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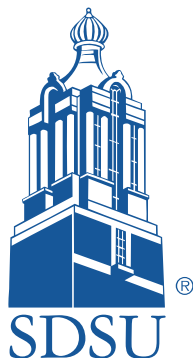
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National Institute of Justice
Research and Development in Forensic Science
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Award: 2014-IJ-CX-K088

**Foundational Research into the Quantification of
the Value of Forensic Evidence for Complex Evidential Forms
arising from Impression and Pattern Evidence**

Award Period: November 1st 2014 – October 31st 2018

Final Summary Overview Report

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Executive summary

During this 4-years basic research project, we aimed to study the validity, accuracy and computational complexity of methods designed to quantify the weight of complex evidence forms, such as pattern evidence and trace evidence. During the first phase of this project, we framed the problem of the identification of source in forensic science using two formal frameworks: the “common source” and “specific source” frameworks. We used these frameworks to explain why scientists may diverge so much from each other when quantifying the weight of forensic evidence. Within these frameworks, we demonstrated that an increasingly popular method to estimate the weight of forensic evidence (i.e., the so-called “score-based likelihood ratio”) was incoherent, inappropriate and could not be relied upon to interpret forensic evidence. During this first phase, we also developed and expanded several methods that alleviate the computational issues related to using a formal Bayes factor in forensic science. Whilst we proved the convergence of the proposed methods to the formal Bayes factor, these methods still suffer from the limitations inherent to the type of random vectors commonly used to characterise forensic evidence.

To assign the value of complex forms of evidence, such as pattern or chemical data, we developed two new classes of models that rely on kernel functions during the second phase of our project. Kernel functions projects complex random vectors into new mathematical spaces, where traditional statistical methods can be used; thus, the proposed models are very flexible and can virtually be used to quantify the evidential value of any pattern or chemical data. One of these models is particularly suited to be used in conjunction with database search algorithms and can easily be deployed on system such as AFIS or NIBIN. The proposed models were tested on FTIR spectra of paint samples and on fingerprints and their reliability was investigated.

Overall, this award, together with two associated NIJ awards, resulted in the training of six graduate students in the interpretation of forensic evidence, including two PhD graduates and four MSc graduates (three of whom are pursuing their PhD on topics related to forensic statistics). This award directly resulted in 1 PhD dissertation, 4 published papers, 2 submitted papers and 3 papers that are nearly ready to be submitted. Additionally, another PhD dissertation, 1 MSc dissertation, 1 accepted paper, 1 submitted paper and 1 paper near completion are related to this award.

Purpose of the project

During this 4-year basic research project, we aimed to study the validity, accuracy and computational complexity of methods designed to quantify the weight of complex evidence forms, such as pattern evidence and trace evidence. This project was divided into four phases:

1. The validity, accuracy and computational complexity of several methods for quantifying uncertainty in forensic conclusions was studied using a toy glass example, where the ground truth weight of evidence was known;
2. Two frameworks for using similarity measures (as a means to reduce the complexity of the problem) in the quantification of the weight of evidence were developed, and their reliability and accuracy were studied;
3. The two frameworks were applied to a variety of complex forms of evidence, such as fingerprints and FTIR analyses of fibres, and are currently being implemented for handwriting and powder residues;
4. The reliability of the proposed frameworks, when applied to these evidence types, was investigated.

Project design

The project relies on a mixture of analytical proofs and statistical simulations to develop and study numerical methods to assign the weight of forensic evidence. The design of our project involves a significant paradigm change: when compared to previously published models, our project explicitly take into account the entire set of evidence available to the scientist. Instead of just considering the observations made on the trace and control material, we also explicitly consider the observations made on samples from the population of potential sources. This paradigm change allows for a more formal development of numerical methods aimed at quantifying the weight of forensic evidence and for a more rigorous study of their convergence.

