\left(A_{(i-1)}\right)\right)$$

where $X = \{x_1, \dots, x_N\}$ has been sorted such that $h\left(x_{(1)}\right) \geq h\left(x_{(2)}\right) \geq \cdots \geq h\left(x_{(N)}\right)$, $A_{(i)} = \left\{x_{(1)}, \dots, x_{(i)}\right\}$ and $g\left(A_{(0)}\right) = 0$. Note, other formulations such as the Mobius fuzzy integral exist in which sorting is not required. This finite realization of the actual definition highlights the fact that the Sugeno integral represents the *best pessimistic agreement* between the objective evidence in support of a hypothesis (the h function) and the subjective worth of the supporting evidence (the fuzzy measure g). That is, we compute an intersection (∧, a t-norm in fuzzy set theory) on each of the inner equations terms (a pessimistic operation). We can think of this as not believing more in what a particular input tells us than what we believe in that subset of sources. We then union (⋁, a t-conorm in fuzzy set theory) of those results (an optimistic operation). Thus, we take the "best" (optimistic) of the "smallest" (a pessimistic operation) of these computed values. On the other hand, the Choquet integral is a direct extension of the Lebesque integral--and the Lebesque integral is "recovered" for an additive measure. There is not a similar story for the Choquet integral. It is more grounded in classical integral theory (from Calculus). It is important to note that these two integrals do not necessarily provide the same answer. The min and max (intersection and union) operations of the Sugeno restrict the result to be one of the inputs or one of the fuzzy measure values. However, the Choquet integral can, and does, return any value between the min and max of the inputs (depending on the selection of the fuzzy measure).

Extensions of the Fuzzy Integral

In our work directly leading up to this grant and subsequent works supported both fully and partially from this grant, a total of twelve publications appeared from our research team (Anderson et al. 2011, 2012, 2014, 2015, Havens et al. 2013, 2014, 2015, Hu et al. 2013, 2014, Price et al., 2013, Wagner and Anderson 2013, Wagner et al 2013). The majority of these works are focused on the formal extension of the fuzzy integral to uncertain information, different application domains and skeletal age-at-death estimation. The extensions most relevant to this particular grant are summarized in Algorithms 1-4.

Algorithm 1 is the subnormal fuzzy integral (SuFI). SuFI is the way to calculate the fuzzy integral when the integrands (our "h" information) are fuzzy set valued and do not have a height of one--note a fuzzy set is called ‘normal’ (height of 1) if there exists at least one element for which the fuzzy set is equal to 1. For example, in skeletal age-at-death estimation, a fuzzy set captures the uncertainty in the degree to which a particular age (the “domain”) belongs to a target concept (e.g. aging method Y says that the individual died at this age). As we outline in Algorithm 4, our inputs are not always normal fuzzy sets. That is, their height can, and often is, less than 1. However, before we can address such a complicated scenario of extending the integral we must start with a simple scenario such as all inputs are normal fuzzy sets.

Algorithm 1 Computation of the SuFI algorithm

Input the $\Re$-valued FM g ▷ *e.g., use the Sugeno λ-FM, learn g from data, manually specify g, etc.*
Input partial support function $\hat{\hat{H}}$ ▷ *sub-normal FN*
Calculate $\beta = \bigwedge_{i=1}^{N} Height(\hat{\hat{H}}_i))$ ▷ *minimum height of partial support FSs*
for each $\alpha \in (0, \beta]$ **do** ▷ *note, this step is discretized in practice*
  $[\int \hat{\hat{H}} \circ g]_\alpha = [\int [^\alpha \hat{\hat{H}}]^- \circ g, \int [^\alpha \hat{\hat{H}}]^+ \circ g]$ ▷ *the $\Re$-based (integrand and measure) FI*
end for

Grabisch and Dubois and Prade were responsible for proposing and proving SuFI. The next algorithm our group extended was the generalized fuzzy integral (gFI), Algorithm 2. The gFI is capable of fusing sets that are not normal (i.e., do not have a height of 1). The gFI can even handle non-convex fuzzy sets, whereas SuFI could only deal with normal and convex sets (which are referred to in the fuzzy set community as a fuzzy number).

Algorithm 2 Algorithm to calculate the *generalized FI* (gFI)

Input the $\Re$-valued FM g ▷ *e.g., use the Sugeno λ-FM, learn g from data or manually specify g, etc.*
Input FS-valued partial support function H ▷ *i.e., H_i for $i = \{1, ..., N\}$*
Calculate $\beta = \bigwedge_{i=1}^{N} Height(H_i)$ ▷ *minimum height of partial support FSs*
for each $\alpha \in (0, \beta]$ **do** ▷ *note, this step is discretized in practice*
  $[\int H \circ g]_\alpha = \left(\int [^\alpha H]_1 \circ g\right)$ ▷ *first gFI calculation, I-based integrand and $\Re$-based FM FI*
  for k = 2 to $\mathcal{M}_\alpha$ **do** ▷ *all discontinuous interval combinations at α*
    $[\int H \circ g]_\alpha = [\int H \circ g]_\alpha \bigcup \left(\int [^\alpha H]_k \circ g\right)$ ▷ *the I-based integrand and $\Re$-based FM FI*
  end for
end for

The third algorithm we extended is the non-direct fuzzy integral (NDFI). It is called non-direct because we did not use Zadeh’s Extension Principle (which helps us extend functions from real-valued inputs to set-valued inputs) to extend the integral. It is still a legit extension, however its extension is not done in a “conventional” manner.

Algorithm 3 Algorithm to calculate the *non-direct FI* (NDFI)

Input the $\Re$-valued FM g ▷ *e.g., use the Sugeno λ-FM, learn g from data, manually specify g, etc.*
Input the FS-valued partial support function H ▷ *i.e., H_i for $i = \{1, ..., N\}$*
Discretize the output domain, $D = \{d_1, ..., d_{|D|}\}$ ▷ *e.g., $D = \{0, 0.01, ..., 1\}$*
Initialize the (FS) result to $R[d_k] = 0$
for each $d_k \in D$ **do** ▷ *for each output domain location*
    for each $i \in \{1, ..., N\}$ **do** ▷ *for each input*
        Let $z_i = H(d_k)$ ▷ *i.e., $\mathbf{z}$ is the vector of memberships at d_k*
        $R[d_k] = \int \mathbf{z} \circ g$ ▷ *calculated using the $\Re$-valued (integrand and measure) FI of $\mathbf{z}$ with g*
    end for
end for

However, SuFI, gFI and NDFI do not directly solve skeletal age-at-death estimation. Last, we extended NDFI to skeletal age-at-death estimation in Algorithm 4.

Algorithm 4 NDFI algorithm for skeletal age-at-death estimation for forensic anthropology

Input fuzzy measure g ▷ *e.g., use the Sugeno λ-FM, learn g from data, manually specify g, etc.*
Input bone quality weathering values, $\{q_1, ..., q_N\}$ ▷ *Where $q_i \in [0, 1]$*
Input age-at-death intervals for each aging method, $\{\bar{v}_1, ..., \bar{v}_N\}$ ▷ *Where $\bar{v}_i$ is an age interval, e.g., $\bar{v}_i = [5, 20]$ years*
Discretized the output domain, $D = \{d_1, ..., d_{|D|}\}$ ▷ *e.g., $D = \{1, 2, ..., 110\}$*
Initialize the (FS-valued) result to $R[d_k] = 0$
for each $d_k \in D$ **do** ▷ *i.e., each discrete age*
    for $i = 1$ to N **do** ▷ *Calculate the partial support function at d_k*
        if $d_k \geq v_i^-$ and $d_k \leq v_i^+$ **then** ▷ *Where $-$ and $+$ are the left and right endpoints, e.g., $[v_i^-, v_i^+]$*
            $z_i = q_i$ ▷ *Age method i indicates possible age-at-death, use bone quality q_i*
        else
            $z_i = 0$ ▷ *Age method i indicates not a possible age-at-death, so no support in the hypothesis*
        end if
    end for
    $R[d_k] = \int \mathbf{z} \circ g$ ▷ *Fuzzy membership at d_k is the $\Re$-valued (integrand and measure) FI of $\mathbf{z}$ with g*
end for

Note, the reader can refer to the list of disseminations from this project for full mathematical details, proofs, etc. We would like to note that while gFI is a “classical extension”, we showed that NDFI actually produces results that are more intuitive and useful for age-at-death estimation.

RESULTS: GRAPHIC USER INTERFACE

One of the main goals of the project was to develop a user-friendly graphical user interface for combining multiple, commonly used indicators of age-at-death. The current web-based GUI and accompanying user manual will be freely available to forensic scientists. A stand-alone desktop-based interface will also become available in the future. The GUI allows forensic investigators to enter age-at-death stages and composite scores selected based on their method preference and the skeletal remains available. The output is a multi-factorial age-at-death estimation or interval, graphical representation, and linguistic interpretation. The procedure for using the interface is outlined below using the selection of the Suchey-Brooks pubic symphysis and Meindl ectocranial suture closure methods as an example. To use the interface the investigator performs the following:

5) Open the webpage (Figure 7).