During phase 1, we decomposed the issue of quantifying the weight of forensic evidence into two

broad classes of propositions: specific source identification vs. common (but unknown) source identification. For both types of classes, we formally developed or expanded several paradigms. Once developed, we studied the benefits and limitations of these different paradigms, as well as their convergence with each other when appropriate. We also tested the convergence of an increasingly popular method proposed in the forensic literature, namely the so-called “score-based likelihood ratios”.

Most of the methods developed and studied during phase 1 are not directly suitable for the type of data observed on complex evidence forms. In most cases, the dimension and heterogeneity of the random vectors measured on forensic objects prevent the use of finite dimensional statistics. During phase 2, we further investigated the use of kernel functions (which are used in the “score-based likelihood ratios”) and we developed two classes of models to formally quantify the value of evidence when the likelihood structure of the observations does not exist. The first one of these models assigns a likelihood structure to the set of all pairwise comparisons between the observations made on the complete set of trace, control and reference objects. The second of these models relies on Approximate Bayesian Computation for model selection.

During phases 3 and 4, these two classes of models were applied to a variety of complex forms of evidence, such as fingerprint and FTIR analyses of fibres. The performance of these methods was assessed by means of large-scale testing and simulations to measure rates of misleading evidence, power of the test, and random match probabilities.

Results

Phase 1

During phase 1, we formally developed or expanded several methods to quantify the value of the evidence using the full set of observations made on the trace, control and reference objects. Let the evidence be denoted as $e = \{e_s, e_a, e_u\}$, where

e_u : represents the sets of samples from the trace.;

e_s : represents any sets of samples collected from the putative source;

e_a : represents the sets of samples originating from a population of other potential sources.

Our main concern is the inference of the source of the samples in e_u . In the interest of shortening this report, we mainly present results obtained in the context of the specific source identification problem. A motivation for defining two separate interpretation frameworks was published by the research team in Law, Probability and Risk [Ommen and Saunders, 2018a]. The mathematical developments of the Bayes factors in the two frameworks were published by the research team in the Journal of Statistical Computation and Simulation [Ommen et al., 2017].

We define the following probability models for how the samples are generated under two competing propositions, H_p and H_d , which respectively state that the trace and control objects originate from the same source versus that they originate from different sources:

M_p : e_u and e_s are simple random samples from the specific source considered under proposition H_p ;

M_d : e_s is a simple random sample from the specific source considered under proposition H_p , but e_u is a sample from a randomly selected source in the population of other potential sources.

The first method that we considered is the formal Bayes factor originally proposed for forensic science by Dennis Lindley in the late 1970s. Since Lindley’s Bayes factor addressed the common source problem, we redeveloped the Bayes factor to address the specific source problem to obtain:

$$V = \frac{\int_{\Theta_p} f(e|\theta, M_p) d\Pi(\theta|M_p)}{\int_{\Theta_d} f(e|\theta, M_d) d\Pi(\theta|M_d)}$$

This Bayes factor is the ideal method to formally and coherently assign weight to forensic evidence. However, it is challenging to implement in the forensic context due to (a) the need to define a likelihood structure for the observations in e , (b) our inability to assign a posterior distribution to the model’s parameters, and (c) issues associated with the Monte-Carlo integration required to calculate the marginal likelihood. In this project, we propose an ad-hoc estimate of the formal Bayes factor. Our estimate is inspired by the Neyman-Pearson lemma for nested model selection. The likelihood ratio proposed by Neyman-Pearson has been modified to take into account the non-nested nature of model selection in forensic science. This ad-hoc method uses parameter estimates

obtained by numerical optimisation and avoids the use of intractable probability distributions over the parameter space; furthermore, it does not require numerical integration. Our approximation is defined as:

$$V = \frac{\max_{\theta_s, \theta_a} f(e_u, e_s | \theta_s) f(e_a | \theta_a)}{\max_{\theta_s, \theta_a} f(e_s | \theta_s) f(e_u, e_a | \theta_a)}.$$