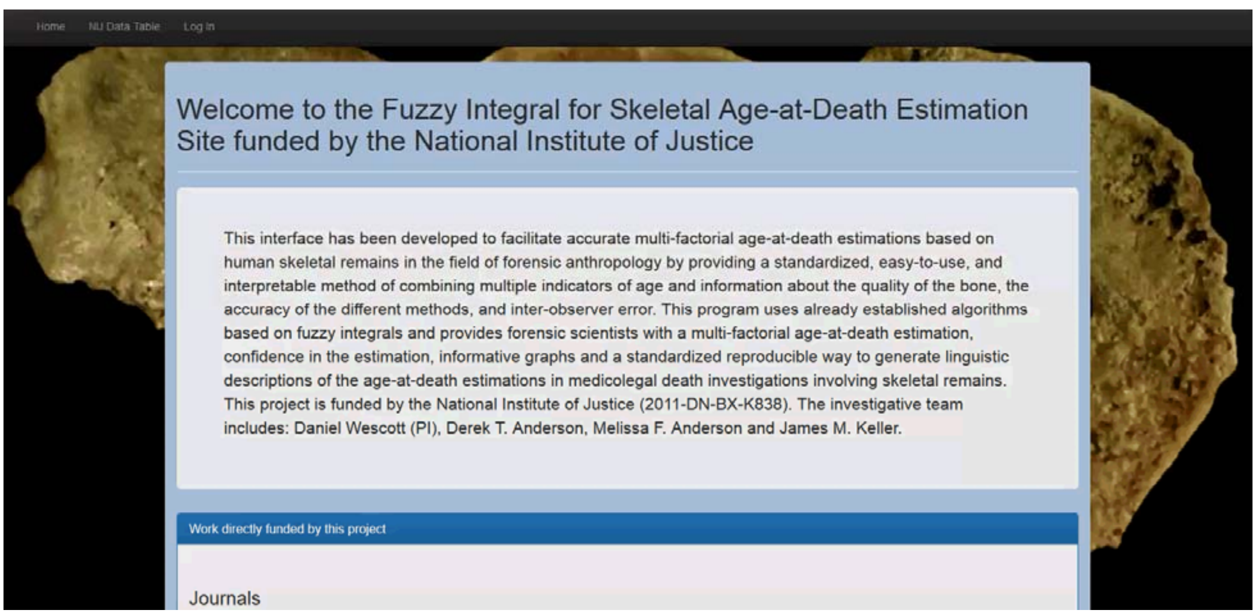

Figure 7. GUI homepage.

6) Click on the Data Table tab to view a table with all the methods available for selection.

7) Select the chosen methods and input the stage using the dropdown tab (Figures 8 and 9).