Our first success to partially overcome the issues associated with the Monte-Carlo integration involved being able to rewrite the Bayes factor using one single integral as:

$$V = \int \frac{f(e_u | \theta_s)}{f(e_u | \theta_a)} d\Pi(\theta | e_u, e_s, e_a, M_d)$$

This new form of the Bayes factor allows us to use Monte-Carlo approximation to estimate the Bayes factor as an average rather than the ratio of two averages. This also enables us to study the convergence of the approximation as the number of samples in e increases. Discussions on this topic have been published in the Journal of Statistical Simulation and Computation [Ommen et al., 2017] and Science & Justice [Ommen et al., 2016]. Our second success overcame the issues associated with assigning posterior distributions to the model's parameters under H_p and H_d . In most situations, the vectors of parameters are too complex and these distributions would not exist. Thus, we would not be able to assign Bayes factors for these models. Our solution consists in replacing these posterior distributions by multivariate normal distributions using maximum likelihood estimates for the mean vectors and covariance matrices. We called this solution the Bernstein-von Mises Bayes factor due to the use of the Bernstein-von Mises theorem to build our approximation. Dr. Ommen, who was a PhD candidate supported by this grant, showed in her dissertation [Ommen, 2017] that the Neyman-Pearson likelihood ratio and the Bernstein-von Mises Bayes factor both converge to the formal Bayes factor as the number of observations in e tends towards infinity. The major implications of this series of proofs are that the Neyman-Pearson likelihood ratio and the Bernstein-von Mises Bayes factor can be considered as universal approximations for the Bayes factor. A summary of these results are currently being drafted by Dr. Ommen and has been submitted to the Annals of Statistics [Ommen and Saunders, 2018b].

The methods presented above only apply directly to low dimensional data, which is commonly encountered when measuring the elemental composition of glass fragments or profiling DNA in the

forensic context. Unfortunately, most forms of evidence are usually characterised by high dimensional (e.g., ≥ 50) heterogeneous (i.e., mixture of different types of random variables) random vectors. For example, fingerprint features are commonly characterised by their type (nominal random variable), their direction (radial random variable) and their location on the x, y -plane (continuous random variable). A common approach to reduce the complexity of pattern recognition problems relies on the use of kernel functions that maps the level of similarity between pairs of objects to the real line. Kernel functions allow for assigning likelihood structures to the similarity/distance between pairs of observations instead of to the original observations. During this project, we demonstrated that most techniques proposed in the forensic literature that attempt to quantify the weight of forensic evidence using similarity measures (known as “score-based likelihood ratios”) are logically incoherent and violate the laws of probability. These results are currently being drafted and will be submitted to a peer-reviewed statistical or forensic journal. In order to leverage the power and flexibility of kernel functions, while satisfying the axioms of probability, we developed two new “kernel-based” methods for quantifying the value of the evidence. The first method involves assigning a likelihood structure to the entire set of pairwise scores between all objects considered in the evidence (i.e., trace, control and reference objects). The second method relies on Approximate Bayesian Computation. These methods were developed during phase 2 of the project and are described in more details below.

Phases 2, 3 and 4

The first “kernel-based” method that we developed spawned from a previous NIJ funded effort (Award 2009-DN-BX-K234). This original model was intended to estimate random match probabilities. During the current effort, we extended the model in three directions. In the first direction, we demonstrated how to use the original model for closed-set identifications. The results of this work were published by the research team in Chemometrics and Intelligent Laboratory Systems [Armstrong et al., 2017]. In the second direction, we propose to use the original model to measure the atypicality of a set of trace objects with respect to a set of control objects obtained from a specific source during the first stage of the so-called “two-stage approach” originally proposed by Parker for glass evidence. As far as we know, the proposed model is the only model that permits

to conduct classical hypothesis testing in “many vs. many” high-dimensional settings with small sample sizes (i.e., so-called “machine learning” algorithms can only classify one object at the time, while forensic scientists are most often concerned with determining whether a set of fragments, bullets or handwritten letters is from the same source as another set of similar objects, hence the “many vs. many” definition of the problem). The results of this work were published as a technical report on arXiv.org [Ausdemore et al., 2018b] and are being completed with comprehensive testing to characterise the performance of the methodology on FTIR paint data before being submitted to a peer-reviewed statistical journal. This work is used a means to resolve the current debate in the OSAC relative to the lack of statistical methodology supporting the comparison of chemical spectra. In the third direction, we extended to the original model to account for pairwise comparisons between trace, control and all reference objects. The resulting model is based on a piecewise random effect linear score model that takes the following form (for four object indices i, j, k, l):

$$s_{ijkl} = \begin{cases} \theta_b + b_i + b_k + d_{ik} + t_{i:ij} + t_{i:kl} + t_{k:ij} + t_{k:kl} + w_{ij} + w_{kl} + e_{ijkl}^b & \text{if } i \neq k \\ \theta_w + w_{ij} + w_{kl} + e_{ijkl}^w & \text{if } i = k \end{cases}$$

where θ_b, θ_w are mean effects, and $b_i \sim N(0, \sigma_b^2)$, $d_{ik} \sim N(0, \sigma_d^2)$, $w_{ij} \sim N(0, \sigma_w^2)$, $t_{i:ij} \sim N(0, \sigma_t^2)$, $e_{ijkl}^b \sim N(0, \sigma_{eb}^2)$, and $e_{ijkl}^w \sim N(0, \sigma_{ew}^2)$.

By assumption, a vector of all pairwise scores between the objects in e has a multivariate normal probability distribution. Dr. Douglas Armstrong showed [Armstrong, 2017], in a subsequent project funded under NIJ Award 2015-R2-CX-0028, that the parameters of that distribution have closed-form estimates, which allowed us to use this likelihood structure to design a Bayes factor that can

be used for model selection in a common source identification framework:

$$\begin{aligned}
 KBF &= \frac{\int f(\mathbf{s}|\psi, \mathcal{M}_p) d\Pi(\psi)}{\int f(\mathbf{s}|\psi, \mathcal{M}_d) d\Pi(\psi)} \\
 &= \frac{\int f\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi, \mathcal{M}_p\right) f(\mathbf{s}_c|\psi) d\Pi(\psi)}{\int f\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi, \mathcal{M}_d\right) f(\mathbf{s}_c|\psi) d\Pi(\psi)} \\
 &= \frac{\int f_p\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right) d\Pi(\psi|\mathbf{s}_c)}{\int f_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right) d\Pi(\psi|\mathbf{s}_c)} \\
 &= \int \frac{f_p\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)}{m_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c\right)} \times \frac{f_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)}{f_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)} d\Pi(\psi|\mathbf{s}_c) \\
 &= \int \frac{f_p\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)}{f_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)} \times \frac{f_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)}{m_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c\right)} d\Pi(\psi|\mathbf{s}_c) \\
 &= \int \frac{f_p\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)}{f_d\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right)} d\Pi\left(\psi|\mathbf{s}_c, \begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix}, \mathcal{M}_d\right) \\
 &= \int \lambda\left(\begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \end{pmatrix} | \mathbf{s}_c, \psi\right) d\Pi(\psi|\mathbf{s}, \mathcal{M}_d)
 \end{aligned}$$

where

1. $\mathbf{s}_m = \kappa(\mathbf{x}_{ij}, \mathbf{x}_{kl})$ s.t. $\mathbf{x}_{ij} \in X_{u1}$, $\mathbf{x}_{kl} \in X_{u2}$ (This represents the comparisons between objects from the sources being tested)
2. $\mathbf{s}_n = \kappa(\mathbf{x}_{ij}, \mathbf{x}_{kl})$ s.t. $\mathbf{x}_{ij} \in \{X_{u1}, X_{u2}\}$, $\mathbf{x}_{kl} \in X_a$ (This represents the comparisons between objects from the sources being tested and objects sampled from the population)
3. $\mathbf{s}_c = \kappa(\mathbf{x}_{ij}, \mathbf{x}_{kl})$ s.t. $\mathbf{x}_{ij}, \mathbf{x}_{kl} \in X_a$ (This represents the comparisons between objects sampled from the population)

$$4. \mathbf{s} = \begin{pmatrix} \mathbf{s}_m \\ \mathbf{s}_n \\ \mathbf{s}_c \end{pmatrix}$$