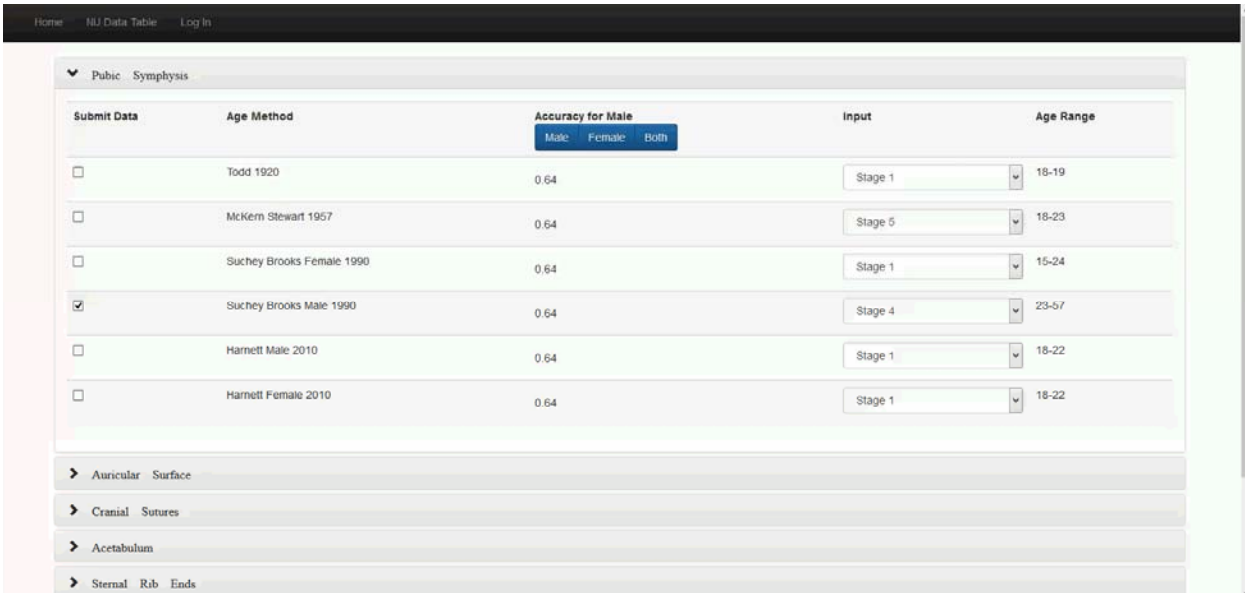

Figure 8. Data Table showing methods and the selection of the Suchey-Brooks method.

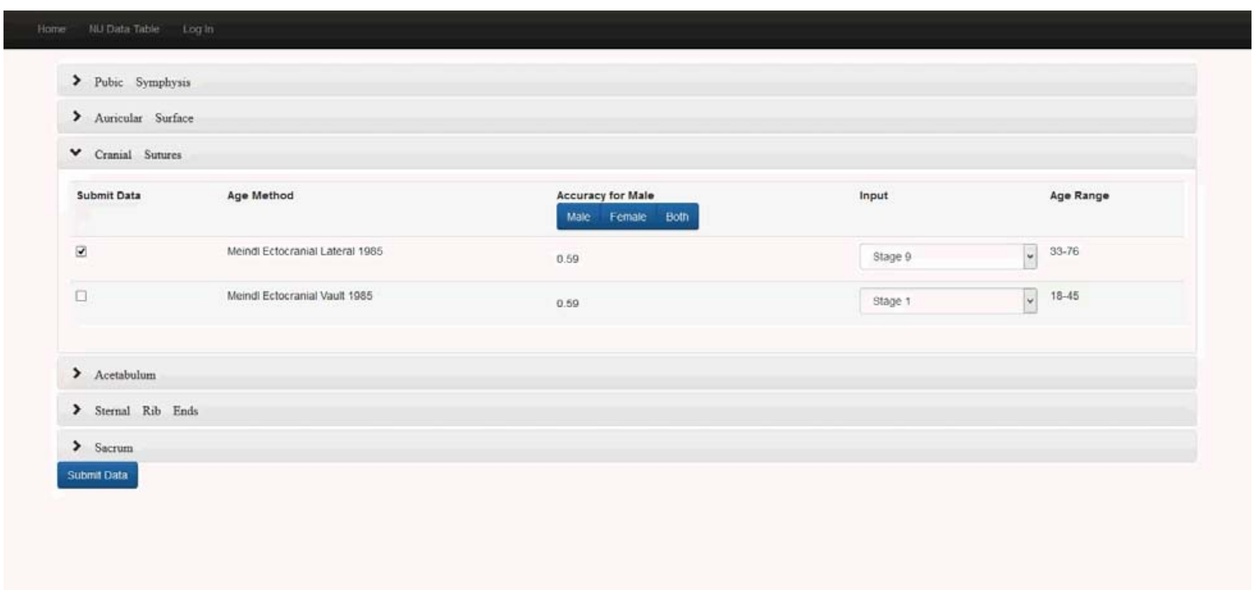

Figure 9. Data Table showing methods and the selection of the Meindl ectocranial suture closure method.

8) Click “Submit Data.” The results will display a multi-factorial age range, graph, linguistic data, and Tuple output (Figure 10).