Note that the above development of the Bayes factor is result from Dr. Ommen's work under phase

1. Furthermore, we showed that the main assumption of the model (i.e., the multivariate normal distribution of the score vectors) is approximately correct as the intrinsic number of dimensions of

the original objects increases, regardless of the number of objects considered. This development is being tested on forensic ink data and will be submitted for publication in a peer reviewed statistical journal. The development of this model is pursued by PhD student Madeline Ausdemore under NIJ Award 2018-R2-CX-0010.

The second “kernel-based” model that we developed relies on Approximate Bayesian Computation (ABC). The ABC approach can be used as a likelihood-free Bayesian model selection method. The ABC algorithm uses data that are very similar to the sets of scores obtained by (a) comparing a single trace objects with multiple control objects and (b) with reference objects. Our implementation of the ABC algorithm for forensic science addresses the shortcomings of the traditional ABC algorithm. Our implementation relies on the use of Receiver Operating Characteristics (ROC) curve and bypasses the need to partition the sets of scores with respect to a threshold. One important result of the proposed method is that we proved that our ABC ROC-based Bayes factor converges to the Bayes factor as the number of observations tends to infinity and if the inputs of the scoring function are sufficient summary statistics of the observed data. Our model was implemented in the context of fingerprint evidence. The performances of the model have been tested by comparing latent prints with control impressions from their true sources, with close-non-matching impressions and with impressions from random sources. The following figures summarise the model’s performances. The proposed ABC ROC-based Bayes factor is flexible and can be applied to virtually any form of forensic evidence. It is currently limited to quantifying the weight of a single trace observation; however, its generalisation to sets of trace observations is straightforward. The theoretical development of the ABC-ROC Bayes factor was part of the MSc. in Mathematics with specialisation in Statistics project of Ms. Jessie Hendricks, and is under revision following comments from reviewers from the Journal of the Royal Statistical Society C. The current draft paper has been published as a technical report on arXiv.org [Hendricks et al., 2018].