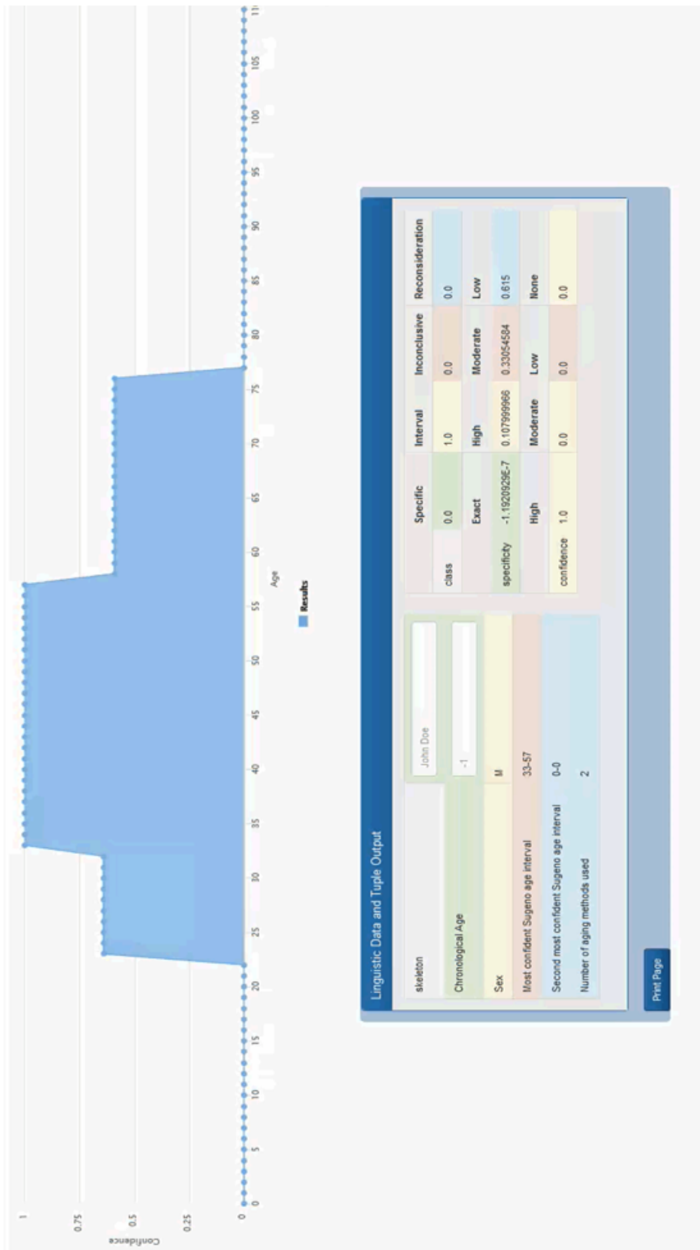

Figure 10. Results based on data input for the pubic symphysis and cranial suture.

The results of the fuzzy integral procedure is a set of confidences, one for each age tested (i.e., 1 to110 years). The output provides reproducible vocabulary (fuzzy sets) about 1) the support of membership of the age-at-death fuzzy set into one of four classes (i.e., specific, interval, inconclusive, and reconsideration), 2) the specificity or information about how specific the fuzzy set is with respect to being able to narrow the age-at-death estimation to a single value, and 3) confidence in the age-at-death estimation. This approach can be used as a standard that will allow different forensic scientists or anthropologists to make important assertions in a natural language about skeletal remains in a quantifiable scientific way. An example linguistic description is "There is a *high* confidence in the age-at-death estimation. There is a *high* confidence that the estimation is of type *interval*. However, the estimation has a *very low* specificity, which ultimately makes it hard to narrow the age-at-death to a single age".

The graph *class* is used to simplify interpretation of the age-at-death estimation (Anderson et al. 2010). Four graph classes are defined: specific, interval, inconclusive, and reconsideration (Figure 11). A fuzzy set that has high membership into a "specific" class graph indicates that the age-at-death methods have a high degree of confidence for a specific age. A more common type of graph, the "interval" graph suggests that the procedure identified a range or interval of possible ages. That is there is similar confidence for a consecutive range of ages but the length of the plateau is less than 30 years (Anderson et al 2010). If the length of the plateau in an interval graph is greater than 30 years then the fuzzy set is considered inconclusive. Finally, if there are two or more peaks with similar confidence the graph is placed in the "reconsideration" class. Reconsideration graphs suggest that the results of the methods selected are too variable to make a conclusion about age-at-death. That is, the age-at-death methods are in extreme disagreement. In these cases the investigator needs to decide how to resolve the conflict.

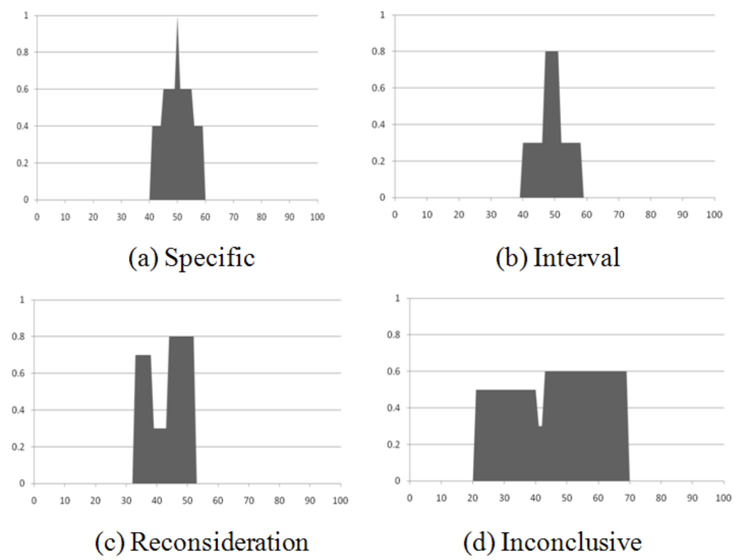

Figure 11. Four classes of graphs. (a) Type specific indicates a single age, (b) type interval indicates a range of possible ages, (c) reconsideration indicates multiple possible age ranges and (d) inconclusive indicates that there is very little evidence that any decision can be reached.

The *specificity* provides information how specific the fuzzy set is with regards to being able to narrow down the age range to a single value (Anderson et al. 2011). The specificity can be exact, high, moderate, or low. As the maximum fuzzy integral value decreases, so does the confidence in the amount of attainable specificity for the fuzzy set.

Finally, the confidence in the age-at-death estimation can be classified as high, moderate, low, or no support (Anderson et al. 2011). This is an interpretation of the fuzzy integral output degree.

In the example above in Figure 10, the results show that the most confident age estimation is 33 to 57 years of age. The graph is an “interval” class with low specificity but high confidence.

Figure 12 shows an example of a skeleton with a most confident age interval between 23 and 24 years and a second interval of 26 to 28 years but there is mainly low specificity and only moderate confidence in the age estimation. While the age range is relatively small (23-28 years) in this case the different methods selected are not in total agreement (no overlap), and the forensic anthropologists might reconsider the methods used or examine the input data for errors.