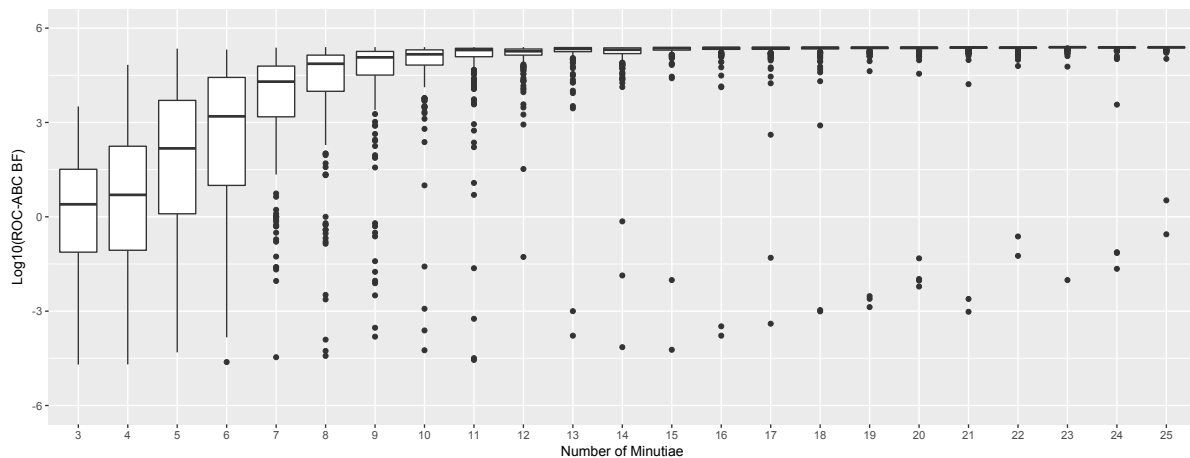


Figure 1: True source, empirical ROC method

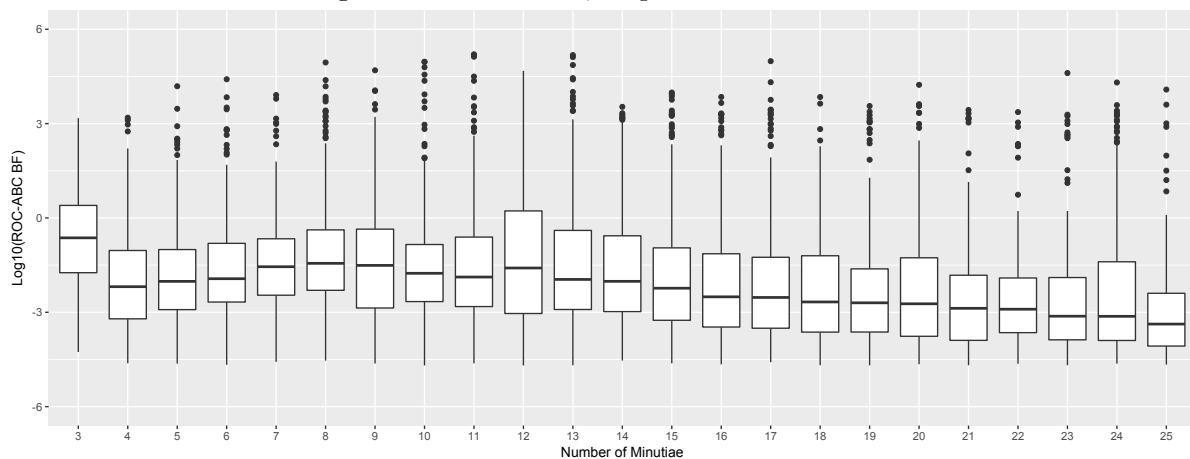


Figure 2: Close non-match, empirical ROC method

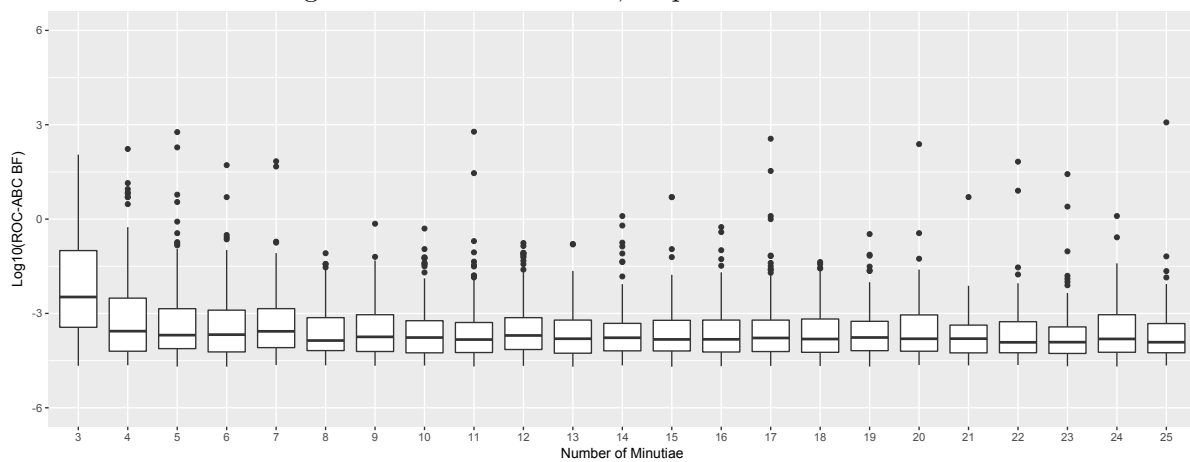


Figure 3: Random source, empirical ROC method

Related activities

While leading the project funded under NIJ Award 2014-IJ-CX-K088, the two principle investigators, Dr. Saunders and Dr. Neumann, mentored promising graduate students and led parallel projects on the interpretation of forensic evidence. The goals of these parallel projects were not part of the primary objectives of the current project. However, they did take advantage of results and competencies developed as part of the current project. These parallel projects were:

1. The development of a model to resolve dust mixtures as part of the MSc. in Mathematics with specialisation in Statistics project of Ms. Madeline Ausdemore, advised by Dr. Neumann. This project was funded under NIJ Award 2016-DN-BX-0146. A draft paper reporting the model and its performances is currently being revised based on comments from reviewers from the Journal of the Royal Statistical Society C. The current draft of the paper has been published as a technical report on arXiv.org [Ausdemore and Neumann, 2018].
2. The development of a U-statistic design to estimate random match probabilities as a function of the amount of handwriting available to an expert system. This research took advantage of the development made during the current award to characterise the likelihood structure of pairwise comparisons. This project was part of the MSc. in Mathematics with specialisation in Statistics project of Ms. Cami Fuglsby, advised by Dr. Saunders [Fuglsby, 2017].
3. A contribution to the controversy surrounding the estimation of error rates in fingerprint examination based on the data acquired by the Miami Dade Police Department under NIJ Award 2010-DN-BX-K268. A paper reporting our findings has been accepted by the Journal of Forensic Identification [Ausdemore et al., 2018a]. Due to the importance of the topic, this paper will be published together with comments from selected members of the forensic, statistical, and legal communities.

Ms. Fuglsby, Ausdemore and Hendricks are currently pursuing their PhDs in Mathematical Statistics on foundational issues related to the interpretation of forensic evidence under the supervision of Drs. Saunders (Fuglsby) and Neumann (Ausdemore and Hendricks).

Implications for the criminal justice system in the U.S.

During this project, we have extended the ability of forensic statisticians to quantify the value of complex evidence forms in the rigorous, logical and coherent manner advocated by legal and scientific scholars for the past four decades. Firstly, we have partitioned the question of the identification of source in forensic science using two distinct formal frameworks. This decomposition helps understand why different scientists potentially construct radically different models for the interpretation of the same evidence type, and, by extension, provides these scientists with rigorous foundations for interpreting and presenting the value of forensic evidence.

Secondly, we have shown that an increasingly popular method proposed to “quantify” the weight of forensic science, namely “score-based likelihood ratios” was not appropriate and could result in extremely misleading evidence.

Thirdly, we propose different methods to address the issues related to calculate formal Bayes factors, namely the difficulty to assign posterior probabilities to models’ parameters and the error associated with Monte-Carlo integration. We show that these methods converge to the formal Bayes factor, and therefore, that they are acceptable substitutes for quantifying the weight of forensic evidence.

Fourthly, since the methods proposed above only apply to situations where the evidence can be characterised in a low dimensional feature space, we propose two methods that leverage the power and flexibility of kernel functions to facilitate the probabilistic interpretation of forensic evidence. One of these methods assign a likelihood structure to the entire set of pairwise comparisons between trace, control and reference objects. This method is currently being validated in order to provide an answer to the controversy that rose within the OSAC regarding the lack of statistical support for the comparison of spectral data. The second method entirely removes the need for defining a tractable likelihood structure. This implies that our method is particularly suited for interpreting pattern evidence. A prototype expert system is currently being researched by a large biometric company.

Finally, this project allowed for training 6 PhD and MSc level graduate students in the interpretation of forensic evidence using probabilistic models. One of the PhD graduate students trained during this research effort is currently a tenure-track Assistant Professor of Statistics at Iowa State

University. Three of the MSc graduates are currently pursuing their PhD in Mathematical Statistics on topics related to the interpretation of forensic evidence.

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