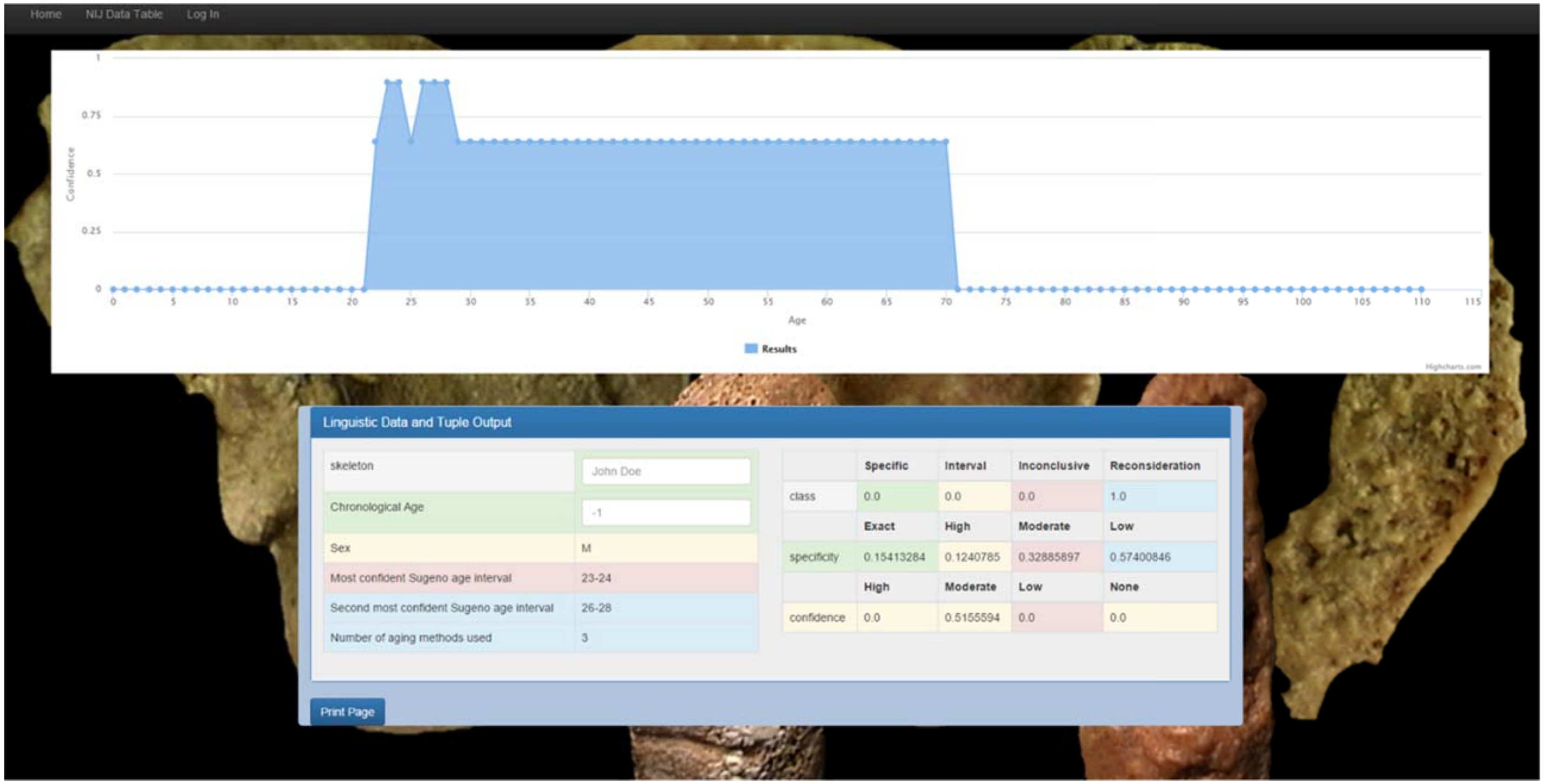

Figure 12. Example of GUI results output showing graph and linguistic interpretations for a reconsideration class graph

CONCLUSIONS

Discussion of Findings

The primary goal of this project was to develop a user-friendly graphical user interface that utilizes algorithms based on fuzzy integrals to provide forensic scientists a standardized procedure for developing a multi-factorial age-at-death estimation. To complete this task, several objectives had to be completed including determining the methods to include in the beta-version of the GUI, developing the necessary core libraries need for the age-at-death algorithm, and extend the fuzzy measure procedure to discover the worth of combinations of different methods. During the grant period, we developed a Java library for fuzzy measures and integrals that can be called via a webpage. We also developed an initial database for skeletal age-at-death methods and known reliabilities. Through the course of the research, our team focused on the extension of the fuzzy integral to uncertain information and different application domains and skeletal age-at-death estimations. The most important extensions are the subnormal fuzzy integral (SuFI), the generalized fuzzy integral (gFI), the non-direct fuzzy integral (NDFI), and the application of the NDFI to skeletal age-at-death estimating. Explanations and proofs for the extensions of the fuzzy integral are presented in a total of twelve publications (Anderson et al. 2011, 2012, 2014, 2015, Havens et al. 2013, 2014, 2015, Hu et al. 2013, 2014, Price et al., 2013, Wagner and Anderson 2013, Wagner et al 2013).

For the graphical user interface, the research team developed a website that pulls from the age-at-death/reliability database and populates a website that the user can interact with and input their skeletal information. The results of the integrals can then be visualized on the website. The output provides reproducible vocabulary (fuzzy sets) about 1) graph class, 2) the specificity of the fuzzy set, and 3) a confidence in the age-at-death estimation. These class definitions for age-at-death fuzzy sets allow forensic scientists to better understand how to interpret the uncertainty in the resultant fuzzy sets. As the GUI is used by professionals and we receive feedback, it can be modified to include different methods, include new measures of uncertainty (e.g., inter-observer error, effects of body mass, asymmetry, and others), and expert derived fuzzy measures.

Implications for Policy and Practice

Age estimation is a crucial component of the biological profile in forensic anthropological case work. Best practices in anthropology recommend utilizing multiple indicators of age when making an estimation based on skeletal remains (SWGANTH 2010). However, SWGANTH provides no specific guidelines for the best way to combine the various methods used. Forensic anthropologists are recommended to synthesize the available information keeping in mind the reference sample for the method, the forensic anthropologist's skill at the method, and the condition of the remains. However, with the call for strengthening the forensic sciences and using reliable and reproducible methods, there is a strong need for an standardized method for conducting multi-factorial age-at-death estimation that utilize methods commonly employed by forensic anthropologists.

We develop a user-friendly graphical user interface (GUI) that employs algorithms based on well-established fuzzy integrals for providing a multi-factorial age-at-death estimation, confidence in the estimation and a reproducible, grounded linguistic way to interpret the results. The GUI will be made freely available to forensic scientists and it will allow them to input skeletal data into the GUI and receive results that provide an age-at-death estimation, a measure of the confidence in the estimation, and graphs that can be presented in a report and during testimony.

Fuzzy integrals are used to produce a confidence in skeletal age-at-death, and the resulting age-at-death fuzzy sets are linguistically interpreted using fuzzy set theory in order to provide results about the age-at-death estimation that are reproducible and can be understood by different scientists. Unlike other multi-factorial methods, the current approach allows investigators to estimate age-at-death for a single skeleton using the age-at-death methods they are comfortable with and that are available to them based on the bones present, the condition of the bones, and the equipment available.

Implications for Further Research

While we have accomplished a great amount of research on extensions of fuzzy integrals and their role in skeletal age-at-death estimation to date, a number of questions and future work remains. The following in a non-comprehensive list of work we are already performing or plan to perform in the near future based in part on the support of this grant.

1. *Learning the fuzzy measure*: In our research, we used published correlation coefficients from the anthropology literature for the beta version. These correlation coefficients are the values of our densities (the fuzzy measure values for each singleton, i.e., the "worth" of each individual input). We then used the Sugeno lambda-fuzzy measure to impute the remainder of the lattice (acquire the other fuzzy measure values beyond the densities). In several publications we have started new research to learn the entire fuzzy measure from data. We believe that we have now laid the foundation (the core mathematics) and our next step is to apply this to skeletal age-at-death estimation. This can potentially be done a number of ways. We can learn the fuzzy measure from a set of generic skeletal data or we can do this specifically for a subset of age-at-death data believed to be closely connected to a particular sample.
2. *Expert derived fuzzy measure*: We have completed the mathematics and are near completion with writing the manuscript for a new way to acquire the fuzzy measure (which ultimately drives the fusion) based on high-level expert knowledge. In this approach, a set of experts provide their expert opinions of different age-at-death estimation methods (in the form of rank ordered preferences) and we create, using Belief theory, a set of fuzzy measures, one for each expert, and we aggregate these fuzzy measures to acquire a single combined expert

fuzzy measure. We have laid the mathematics to do this procedure and are now exploring how to carry this out for skeletal age-at-death estimation. Our current thought is that we will need to create a web-based interface to collect data in the form of a questionnaire from domain experts.

3. *Data collection via the user interface*: Another idea that we will explore is the ability of the interface to collect new data from different researchers in order to compile a larger skeletal age-at-death estimation database and to help with the learning of the fuzzy measure.
4. *Transition analysis:* To date, the method known as "transition analysis" is used by many for skeletal age-at-death estimation. While there are numerous advantages of our procedure, transition analysis nevertheless has important information and operates in a different way. We would like to try incorporate informed priors into our algorithm.
5. *Life tables and mortality tables*: One piece of information that we have not used to date are published life tables and mortality tables. We have already begun investigations into how to incorporate this into the fuzzy integral in terms of the integrand (h), the fuzzy measure (g) or a latter (post-fuzzy integral) combination step.
6. *Fuzzification of aging methods*: Currently, we are using known aging methods which yield a crisp interval for age-at-death. One of our longer term goals is to explore the possibility of "fuzzifying" the individual aging methods to reflect the researchers confidence in the age interval selected and/or the age interval end points (and distribution in that interval at that). This is a longer term goal, but if successful then we would in effect better characterize the uncertainty in the domain and have better information to compute with.

7. We would like to provide a method for forensic anthropologists to weight the age method based on the reference sample. For example, is the skeleton from the population/sex used to develop the method.
8. *Effect of obesity on age-at-death estimation*: In a project related to this grant we (Wescott and Drew 2015) demonstrated obese individuals exhibit greater bias and inaccuracy in age estimation. Future work may allow us to incorporate information about body mass into the confidence of age estimation.

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DISSEMINATION OF RESEARCH FINDINGS

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This grant has also been of great help in partially supporting related work that will be used in the future for skeletal age-at-death estimation and incorporated into the GUI.

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Technology Transfer

The GUI and associated user manual will be made freely available for forensic scientists to use. Some of the age-at-death data from the Texas State Documented Skeletal Collection have been submitted to the Forensic Data Bank. Upon completion of final publications associated with this grant, age data along with other biometric information on the remainder individuals will be submitted to the Forensic Data Bank.